Cluster Analysis for Business - Course paper

In [28]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from scipy.cluster.hierarchy import dendrogram, linkage, fcluster
from sklearn.metrics import silhouette_score, davies_bouldin_score
from sklearn.cluster import KMeans
import tensorflow as tf
from tensorflow.keras import layers
from tensorflow.keras.models import Sequential
import random
import time
from sklearn.manifold import TSNE
import seaborn as sns
from numpy import linalg as LA
from sklearn.cluster import DBSCAN
import tensorflow as tf
import os
import warnings
```

1) Clustering and Representation Learning

i)

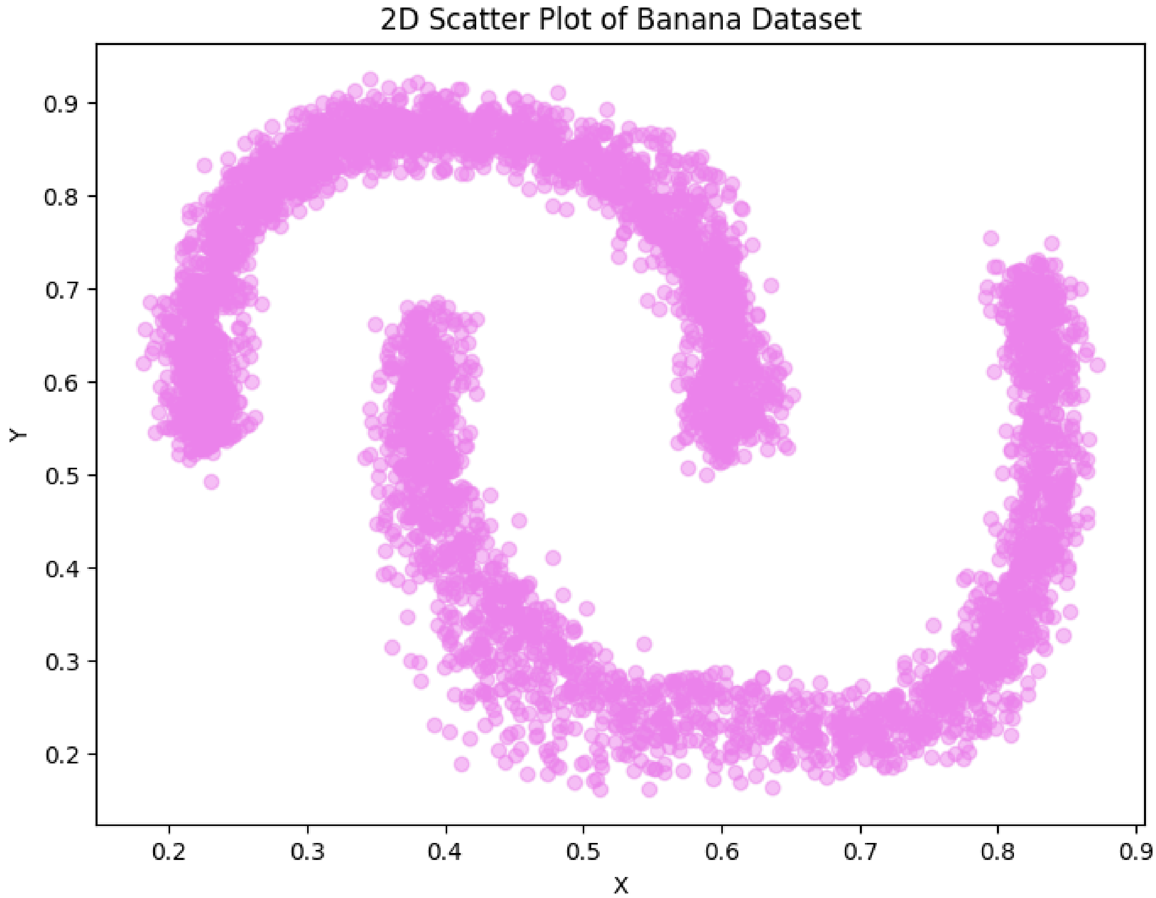
In [37]:
```
# Loading the data
path ='https://raw.githubusercontent.com/milaan9/Clustering-Datasets/refs
data = pd.read_csv(path)
data.head()
```

Out[37]:

	x	y	class
0	0.228	0.559	1
1	0.216	0.528	1
2	0.221	0.552	1
3	0.215	0.538	1
4	0.224	0.548	1

a) Visualize the data in a 2-dimensional scatter plot

In [38]:
```
plt.figure(figsize=(8, 6))
plt.scatter(data['x'], data['y'], c='violet', alpha=0.5)
plt.xlabel('X')
plt.ylabel('Y')
plt.title('2D Scatter Plot of Banana Dataset')
plt.show()
```



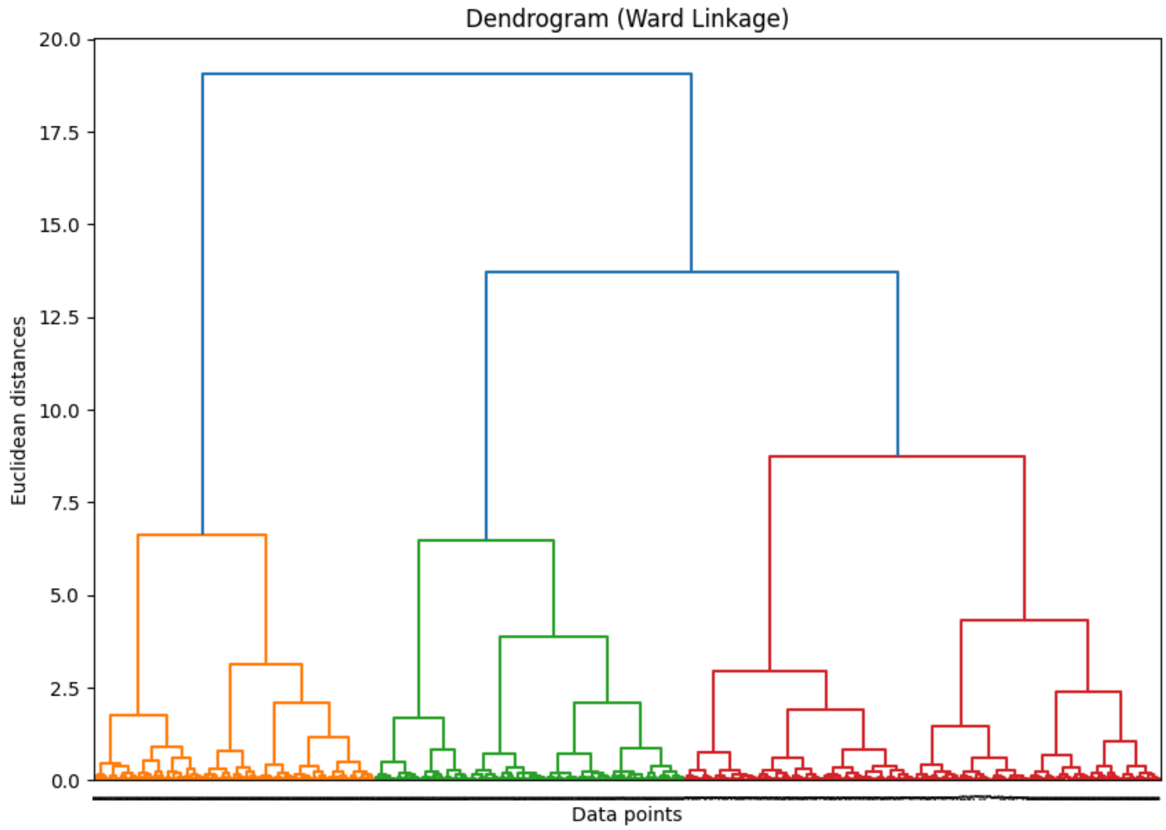
c) Use the Euclidean distance and the ward and average linkage methods and plot the dendrogram for both linkage methods. Discuss the number of clusters that the dendrograms suggest and contrast the result with your answer in b)

In [43]:

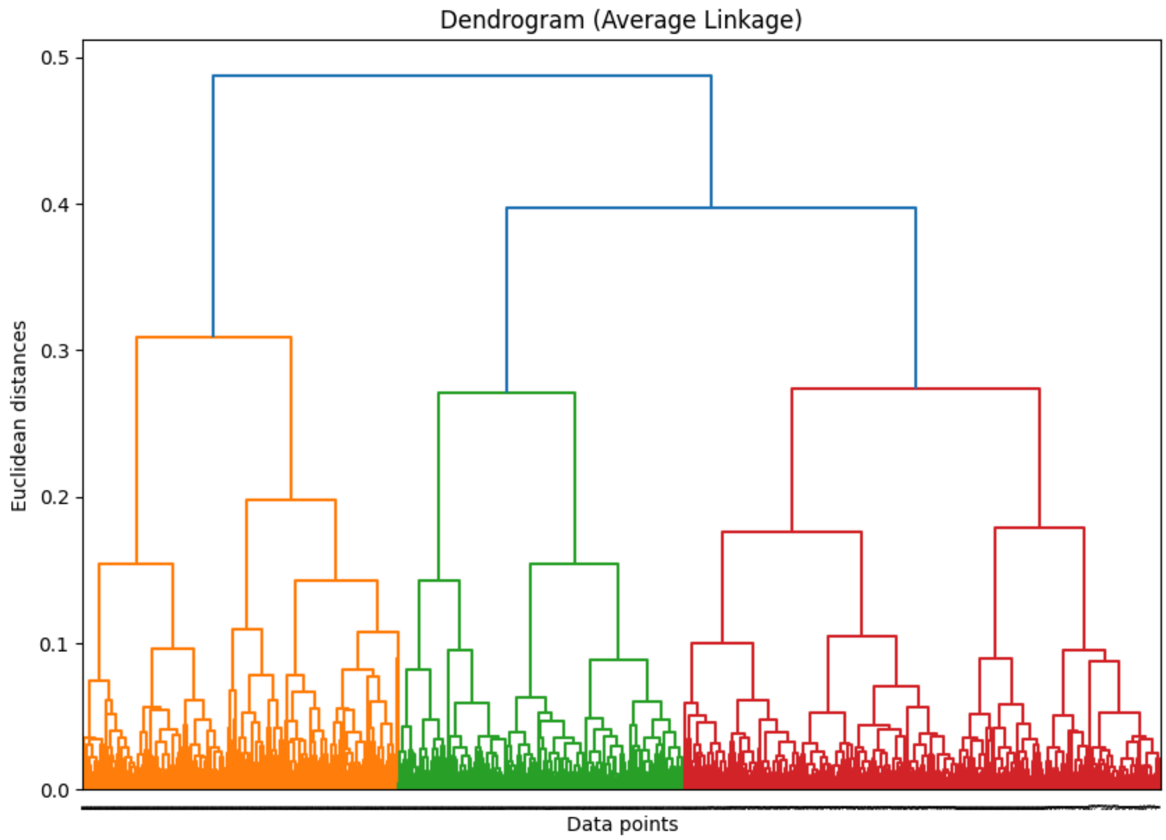
```
x = data[['x', 'y']].values
cl_ward = linkage(x, method='ward', metric='euclidean')
```

In [7]:

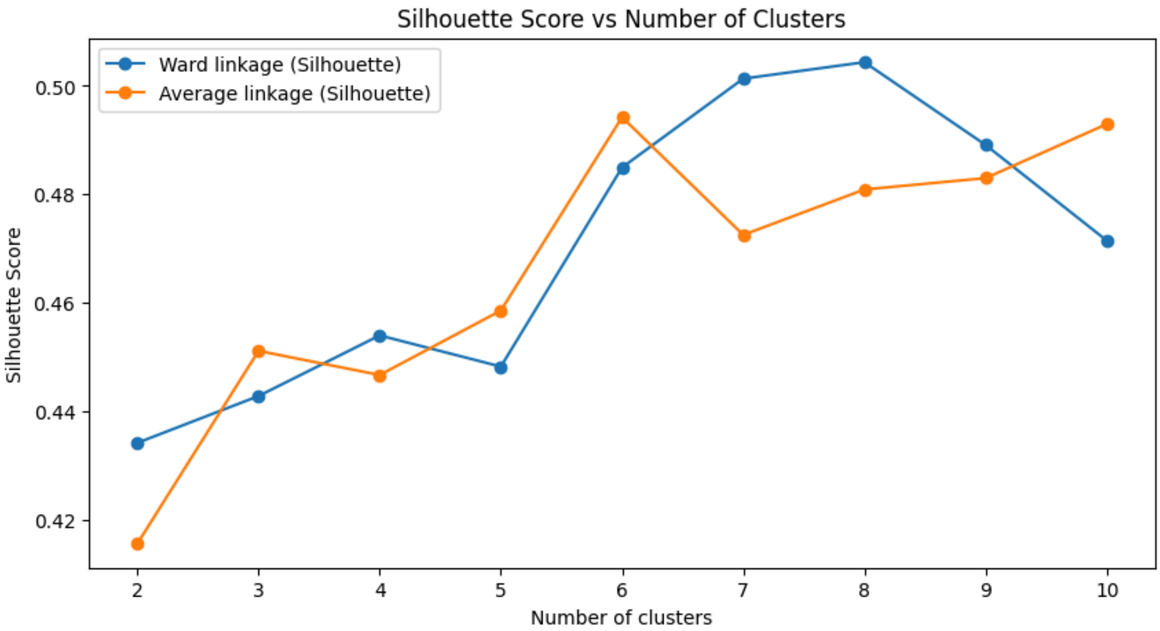
```
# Dendogram with Ward Linkage
plt.figure(figsize=(10, 7))
plt.title('Dendrogram (Ward Linkage)')
dendrogram(cl_ward, leaf_font_size=0)
plt.xlabel('Data points')
plt.ylabel('Euclidean distances')
plt.show()
```

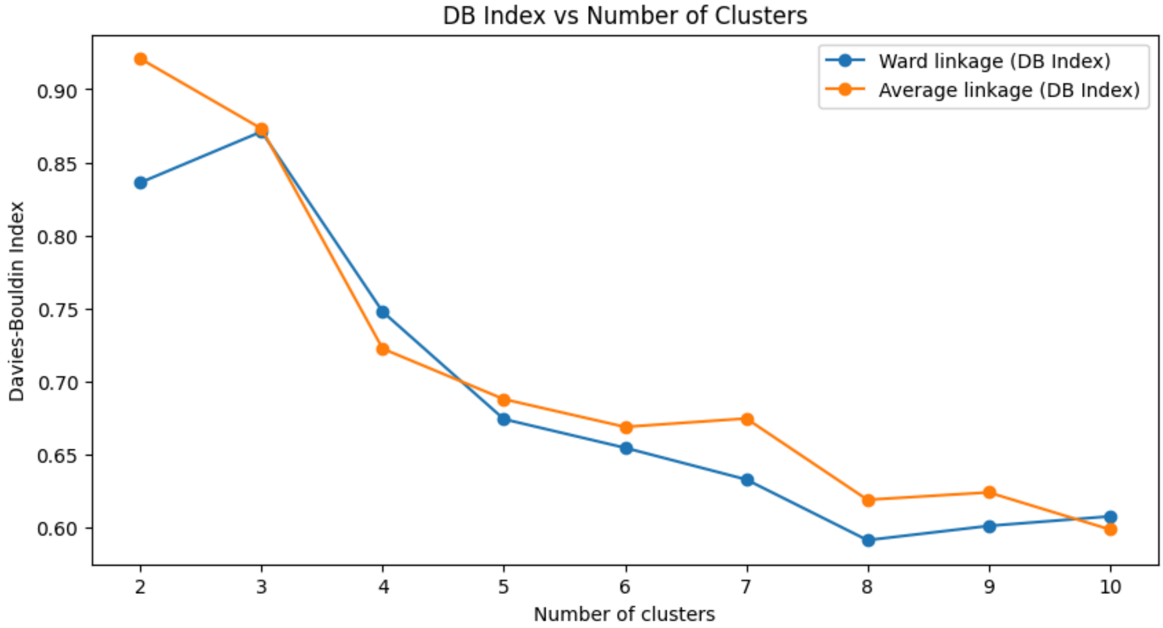


In [8]:
```
# Dendogram with Single Linkage
cl_average = linkage(x, method='average', metric='euclidean')
plt.figure(figsize=(10, 7))
plt.title('Dendrogram (Average Linkage)')
dendrogram(cl_average, leaf_font_size=0)
plt.xlabel('Data points')
plt.ylabel('Euclidean distances')
plt.show()
```



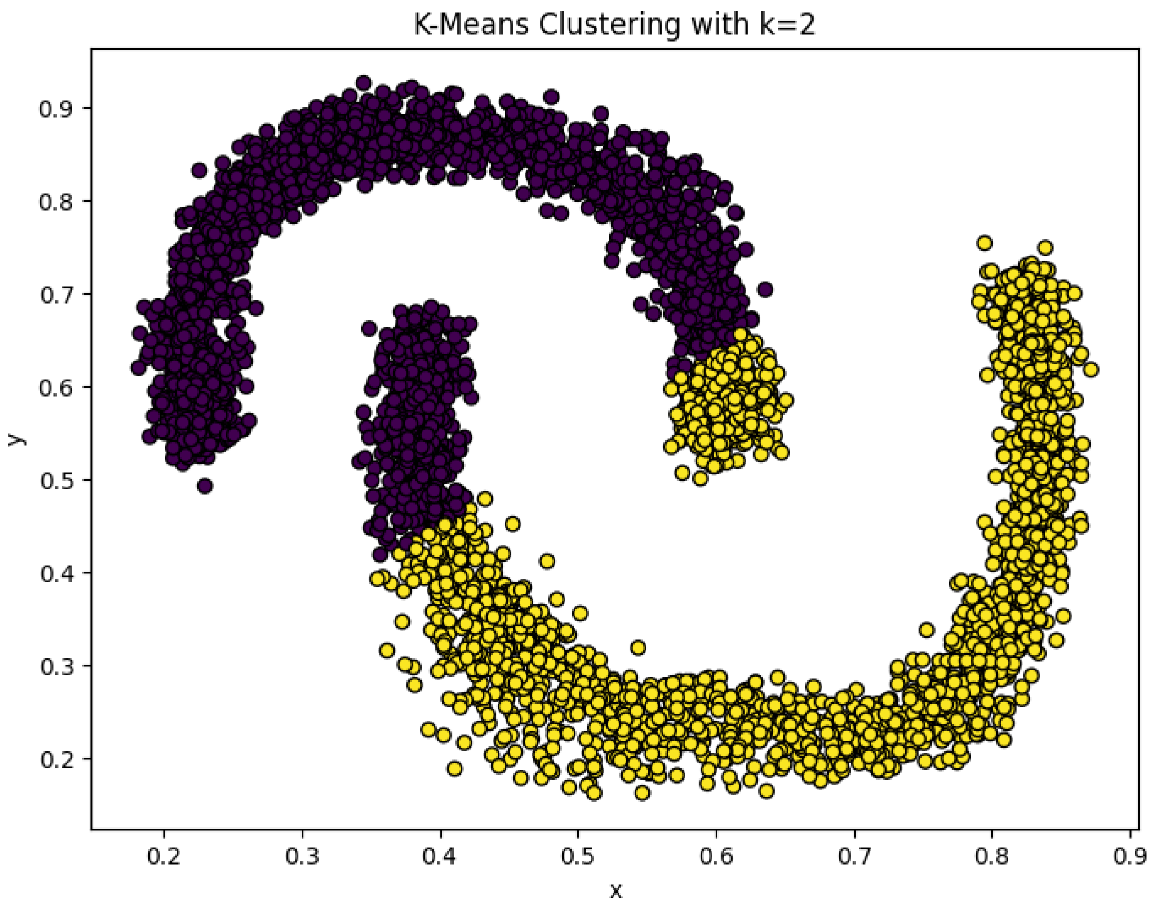
In [9]:
```python
# Silhouette Score and Davies-Bouldin Index
k_range = range(2, 11) # evaluate the number of clusters from 2 to 10

silhouette_ward_scores = []
db_ward_scores = []
silhouette_average_scores = []
db_average_scores = []

for k in k_range:
    labels_ward = fcluster(cl_ward, k, criterion='maxclust')
    silhouette_ward_scores.append(silhouette_score(x, labels_ward))
    db_ward_scores.append(davies_bouldin_score(x, labels_ward))

    labels_average = fcluster(cl_average, k, criterion='maxclust')
    silhouette_average_scores.append(silhouette_score(x, labels_average))
    db_average_scores.append(davies_bouldin_score(x, labels_average))

plt.figure(figsize=(10, 5))
plt.plot(k_range, silhouette_ward_scores, label='Ward linkage (Silhouette
plt.plot(k_range, silhouette_average_scores, label='Average linkage (Silh
plt.xlabel('Number of clusters')
plt.ylabel('Silhouette Score')
plt.title('Silhouette Score vs Number of Clusters')
plt.legend()
plt.show()

plt.figure(figsize=(10, 5))
plt.plot(k_range, db_ward_scores, label='Ward linkage (DB Index)', marker
plt.plot(k_range, db_average_scores, label='Average linkage (DB Index)',
plt.xlabel('Number of clusters')
plt.ylabel('Davies-Bouldin Index')
plt.title('DB Index vs Number of Clusters')
plt.legend()
plt.show()
```





d) Select the K parameter based on your answer in exercise b), run the K-means algorithm, and visualize a 2-dimensional scatter plot where the K-means cluster labels are used as colors in the scatters. Comment on the results.

In [44]:
```
kmeans = KMeans(n_clusters=2, random_state=42)
kmeans.fit(x)
labels = kmeans.labels_

plt.figure(figsize=(8, 6))
plt.scatter(data['x'], data['y'], c=labels, cmap='viridis', marker='o', e
plt.title('K-Means Clustering with k=2')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



ii) - v)

In [45]:
```
# Autoencoder definition
labels = data['class'].copy().to_numpy()
y = tf.keras.utils.to_categorical(labels-1,num_classes=2)
class AutoEncoder(tf.keras.Model):
    def __init__(self, enc, dec, name='autoencoder'):
      super().__init__()
      self.encoder = enc
      self.decoder = dec
      self.cross_entropy = tf.keras.losses.BinaryCrossentropy()

      self.params = encoder.trainable_variables + decoder.trainable_varia

    def call(self, inputs):
      x,y = inputs
      z = self.encoder(x)
      x_hat = self.decoder(z)

      self.loss = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(

    @tf.function
    def train(self, x, optimizer):
        with tf.GradientTape() as tape:
            enc_dec = self.call(x)
        gradients = tape.gradient(self.loss, self.params)
        optimizer.apply_gradients(zip(gradients, self.params))

        return self.loss

    def rep_learning(self, x):
```

```
      z = self.encoder(x)

      return z

    def reconstruct(self, x):
      z = self.rep_learning(x)
      x_hat = self.decoder(z)

      return x_hat
```

In [47]:
```
print(f'x shape: {x.shape}')
```

```
x shape: (4811, 2)
```

In [48]:
```
# Hyperparameters
input_shape = (2, )
activation = 'tanh'
units = 10
z_dim = 2
```

In [49]:
```
# Encoder and decoder architecture
encoder = Sequential([
    layers.InputLayer(input_shape=(input_shape)),
    layers.Dense(units, activation=activation),  # 1st hidden layer
    layers.Dense(units, activation=activation),  # 2nd hidden layer
    layers.Dense(units, activation=activation),  # 3rd hidden layer
    layers.Dense(z_dim, activation='linear'),    # output layer (latent s
])

decoder = Sequential([
    layers.InputLayer(input_shape=(z_dim,)),
    layers.Dense(units, activation=activation),  # 1st hidden layer
    layers.Dense(units, activation=activation),  # 2nd hidden layer
    layers.Dense(units, activation=activation),  # 3rd hidden layer
    layers.Dense(2, activation='linear'),  # output layer with linear act
])

warnings.filterwarnings("ignore", category=UserWarning, message="Argument
```

In [50]:
```
# Training the autoencoder
seed = 42
random.seed(seed)
np.random.seed(seed)
tf.random.set_seed(seed)

autoencoder = AutoEncoder(encoder, decoder)
batch_size = 256
tr_data = tf.data.Dataset.from_tensor_slices((x,y)).shuffle(x.shape[0]).b
optimizer = tf.keras.optimizers.Adam(learning_rate=0.01)

counter = 0
epochs  = 1000
start = time.time()
losses = []
while counter < epochs:
  for i, x_batch in enumerate(tr_data):
    loss = autoencoder.train(x_batch, optimizer)
    losses.append(loss.numpy())  # storing loss
  counter+=1
```

```
  if counter%10==0:
    print('Model has been trained for {} epochs'.format(counter))
elapsed_time = time.time() - start
minutes, seconds = divmod(elapsed_time, 60)
print('Elapsed time: {}m {:.1f}s'.format(int(minutes), seconds))

os.environ['TF_CPP_MIN_LOG_LEVEL'] = '3'
tf.get_logger().setLevel('ERROR')
```

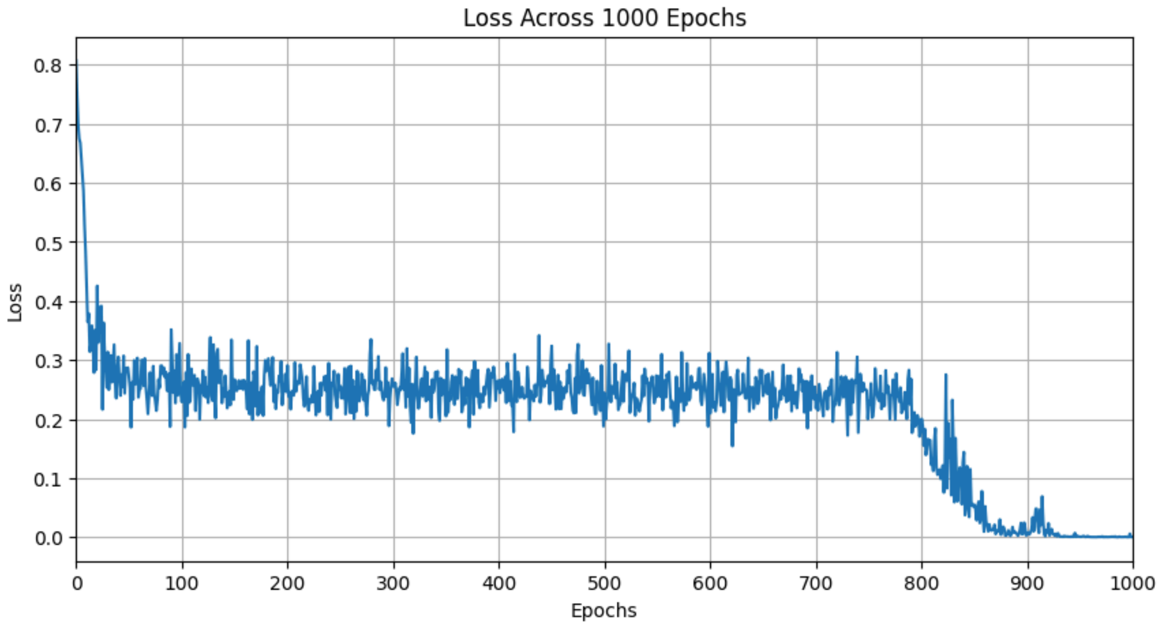
```
Model has been trained for 10 epochs
Model has been trained for 20 epochs
Model has been trained for 30 epochs
Model has been trained for 40 epochs
Model has been trained for 50 epochs
Model has been trained for 60 epochs
Model has been trained for 70 epochs
Model has been trained for 80 epochs
Model has been trained for 90 epochs
Model has been trained for 100 epochs
Model has been trained for 110 epochs
Model has been trained for 120 epochs
Model has been trained for 130 epochs
Model has been trained for 140 epochs
Model has been trained for 150 epochs
Model has been trained for 160 epochs
Model has been trained for 170 epochs
Model has been trained for 180 epochs
Model has been trained for 190 epochs
Model has been trained for 200 epochs
Model has been trained for 210 epochs
Model has been trained for 220 epochs
Model has been trained for 230 epochs
Model has been trained for 240 epochs
Model has been trained for 250 epochs
Model has been trained for 260 epochs
Model has been trained for 270 epochs
Model has been trained for 280 epochs
Model has been trained for 290 epochs
Model has been trained for 300 epochs
Model has been trained for 310 epochs
Model has been trained for 320 epochs
Model has been trained for 330 epochs
Model has been trained for 340 epochs
Model has been trained for 350 epochs
Model has been trained for 360 epochs
Model has been trained for 370 epochs
Model has been trained for 380 epochs
Model has been trained for 390 epochs
Model has been trained for 400 epochs
Model has been trained for 410 epochs
Model has been trained for 420 epochs
Model has been trained for 430 epochs
Model has been trained for 440 epochs
Model has been trained for 450 epochs
Model has been trained for 460 epochs
Model has been trained for 470 epochs
Model has been trained for 480 epochs
Model has been trained for 490 epochs
Model has been trained for 500 epochs
Model has been trained for 510 epochs
Model has been trained for 520 epochs
Model has been trained for 530 epochs
Model has been trained for 540 epochs
Model has been trained for 550 epochs
Model has been trained for 560 epochs
Model has been trained for 570 epochs
Model has been trained for 580 epochs
Model has been trained for 590 epochs
Model has been trained for 600 epochs
```

```
Model has been trained for 610 epochs
Model has been trained for 620 epochs
Model has been trained for 630 epochs
Model has been trained for 640 epochs
Model has been trained for 650 epochs
Model has been trained for 660 epochs
Model has been trained for 670 epochs
Model has been trained for 680 epochs
Model has been trained for 690 epochs
Model has been trained for 700 epochs
Model has been trained for 710 epochs
Model has been trained for 720 epochs
Model has been trained for 730 epochs
Model has been trained for 740 epochs
Model has been trained for 750 epochs
Model has been trained for 760 epochs
Model has been trained for 770 epochs
Model has been trained for 780 epochs
Model has been trained for 790 epochs
Model has been trained for 800 epochs
Model has been trained for 810 epochs
Model has been trained for 820 epochs
Model has been trained for 830 epochs
Model has been trained for 840 epochs
Model has been trained for 850 epochs
Model has been trained for 860 epochs
Model has been trained for 870 epochs
Model has been trained for 880 epochs
Model has been trained for 890 epochs
Model has been trained for 900 epochs
Model has been trained for 910 epochs
Model has been trained for 920 epochs
Model has been trained for 930 epochs
Model has been trained for 940 epochs
Model has been trained for 950 epochs
Model has been trained for 960 epochs
Model has been trained for 970 epochs
Model has been trained for 980 epochs
Model has been trained for 990 epochs
Model has been trained for 1000 epochs
Elapsed time: 0m 21.2s
```

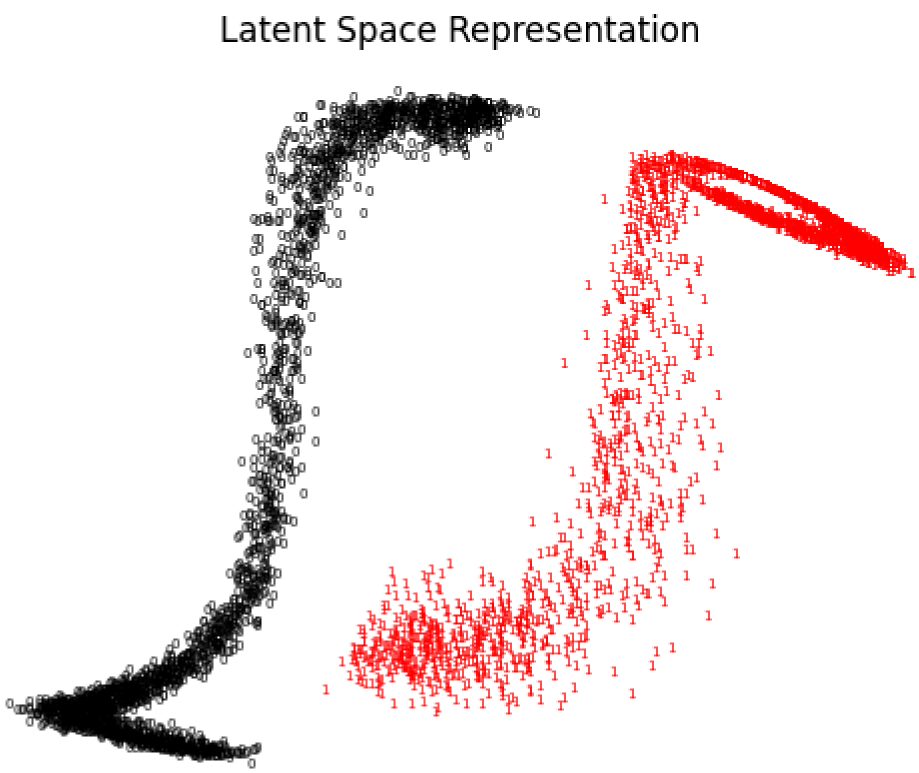
e) Plot the loss across number of epochs and generate the latent representation for all data using the function rep_learning and visualize it in a 2-dimensional scatter plot

In [13]:
```
plt.figure(figsize=(10, 5))
plt.plot(losses[:epochs])
plt.xlim([0, epochs])
plt.xticks(range(0, epochs + 1, 100))
plt.title('Loss Across 1000 Epochs')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.grid(True)
plt.show()

def nice_scatter(z_tsne, y_sample, title):
    print('plotting nice scatter...')
    COLORS=[[1.0000, 0,      0     ],
```

```
              [0,      1.0000, 0     ],
              [0,      0,      1.0000],
              [1.0000, 0,      1.0000],
              [0.9569, 0.6431, 0.3765],
              [0.4000, 0.8039, 0.6667],
              [0.5529, 0.7137, 0.8039],
              [0.8039, 0.5882, 0.8039],
              [0.7412, 0.7176, 0.4196],
              [0,      0,      0     ]]

    fig, ax = plt.subplots()
    for i, ((x,y),) in enumerate(zip(z_tsne)):
        #rot = random.randint(0,0) # in case you want randomly rotated nu
        rot = 0
        ax.text(x, y, y_sample[i], color=COLORS[y_sample[i]-1], ha="cente
        ax.plot(x,y, alpha=0.0)
        ax.axis('off')
        plt.title(title)

z = autoencoder.rep_learning(x)
y_sample = np.argmax(y, axis=1)
nice_scatter(z, y_sample, title="Latent Space Representation")
```



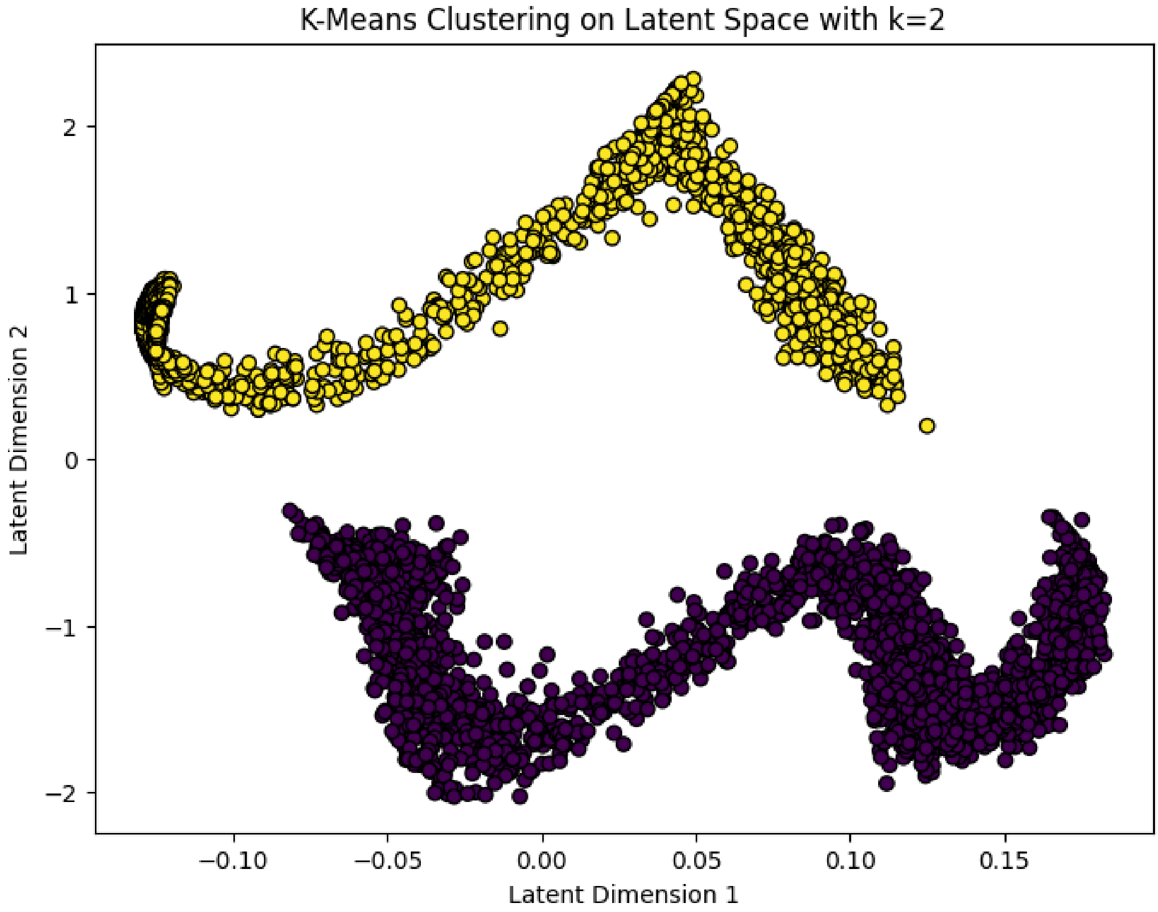

```
plotting nice scatter...
```



In [27]:
```
### DO NOT RUN AGAIN !!! (already savedo)
# np.save('lentent_z.npy',z)
```

f) Use K-means again, but this time you will cluster the latent representations that you obtained in exercise f). Visualize the latent representation and use the K-means cluster labels as colors in the scatters. Comment on the results.

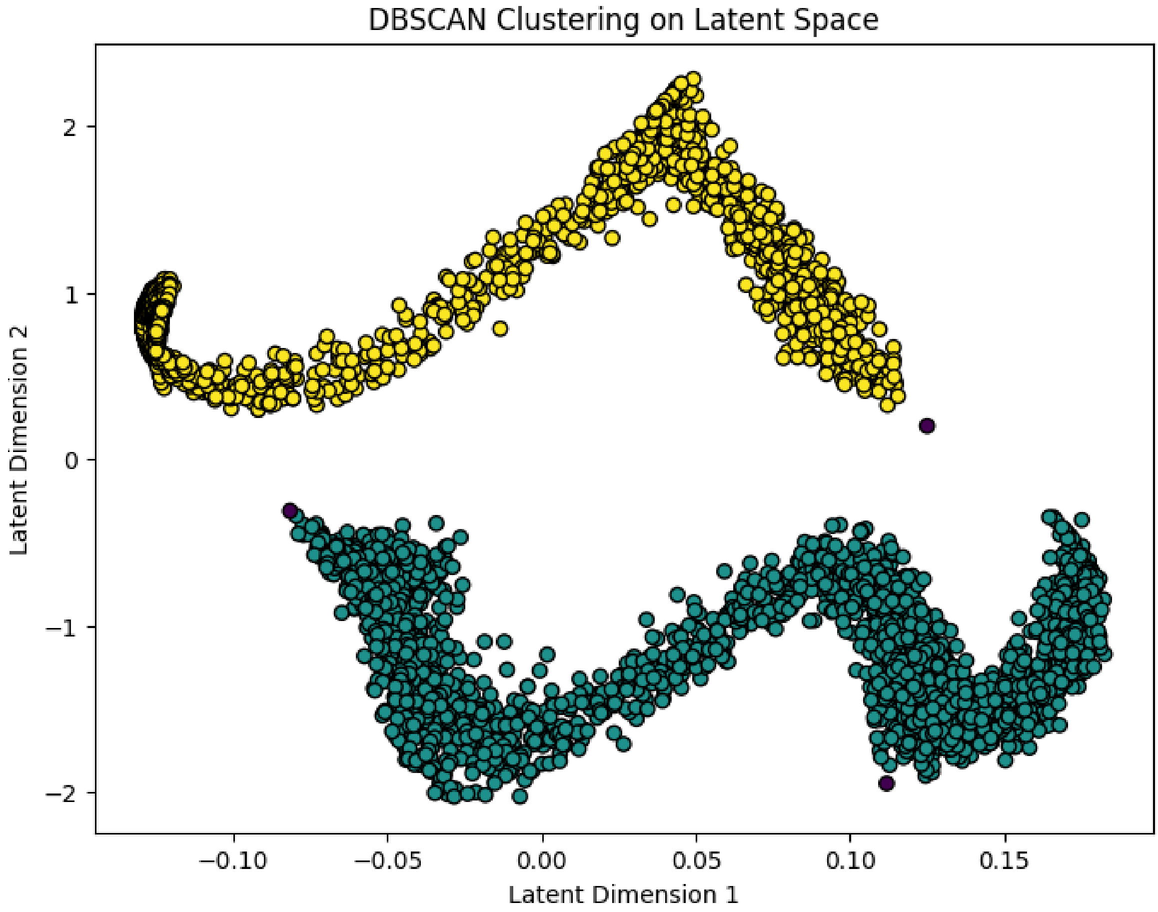
In [14]:
```
latent_z = np.load('lentent_z.npy')

kmeans = KMeans(n_clusters=2, random_state=42)
kmeans.fit(latent_z)
labels = kmeans.labels_

plt.figure(figsize=(8, 6))
plt.scatter(latent_z[:, 0], latent_z[:, 1], c=labels, cmap='viridis', mar
plt.title('K-Means Clustering on Latent Space with k=2')
plt.xlabel('Latent Dimension 1')
plt.ylabel('Latent Dimension 2')
plt.show()
```



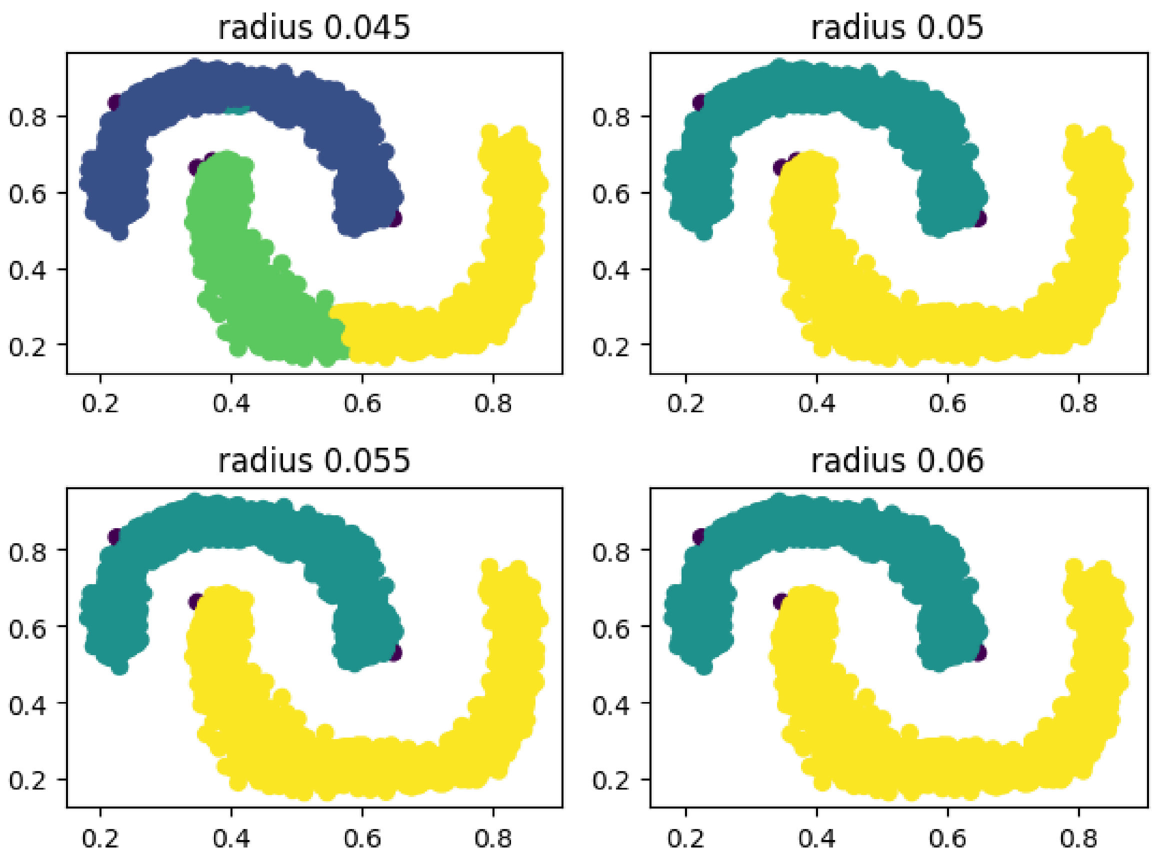
g) Can you think of a clustering algorithm that is better suited for this data set? Explain the mechanism behind such an algorithm

In [15]:

```
dbscan = DBSCAN(eps=0.055, min_samples=5)
labels_dbscan = dbscan.fit_predict(latent_z)

plt.figure(figsize=(8, 6))
plt.scatter(latent_z[:, 0], latent_z[:, 1], c=labels_dbscan, cmap='viridi
plt.title('DBSCAN Clustering on Latent Space')
plt.xlabel('Latent Dimension 1')
plt.ylabel('Latent Dimension 2')
plt.show()
```



In [16]:
```
# DBSCAN with different radius
radius = [0.045, 0.05, 0.055, 0.06]
fig, ax = plt.subplots(2,2)
c=0
for i in range(2):
  for j in range(2):
    dbscan = DBSCAN(eps=radius[c], min_samples=5, metric='euclidean')
    labels = dbscan.fit_predict(latent_z)
    ax[i,j].scatter(x[:,0],x[:,1],c=labels)
    ax[i,j].set_title('radius {}'.format(radius[c]))
    c+=1
plt.tight_layout()
plt.show()
```



2) Retail Store Business Case

i)

In [51]:
```
# Loading the data
path = 'retail_store_data.csv'
original_df = pd.read_csv(path)
original_df.head()
```

Out[51]:

	Education	Marital_Status	Income	Kids	Days_is_client	Recency	Expenses
0	Graduate	Single	58138.0	0	663	58	1617
1	Graduate	Single	46344.0	2	113	38	27
2	Graduate	Partner	71613.0	0	312	26	776
3	Graduate	Partner	26646.0	1	139	26	53
4	Postgraduate	Partner	58293.0	1	161	94	422

ii)

In [52]:
```
# Checking for missing values in each feature
display(original_df.isnull().any())
```

```
Education            False
Marital_Status       False
Income               False
Kids                 False
Days_is_client       False
Recency              False
Expenses             False
CustomerAge          False
TotalNumPurchases    False
TotalAcceptedCmp     False
Complain             False
Response             False
dtype: bool
```

In [54]:
```
print(f'retail_store_data shaper: {original_df.shape}')
```

```
retail_store_data shaper: (2216, 12)
```

In [55]:
```
# Checking for duplicates
print(f'Duplicates before removal: {original_df.duplicated().sum()}')

# Removing duplicates
df = original_df.drop_duplicates()
print(f'Duplicates after removal: {df.duplicated().sum()}')
```

```
Duplicates before removal: 185
Duplicates after removal: 0
```

In [56]:
```
# Identifying the categorical and numerical variables
cat_cols = df.select_dtypes(include=['object']).columns
num_cols = df.select_dtypes(include=np.number).columns.tolist()

print(f"Categorical Variables: {list(cat_cols)}")
print(f"Numerical Variables: {list(num_cols)}")
```

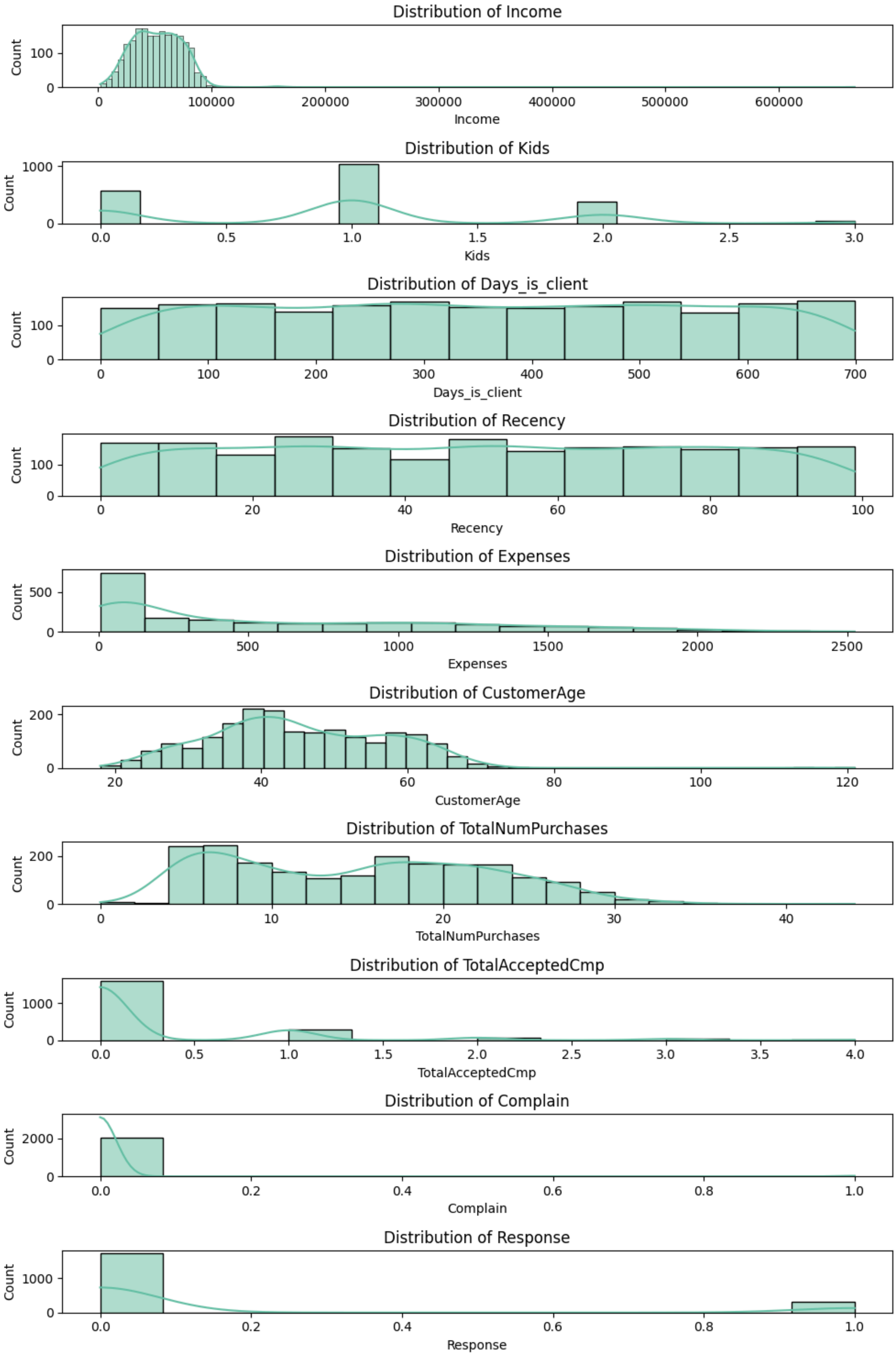
```
Categorical Variables: ['Education', 'Marital_Status']
Numerical Variables: ['Income', 'Kids', 'Days_is_client', 'Recency', 'Expe
nses', 'CustomerAge', 'TotalNumPurchases', 'TotalAcceptedCmp', 'Complain',
'Response']
```

In [57]:
```
df.describe(include='number').round(2).T
```

Out[57]:

	count	mean	std	min	25%	50%	75%
Income	2031.0	52376.80	25539.84	1730.0	35533.5	51563.0	68557.0
Kids	2031.0	0.95	0.75	0.0	0.0	1.0	1.0
Days_is_client	2031.0	352.56	202.34	0.0	178.0	352.0	528.0
Recency	2031.0	48.89	28.97	0.0	24.0	49.0	74.0
Expenses	2031.0	608.26	603.75	5.0	69.0	397.0	1048.5
CustomerAge	2031.0	45.21	11.98	18.0	37.0	44.0	55.0
TotalNumPurchases	2031.0	14.89	7.66	0.0	8.0	15.0	21.0
TotalAcceptedCmp	2031.0	0.30	0.68	0.0	0.0	0.0	0.0
Complain	2031.0	0.01	0.10	0.0	0.0	0.0	0.0
Response	2031.0	0.15	0.36	0.0	0.0	0.0	0.0

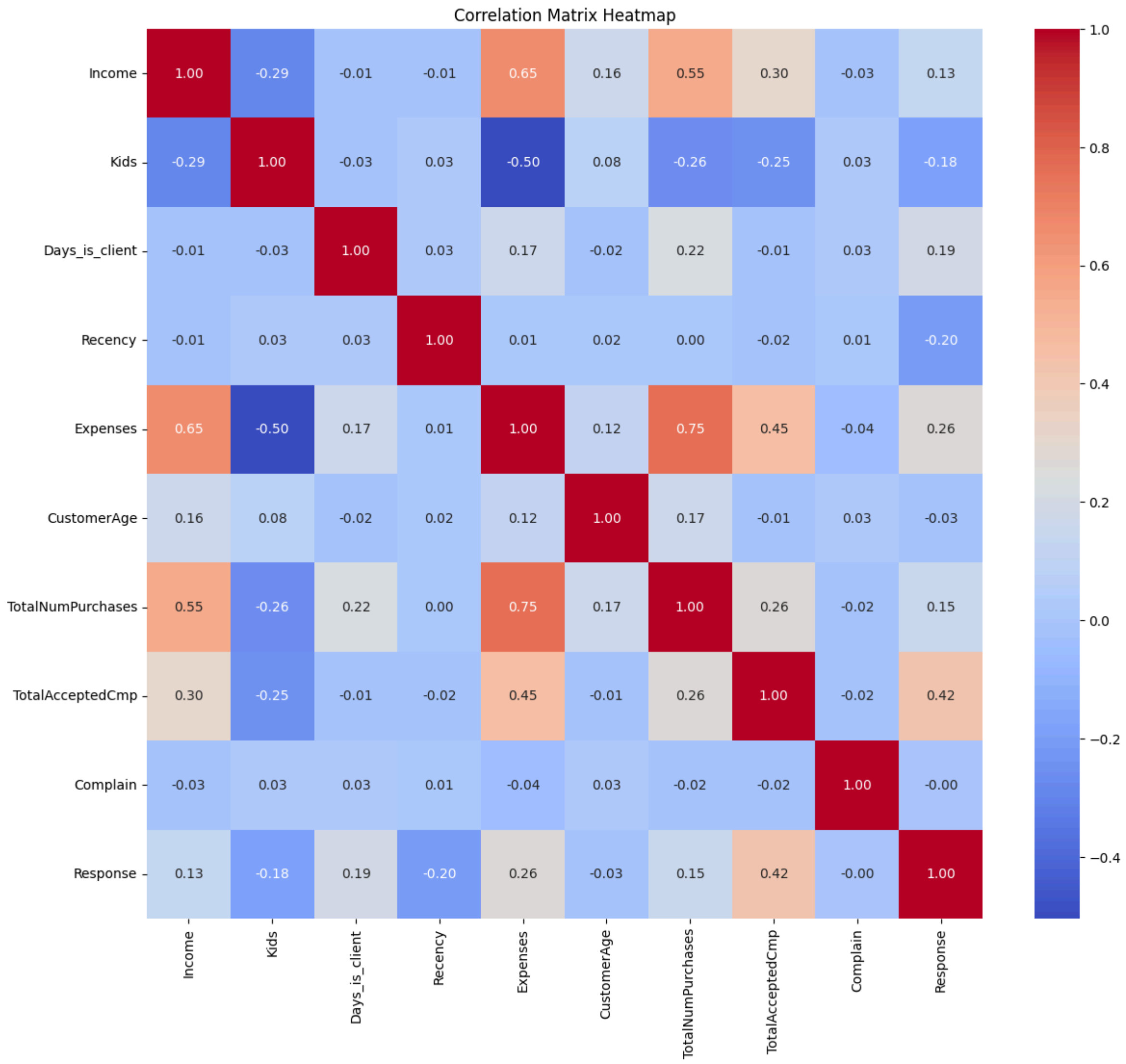
In [83]:
```
# Plotting the distribution of the numerical variables
fig, axes = plt.subplots(len(num_cols), 1, figsize=(10, 15))

for i, col in enumerate(num_cols):
        sns.histplot(data=df, x=col, ax=axes[i], kde=True)
        axes[i].set_title(f'Distribution of {col}')
plt.tight_layout()
plt.show()
```



In [148…

```
# Plotting the correlation matrix between numerical variables as a heatma
num_cols = df.select_dtypes(include=np.number).columns.tolist()
correlation_matrix = df[num_cols].corr()
plt.figure(figsize=(14, 12))
sns.heatmap(correlation_matrix, annot=True, fmt=".2f", cmap="coolwarm", c
plt.title("Correlation Matrix Heatmap")
```

Out[148… Text(0.5, 1.0, 'Correlation Matrix Heatmap')

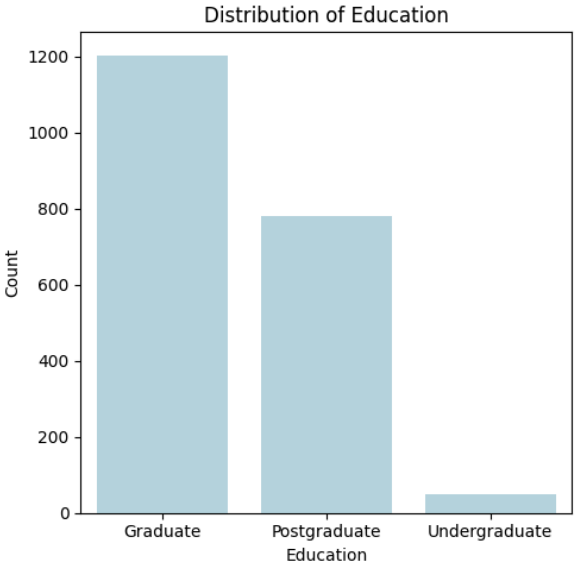


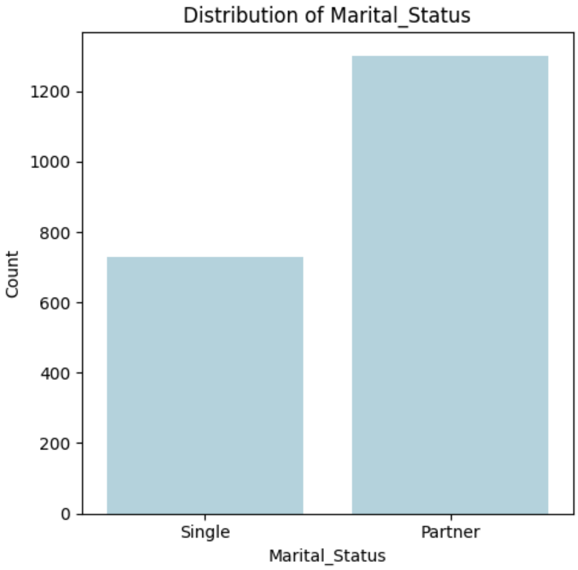
In [16]:
```
df.describe(include='object').round(2).T
```

Out[16]:

	count	unique	top	freq
Education	2031	3	Graduate	1203
Marital_Status	2031	2	Partner	1302

In [17]:
```
# Plotting the distribution of the categorical variables
fig, axes = plt.subplots(1, len(cat_cols), figsize=(10, 5))

for i, col in enumerate(cat_cols):
    sns.countplot(data=df, x=col, ax=axes[i], color='#ADD8E6')
    axes[i].set_title(f'Distribution of {col}', fontsize=12)
    axes[i].set_xlabel(col)
    axes[i].set_ylabel('Count')

plt.tight_layout()
plt.show()
```





a) What can you say about the data? Recall that it will be used to identify clusters, or groups of customers, in the retail store

iii)

In [58]:
```python
df_encoded = pd.get_dummies(df, columns=cat_cols, dtype =int)
df_encoded.head()
```

Out[58]:

	Income	Kids	Days_is_client	Recency	Expenses	CustomerAge	TotalNumPurc
0	58138.0	0	663	58	1617	57	
1	46344.0	2	113	38	27	60	
2	71613.0	0	312	26	776	49	
3	26646.0	1	139	26	53	30	
4	58293.0	1	161	94	422	33	

iv)

In [59]:
```python
# PCA transformation
df_mean = df_encoded.mean(axis=0)
df_encoded -= df_mean
C = np.cov(df_encoded.T)

w, v = LA.eig(C)

inx = w.argsort()[::-1]
w, v = w[inx], v[:, inx]

m = 15
w = w[:m]
v = v[:, :m]
print(f'Eigenvectors shape: {v.shape}')
```

```python
pca = df_encoded.dot(v)
print(f'PCA shape: {pca.shape}')
```

```
Eigenvectors shape: (15, 15)
PCA shape: (2031, 15)
```

b) Explain what is the reason for using PCA and not reducing the number of dimensions in the input data

c) For K = 1, 2, · · · , 10, plot the Silhouette and Davies-Bouldin scores for both hierarchical clustering and K-means, i.e. you are supposed to show 4 plots

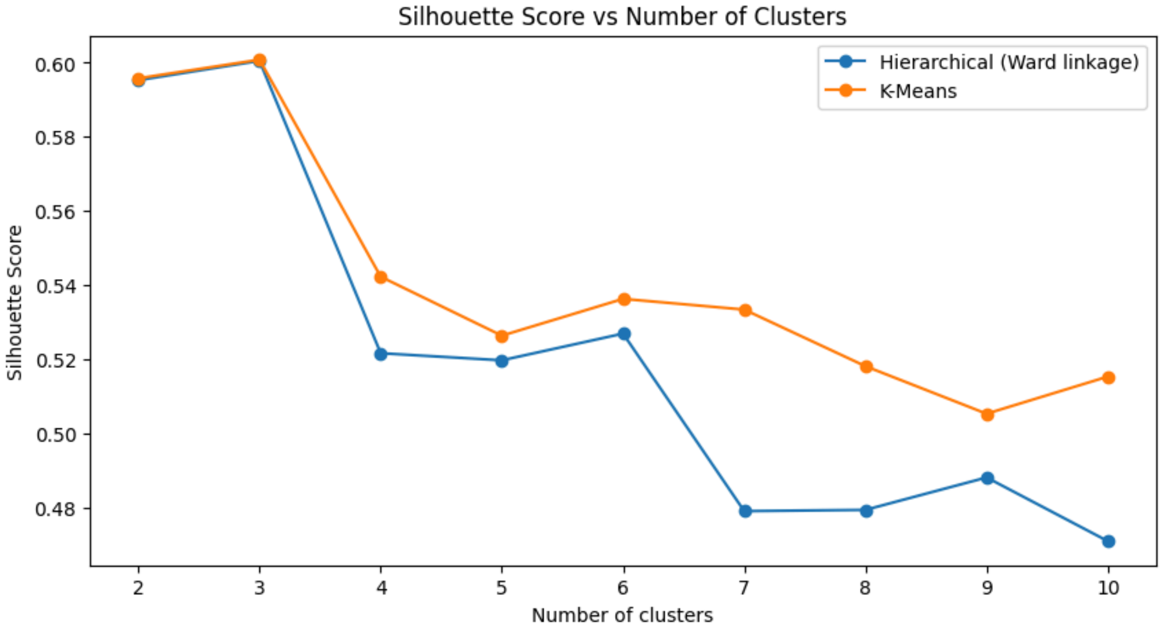
In [60]:

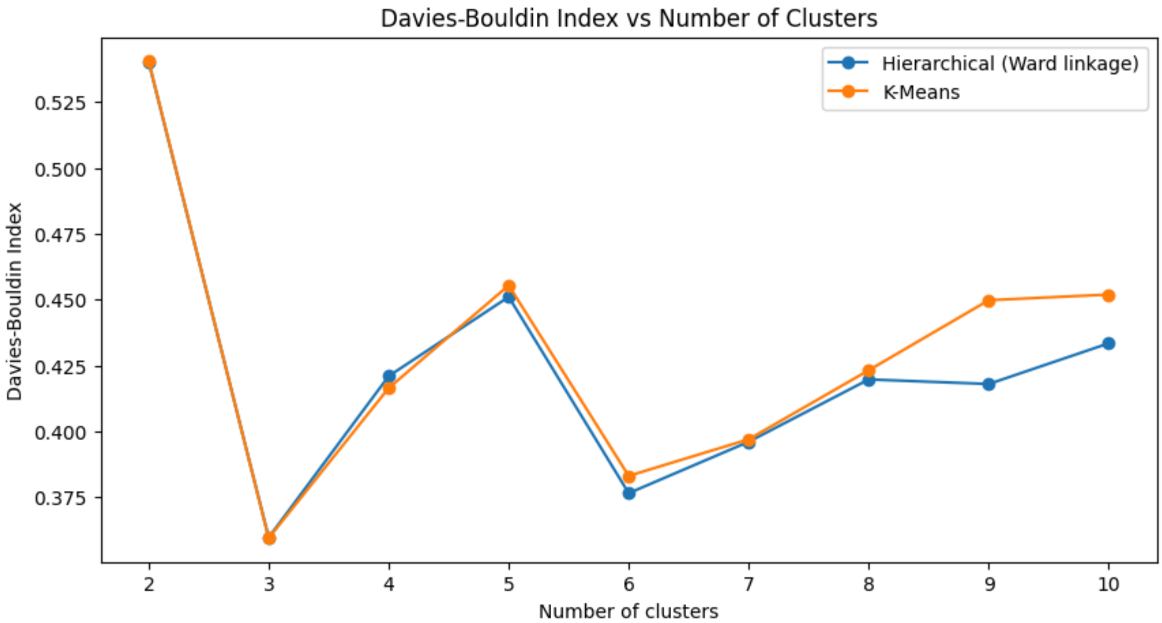
```python
K = 10 #how many times we repeat the calculations

hierarchical_cv = {}
kmeans_cv = {}
hierarchical_silhouette = []
hierarchical_db = []
kmeans_silhouette = []
kmeans_db = []

for k in range(2, K+1): # we start at 2, since clustering for K=1 doesnt
  # First hierarchical
  P = linkage(pca, method='ward', metric='euclidean')
  cluster_labels = fcluster(P, k, criterion='maxclust')
  silhouette = silhouette_score(pca, cluster_labels)
  db = davies_bouldin_score(pca,cluster_labels)

  # save results
  temp_dict = {'silhouette':silhouette,'db':db}
  hierarchical_cv[k] = temp_dict
  hierarchical_silhouette.append(silhouette)
  hierarchical_db.append(db)

  # Second kmeans
  kmeans = KMeans(n_clusters=k, n_init="auto")
  cluster_labels = kmeans.fit_predict(pca)
  silhouette = silhouette_score(pca, cluster_labels)
  db  = davies_bouldin_score(pca, cluster_labels)

  # save results
  temp_dict = {'silhouette':silhouette,'db':db}
  kmeans_cv[k] = temp_dict
  kmeans_silhouette.append(silhouette)
  kmeans_db.append(db)
```

In [25]:

```python
k_range = range(2, K+1)

# Plot the silhouette scores for hierarchical and k-means clustering
plt.figure(figsize=(10, 5))
plt.plot(k_range, hierarchical_silhouette, label='Hierarchical (Ward link
plt.plot(k_range, kmeans_silhouette, label='K-Means', marker='o')
plt.xlabel('Number of clusters')
plt.ylabel('Silhouette Score')
plt.title('Silhouette Score vs Number of Clusters')
plt.legend()
plt.show()
```

```
# Plot the Davies-Bouldin scores for hierarchical and k-means clustering
plt.figure(figsize=(10, 5))
plt.plot(k_range, hierarchical_db, label='Hierarchical (Ward linkage)', m
plt.plot(k_range, kmeans_db, label='K-Means', marker='o')
plt.xlabel('Number of clusters')
plt.ylabel('Davies-Bouldin Index')
plt.title('Davies-Bouldin Index vs Number of Clusters')
plt.legend()
plt.show()
```





d) What are the number of clusters suggested by your cross-validation? Does the Silhouette and Davies-Bouldin scores agree with each other?

v) - vii)

In [61]:
```
# Autoencoder definition
class AutoEncoder(tf.keras.Model):
```

```
    def __init__(self, enc, dec, name='autoencoder'):
      super().__init__()
      self.encoder = enc
      self.decoder = dec
      self.cross_entropy = tf.keras.losses.BinaryCrossentropy()

      self.params = encoder.trainable_variables + decoder.trainable_varia

    def call(self, inputs):
      z = self.encoder(inputs)
      x_hat = self.decoder(z)

      self.loss = tf.reduce_mean(tf.keras.losses.MSE(inputs,x_hat))

    @tf.function
    def train(self,x, optimizer):
        with tf.GradientTape() as tape:
            enc_dec = self.call(x)
        gradients = tape.gradient(self.loss, self.params)
        optimizer.apply_gradients(zip(gradients, self.params))

        return self.loss

    def rep_learning(self, x):
      z = self.encoder(x)

      return z

    def reconstruct(self, x):
      z = self.rep_learning(x)
      x_hat = self.decoder(z)

      return x_hat
```

In [62]:

```
# Hyperparameters
input_shape = (15, )
activation = 'tanh'
units = 35
z_dim = 10
```

In [63]:

```
# Encoder and decoder architecture
encoder = Sequential([
    layers.InputLayer(input_shape=(input_shape)),
    layers.Dense(units, activation=activation),  # 1st hidden layer
    layers.Dense(units, activation=activation),  # 2nd hidden layer
    layers.Dense(units, activation=activation),  # 3rd hidden layer
    layers.Dense(z_dim, activation='linear'),    # output layer (latent s
])

decoder = Sequential([
    layers.InputLayer(input_shape=(z_dim,)),
    layers.Dense(units, activation=activation),  # 1st hidden layer
    layers.Dense(units, activation=activation),  # 2nd hidden layer
    layers.Dense(units, activation=activation),  # 3rd hidden layer
    layers.Dense(15, activation='linear'),  # output layer with linear ac
])

warnings.filterwarnings("ignore", category=UserWarning, message="Argument
```

In [64]:
```
# Training the autoencoder
seed = 42
random.seed(seed)
np.random.seed(seed)
tf.random.set_seed(seed)

autoencoder = AutoEncoder(encoder, decoder)
batch_size = 256
tr_data  = tf.data.Dataset.from_tensor_slices(pca).shuffle(pca.shape[0]).
optimizer = tf.keras.optimizers.Adam(learning_rate=0.01)

counter = 0
epochs  = 1000
start = time.time()
losses = []
while counter < epochs:
  for i, x_batch in enumerate(tr_data):
    loss = autoencoder.train(x_batch, optimizer)
    # losses.append(loss.numpy())  # storing loss
  counter+=1
  if counter%10==0:
    print('Model has been trained for {} epochs'.format(counter))
elapsed_time = time.time() - start
minutes, seconds = divmod(elapsed_time, 60)
print('Elapsed time: {}m {:.1f}s'.format(int(minutes), seconds))
```

```
Model has been trained for 10 epochs
Model has been trained for 20 epochs
Model has been trained for 30 epochs
Model has been trained for 40 epochs
Model has been trained for 50 epochs
Model has been trained for 60 epochs
Model has been trained for 70 epochs
Model has been trained for 80 epochs
Model has been trained for 90 epochs
Model has been trained for 100 epochs
Model has been trained for 110 epochs
Model has been trained for 120 epochs
Model has been trained for 130 epochs
Model has been trained for 140 epochs
Model has been trained for 150 epochs
Model has been trained for 160 epochs
Model has been trained for 170 epochs
Model has been trained for 180 epochs
Model has been trained for 190 epochs
Model has been trained for 200 epochs
Model has been trained for 210 epochs
Model has been trained for 220 epochs
Model has been trained for 230 epochs
Model has been trained for 240 epochs
Model has been trained for 250 epochs
Model has been trained for 260 epochs
Model has been trained for 270 epochs
Model has been trained for 280 epochs
Model has been trained for 290 epochs
Model has been trained for 300 epochs
Model has been trained for 310 epochs
Model has been trained for 320 epochs
Model has been trained for 330 epochs
Model has been trained for 340 epochs
Model has been trained for 350 epochs
Model has been trained for 360 epochs
Model has been trained for 370 epochs
Model has been trained for 380 epochs
Model has been trained for 390 epochs
Model has been trained for 400 epochs
Model has been trained for 410 epochs
Model has been trained for 420 epochs
Model has been trained for 430 epochs
Model has been trained for 440 epochs
Model has been trained for 450 epochs
Model has been trained for 460 epochs
Model has been trained for 470 epochs
Model has been trained for 480 epochs
Model has been trained for 490 epochs
Model has been trained for 500 epochs
Model has been trained for 510 epochs
Model has been trained for 520 epochs
Model has been trained for 530 epochs
Model has been trained for 540 epochs
Model has been trained for 550 epochs
Model has been trained for 560 epochs
Model has been trained for 570 epochs
Model has been trained for 580 epochs
Model has been trained for 590 epochs
Model has been trained for 600 epochs
```

```
Model has been trained for 610 epochs
Model has been trained for 620 epochs
Model has been trained for 630 epochs
Model has been trained for 640 epochs
Model has been trained for 650 epochs
Model has been trained for 660 epochs
Model has been trained for 670 epochs
Model has been trained for 680 epochs
Model has been trained for 690 epochs
Model has been trained for 700 epochs
Model has been trained for 710 epochs
Model has been trained for 720 epochs
Model has been trained for 730 epochs
Model has been trained for 740 epochs
Model has been trained for 750 epochs
Model has been trained for 760 epochs
Model has been trained for 770 epochs
Model has been trained for 780 epochs
Model has been trained for 790 epochs
Model has been trained for 800 epochs
Model has been trained for 810 epochs
Model has been trained for 820 epochs
Model has been trained for 830 epochs
Model has been trained for 840 epochs
Model has been trained for 850 epochs
Model has been trained for 860 epochs
Model has been trained for 870 epochs
Model has been trained for 880 epochs
Model has been trained for 890 epochs
Model has been trained for 900 epochs
Model has been trained for 910 epochs
Model has been trained for 920 epochs
Model has been trained for 930 epochs
Model has been trained for 940 epochs
Model has been trained for 950 epochs
Model has been trained for 960 epochs
Model has been trained for 970 epochs
Model has been trained for 980 epochs
Model has been trained for 990 epochs
Model has been trained for 1000 epochs
Elapsed time: 0m 11.7s
```

e) Generate the latent representation for all data using the function rep_learning and visualize it in a 2-dimensional scatter plot using the t-SNE dimensionality reduction method

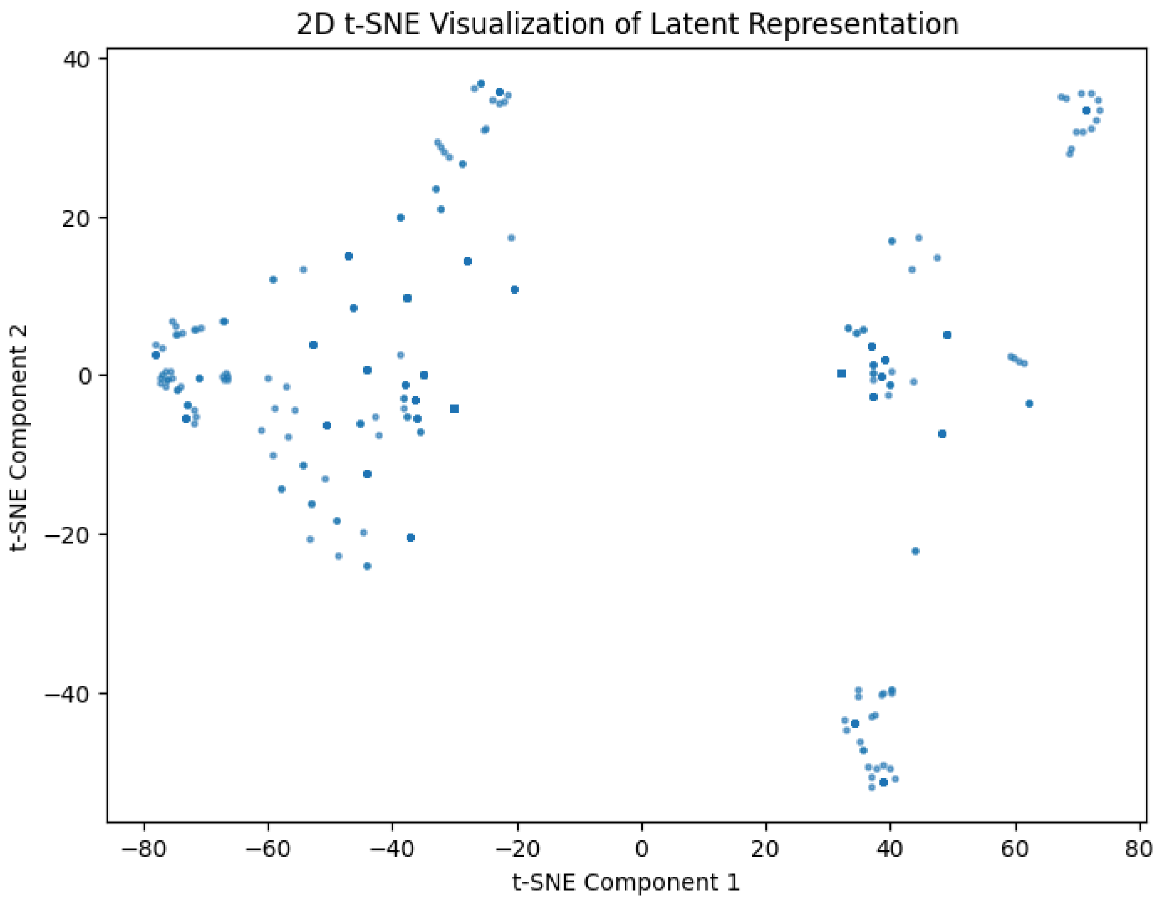
In [157…

```
z = autoencoder.rep_learning(pca)
z_2d = TSNE(n_components=2, n_jobs=-1).fit_transform(z)

print(f"Shape of 2D representation: {z_2d.shape}")
```

```
Shape of 2D representation: (2031, 2)
```

In [353…

```
plt.figure(figsize=(8, 6))
plt.scatter(z_2d[:, 0], z_2d[:, 1], s=5, alpha=0.6)
plt.title("2D t-SNE Visualization of Latent Representation")
plt.xlabel("t-SNE Component 1")
plt.ylabel("t-SNE Component 2")
plt.show()
```



f) How many clusters can you identify in the scatter plot? Is it similar to the number of clusters that you identified in exercise d)?

viii)

In [65]:
```
# K-means clustering
kmeans_pca = KMeans(n_clusters=3, random_state=42)
kmeans_pca.fit(pca)
labels = kmeans_pca.labels_
```

g) Write the executive summary for the retail company analyzing each of the clusters. What are the features that distinguish each cluster? Use any method that you consider appropriate, e.g. average values, standard deviations, histograms etc.

In [66]:
```
pca_clusters = pca.assign(Cluster=labels)
pca_clusters.head()
```

Out[66]:

	0	1	2	3	4	5
0	5776.069014	951.164104	194.498201	-7.078760	11.116068	-0.378839
1	-6041.035925	-514.075150	-177.812244	9.386709	15.133806	-3.749332
2	19236.501193	-133.282218	-22.288858	22.384152	2.780221	3.944302
3	-25736.286918	-183.307294	-195.013501	22.308189	-13.257693	0.309136
4	5912.635116	-299.094190	-154.821140	-46.064192	-12.728603	7.016106

In [67]:
```
# Assigning the cluster labels we obtained from K-means to the original d
df_clusters = df.assign(Cluster=labels)
df_clusters.head()
```

Out[67]:

	Education	Marital_Status	Income	Kids	Days_is_client	Recency	Expenses
0	Graduate	Single	58138.0	0	663	58	1617
1	Graduate	Single	46344.0	2	113	38	27
2	Graduate	Partner	71613.0	0	312	26	776
3	Graduate	Partner	26646.0	1	139	26	53
4	Postgraduate	Partner	58293.0	1	161	94	422

In [69]:
```
cluster_counts = df_clusters['Cluster'].value_counts().sort_index()

# Display the number of observations in each cluster
cluster_counts_table = pd.DataFrame(cluster_counts).reset_index()
cluster_counts_table.columns = ['Cluster', 'Number of Observations']
cluster_counts_table
```

Out[69]:

	Cluster	Number of Observations
0	0	544
1	1	710
2	2	777

In [70]:
```
# Displaying cluster statistics
cat_cols = df_clusters.select_dtypes(include=['object']).columns
num_cols = df_clusters.select_dtypes(include=np.number).columns

for cluster in df_clusters['Cluster'].unique():
    print(f"\nCluster {cluster}")
    cluster_data = df_clusters[df_clusters['Cluster'] == cluster]

    # For numerical features
    print("Numerical Features Statistics:")
    display(cluster_data.describe(include='number').round(2).T)
```

```python
    # For categorical features
    print("Categorical Features Mode:")
    display(cluster_data.describe(include='object').round(2).T)
```

```
Cluster 2
Numerical Features Statistics:
```

	count	mean	std	min	25%	50%	75%
Income	777.0	54454.54	7485.15	41713.0	47889.0	54450.0	61180.0
Kids	777.0	1.16	0.68	0.0	1.0	1.0	2.0
Days_is_client	777.0	354.56	199.83	2.0	188.0	352.0	526.0
Recency	777.0	49.44	28.86	0.0	25.0	50.0	74.0
Expenses	777.0	557.37	409.93	15.0	222.0	468.0	858.0
CustomerAge	777.0	48.18	10.93	22.0	40.0	48.0	57.0
TotalNumPurchases	777.0	16.95	6.97	4.0	12.0	17.0	22.0
TotalAcceptedCmp	777.0	0.21	0.50	0.0	0.0	0.0	0.0
Complain	777.0	0.01	0.09	0.0	0.0	0.0	0.0
Response	777.0	0.11	0.31	0.0	0.0	0.0	0.0
Cluster	777.0	2.00	0.00	2.0	2.0	2.0	2.0

```
Categorical Features Mode:
```

	count	unique	top	freq
Education	777	2	Graduate	436
Marital_Status	777	2	Partner	500

```
Cluster 0
Numerical Features Statistics:
```

	count	mean	std	min	25%	50%	75%
Income	544.0	79970.45	27781.53	67225.0	71840.00	77261.5	82356.25
Kids	544.0	0.39	0.60	0.0	0.00	0.0	1.00
Days_is_client	544.0	347.74	205.61	0.0	166.75	352.0	525.00
Recency	544.0	48.48	29.68	0.0	23.00	48.0	73.25
Expenses	544.0	1343.64	466.14	6.0	1015.00	1347.0	1674.50
CustomerAge	544.0	46.34	13.03	19.0	37.00	45.0	56.00
TotalNumPurchases	544.0	20.85	5.02	0.0	17.00	21.0	24.00
TotalAcceptedCmp	544.0	0.70	1.01	0.0	0.00	0.0	1.00
Complain	544.0	0.00	0.06	0.0	0.00	0.0	0.00
Response	544.0	0.26	0.44	0.0	0.00	0.0	1.00
Cluster	544.0	0.00	0.00	0.0	0.00	0.0	0.00

```
Categorical Features Mode:
```

	count	unique	top	freq
Education	544	2	Graduate	334
Marital_Status	544	2	Partner	344

```
Cluster 1
Numerical Features Statistics:
```

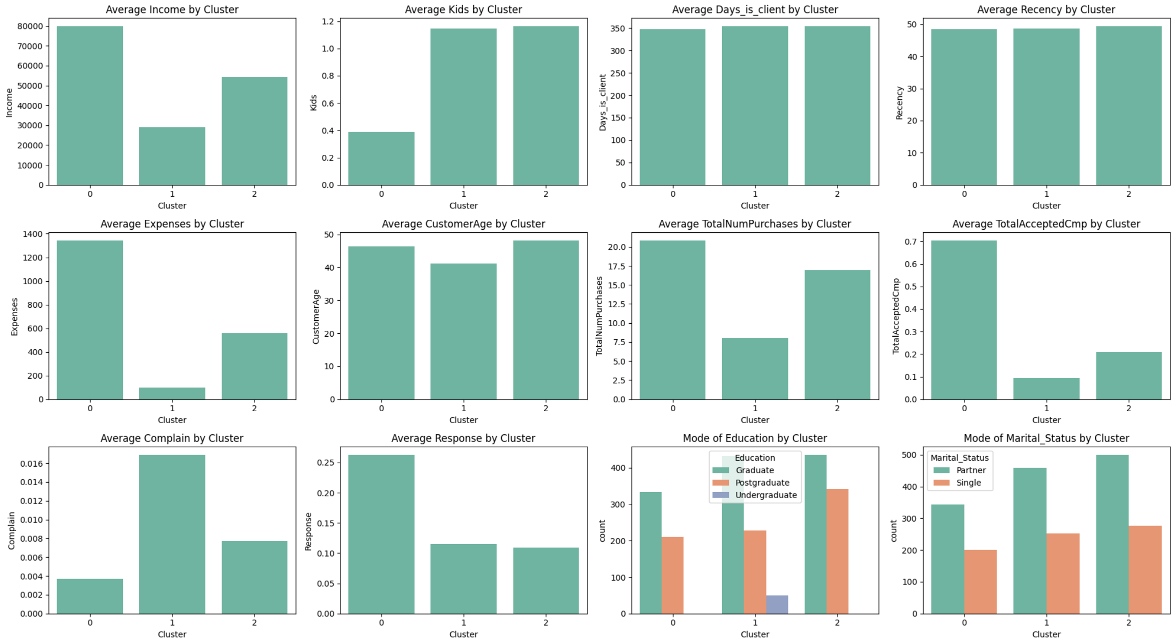
	count	mean	std	min	25%	50%	75%
Income	710.0	28960.82	8783.88	1730.0	23239.0	30514.5	36136.00
Kids	710.0	1.15	0.69	0.0	1.0	1.0	2.00
Days_is_client	710.0	354.06	202.76	0.0	175.5	351.5	531.75
Recency	710.0	48.60	28.57	0.0	24.0	49.0	74.00
Expenses	710.0	100.50	131.79	5.0	35.0	57.0	101.00
CustomerAge	710.0	41.09	11.05	18.0	33.0	40.0	47.00
TotalNumPurchases	710.0	8.08	4.12	0.0	5.0	7.0	10.00
TotalAcceptedCmp	710.0	0.09	0.30	0.0	0.0	0.0	0.00
Complain	710.0	0.02	0.13	0.0	0.0	0.0	0.00
Response	710.0	0.12	0.32	0.0	0.0	0.0	0.00
Cluster	710.0	1.00	0.00	1.0	1.0	1.0	1.00

```
Categorical Features Mode:
```

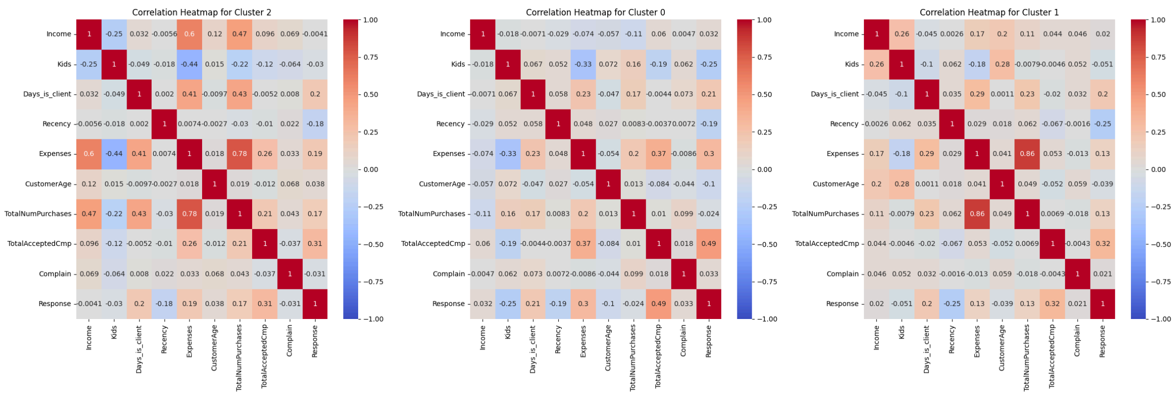
	count	unique	top	freq
Education	710	3	Graduate	433
Marital_Status	710	2	Partner	458

In [97]:

```python
# Plotting the distribution of the numerical features by cluster
plt.style.use('default')
sns.set_palette("Set2")

fig, axes = plt.subplots(3, 4, figsize=(20, 12))
axes = axes.ravel()

# Plot numerical features (average by cluster)
for i, feature in enumerate(num_cols):
    if feature == 'Cluster':
        continue
    sns.barplot(data=df_clusters, x='Cluster', y=feature, errorbar=None,
    axes[i].set_title(f"Average {feature} by Cluster")

# Plot categorical features (mode by cluster)
for j, feature in enumerate(cat_cols, start=len(num_cols)-1):
    sns.countplot(data=df_clusters, x='Cluster', hue=feature, ax=axes[j])
    axes[j].set_title(f"Mode of {feature} by Cluster")
    axes[j].legend(title=feature)

plt.tight_layout(rect=[0, 0.03, 1, 0.95])
plt.show()
```



In [98]:
```python
# Plotting the correlation heatmap for each cluster
num_cols_without_cluster = [col for col in num_cols if col != 'Cluster']
fig, axes = plt.subplots(1, 3, figsize=(24, 8))

for idx, cluster in enumerate(df_clusters['Cluster'].unique()):
    sns.heatmap(df_clusters[df_clusters['Cluster'] == cluster][num_cols_w
                annot=True, cmap="coolwarm", vmin=-1, vmax=1, ax=axes[idx
    axes[idx].set_title(f"Correlation Heatmap for Cluster {cluster}")

plt.tight_layout()
plt.show()
```



In [99]:
```python
# Plotting the income vs. expenses by cluster
plt.figure(figsize=(6, 4))
sns.scatterplot(data=df_clusters, x='Income', y='Expenses', hue='Cluster'
plt.title("Income vs. Expenses by Cluster")
plt.xlabel("Income")
plt.ylabel("Expenses")
plt.legend(title="Cluster")
plt.show()
```

In [103…
```
df_melted = pd.melt(df_clusters, id_vars='Cluster', value_vars=['TotalNum
                    var_name='EngagementMetric', value_name='Count')

plt.figure(figsize=(6, 4))
sns.barplot(data=df_melted, x='Cluster', y='Count', hue='EngagementMetric
plt.title("Total Purchases and Accepted Campaigns by Cluster")
plt.xlabel("Cluster")
plt.ylabel("Total Count")
plt.legend(title="Engagement Metric")
plt.show()
```

In [105…

```
# Plotting the number of kids by cluster
plt.figure(figsize=(6, 4))
sns.countplot(data=df_clusters, x='Cluster', hue='Kids', palette="Set2")
plt.title("Number of Kids by Cluster")
plt.xlabel("Cluster")
plt.ylabel("Count")
plt.legend(title="Kids")
plt.show()
```