Compact formats for storing directed graphs

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Representing digraphs

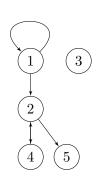
How do we represent this digraph?

► Edge set: {(1,1),(1,2),(2,4),(2,5),(4,2)}

► Adjacencies: ({1,2}, {4,5}, {}, {2}, {})

► Adjacency matrix:

| | 1 | 2 | 3 | 4 | 5 |
|---|----------|----------|---|----------|----------|
| 1 | √ | √ | | | |
| 2 | | | | √ | √ |
| 3 | | | | | |
| 4 | | √ | | | |
| 5 | | | | | |



Storing as text

- ► Edge set: $\{(1,1),(1,2),(2,4),(2,5),(4,2)\}$
- ► Adjacencies list: ({1,2}, {4,5}, {}, {2}, {})
- Adjacency matrix

| Print the edge set: | Print the adjacencies: | Print a matrix: |
|---------------------|------------------------|-----------------|
| 1>1_ | 1,2_ | 11000 |
| 1>2_ | 4,5_ | 00011 |
| 2>4_ | _ | 00000 |
| 2>5_ | 2_ | 01000 |
| 4>2 | | 00000 |
| (19 characters) | (11 characters) | (25 characters) |
| $(\sim 4e)$ | $(\sim 2e + v)$ | $(\sim v^2)$ |

The matrix approach: Digraph6

| 1 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

- ▶ 10000 10010 00000 01000 01000
- ► 100001 001000 000010 000100 000000
- **▶** 33 8 2 4 0
- **▶** 96 71 65 67 63
- ' G A C ?
- +D'GAC? (7 characters)

The adjacencies/edge-set approach: Disparse6

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► Edge set: \{(1,1),(1,2),(2,4),(2,5),(4,2)\}
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- (1,3) (0,0) (0,1) (1,3) (1,4)
- ► 1 3 0 1 1 5 0 0 1 0 1 3 0 1 1 1
- ▶ 1 011 0 001 1 101 0 000 1 000 1 011 0 001 1 001
- ▶ 101100 011101 000010 001011 000110 011111
- ▶ 44 29 2 11 6 31
- **▶** 107 92 65 74 69 94
- ▶ k \ A J E ^
- .Dk\AJE^ (9 characters)

Choosing a representation

- ▶ Digraph6 gives a string of $\sim v^2$ bits.
- ▶ Disparse6 gives a string of $\sim \frac{3}{2}e(\lceil \log_2 v \rceil + 1)$ bits.
- ▶ When writing a graph, calculate both of these and choose the smaller.