CSCD 327: RELATIONAL DATABASE SYSTEMS

RELATIONAL ALGEBRA

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Formal Relational Query Languages

- Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:
  - *Relational Algebra*: procedural, tells you how to process a query, very useful for representing execution plans.
  - *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-procedural, *declarative*.)

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Relational Algebra

- Six basic operators
  - select: σ
  - project: Π
  - union: $\cup$
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- The operators take one or two relations as inputs and produce a new relation as a result.

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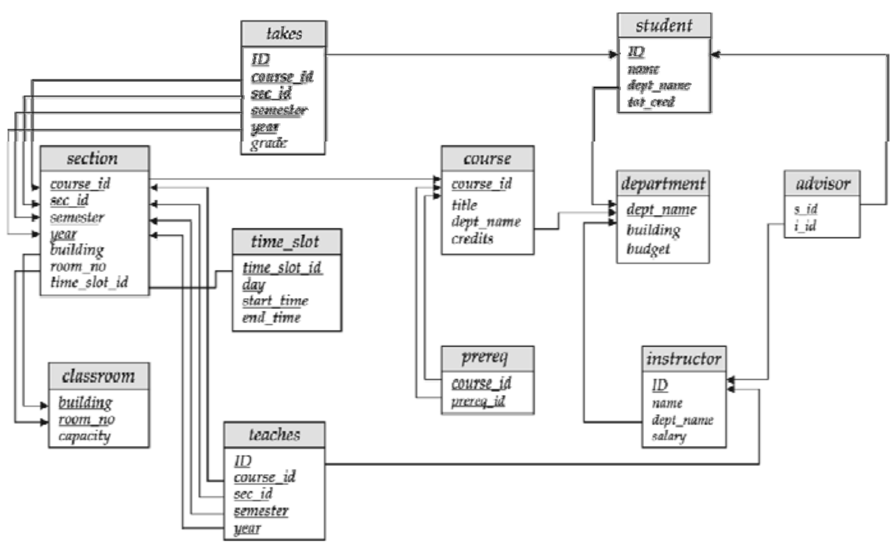
Running Example: Schema Diagram for University Database



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Select Operation

- Notation: $\sigma_p(r)$
- *p* is called the **selection predicate**
- Defined as:

$$\sigma_p(\mathbf{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

Where *p* is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
Each **term** is one of:

<attribute> *op* <attribute> or <constant>

where *op* is one of: $=, \neq, >, \geq. <. \leq$

- Example of selection: Find all instructors who work in Physics dept.

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Project Operation

- Notation: $\prod_{A_1, A_2, \ldots, A_k} (r)$

  where A_1, A_2 are attribute names and r is a relation name.
- The result is defined as the relation of *k* columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the *dept_name* attribute of *instructor*

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Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\prod_{A,C} (r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

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Union Operation

- Notation: $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For $r \cup s$ to be valid.
  1. *r, s* must have the *same* **degree.**
  2. The attribute domains must be **compatible**
- Example: to find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

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Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r \cup s$:

A	B
α	1
α	2
β	1
β	3

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Set Difference Operation

- Notation $r - s$
- Defined as:

  $r - s = \{t \mid t \in r \text{ and } t \notin s\}$

- Set differences must be taken between **compatible** relations.
  - r and s must have the same arity
  - attribute domains of r and s must be compatible
- Example: to find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

Set difference of two relations

- Relations r, s:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r - s$:

A	B
α	1
β	1

Cartesian-Product Operation

- Notation *r* x *s*
- Defined as:

$$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \emptyset$).
- If attributes of *r(R)* and *s(S)* are not disjoint, then renaming must be used.

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Cartesian-Product Operation – Example

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

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Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$
- $r \times s$

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \times s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

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Rename Operation

- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_x(E)$$

returns the expression *E* under the name *X*

- If a relational-algebra expression *E* has degree *n*, then

$$\rho_{x(A_1, A_2, \ldots, A_n)}(E)$$

returns the result of expression *E* under the name *X*, and with the attributes renamed to $A_1, A_2, \ldots, A_n$.

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Example Query

- Find the largest salary in the university
  - Step 1: find instructor salaries that are less than some other instructor salary (i.e. not maximum)
  - Step 2: Find the largest salary

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Example Queries

- Find the names of all instructors in the Physics department, along with the *course_id* of all courses they have taught
  - Solution 1
  - Solution 2

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Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Division
- Assignment

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Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \textbf{ and } t \in s \}$
- Assume:
  - *r*, *s* have the *same arity*
  - attributes of *r* and *s* are compatible
- Note: $r \cap s = r - (r - s)$

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Set-Intersection Operation – Example

- Relation *r, s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r $\cap$ s

A	B
α	2

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Natural-Join Operation

- Notation: r $\bowtie$ s

- Let *r* and *s* be relations on schemas *R* and *S* respectively. Then, r $\bowtie$s is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from s.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on s
- Example:

  R = (*A*, *B*, *C*, *D*)

  S = (*E*, *B*, *D*)

  - Result schema = (*A*, *B*, *C*, *D*, *E*)
  - $r \bowtie s$ is defined as:

$$\prod_{r.A,\ r.B,\ r.C,\ r.D,\ s.E} (\sigma_{r.B\ =\ s.B\ \wedge\ r.D\ =\ s.D}\ (r\ \times\ s))$$

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Natural Join Example

- Relations r, s:

r

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

s

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

- r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

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Another Natural Join Example

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
- Natural join is associative
- Natural join is commutative

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Division Operation

- Notation: $r \div s$
- Suited to queries that include the phrase "for all".
- Let *r* and *s* be relations on schemas *R* and *S* respectively where
  - $R = (A_1, \ldots, A_m, B_1, \ldots, B_n)$
  - $S = (B_1, \ldots, B_n)$

  The result of $r \div s$ is a relation on schema

  $R - S = (A_1, \ldots, A_m)$

  A tuple *t* is in $r \div s$ if and only if both of two conditions hold:

  1. *t* is in $\prod_{R-S}(r)$
  2. For every tuple t_s in *S*, there is a tuple t_r in *R* satisfying both of the following:
     1. $t_r[S] = t_s[S]$
     2. $t_r[R\text{-}S] = t$

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Division Operation – Example

- Relations *r, s*:

A	*B*
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

- $r \div s$:

A
α
β

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Another Division Example

- Relations *r*, *s*:

A	B	C	D	E
α	a	α	a	1
α	a	γ	a	1
α	a	γ	b	1
β	a	γ	a	1
β	a	γ	b	3
γ	a	γ	a	1
γ	a	γ	b	1
γ	a	β	b	1

r

D	E
a	1
b	1

s

- $r \div s$:

A	B	C
α	a	γ
γ	a	γ

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Assignment Operation

- The assignment operation ($\leftarrow$) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
  - a series of assignments
  - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
- Example: Write $r \cap s$ as

$$temp \leftarrow r - s$$
$$result \leftarrow r - temp$$

  - The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.

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