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Derivatives Pricing theory

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1 PDF

The integral of PDF is CDF The derivative of CDF is PDF

The probability density function (pdf) for a normal distribution is given by

$$f(x) = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{1}$$

The equation for the area under the normal curve is...

$$\int_{-\infty}^{+\infty} f(x)\Delta x = 1 \tag{2}$$

Substituting 1 into 2 gives

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\Delta x = 1 \tag{3}$$

Z-score is given by

$$z = \frac{x-\mu}{\sigma} \tag{4}$$

Substituting 4 into 1 gives

$$f(x) = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2} \tag{5}$$

Simplest case of a normal distribution is with $\mu = 0$ and $\sigma = 1$ and it is called standard normal

$$f(x) = \frac{1}{1}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2} \tag{6}$$

2 CDF

cdf for standard normal

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}\Delta x \tag{7}$$

and the derivative of CDF gives back the PDF from 6

$$\frac{\partial F(x)}{\partial x} = f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2} \tag{8}$$

3 Black-Scholes

Developed in 1973 by Fischer Black, Robert Merton, and Myron Scholes

- S_0 current spot rate
  - aaaa
- K strike price
- $F(.)$ cumulative density function for normal distribution (see 7)
- r_d domestic risk-free rate
- r_f foreign risk-free rate
- σ volatility of the FX rate
- T time to maturity (day count convention)

$$d_1 = \frac{\log\left(\frac{S_0}{K}\right) + \left(r_d + r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{9}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{10}$$

Once we have d_1, d_2, we can calculate the value for call/put

$$c = S_0 \exp^{-r_f T} F(d_1) - K \exp^{-r_d T} F(d_2) \tag{11}$$

$$p = K \exp^{-r_d T} F(-d_2) - S_0 \exp^{-r_f T} F(-d_1) \tag{12}$$