

## Week 4: Fitting a Galaxy Spectrum with Stellar Spectra

### Task:

Model the stellar populations which make up a galaxy spectrum by combining a set of spectral templates drawn from optical spectra of Main Sequence and Giant stars of various spectral classes. Write a C program to read in a set of spectral templates found on the ASTR698 class web pages and a spectrum of the galaxy NGC 3512. Scale each spectral template independently to create a combined best-fit model spectrum which matches that of the galaxy. Using the Moore-Penrose inversion technique, derive a least squares solution. For a galaxy spectrum  $F_\lambda$ , we can fit the scalar amplitudes  $s_j$  to the stellar templates  $T_j^\lambda$  as follows. To achieve a best-fit match,

$$F_\lambda = \sum_{j=0}^n s_j T_j^\lambda \quad \text{where } \mathbf{s} = (\mathbf{T}^t \mathbf{T})^{-1} \mathbf{T}^t \mathbf{F}$$

and  $n$  is one less than the number of stellar templates to be used.

Explore the effect of using various subsets and all of the stellar templates. Calculate a reduced  $\chi^2$  value for the fits. How does the use of Giant stars compared to the Main Sequence components? Are there any non-physical repercussions to our choice of analysis technique?

Are there particular galaxy features which are consistently poorly, or well, fit by the stellar templates? Do your fits, and the derived reduced  $\chi^2$  values, make intuitive sense, based on your knowledge of stellar populations? Discuss roughly extrapolated number-weighted counts of various populations, based on the luminosity-weighted amplitudes derived.

Add uniformly distributed noise to the galaxy spectrum and then test your fitting technique for sensitivity to help understand the drivers of the variations in template amplitude.

## Tables:

The tables I included give derived relative luminosities and reduced  $\chi^2$  values for each of the 10 combinations of templates I explored. I added uniformly distributed noise of amplitude 2, 4, and 6 units of flux. Table 1 is the example case using B, G, and K dwarf stellar templates. K dwarfs contribute the most to the luminosity with 69%. Using an average luminosity of  $0.2 L_{\odot}$ , they also are the most numerous type of star. Table 2 uses B dwarf and G and K giant stellar templates. G giants contribute the most to the luminosity with 46% and K giants next with 40%. Since G giants have average luminosity of  $31 L_{\odot}$  and K giants are about  $80 L_{\odot}$ , there are many more G giants than other stars.

Table 3 uses all 10 stellar templates. It has lower  $\chi^2$  values, but an unphysical solution, with more than 100% luminosity from G stars and negative contributions from several other stellar spectral types.

Table 4 uses all the Main Sequence templates. Again, it has an unphysical solution, with negative contributions from the A stars.

Table 5 uses B, F (instead of G), and K dwarf stellar templates. The K dwarf stars comprise 79% of the luminosity. Since average luminosity is  $1,580 L_{\odot}$  for B dwarfs,  $3 L_{\odot}$  for F dwarfs, and  $0.2 L_{\odot}$  for K dwarfs, K dwarfs are the most numerous.

Table 6 uses B, F, and K dwarf and M giant stellar templates. K dwarfs contribute most to the luminosity with 73% and are the most numerous.

Table 7 uses B, K, and M dwarf stellar templates. Again, K dwarfs win.

In Table 8, I used only O and M dwarf templates, and I got ridiculously bad  $\chi^2$  values, as I expected.

Table 9 uses only the giant stellar templates. It has an unphysical solution, with more than 100% luminosity from G giants and negative contributions from K giants.

Table 10 uses the K and M dwarfs, the MS stars with longest lifetimes, and all the giants. It has an unphysical solution, with more than 100% luminosity from G and K giants and negative contributions from K dwarfs and M giants.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	0.164756	-	-	0.157129	0.685943	-	-	-	-	25.768
2 noise	-	0.166588	-	-	0.149241	0.692688	-	-	-	-	26.454
4 noise	-	0.168434	-	-	0.141287	0.699487	-	-	-	-	29.633
6 noise	-	0.170255	-	-	0.133456	0.706188	-	-	-	-	35.304

Table 1: Relative luminosities and reduced  $\chi^2$  using B, G, and K dwarf stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	0.141281	-	-	-	-	-	0.460766	0.405402	-	22.721
2 noise	-	0.143400	-	-	-	-	-	0.450572	0.414055	-	23.434
4 noise	-	0.145543	-	-	-	-	-	0.440248	0.422804	-	26.635
6 noise	-	0.147672	-	-	-	-	-	0.430000	0.431499	-	32.322

Table 2: Relative luminosities and reduced  $\chi^2$  using B dwarf, G giant, and K giant stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	0.027368	0.038984	-0.049064	0.165556	-0.455779	-0.295958	0.188247	1.224201	0.255006	-0.074280	16.132
2 noise	0.030540	0.034674	-0.041460	0.165484	-0.423161	-0.281496	0.186379	1.157241	0.266870	-0.070073	16.870
4 noise	0.033795	0.030512	-0.033069	0.163804	-0.386743	-0.263536	0.184395	1.085870	0.277030	-0.065748	20.052
6 noise	0.036742	0.026807	-0.025130	0.161977	-0.350709	-0.250531	0.182392	1.016084	0.290102	-0.061441	25.651

Table 3: Relative luminosities and reduced  $\chi^2$  using all stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	0.043846	0.049694	-0.040035	0.153712	0.257257	0.431908	0.125119	-	-	-	18.664
2 noise	0.046766	0.044083	-0.032500	0.154532	0.243728	0.438133	0.127599	-	-	-	19.157
4 noise	0.049659	0.038605	-0.024169	0.153819	0.230886	0.444477	0.130033	-	-	-	22.112
6 noise	0.052371	0.033530	-0.016269	0.153019	0.218715	0.450374	0.132522	-	-	-	27.532

Table 4: Relative luminosities and reduced  $\chi^2$  using all Main Sequence stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	0.182100	-	0.032600	-	0.791690	-	-	-	-	27.836
2 noise	-	0.183437	-	0.030058	-	0.793647	-	-	-	-	28.334
4 noise	-	0.184793	-	0.027470	-	0.795630	-	-	-	-	31.333
6 noise	-	0.186151	-	0.024880	-	0.797614	-	-	-	-	36.833

Table 5: Relative luminosities and reduced  $\chi^2$  using B, F, and K dwarf stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	0.175105	-	0.055934	-	0.729752	-	-	-	0.049891	26.747
2 noise	-	0.175944	-	0.055049	-	0.727307	-	-	-	0.053435	27.091
4 noise	-	0.176806	-	0.054113	-	0.724907	-	-	-	0.056967	29.922
6 noise	-	0.177669	-	0.053173	-	0.722512	-	-	-	0.060495	35.239

Table 6: Relative luminosities and reduced  $\chi^2$  using B, F, and K dwarf and M giant stellar templates.



Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	0.198418	-	-	-	0.764170	0.047387	-	-	-	27.029
2 noise	-	0.198916	-	-	-	0.761134	0.051010	-	-	-	27.317
4 noise	-	0.199413	-	-	-	0.758094	0.054638	-	-	-	30.097
6 noise	-	0.199912	-	-	-	0.755047	0.058273	-	-	-	35.369

Table 7: Relative luminosities and reduced  $\chi^2$  using B, K, and M dwarf stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	0.176929	-	-	-	-	-	0.831579	-	-	-	478.448
2 noise	0.177121	-	-	-	-	-	0.832457	-	-	-	476.527
4 noise	0.177312	-	-	-	-	-	0.833336	-	-	-	477.111
6 noise	0.177504	-	-	-	-	-	0.834215	-	-	-	480.197

Table 8: Relative luminosities and reduced  $\chi^2$  using O and M dwarf stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	-	-	-	-	-	-	1.184870	-0.289667	0.144818	57.908
2 noise	-	-	-	-	-	-	-	1.185763	-0.292796	0.148231	59.546
4 noise	-	-	-	-	-	-	-	1.186643	-0.295915	0.151644	63.680
6 noise	-	-	-	-	-	-	-	1.187529	-0.299026	0.155044	70.313

Table 9: Relative luminosities and reduced  $\chi^2$  using only the giant stellar templates.

Star	O dwarf	B dwarf	A dwarf	F dwarf	G dwarf	K dwarf	M dwarf	G giant	K giant	M giant	$\chi^2$
no noise	-	-	-	-	-	-1.975905	0.255119	1.629764	1.217361	-0.098721	28.232
2 noise	-	-	-	-	-	-2.007424	0.254787	1.637248	1.239570	-0.095670	29.289
4 noise	-	-	-	-	-	-2.038273	0.254491	1.644545	1.261016	-0.092664	32.829
6 noise	-	-	-	-	-	-2.070211	0.254160	1.652166	1.283805	-0.089637	38.851

Table 10: Relative luminosities and reduced  $\chi^2$  using K and M dwarf and G, K, and M giant stellar templates.

## Plots:

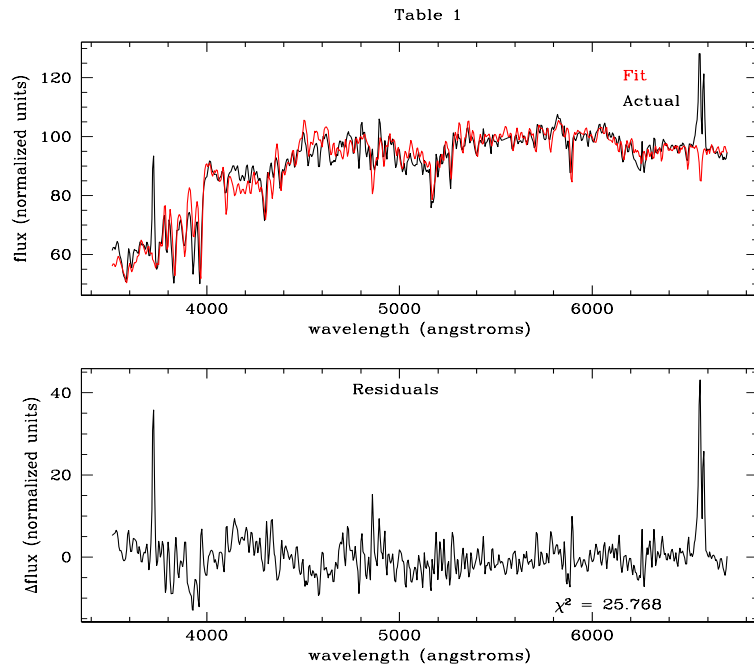


Figure 1: Worked example with B, G, and K dwarfs.

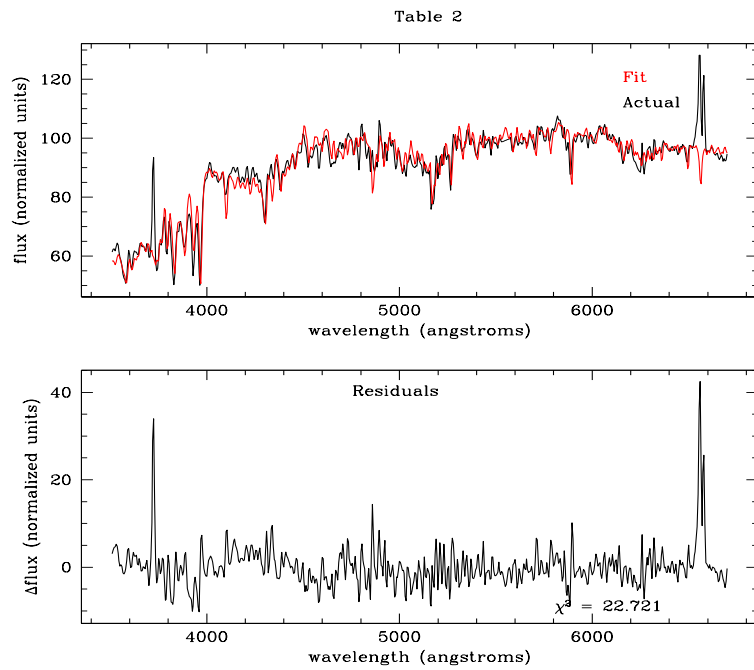


Figure 2: B dwarfs and G and K giants.

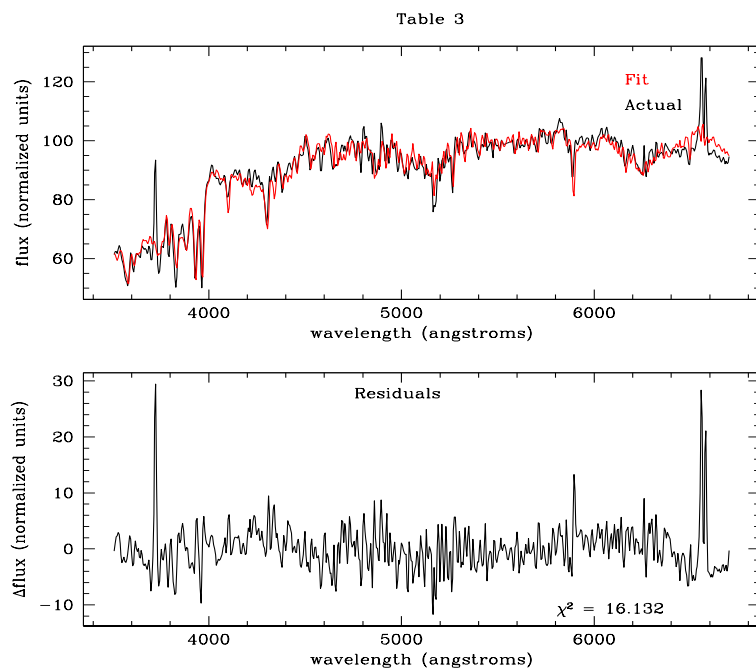


Figure 3: All stellar templates.

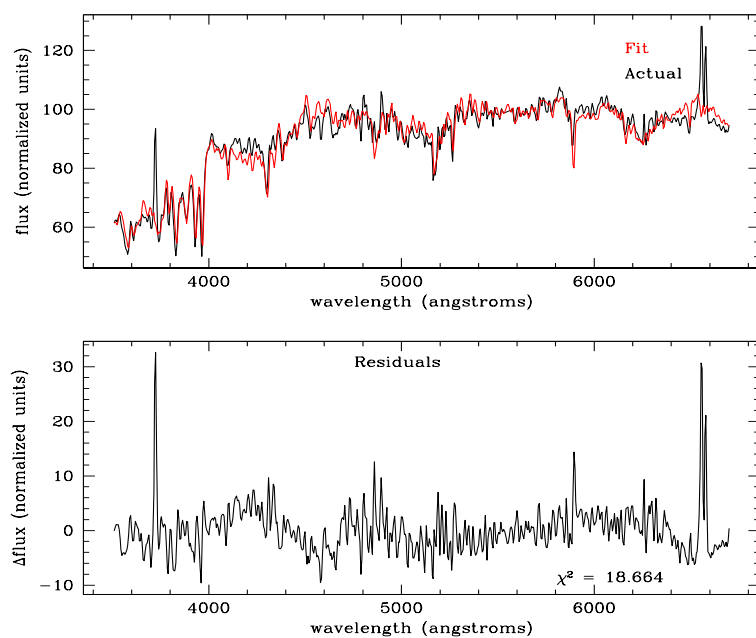


Figure 4: Only Main Sequence templates.

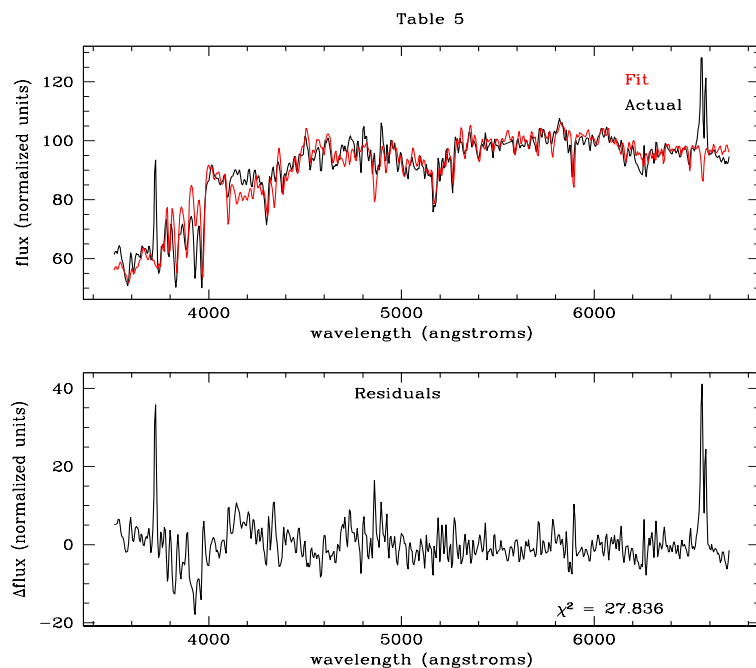


Figure 5: B, F, and K dwarf templates.

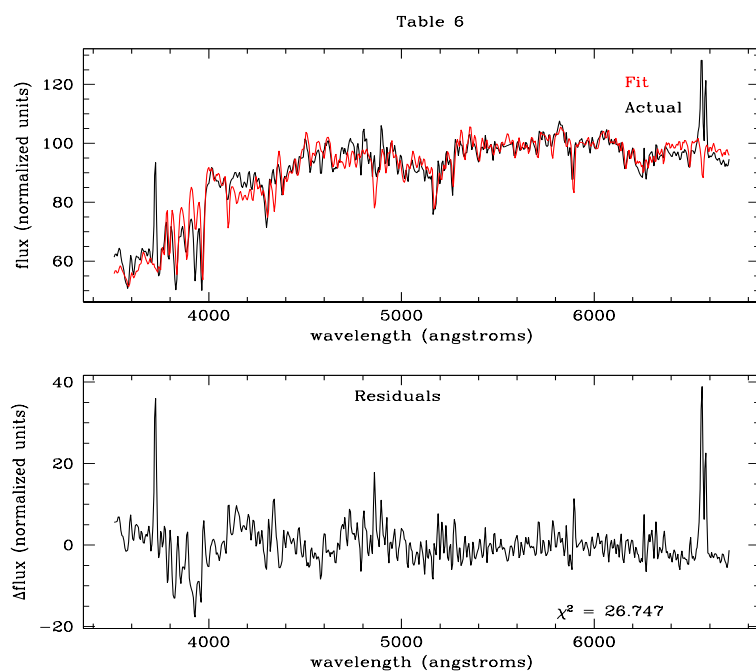


Figure 6: B, F, and K dwarf and M giant templates.

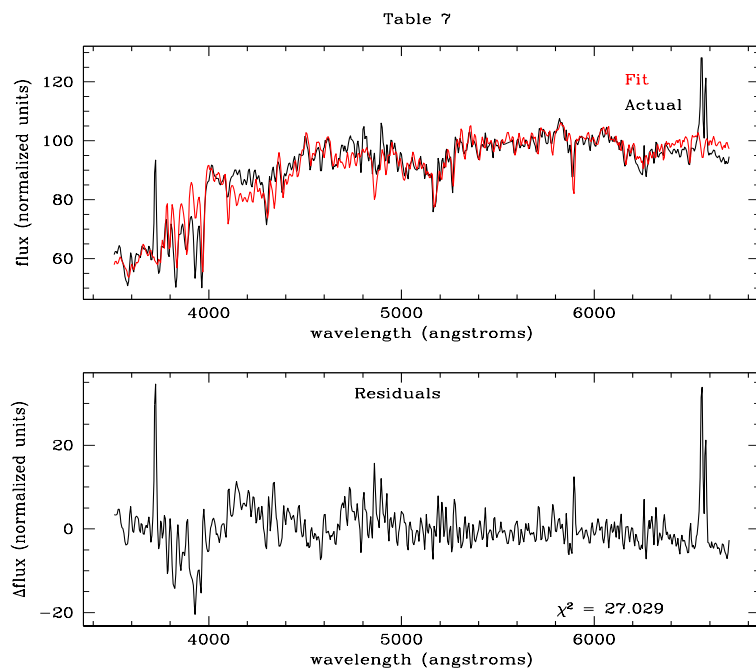


Figure 7: B, K, and M dwarf templates.

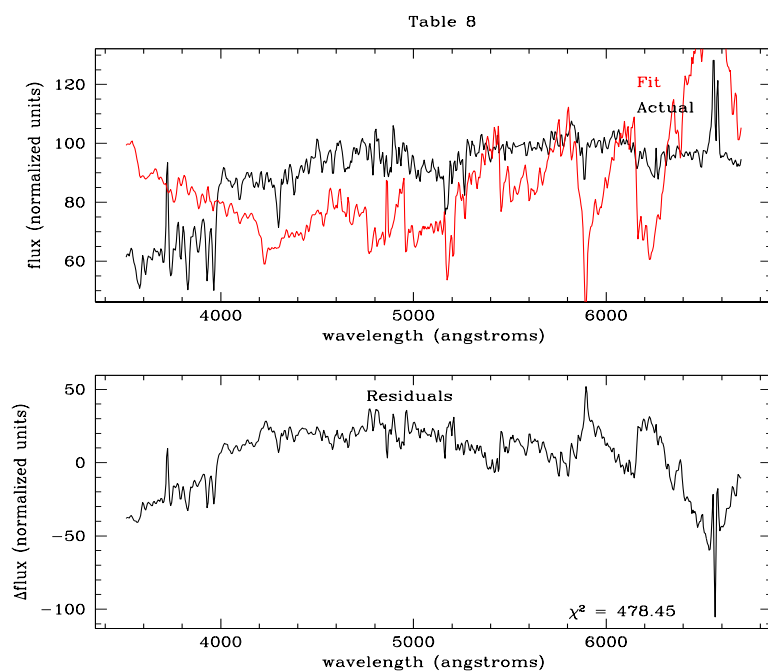


Figure 8: O and M dwarf templates.



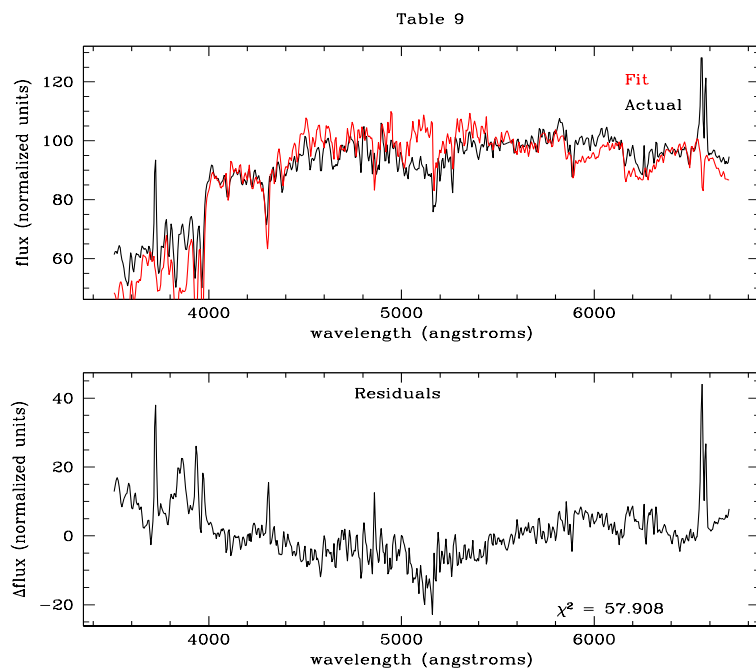


Figure 9: Only giant templates.

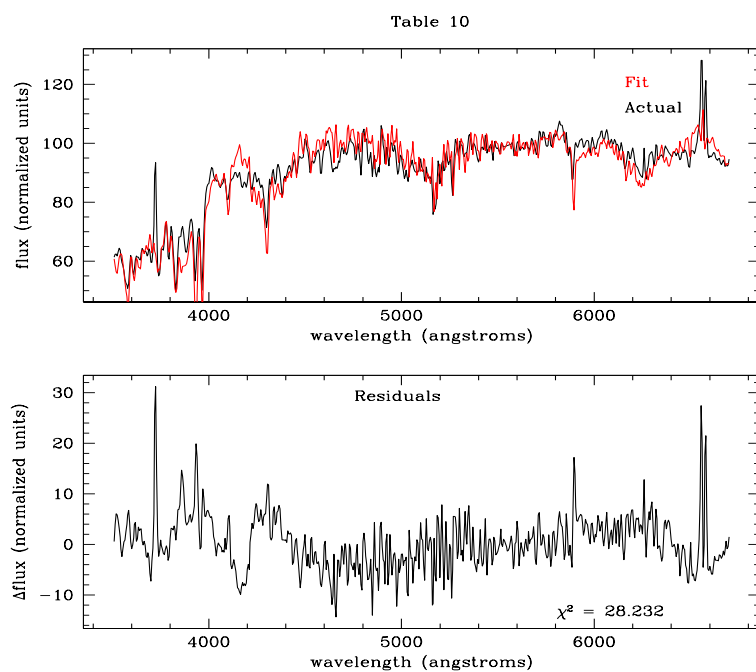


Figure 10: K and M dwarf and all giant templates.