Cluster Analysis and COVID-19 Immigration Policies

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Introduction

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Research Question: Using cluster analysis, can we find demographic patterns in countries' COVID-19 immigration policies?

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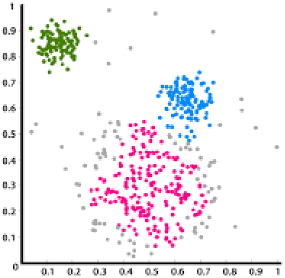

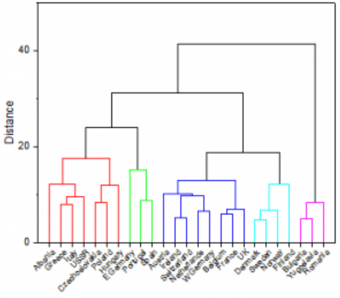

K-Means

- Hartigan-Wong method: less prone to converge to a local optima [1]

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  - Re-assign x_i to the cluster c_ℓ that has the lowest SSE with the inclusion of x_i

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- Deterministic, and does not require a choice of K clusters ahead of time

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- Choosing a linkage criteria with the cophenetic correlation [2]:

$$c = \frac{\sum_{i<j}[d(x_i, x_j) - \bar{d}][t(x_i, x_j) - \bar{t}]}{\sqrt{\sum_{i<j}[x(i,j) - \bar{x}]^2 \sum_{i<j}[t(i,j) - \bar{t}]^2}}$$

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Choosing the Number of Clusters

Gap statistic, the difference of total intra-cluster variation between observed data and reference data [3]

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- Choose the number of clusters as the smallest value of k such that $Gap(k) \geq Gap(k+1) - \sigma(k+1)$.

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$$R = \frac{a+b}{\binom{n}{2}},$$

where for a partition X and Y of some set S of n elements, a is the number of pairs in S that are in the same subset in both X and Y, and b is the number of pairs that are in different subsets in X and Y

Data and Data Cleaning

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- World Bank data: GDP, population, life expectancy, fertility rate, and adult literacy rate (2020)

Chosen Hyperparameters

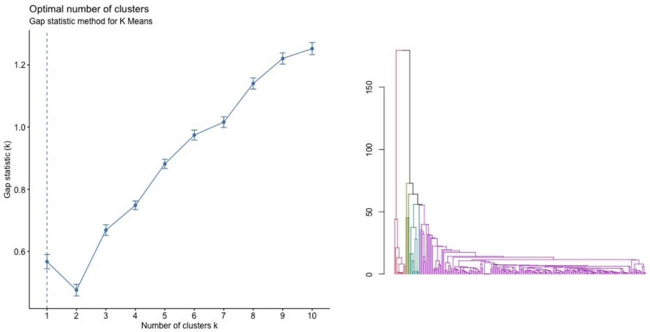


Figure: Gap statistic for K-Means (left), final dendrogram (right).

The cophonetic correlation was highest for minimum linkage (0.862).

ANOVA and Rand Index

Demographic	K-Means	HAC
GDP	0.485	0.334
Population	0.155	0.984
Life Expectancy	**0.00542**	**0.039**
Fertility Rate	**0.0067**	0.089
Literacy Rate	**0.0273**	0.149

Table: Significant p-values are bolded.

Policy Clustering	Continent	Development Level
K-Means	0.640	0.612
HAC	0.300	0.342

Table: Another interesting result: the rand index for K-Means vs HAC was 0.405.

ANOVA

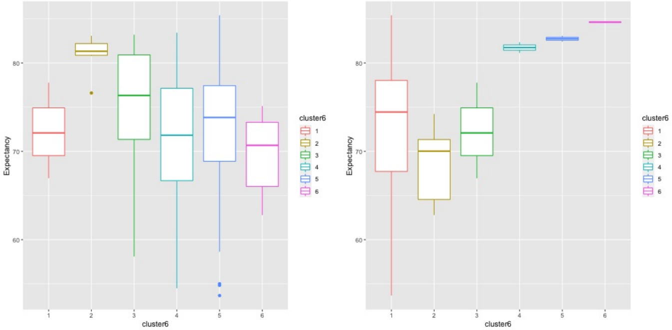
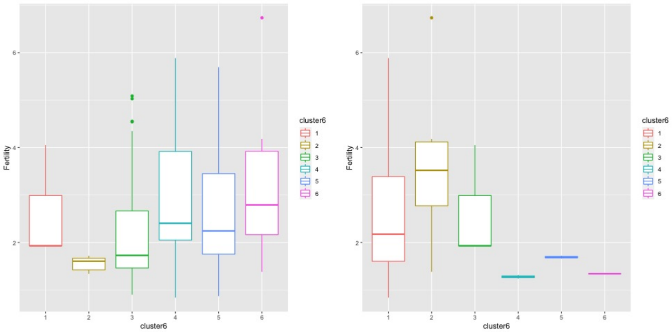


Figure: K-Means (left) and HAC (right)

ANOVA



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- Minimum distance linkage $\implies$ stringier clusters; may merge clusters whose centroids are far apart
- K-Means are more compact; takes into account the size of the clusters and the internal variance
- K-Means had more similar clustering to our "natural" metrics – why?
  - Continents: 43, 41, 50, 15, 40
  - Development level: 33, 29, 48, 61

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- Impact of choosing cluster method