

**TRACK ASSOCIATION USING CORRECTION
FOR BIAS AND MISSING DATA**

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ABSTRACT

One technique for multisensor tracking forms sensor level tracks using measurements received from the individual sensors. Then, the sensor level tracks are combined into a central level trackfile by performing multisensor track-to-track association and track fusion.

Due to difference in sensor resolution, detection capability and coverage, there may be targets for which a track is formed by one sensor but not by the other. Also, tracks formed on the same target by multiple sensors may differ due to multisensor misalignment (or bias) error. This paper addresses these problems by developing a method to perform multisensor track-to-track association under the conditions of intersensor bias and missing track data. An augmentation to the association matrix is developed to account for the fact that each sensor may not contain a full set of tracks for all targets in the field of view. An iterative approach is used to estimate and correct for the bias error. Monte Carlo simulation results are presented to illustrate the methods and close correspondence is found between these results and the theoretical probability of correct association.

I. INTRODUCTION

One technique for multisensor tracking forms sensor level tracks using measurements received from the individual sensors [1]. Then, the sensor level tracks are combined into a central level trackfile by performing multisensor track-to-track association and track fusion. This paper addresses two important practical problems in the track-to-track association process.

Due to differences in sensor resolution, detection capability and coverage, there may be targets for which a track is formed by one sensor but not by the other. This means that an assignment algorithm, such as the standard Munkres algorithm [1], that maximizes the number of allowable assignments made may inherently lead to association errors. Thus, an augmentation to the assignment matrix is required to represent the case of potential missing track data. Following

[1,2], this augmentation and the manner in which it can be conveniently included within the Jonker-Volgenant-Castanon (JVC) algorithm [3] is discussed in Section 3.

A major source of error in the track association process can be the bias (misalignment or registration) error that will cause systematic errors in the measured position of targets as seen by two sensors. Ideally, these biases may be estimated and removed through the use of tracks on targets with known origin [4]. However, in the case of unknown target origin, a combined association and bias estimation process must be performed [5, 6]. Section 3 presents a process whereby association is performed, bias is estimated and the process continued until no further improvement in the association result is obtained.

Kenefic [6] has addressed a similar track-to-track association problem by defining a minimum-description length (MDL) criterion to determine the probability that targets have tracks in both trackfiles. The MDL criterion of [6] and the resulting ad hoc probability calculations are not directly based upon system parameters. The association criterion presented in this paper is derived using the probabilities that a given target will be contained within the trackfiles.

The application considered is for co-located active and electro-optical (EO) sensors. Thus, common estimated states are two angles. However, the approach extends directly to three or more common states, such as the three Cartesian position components and/or rate estimates.

II. TRACK-TO-TRACK ASSOCIATION

Consider the case where two sensors A and B each have trackfiles containing N_A and N_B tracks, respectively. In an ideal case N_A would equal N_B , but in practical conditions this will not be true. Assume that gating is performed [1,7] and that a cost, C_{ij} , is given to all assignments of track i from sensor A to track j from sensor B that pass the gate test. Note that, in effect, an infinite cost is given to all assignments that do not pass the gate test. In general, the cost may either be a statistical distance, d_{ij}^2 , formed only from the current track state estimates or may also be based upon previous track history [6, 8]. For our example, we will use the statistical distance as defined in [1],

$$d_{ij}^2 = y_{ij}^T S_{ij}^{-1} y_{ij} \quad (1)$$

where y_{ij} and S_{ij} are the state vector difference and the covariance of the distance vector for track i from sensor A and track j from sensor B.

Without losing generality, we may assume that $N_B \geq N_A$. Then, following [1,2], an assignment matrix of the form shown in Figure 1 results. Referring to Figure 1, G is a generalized gate similar to that discussed in [1,2]. The form for this problem will be derived below. There are a number of approaches to the solution of the assignment matrix shown in Figure 1. Our approach will be through use of the JVC algorithm [3].

SENSOR A TRACKS	SENSOR B TRACKS				MATRIX AUGMENTATION			
	1	2	...	N _B	1	2	...	N _A
1	d_{11}^2	d_{12}^2		$d_{1N_B}^2$	G	X	X	X
2	d_{21}^2	d_{22}^2		$d_{2N_B}^2$	X	G	X	X
.	.	.		.	X	X	G	X
.	.	.		.				
N _A	$d_{N_A1}^2$	$d_{N_A2}^2$		$d_{N_A N_B}^2$	X	X	X	G

Figure 1. Track-To-Track Assignment Matrix

In order to determine G, first define P_{TA} and P_{TB} to be the probabilities that sensor A and sensor B have a track on given target, respectively. Next, define β_T to be the target density and β_E to be the new (or extraneous) source density. For this application, the new source density is defined to be the density of targets for which sensor B has a track but sensor A does not. Thus,

$$\beta_E = (1 - P_{TA}) P_{TB} \beta_T \quad (2)$$

The maximum likelihood gate for the observation-to-track assignment problem is [1].

$$G = 2 \ln \left[\frac{P_D}{(1 - P_D) (2\pi)^{M/2} \beta_E |S|^{1/2}} \right] \quad (3)$$

For this track-to-track assignment problem, the probability of detection P_D is replaced by P_{TB} , the measurement dimension (M) is two, the new source density is as given in Eq. (2) and the residual covariance matrix, S, will be discussed below. Upon combining Eqs. (2) and (3) and using the correspondences defined above, the gate becomes

$$G = 2 \ln \left[\frac{1}{(1 - P_{TA}) (1 - P_{TB}) 2\pi \beta_T |S|^{1/2}} \right] \quad (4)$$

The residual covariance matrix S defines the statistics of the angular differences between tracks from different sensors on the same target. It is used in the difference calculation of Eq. (1) and the gate of Eq. (4). It contains contributions from the estimation errors of both sensors and

the bias error, as determined from the Kalman filter covariance matrices and the assumed known bias error statistics.

III. ITERATIVE BIAS ESTIMATION

The multisensor track-to-track association problem in the presence of intersensor bias requires that association and bias estimation be performed simultaneously. The bias error can prevent association of tracks which do correspond to the same target and can cause misassociation of tracks corresponding to different targets. This problem was addressed by Bath [5] through the use of a finite set of bias values that were applied and evaluated by a cross correlation process. Here, we present an iterative approach, similar to that of [9], to the combined bias estimation and association problem. The actual bias value will be estimated rather than chosen from a predefined set as in [5].

The bias estimation method will be illustrated through the use of two misalignment angles in azimuth (η) and elevation (ϵ). Two sensors are considered and the misalignment $\delta\eta_B$, $\delta\epsilon_B$ in azimuth and elevation angles, respectively, from one sensor to the other will be estimated.

In general, the angular misalignment estimation problem involves three angles [10]. However, for the closely spaced, long range target scenario considered in this application, one coordinate axis was chosen to point in the direction of the target centroid. Under this condition, it is only necessary to estimate two misalignment angles.

The same basic procedure could be used for the more general problem. However, a batch estimation procedure, similar to those described in [6, 9], should be considered for the estimation of the three bias angles. Then, the same iterative process, described below, would be performed.

The process begins with a solution (at iteration k) to the assignment matrix using bias estimates from the previous ($k-1$) iteration. Given that no prior information exists, for this two angle application the initial bias estimates are taken to be

$$\delta\eta_{B1} = \delta\epsilon_{B1} = 0$$

Then, upon solution of the assignment problem, two lists of associated track pairs are formed by ranking the track pairs according to the differences in azimuth and elevation angle. Estimates of the bias are taken to be the median differences. Finally, the assignment process is repeated (at iteration $k+1$) after correcting for the bias using the estimates

$$\begin{aligned}\delta\eta_{B_{k+1}} &= \text{median}(\delta\eta_k) \\ \delta\epsilon_{B_{k+1}} &= \text{median}(\delta\epsilon_k)\end{aligned}$$

The quantity median ($\delta\eta_k$) is defined to be the median difference formed from ranking the track association pairs according to azimuth angle differences. For example, if five track assignments are made and differences between the estimated azimuth angles from the two sensors were (3, 2, 1, 0, -1), the median value would be 1.

At each iteration, the misalignment angle estimates are subtracted from all angle differences, the gating tests are repeated and the assignment matrix is formed and solved again. This process is continued until a convergence test is satisfied. The convergence test that we have chosen compares the median values as computed upon two successive iterations and terminates the process if both azimuth and elevation median bias values satisfy a difference test.

IV. RESULTS

The techniques presented in this paper have been evaluated through Monte Carlo simulation of typical space interceptor-to-target scenarios. It is assumed that there are 60 targets in a three dimensional target cloud consisting of a reentry vehicle(RV) and decoys. The target angular distribution is determined from the projection of the target cloud upon the focal plane of an EO sensor.

For the purposes of determining the theoretical gate G of Eq. (4), the target distribution will be assumed to be uniform in angle space. However, as discussed below, results indicate that the actual distribution may be somewhat denser at the center of the groups with the result that the theoretical G , computed using the uniformly distributed assumption, is slightly too large.

Two sensors (A,B) are assumed with angular estimation error standard deviations that are considered representative for EO and active sensors. All results will be presented as normalized with respect to the estimation standard deviation of the active sensor (σ_A). The estimation error standard deviation for the EO error (σ_E) was taken to be

$$\sigma_E = 0.2 \sigma_A$$

The same error statistics are used for azimuth and elevation angles.

Results will be presented as a function of the probabilities (P_{TA} , P_{TB}) that a track for a given target exists in the trackfiles of sensors A and B. Results are obtained from 100 Monte-Carlo simulation runs averaging over the effects of missing tracks, track angle estimation error and the intersensor bias error. The results are presented in terms of the probability of correct decision (P_{CD}) as defined

$$P_{CD} = \frac{N_{CA} + N_{CU}}{N_{TR}} \quad (5)$$

where, for a set of N_{TR} tracks from sensor A,

N_{CA} = number of tracks correctly assigned

N_{CU} = number of tracks correctly left unassigned (because of no corresponding

track from the other sensor B).

The bias error is taken to have magnitude B_M and uniform angle such that

B_η = azimuth bias = $B_M \cos u$

B_ϵ = elevation bias = $B_M \sin u$

where u is uniformly distributed over the interval $(0, 2\pi)$.

The target density, β_T , is required in order to determine the theoretical gate value given in Eq. (4). Also, it is useful to compare the probability of correct association (P_{CA}) formed from Monte-Carlo experiment with the theoretical predictions of [11]. As presented below, this comparison will also require β_T . An estimate of β_T can be obtained from the mean target distance, D , from its nearest neighbor, which was computed for the scenario examined, using

$$\beta_T = \frac{1}{4D^2} \quad (6)$$

Figures 2 and 3 show results for a case where P_{TB} is unity. As shown by Eq. (4), this leads to an infinite value for G so that the assignment algorithm will maximize the number of assignments. This is intuitively reasonable since if either sensor can be assumed to have a track on all targets, all tracks should, if possible, be assigned. Also, for this case the average target separation (D) was found to be

$$D = 5.3 \sigma_A$$

Figure 2 shows the probability of correct association P_{CA} , which for this case is the same as P_{CD} , as a function of the bias magnitude B_M and track probability for sensor A (P_{TA}). The bias magnitude is also presented in units of σ_A . For example, for the case where P_{TA} is 0.75, P_{CA} decreases from about .93 with no bias to about 0.74 when the magnitude of B_M is $4\sigma_A$.

Figure 2 also shows that P_{CA} is independent of the bias error magnitude as long as all tracks are present ($P_{TA} = P_{TB} = 1.0$). This results because the assignment process which maximizes the number of assignments is essentially a pattern matching process.

Figure 3 shows results for the same conditions as Figure 2 except that for the results of Figure 3, the bias estimation method is used. The results of Figure 3 show that the bias estimation method has essentially eliminated the effects of bias upon the probability of correct association.

A second case was examined for which the target density was increased so that the average separation was

$$D = 2.9 \sigma_A$$

Also, results were derived as a function of track probability P_T such that

$$P_{TA} = P_{TB} = P_T$$

Again, the bias estimation technique was found to essentially eliminate the effect of bias. Then, the second result of the study was to show as a function of P_T how the probability of correct decision P_{CD} varies with the choice of the gate value G used as an assignment cost. Results shown in Figure 4 are for the maximum bias examined ($B_M = 4\sigma_A$) but results for the other bias value were essentially identical.

Figure 4 shows P_{CD} as a function of G for values of $P_T = 1.0, 0.95, 0.85, 0.75$. The probability of correct decision (as defined in Eq. (5)) includes both correct association when available and no assignment when a correct association is not available. Clearly, P_{CD} can be increased through the proper choice of G and the value of G should decrease with decreasing P_T . This corresponds with the intuitive reasoning that the cost for no assignment should decrease as the probability increases that a given track from one sensor does not have a corresponding track from the other sensor.

Also, P_{CD} for the gate value 1000 was computed to be 0.88, 0.79, 0.76, and 0.73 for four values of P_T (1.0, 0.95, 0.85, 0.75) examined. This essentially corresponds to an infinite gate and is the solution that a standard assignment algorithm which maximizes the number of allowable assignments made would give. Clearly, as P_T decreases there is considerable degradation associated with an assignment algorithm, such as the standard Munkres, that maximizes the number of assignments.

Figure 4 shows that P_{CD} is maximized by decreasing values of G as P_T decreases. The theoretical gate values predicted by Eq. (4) for P_T values of 0.95, 0.85, and 0.75 are 15.3, 10.9, and 8.8, respectively. These are slightly larger than the values for G that maximizes P_{CD} in Figure 4. However, the discrepancy is small and is probably attributable to a slight deviation from a uniform target density.

Finally, it is of interest to compare the probability of correct association (P_{CA}) found in the Monte-Carlo experiment performed here with the theoretical predictions of [11]. For the two dimensional assignment problem the theoretical P_{CA} is

$$P_{CA} = e^{-\pi(\beta + 2\beta_E)\sigma^2} \quad (7)$$

where, for our application,

β = density of targets for which sensor A has a track
 β_E = density of targets for which sensor A has no track (but sensor B does have a track)
 σ^2 = estimation variance = $1.04 \sigma_A^2$

The application of Eq. (7) is only valid for those cases in which sensor B has a track on all targets. However, P_{TA} can assume any value. Then,

$$\beta = P_{TA} \beta_T$$

$$\beta_E = (1 - P_{TA}) \beta_T \quad (8)$$

with β_T as computed from Eq. (6).

Table 1 shows a comparison of the results found using Eqs. (7) and (8) with those found from the Monte-Carlo simulation and taken from Figures 3 and 4. This comparison shows a close correspondence in both magnitude and trend. The fact that the simulation P_{CA} are slightly below the theoretical values is again probably attributable to a deviation from uniform target density.

TABLE 1. COMPARISON OF SIMULATION AND THEORETICAL PROBABILITIES OF CORRECT ASSOCIATION (P_{CA}) ($P_{TB} = 1.0$ FOR ALL CASES)

SCENARIO	P_{TA}	P_{CA} SIMULATION	P_{CA} THEORY (EQ. 8)
1	1.0	0.958	0.971
	0.95	0.950	0.970
	0.85	0.946	0.967
	0.75	0.941	0.964
2	1.0	0.898	0.907

V. CONCLUSION

Techniques have been presented for performing multisensor track-to-track association in the presence of bias(misalignment) error and missing track data. The validity of these techniques has been established through the use of Monte-Carlo simulation and comparison of simulation results with theoretical expressions for the optimal gate size and the probability of correct association. Future study areas include the use of angle rate in the association process and the simulation of more complex scenarios.

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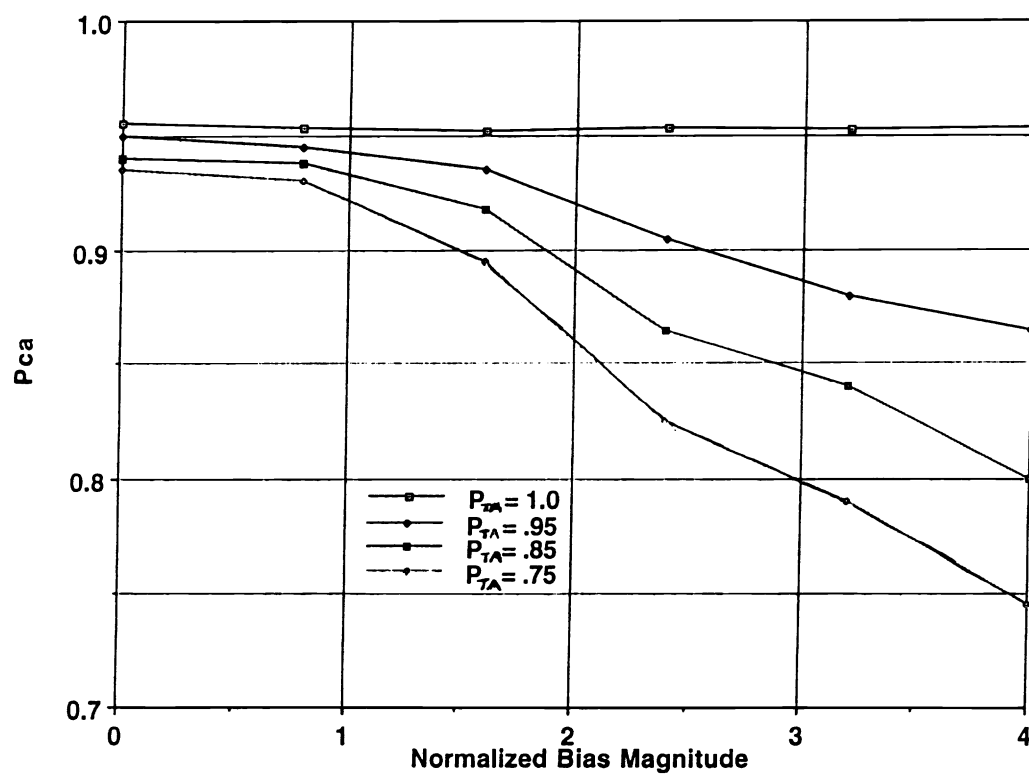


Figure 2. Probability of Correct Association with No Bias Estimation (Scenario 1, $P_{TB} = 1.0$)

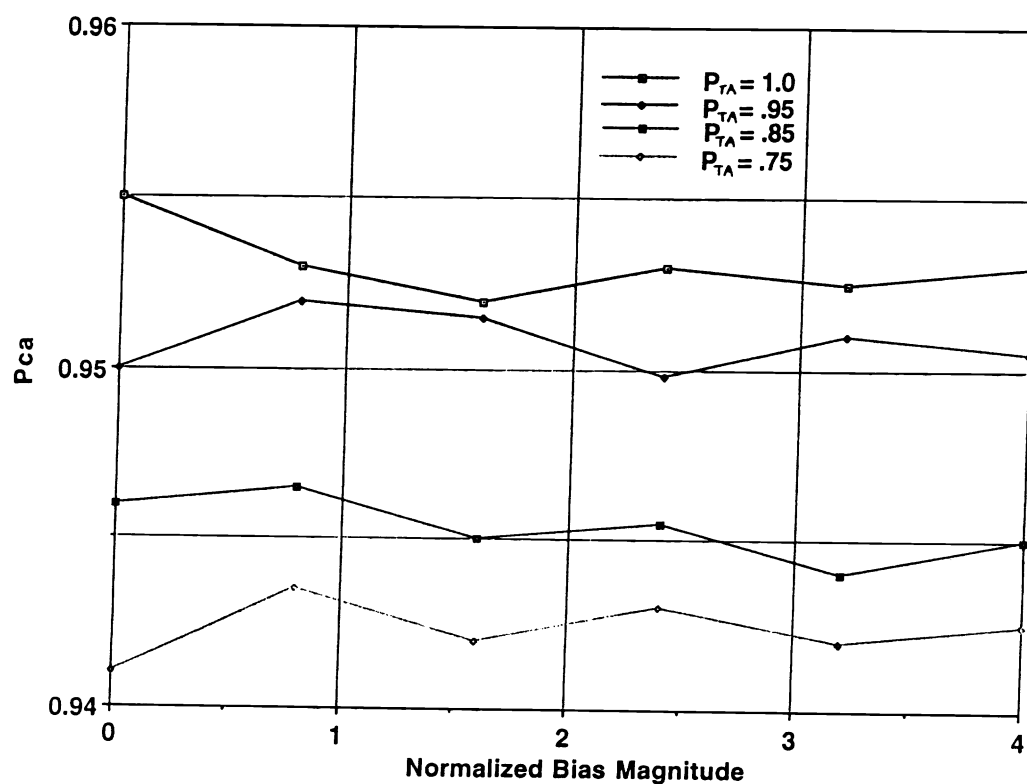


Figure 3. Probability of Correct Association with Bias Estimate (Scenario 1, $P_{TB} = 1.0$)

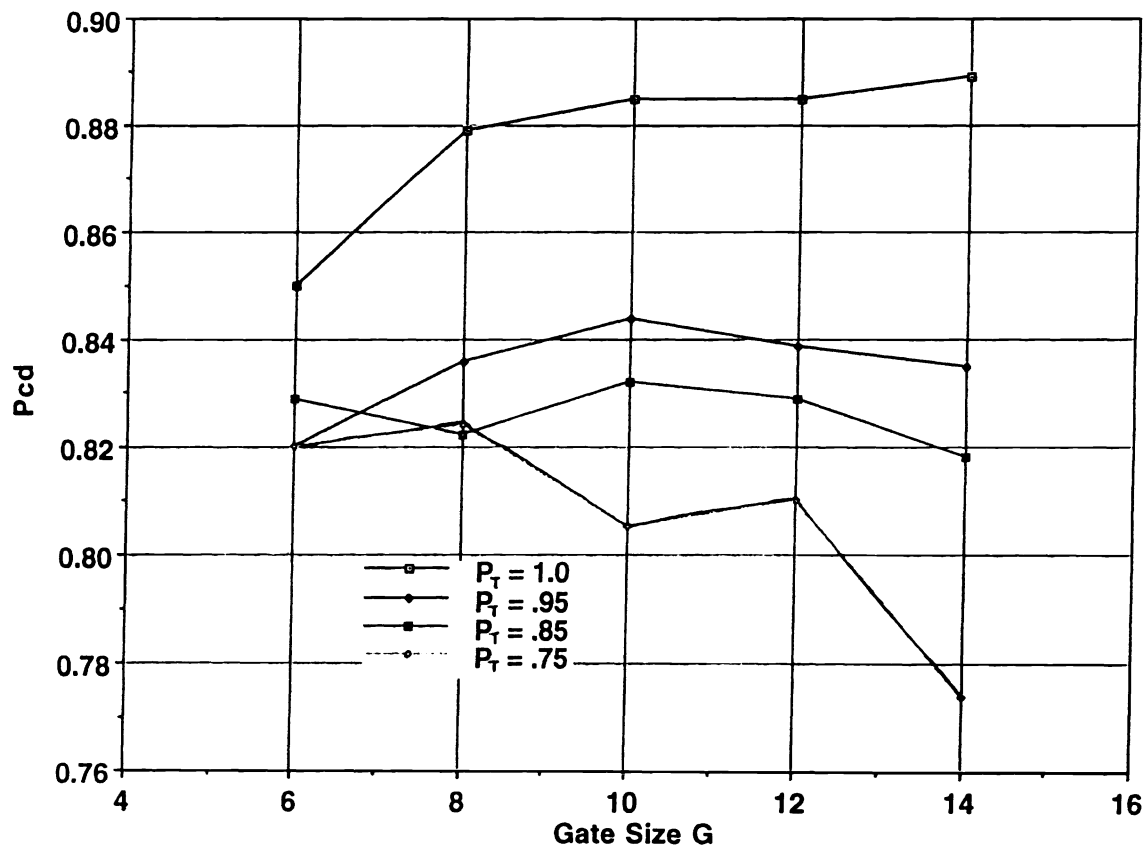


Figure 4. Probability of Correct Decision P_{cd} as a Function of Gate Size G for Various P_r (Scenario 2)