

# Using genetic algorithm to support portfolio optimization for index fund management

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## Abstract

Using genetic algorithm (GA), this study proposes a portfolio optimization scheme for index fund management. Index fund is one of popular strategies in portfolio management that aims at matching the performance of the benchmark index such as the S&P 500 in New York and the FTSE 100 in London as closely as possible. This strategy is taken by fund managers particularly when they are not sure about outperforming the market and adjust themselves to average performance. Recently, it is noticed that the performances of index funds are better than those of many other actively managed mutual funds [Elton, E., Gruber, G., & Blake, C. (1996). Survivorship bias and mutual fund performance. *Review of Financial Studies*, 9, 1097–1120; Gruber, M. J. (1996). Another puzzle: the growth in actively managed mutual funds. *Journal of Finance*, 51(3), 783–810; Malkiel, B. (1995). Returns from investing in equity mutual funds 1971 to 1991. *Journal of Finance*, 50, 549–572]. The main objective of this paper is to report that index fund could improve its performance greatly with the proposed GA portfolio scheme, which will be demonstrated for index fund designed to track Korea Stock Price Index (KOSPI) 200.

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**Keywords:** Index fund; Genetic algorithm; Portfolio

## 1. Introduction

Index funds are popular investment tools being used in modern portfolio management. Index funds are designed to mimic the behavior of the given benchmark market indices (e.g. the S&P 500 in New York, the FTSE 100 in London, the KOSPI 200 in Seoul, etc.). Thus, index funds are generally regarded as relatively stable and efficient investment tool compared with other mutual funds (Jensen, 1968; Sharpe, 1966).

The index fund strategy is based on the concept of the passive investment management. There are several interesting papers reporting the superior performance of the index funds compared with other actively managed portfolios (Elton, Gruber, & Blake, 1996; Gruber, 1996; Malkiel, 1995). In addition to the performances of index funds in terms of risk and return, index funds are also considered cost effective investment tool in the capital market (Hogan, 1994).

Index funds are composed of relatively small number of stocks. If we want to set up a perfect index fund, we need put every company included in the index into the index fund portfolio (e.g. 500 companies for S&P 500, 100 companies for FTSE 100, and 200 companies for KOSPI 200). However, it is costly and not practical to include every company in the index fund portfolio. Thus, index funds try to replicate the movement of the indices with a relatively small number of stocks.

This article proposes a genetic algorithm (GA) portfolio scheme for the index fund optimization. The scheme exploits GA and provides the optimal selection of stocks utilizing fundamental variables—standard error of portfolio beta given by formula (1), average trading amount, and average market capitalization. These fundamental variables are well-known core factors frequently used in analyzing and forecasting the stock market. Roughly speaking, the GA portfolio scheme consists of two steps. First, the stocks for the index fund are selected through working with the fundamental variables in each industry sector of the benchmark index. Second, the relative weights of the selected stocks are optimized through the GA process.

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It will be shown that the portfolio scheme efficiently replicates the benchmark index with a relatively small number of stocks. Notice that the business of the efficient index fund management relies on the technique of replicating the benchmark index.

The proposed GA scheme is applied to Korea stock price index (KOSPI) 200 from Jan 1999 to Dec 2001. KOSPI 200 includes 200 major companies in 22 industry sectors, which are currently listed on the Korean Stock Exchange. The 200 companies cover general spectrum on the Korea Stock Exchange and KOSPI 200 is also the base index of KOSPI 200 futures contract, which is the most active futures contract on the Korea Stock Exchange.

This paper consists of five sections. Following this section, Section 2 provides a brief survey about portfolio theory, index funds, and GA. Section 3 discusses the detailed procedure of the proposed scheme and Section 4 reports empirical experiment results. Finally, Section 5 is devoted to the concluding remarks.

## 2. Literature review

### 2.1. Portfolio theory, index funds and index tracking

Modern portfolio theory provides a well-developed paradigm to form a portfolio with the highest expected return for a given level of risk tolerance. Markowitz (1952, 1959), a creator of modern portfolio theory, originally formulated the fundamental theorem of mean–variance portfolio framework, which explains the trade-off between mean and variance each representing expected returns and risk of a portfolio, respectively. Although Markowitz's theory uses only mean and variance to describe the characteristics of return, his theory about the structures of a portfolio became a cornerstone of modern portfolio theory (Fama, 1970; Hakansson 1970, 1974; Merton, 1990; Mossin, 1969). After mean–variance portfolio theory, there was an enormous progress on portfolio theory and practice which include various practical applications introduced in portfolio formulation. Recently, it is found that low-cost passively managed index funds actually deliver the highest risk-adjusted returns in each category of mutual funds (Bogle, 1998).

Index fund management is a stock-allocation strategy equipped with index tracking skill which attempts to replicate the behavior of a given benchmark index. Index funds usually do not include every stock comprising the index. However, they are designed to copy the benchmark index with relatively small number of stocks, which can be easily managed and controlled in the capital market. Thus, the performance of the index fund critically depends on how well the index tracking skill replicates the benchmark index with only a subset of the stocks.

Index tracking, of course, involves tracking error (TE) which is measured by TE volatility, the sum of

the deviations of returns of the replicating portfolio from the benchmark index. When fund managers formulate an index fund, they try to minimize the TE volatility level since it would produce as close as possible returns to the benchmark returns (Clarke, Krase, & Statman, 1994; Sharpe, 1971; Konno & Yamazaki, 1991). In general, there are several types of TE measures available, i.e. quadratic, linear and absolute, among which quadratic measure are preferred since it possesses a number of desirable statistical properties (Roll, 1992). Throughout this study, quadratic measure is employed.

### 2.2. Genetic algorithm

GA is a stochastic optimization technique invented by Holland (1975) and a search algorithm based on survival of the fittest among string structures (Goldberg, 1989). They applied the idea from biology research to guide the search to an (near-) optimal solution (Wong & Tan, 1994). The general idea was to maintain an artificial ecosystem, consisting of a population of chromosomes. In this study, each chromosome represents the weight of individual stock of portfolio and is optimized to reach a possible solution. Attached to each chromosome is a fitness value, which defines how good a solution the chromosome represents. By using mutation, crossover values, and natural selection, the population will converge to one containing only chromosomes with good fitness (Adeli & Hung, 1995). Recently, GA attracts much attention in portfolio formulations (Orito, Yamamoto, & Yamazaki, 2003; Xia, Liu, Wang, & Lai, 2000).

## 3. Scheme specification

### 3.1. Related variables

The proposed scheme is based on three fundamental variables: portfolio beta, trading amount, and market capitalization. These three variables are frequently used in portfolio management area, among which portfolio beta is especially the most important variable (Chang, 2004; Keim, 1999).

#### 3.1.1. Portfolio beta

Let  $\beta_p$  be portfolio beta for a given portfolio  $p$  defined by

$$\beta_p = \frac{\text{Cov}(r_p, I_m)}{\text{Var}(I_m)}, \quad (1)$$

where  $r_p$  is the rate of return of the portfolio  $p$  and  $I_m$  is the rate of return of the benchmark index or the capital market  $m$ . It is well known that the portfolio beta measures portfolio volatility relative to the benchmark index or the capital market. Indeed, if a portfolio is well chosen such that returns of portfolio and benchmark index are highly correlated, then portfolio beta becomes the volatility ratio between the

Table 1  
Standard deviation of absolute values of TEs starting Aug 9, 2001

Weight of standard error of beta ( $v_1$ )	Weight of average trading amount ( $v_2$ )	Weight of average market capitalization ( $v_3$ )	Standard deviation of TEs	
			SIM_IF	GA_IF
1	1	1	0.3686	0.2093
1	1	2	0.3386	0.2249
1	2	1	0.3111	0.2011
1	2	2	0.3356	0.2207
2	1	1	0.2369	0.2043
2	1	2	0.3279	0.2301
2	2	1	0.3497	0.2127
Average			0.3241	0.2147

portfolio and the benchmark index approximately. Thus, portfolio beta usually implies the portfolio sensitivity relative to the fluctuations of the market index. For further discussions of our portfolio scheme, denote portfolio beta for the  $j$ th stock among stocks that comprises a market index  $m$  by  $\beta_j$ , i.e.

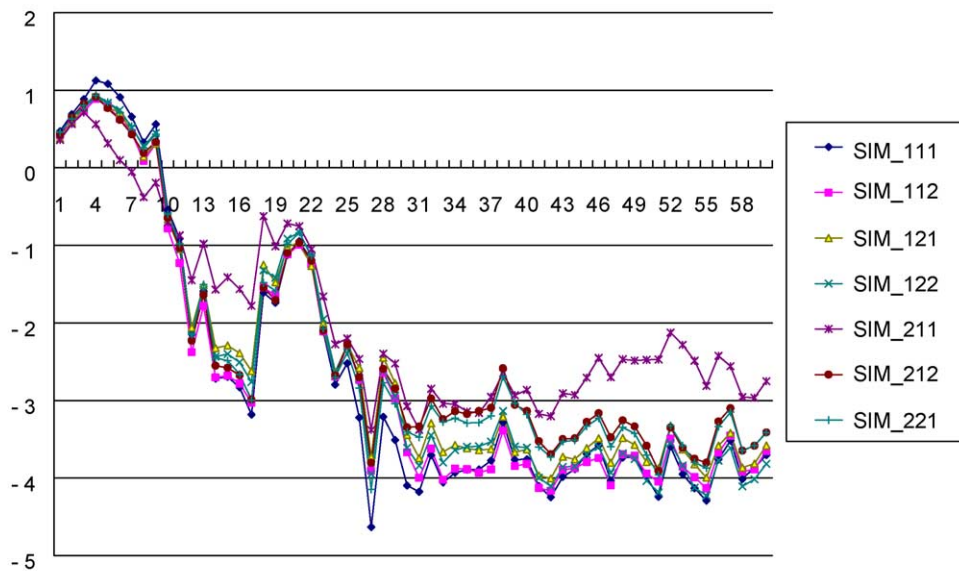
$$\beta_j = \frac{\text{Cov}(r_j, I_m)}{\text{Var}(I_m)}. \quad (2)$$

where  $r_j$  is the rate of return of  $j$ th stock.

### 3.1.2. Trading amount and market capitalization

Trading amount and market capitalization for each stock are calculated through multiplying stock price by its trading

(a) TEs of SIM\_IF



(b) TEs of GA\_IF

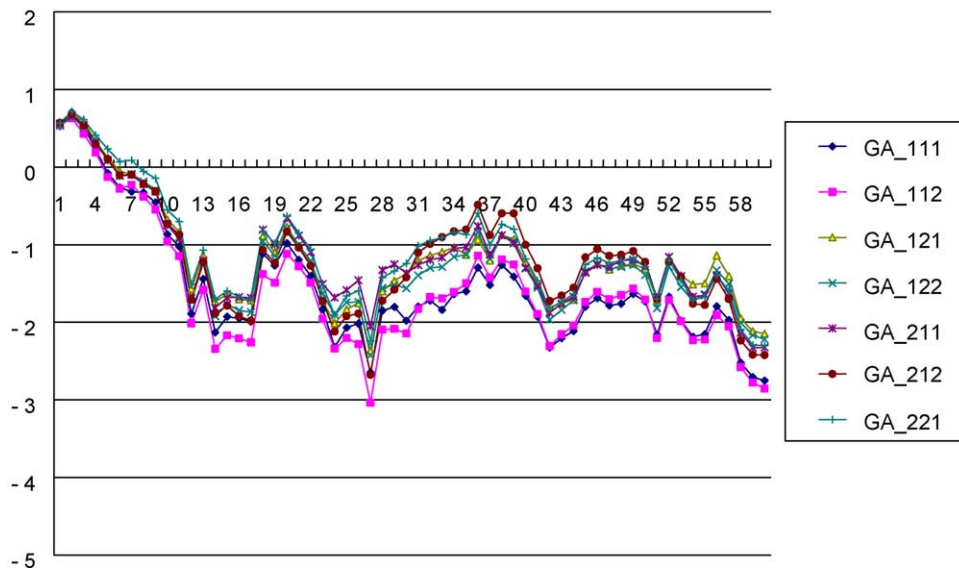


Fig. 1. TEs for 60 days. For example, SIM\_121 and GA\_211 mean SIM\_IF and GA\_IF having 1:2:1 and 2:1:1 for  $v_1:v_2:v_3$ , respectively.

Table 2  
Standard deviation of absolute values of TEs starting Aug 9, 2001

The number of stock ( $l$ )	Training interval of parameters ( $T$ )	Standard deviation of tracking errors	
		SIM_IF	GA_IF
20	20	0.9261	0.7037
20	60	1.0745	0.6558
20	120	0.9366	0.6025
40	20	0.8758	0.4402
40	60	0.9920	0.4871
40	120	1.0127	0.5570
60	20	1.0767	0.8437
60	60	1.1159	0.8836
60	120	0.9568	0.7108
Average		0.9964	0.6538

volume and by the total number of stocks listed in the stock market, respectively. Trading volume indicates the liquidity and the marketability of a stock. Market capitalization indicates the size of the company, which issued the stock. Both of the variables are considered importantly when an

investor evaluates stock in the market for selection. Thus, they are included in this study.

### 3.2. Portfolio scheme for index fund management

Let  $n$  and  $l$  be the numbers of stocks for the benchmark index and its index fund portfolio, respectively ( $l < n$ ). Further, let  $c_k$  ( $k = 1, 2, \dots, l$ ) be the serial code of  $k$ th stock to be included in the index fund portfolio. In other words, index fund portfolio set is  $\Phi_p = \{c_1, c_2, \dots, c_l\}$  which is to be selected from the entire  $n$  stocks. Let  $s$  denote the number of industry sectors comprising the benchmark index and  $d_i$  the number of stocks comprising  $i$ th industry sectors (i.e.  $\sum_{i=1}^s d_i = n$ ). In addition, for each  $j$ th stock of  $i$ th industry sector ( $j = 1, 2, \dots, d_i$  and  $i = 1, 2, \dots, s$ ), suppose the portfolio beta is given by  $\beta_{i(j)}$  where the subscript  $i(j)$  is used to stress dependence of  $j$  on  $i$ . For that specific stock, let  $r_{i(j)}(t)$ ,  $A_{i(j)}(t)$  and  $M_{i(j)}(t)$  denote rate of return, trading amount, and market capitalization at time  $t$ , respectively. Further,  $I_m(t)$

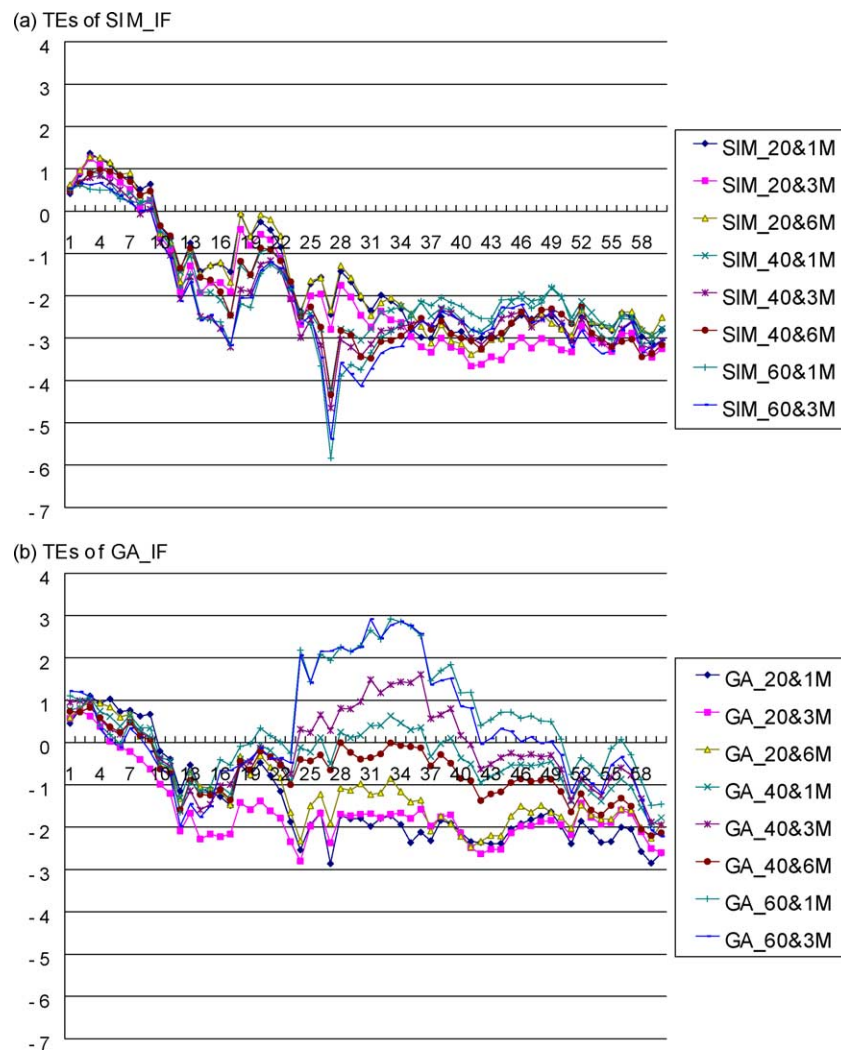


Fig. 2. TEs for 60 days. SIM\_ $x$  &  $y$  M and GA\_ $x$  &  $y$  M denote SIM\_IF and GA\_IF, respectively, under the condition that the number of stock is  $x$  and the training interval is  $y$ .

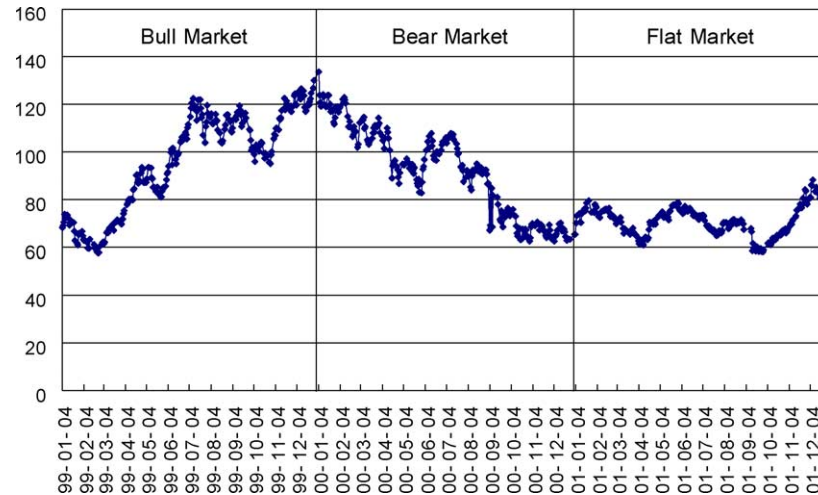


Fig. 3. The flow of KOSPI 200 from 1999 to 2001.

denotes the rate of return of the benchmark index  $m$  at  $t$ . Now, let us define priority  $P_{i(j)}$  for each stock (i.e.  $i = 1, 2, \dots, s$  and  $j = 1, 2, \dots, d_i$ ). Below, unless otherwise stated,  $\bar{X}$  means  $\sum_{t \in E} X(t)/T$  where  $E$  is a training period of the scheme given by  $\{a_0, a_0 + 1, \dots, a_0 + T\}$  with starting point  $a_0$  and size  $T$ .

$$P_{i(j)} = v_1 \{B_{i(j)}\}^{-1} + v_2 \bar{A}_{i(j)} + v_3 \bar{M}_{i(j)}, \quad (3)$$

where  $v_1$ ,  $v_2$  and  $v_3$  are pre-assigned positive weights and  $B_{i(j)} = \frac{\{\sum_{t \in E} (r_{i(j)}(t) - \bar{r}_{i(j)})^2 / (T-2)\}^{1/2}}{\{\sum_{t \in E} (I_m(t) - \bar{I}_m)^2\}^{1/2}}$ , which is the standard deviation of  $\hat{\beta}_{i(j)}$  ( $\hat{\beta}_{i(j)}$  is an estimate of  $\beta_{i(j)}$ ). In experiments,  $\{\beta_{i(j)}\}^{-1}$ ,  $\bar{A}_{i(j)}$  and  $\bar{M}_{i(j)}$  are scaled from 0 to 1.

Note that  $P_{i(j)}$  eventually indicates the relative importance of individual stock in the portfolio.

Now, the detailed procedure of the GA portfolio index fund scheme is given as follows. First, calculate market

capitalization for each industry sector, say  $(mc_{1(1)}, mc_{2(1)}, \dots, mc_{s(1)})$ .

*Step 1.* Repeat the procedure below for  $k = 1, 2, \dots, l$  until this step selects  $l$  stocks for the portfolio.

Select industry sector  $i_k$  having the largest amount of market capitalization (i.e.  $mc_{i_k(k)} = \max_{i=1,2,\dots,s} mc_{i(k)}$ ). For the selected industry sector  $i_k$ , calculate  $P_{i_k(j)}$  for  $j = 1, 2, \dots, d_{i_k}$  and choose the stock  $i_k(j_k)$  having the highest priority (i.e.  $P_{i_k(j_k)} = \max_{j=1,2,\dots,d_{i_k}} P_{i_k(j)}$ ) to add to the portfolio and remove from the selected industry sector  $i_k$ . Then, without the stock  $i_k(j_k)$ , update market capitalization as  $(mc_{1(k+1)}, mc_{2(k+1)}, \dots, mc_{s(k+1)})$ . Return to the starting point of this step.

For  $\Phi_p = \{c_1, c_2, \dots, c_l\}$  established by Step 1, let  $w_k^m$  ( $k = 1, 2, \dots, l$ ) be the market capitalization of  $c_k \in \Phi_p$  scaled by the entire market capitalization. Note that  $\sum_{k=1}^l w_k^m < 1$  if  $l < n$ .

Table 3

Standard deviation of absolute values of TEs for various starting dates by the classification of market status for given parameters

$v_1:v_2:v_3$	The number of stock ( $l$ )	Training interval of parameters ( $T$ )	Starting date (yy-mm-dd)	Standard deviation of TEs	
				SIM_IF	GA_IF
<i>(a) 1999 (bull market)</i>					
2:1:1	40	20	99-05-12	0.9336	0.6694
2:1:1	40	20	99-06-28	0.8789	0.7215
2:1:1	40	20	99-11-17	2.6957	0.9488
Average				1.5027	0.7799
<i>(b) 2000 (bear market)</i>					
2:1:2	20	60	00-02-18	0.7555	0.6680
2:1:2	20	60	00-07-04	1.4169	1.1480
2:1:2	20	60	00-12-18	2.8576	0.7840
Average				1.6767	0.8667
<i>(c) 2001 (flat market)</i>					
1:2:2	30	40	01-01-09	1.1321	0.8840
1:2:2	30	40	01-04-24	1.3735	1.2743
1:2:2	30	40	01-08-02	0.9380	0.7408
Average				1.1479	0.9664



Step 2. Assign the optimal weights  $\{w_k^p : \sum_{k=1}^l w_k^p = 1, k=1, 2, \dots, l\}$  to each stock in the portfolio which minimizes (4) through GA process.

$$Q(w_1, \dots, w_l) = \sum_{k=1}^l (w_k - w_k^m)^2 \sigma_k^2, \quad (4)$$

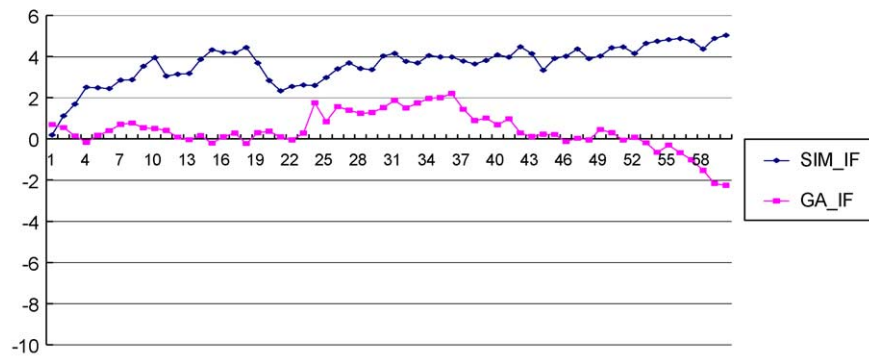
where  $\sigma_k^2 = \frac{\sum_{t \in E} (r_k(t) - \bar{r}_k)^2}{\sum_{t \in E} (I_m(t) - \bar{I}_m)^2} / (T-2)$ , which is the variance of  $\hat{\beta}_k$  for  $k^{\text{th}}$  stock constituting the portfolio set, and  $\beta_p$  is restricted to be close to 1 (i.e.,  $0.995 \leq \beta_p \leq 1.005$ ).

Step 1 describes stock selection process for the portfolio while Step 2 optimizes the selected stocks by GA under the condition  $\beta_p$  is close to 1.

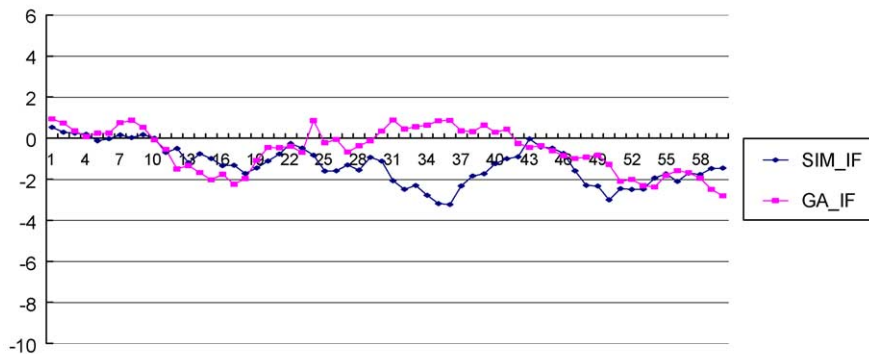
#### 4. Empirical studies

To demonstrate the usefulness of the proposed GA portfolio scheme, KOSPI 200 (a major benchmark index in Korea Stock Exchange) from Jan 1999 to Dec 2001 is used. For the comparison with conventional weight optimization algorithms, we used an algorithm labeled which optimizes the weights by minimizing (4) over 1,000,000 random generations. In the process of GA, the crossover and mutation rates are changed to prevent the output from falling into local optima. The crossover rate runs from 0.5 to 0.8 and the mutation rate runs from 0.05 to 0.06, which uses 50 organisms in the population. The GA automatically stops when there is no

(a) May 12, 1999



(b) Jun. 28, 1999



(c) Nov. 17, 1999

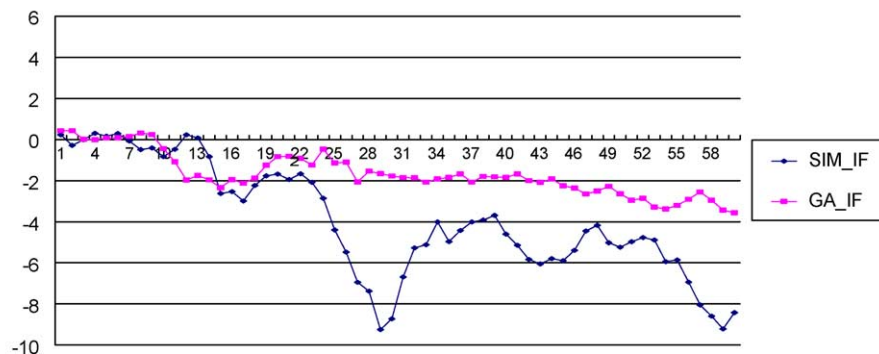


Fig. 4. TEs for 60 days in 1999 (Bull market).

improvement over 1% at the last 5000 trials. In this section, the GA algorithm and the conventional algorithm are labeled GA\_IF and SIM\_IF, respectively.

We carefully examined the variables commonly involved in two algorithms GA\_IF and SIM\_IF which may influence their performances. First, the weights of three fundamental variables (standard error of beta, average trading amount, and average market capitalization) ( $v_1$ ,  $v_2$ ,  $v_3$ ) in (3), are examined since they are crucial parameters influencing the algorithm performance. Second, the number of stocks in portfolio  $l$  ( $l < n$ ) is investigated since a large  $l$  usually implies decreased tracking error (TE), but at the same time requires more rebalancing costs (see Aiello & Chieffe (1999); Chang (2004); Keim (1999) for importance of rebalancing costs). Third, the training period  $E = \{a_0, a_0 + 1, \dots, a_0 + T\}$  with

starting point  $a_0$  and size  $T$  is investigated since it may affect the algorithm training efficiency. Finally, market situation effect (i.e. bull, bear, and flat market effect) is examined.

#### 4.1. Experiments

For the comparison of two algorithms,  $l$  is chosen to be not over 60, and  $T$  is set to be less than 60 days since Korea futures market connected to KOSPI 200 expires every 3 months. The starting date  $a_0$  is chosen such that the training period contains Sep 11, 2001, the day of terrorists attack on the World Trade Center in New York, since performance of the algorithm would be checked better against such an unexpected shock to the financial market. For example,  $a_0$ s in the algorithms are randomly selected among the trading

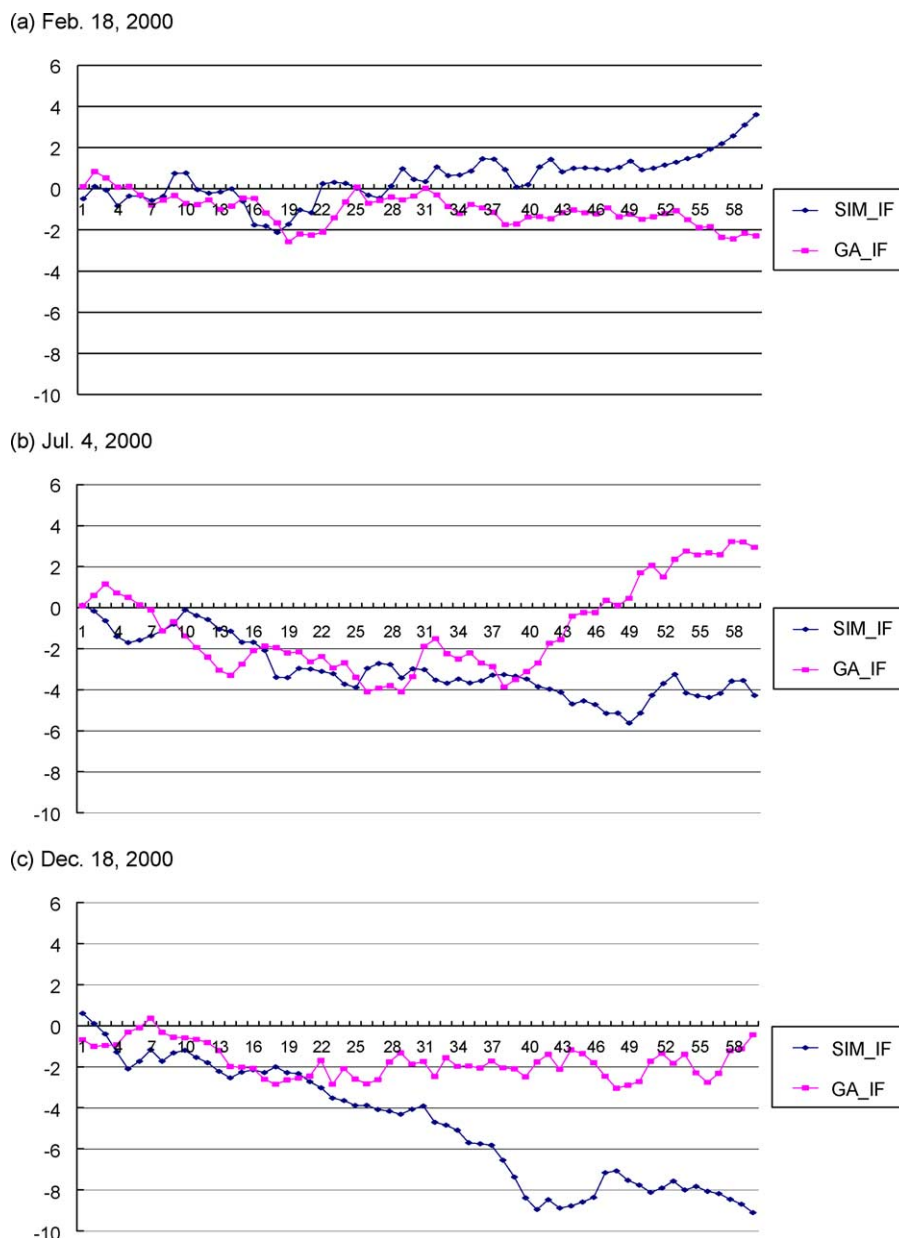


Fig. 5. TEs for 60 days in 1999 (Bear market).

days of Aug, 2001. After the training is done, the trained algorithm is immediately applied to the following next 60 days for its performance evaluation (i.e. the next 60 days following the training period is test period.)

Experiments are first conducted for various  $v_1$ ,  $v_2$  and  $v_3$  with fixed  $l=30$ ,  $T=60$ , and  $a_0$  being Aug 9, 2001. Standard deviation of absolute values of TE and movements of TE itself during the test period are provided in Table 1 and Fig. 1, respectively. It can be noticed that standard errors of GA\_IF are uniformly less than those of SIM\_IF. In particular, GA\_IF seems to be less sensitive than SIM\_IF to changes in  $(v_1, v_2, v_3)$ .

Next experiments examine  $l$  and  $T$  with fixed  $a_0$  and  $(v_1, v_2, v_3)$ . Indeed, when  $a_0$  and  $(v_1, v_2, v_3)$  are given by Aug 9, 2001 and (2,1,1), for various  $l$  and  $T$ , experiments are done

which yields Table 2 and Fig. 2 during the test period. Again, TEs of GA\_IF are uniformly less than SIM\_IF. We also find that the results of GA\_IF are more sensitive to choices of  $l$  and  $T$  than those of SIM\_IF. Through the experiment, it is easy to find that that GA\_IF outperforms the conventional algorithm SIM\_IF.

Third experiment is performed to investigate the market effect. We do this to see if the GA performances are robust under various market conditions. Fig. 3 depicts the movement of KOSPI 200 from 1999 to 2001, which clearly classify the market conditions of 1999–2001 as bull, bear and flat, respectively. Notice that, on the average, KOSPI 200 tends to rise in the bull market, fall in the bear market, and stay in a relatively narrow range between 60 and 80 points in the flat market. GA\_IF and SIM\_IF are compared

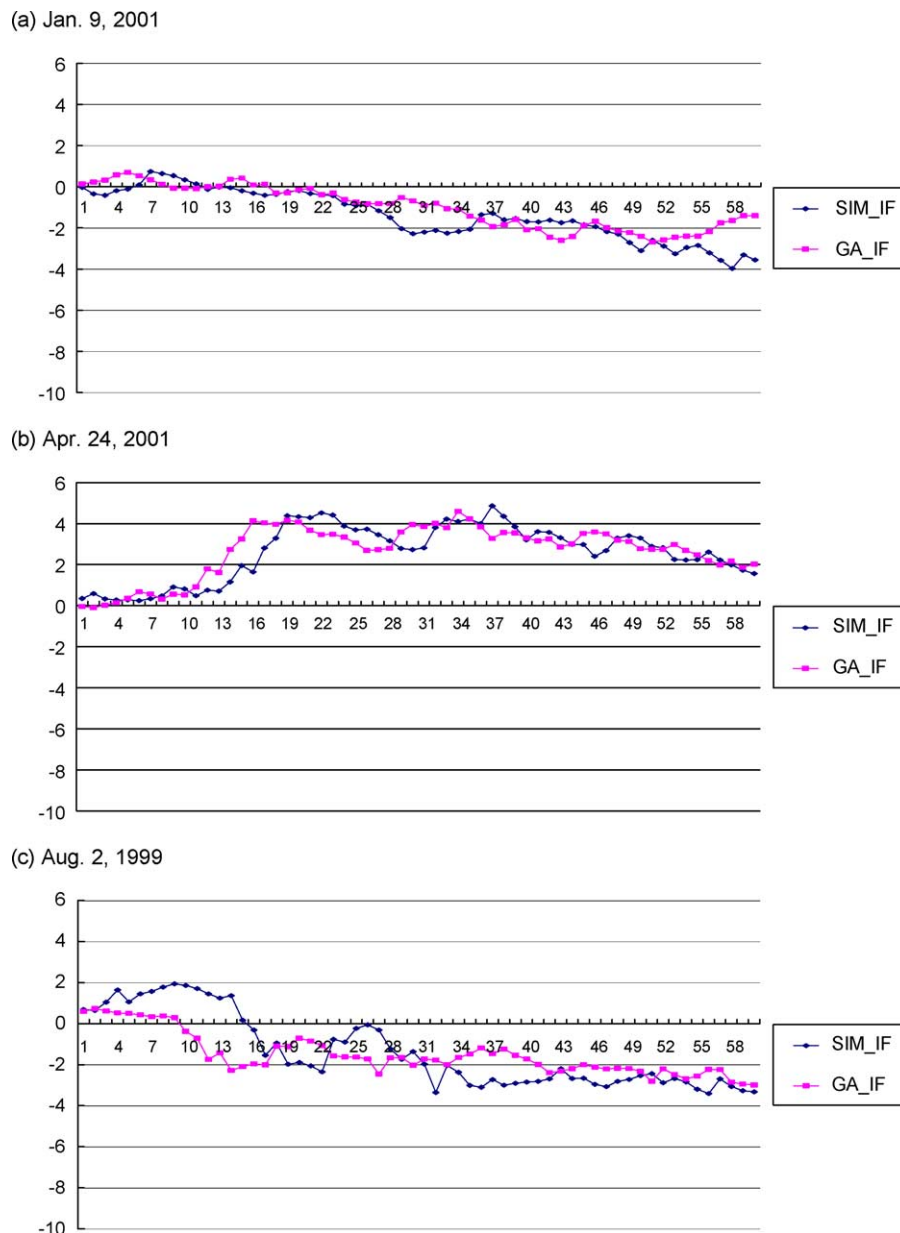


Fig. 6. TEs for 60 days in 1999 (Flat market).



under various market situations. Table 3 provides standard deviations of absolute values of TEs and Figs. 4–6 present movements of TE itself during the test period under various market situations. Considering the degree that GA\_IF dominates over the SIM\_IF in the bull and bear market, the performance of the GA\_IF does not appear to be meaningfully different from those of the SIM\_IF in the flat market.

Indeed, in the bull and bear markets, TEs for GA\_IF (TEs for SIM\_IF) are scattered mostly inside (outside)  $-2$  and  $2$  while, in the flat market, TEs of GA\_IF and SIM\_IF do not show significant differences.

#### 4.2. Discussions

The GA experiment provides useful information for an efficient portfolio scheme. Indeed, GA\_IF scheme shows an improved performance over conventional algorithm. Further, GA\_IF is less sensitive to changes in  $(v_1, v_2, v_3)$ , but more sensitive to changes to  $l$  and  $T$  than conventional one. The results altogether support the superiority of the GA. Notice that  $l$  and  $T$  are mostly internal components of the algorithm and hence can be controlled easily. However, the variables  $(v_1, v_2, v_3)$  are to be determined by external factors outside the algorithm. It is also interesting to observe that GA appears to become quite effective when market energy or volatility has increased.

#### 5. Concluding remarks

Recently, index funds tied to the well-known benchmark indices have become popular among investors. This study proposes the index fund management scheme that uses GA to support portfolio optimization process. Index TEs of the GA portfolio scheme are examined through empirical experiments with KOSPI 200 under various settings. Our results strongly suggest that GA process has outstanding advantages over the conventional portfolio mechanism. Indeed, GA portfolio reports the dominating performances in addition to more desirable properties. Also, it is interesting to notice that GA portfolio reports a mediocre performance when the market is flat.

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