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Vehicles Platooning in Urban Environment: Consensus-based Longitudinal Control with Limited Communications Capabilities

Ahmed Khalifa, Olivier Kermorgant, Salvador Dominguez, and Philippe Martinet

Abstract— **In this research, a general control framework for platooning in urban environment is proposed. A consensus-based control law is described taking into account the nature of traveling in urban environment, that is the human driven leader travels with variable velocity. In addition, the proposed control law does not depend on the predecessor velocity, which in turn allows us to utilize a low cost limited bandwidth communication module by using a sensor-based link for predecessor distance and a communication-based link for leader's information. A constant-spacing policy is used to get a high capacity flow of vehicles. The control system is analyzed and conditions for both internal and string stability are set. The efficiency of the proposed framework and control law is verified via numerical analysis.**

I. INTRODUCTION

Shared transportation systems in urban environments are the current trend to overcome the transportation problems toward an eco-friendly city. One of the recent trends is to use a car-sharing system. However, one of the main problems of the car-sharing system is related to the relocation/redistribution strategies such that the cars are always available and well distributed in all stations. These relocation strategies require more sophisticated techniques to be implemented on cities. An alternative is to guide the cars to move in platoon to relocate them in the available stations.

One of the first and most popular platooning application is developed within the California Partners for Advanced Transit and Highways (PATH) program [1]. Other projects are also proposed in [2], [3] that address the problem of platooning in Highways. Unlike the Highways environment, In urban environments each vehicle nevertheless whole the vehicles in the platoon intend to follow a path. Thus, for vehicle control, localization has to be with respect to a path to be followed. So, the path coordinates can be used instead to represent the vehicle model and control law.

The VALET project proposes a novel approach for solving car-sharing vehicles redistribution problem using vehicle platoons guided by professional drivers who comes to pick up and drop off vehicles over the stations.

The authors are with the Laboratoire des Sciences du Numérique de Nantes (LS2N), Ecole Centrale de Nantes, 1 rue de la Noë, 44321 Nantes, France, ahmed.khalifa@ec-nantes.fr, olivier.kermorgant@ec-nantes.fr, salvador.dominguez-quijada@ec-nantes.fr, philippe.martinet@ec-nantes.fr.

A. Khalifa is on leave from the Department of Industrial Electronics and Control Engineering, Faculty of Electronic Engineering, Menoufia University, Egypt, ahmed.khalifa@el-eng.menofia.edu.eg.

P. Martinet is with the Inria Sophia Antipolis, 06902 Sophia Antipolis, France, philippe.martinet@inria.fr.

In general, the global architecture of platooning has main three layers, including Management, Communication, and Guidance layers. The Guidance layer consists of two sub-layers: Intelligent Sensing sub-layer that is a set of sensors and algorithms to determine the vehicle's actual state and Vehicle Control sub-layer that brings the vehicle at the desired states, which consists of two modules, including Longitudinal control and lateral control modules. Each of them has two sub-modules, including Upper-Level Controller (ULC) and Lower-Level Controller (LLC). In this study, we are interested in designing the Longitudinal ULC that provides the desired vehicle linear acceleration that the car has to follow by the Longitudinal LLC to achieve the desired longitudinal states.

The vehicle longitudinal dynamics are inherently nonlinear. For platoon modeling, different models are used in the literature, including linear and nonlinear models. The linear models are frequently used. Several linear models are commonly used. First, single integrator model, which is the simplest one and it is widely different from the real vehicle dynamics [4]. The second model is the double integrator model [5]. The third approach is to use a third order model by adding another state to model approximately the input/output behavior of the power-train which adds complexity to the control design and analysis [6]. To the best of our knowledge, despite huge amount of relevant literature to date, few studies in the literature have handled the problem of platooning in the urban environment/path coordinates, for instance in [7]. In these studies, the authors uses the first order model of the platoon which has some limitations as it is mentioned before. Therefore, in this research, a second order longitudinal dynamic model is used and its corresponding one in the path coordinates is derived such that one can design a controller to achieve control objectives in the path (operational) coordinates.

The simplest platoon controller is proposed in [8] that is called Adaptive Cruise Control (ACC), which controls only the vehicle's gas and brake throttle, to maintain a safe distance with the front vehicle. Recently, platoon control is based on a distributed controller known as Cooperative Adaptive Cruise Control (CACC) [9]. In the literature there are four main approaches in designing a platoon controller, including Linear, Optimal, $\mathcal{H}_\infty$-based, Sliding Mode, Model predictive, and Consensus-based controller. Different types of Linear Controllers (LC) are widely used in the literature [10]. Some studies propose a sliding mode controller to achieve the string stability of platoon as in [11]. In [12], $\mathcal{H}_\infty$ controller is proposed to achieve system robustness.

The last type is what is called consensus-based controller. Consensus problems of multi-agent systems have attracted an ever-increasing interest in the control community due to their great potential in various applications such as cooperative unmanned air vehicles, automated highway systems scheduling, air traffic control, sensor networks [13]. Recently, CACC control problems using consensus-based algorithms have a tremendous surge of interest among researchers and various algorithms have been developed. Several studies in the literature propose a consensus-based control for platooning in Highways applications as in [14] for example. However, non of theses works consider the control in the path coordinates. In addition, they assume that the leader travel with constant velocity that is not applicable to platooning in urban environment in which, the leader is driven by human, so it may travel with variable velocities. Thus, in this study, the authors propose a controller that can achieve asymptotic stability of the tracking errors and the platoon string stability as well even if the leader travels with variable velocity.

The Information Flow Topology (IFT) represents the Inter-vehicles communication topology that the vehicles can utilize to acquire the information from its surrounding vehicles. It has a significant impact on the platoon performance. Thanks to the rapid advancement of vehicle-to-vehicle (V2V) communication technology, various IFTs are developed, including Predecessor Following (PF), Predecessor - Leader Following (PLF), Bidirectional (BD), etc. Several approaches have been proposed to improve string stability of a platoon. One of them is to broadcast the leader information to every following vehicle, resulting in the PLF topology [15]. This approach is commonly used in the literature in which the leader communicates with all the vehicles in broadcast, and every other vehicle also considers information from its predecessor to compute the control action. However, for low coast communication solution, due to limited bandwidth capabilities of the on-board communication module two channel communication is not feasible. Thus, in this study, the proposed control law depends on a hybrid PLF topology. That is, the leader broadcasts its information (position, velocity, and acceleration) to all the vehicles via a communication-based link. For the inter-followers communication, the inter-vehicle distance is measured by a distance sensor, e.g., laser, i.e., by a sensor-based link. In addition, the proposed control law does not depend on the predecessor velocity.

To sum up, the main contributions of this research are,

- developing a second-order longitudinal platoon control framework in the path coordinates;
- designing a consensus-based controller that is applicable to both motion in urban environment and limited communication capabilities of the vehicles;
- driving conditions for the platoon stability (internal and string).

II. PROBLEM STATEMENT

A. Car-like Vehicle Kinematic Model

Simplifying the car-vehicle system model to a kinematic bicycle model is a common approximation [16]. Consider a car-like vehicle shown in Fig. 1. The kinematic bicycle model combines the left and right wheels into a pair of single wheels at the center of the front and rear axles. The wheels are assumed to have no lateral slip and only the front wheel is steerable.

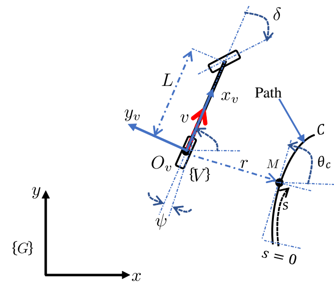


Fig. 1: Kinematic model in Cartesian & Curvilinear coordinates.

1) Cartesian Coordinates Model: Let $\{V\}$, $O_v-\ x_v\ y_v$, denotes the vehicle body-fixed reference frame with its x-axis in the vehicle's forward direction and its origin at the center of the rear axle, O_v, see Fig. 1. The configuration of the vehicle, with respect to the world-fixed inertial reference frame, $\{G\}$, $O-\ x\ y$, is represented by the generalized coordinates $z = [x\ y\ \theta]$, where its position is given by $p = [x\ y]^T \in \mathbb{R}$, while its orientation in the global frame is represented by θ. δ is the steering angle in the body frame. The vehicle's velocity is by definition v in the vehicle's x-direction, x_v, and zero in the y-direction, y_v, since the wheels cannot slip sideways. If the front wheel is located at distance L from the rear wheel along the orientation of the vehicle, then the kinematic model is given by

$$\dot{x} = v\cos(\theta), \tag{1a}$$

$$\dot{y} = v\sin(\theta), \tag{1b}$$

$$\dot{\theta} = \frac{v}{L}\tan(\delta). \tag{1c}$$

2) Path Coordinates Model: Let us define the curvilinear coordinates $(s\ r\ \psi)^T$, as shown in Fig. 1. The tracking path, C, defined in the Global Frame, can be represented as a function of its length s (Curvilinear abscissa) at the closest point M to O_v, the relative angle (angular deviation), $\psi = \theta - \theta_c$, of the vehicle with respect to the path, where θ_c is the angle between the path tangent at M and the x-axis, and finally, the lateral distance (lateral deviation), r, which is the signed orthogonal distance from the center of the rear axle, O_v, to the closest point on the path, M. From the geometry in Fig. 1, the kinematic model in the path coordinates is given by

$$\dot{s} = v\frac{\cos(\psi)}{1 - r\kappa(s)}, \tag{2a}$$

$$\dot{r} = v\sin(\psi), \tag{2b}$$

$$\dot{\psi} = v\left(\frac{\tan(\delta)}{L} - \frac{\kappa(s)\cos(\psi)}{1 - r\kappa(s)}\right), \tag{2c}$$

where $\kappa(s)$ is the curvature of path at point M.

3) Lateral Controller: Motion control of autonomous vehicles moving either alone or in platoon requires design of lateral and longitudinal control to achieve performance objectives in the lateral and longitudinal directions. The lateral controller will provide the desired steering angle as the control signal. The vehicle lateral control will be designed to achieve the lateral control objectives which are: independently from the longitudinal states/controller, $r \to 0$ and $\psi \to 0$ as $t \to \infty$. An interesting and useful method for controlling kinematic models of car-like vehicle systems given by (2) can be found in [17] which is based on the chained forms. In this work, the lateral control law is given by

$$\delta = \operatorname{atan}\Bigg(L\bigg(\frac{\cos^3(\psi)}{(1-r\kappa)^2}\Big(\frac{d\kappa}{ds} r\tan(\psi) - k_d(1-r\kappa)\tan(\psi) - k_p r + \kappa(1-r\kappa)\tan^2(\psi)\Big) + \frac{\kappa\cos(\psi)}{1-r\kappa}\bigg)\Bigg), \tag{3}$$

where k_p and k_d are the controller parameters to be tuned for a desired system response.

B. Car-like Vehicle Longitudinal Dynamic Model

For each vehicle, several factors determine its longitudinal dynamic behavior, which is inherently nonlinear, including the engine, drive line, brake system, aerodynamics drag, tire friction, rolling distance, and gravitational force, . . ., etc [18]. To make a balance between accuracy and conciseness, it is assumed that:

Assumption 1. *The vehicle body is considered to be rigid and symmetric.*

Assumption 2. *The driving and braking inputs are integrated into one control input.*

Assumption 3. *The influence of pitch and yaw motions is considered to be neglected.*

Therefore, the longitudinal dynamic equation can be represented by

$$M_v a + C_v v + G_v = F_v, \tag{4}$$

where $a = \dot{v}$ denotes the vehicle's acceleration in the vehicle frame, and M_v, C_v, G_v, and F_v are the vehicle's Inertia effect, Coriolis effect, Gravity effect, and input force, respectively.

The inverse model compensation technique is frequently used to eliminate the non-linearities in longitudinal dynamics for the purpose of controller design [19].

The control law of the inverse dynamics technique is given by

$$F_v = M_v \mu + C_v v + G_v, \tag{5}$$

where μ is the new input signal after system linearization which has to be designed such that the control objectives can be achieved. By applying this lower-level controller, one can obtain a linear model for vehicle longitudinal dynamics as

$$a = \mu, \tag{6}$$

Most of the research consider the Cartesian coordinates for control design. However, as we intended to platoon in the urban environment, then the relation between the dynamics in the path and Cartesian coordinates will be derived such that the controller can be designed in the path coordinates.

Let us recast (2a) as

$$\dot{s} = Tv, \tag{7}$$

where T is given by

$$T = \frac{\cos(\psi)}{(1-r\kappa)}. \tag{8}$$

Vehicle's acceleration in the path coordinates can be found from the differentiation of (7) with respect to (w.r.t) time as follows

$$\eta = Ta + \dot{T}v, \tag{9}$$

where $\eta = \dot{q} = \ddot{s}$ and $q = \dot{s}$ are the vehicle's acceleration and velocity in the path coordinates respectively.

Consequently, if the mapping from the control signal (i.e., desired acceleration) in the path coordinates, u, to that in the Cartesian coordinates, μ, is given by

$$\mu = \frac{1}{T}(u - \dot{T}v), \tag{10}$$

then, the vehicle longitudinal dynamic model in the path coordinates can be represented by

$$\dot{s} = q, \tag{11a}$$
$$\dot{q} = \eta = u. \tag{11b}$$

C. Platoon Longitudinal Model

From a control viewpoint, the main goal is to create the platoon and then maintain a spacing policy in the presence of perturbations. Consider $N+1$ vehicles are required to move in a platoon, as illustrated in Fig. 2, including a leading vehicle (noted as the leader with index 0) and N following vehicles (noted as followers with index i; $i = 1, 2, \ldots, N$). The platoon runs on urban and flat road, and the vehicles can share information either sensor-based or communication-based way. All the vehicles are assumed to have equal

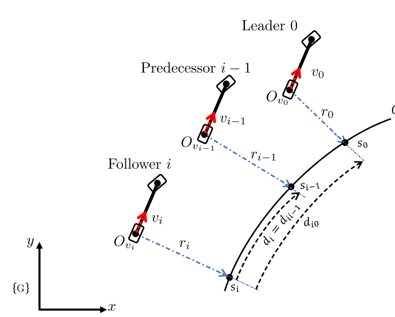


Fig. 2: Platoon representation in the path coordinates.

dynamics. The position, velocity, acceleration, and control input (i.e., desired acceleration.) of vehicle i are denoted with s_i, q_i, η_i, and u_i in the path coordinates. Consequently, the dynamics can be written in the path coordinates as:

$$\dot{s}_i = q_i, \tag{12a}$$
$$\dot{q}_i = u_i, \tag{12b}$$

where the transformation between the two coordinates can be implemented via (2a) and (10).

For the string of vehicles, the inter-vehicle distance, $d_i = s_j - s_i$, is the actual curvilinear distance between vehicle i and its predecessor $j = i - 1$, and $d_{r,i}$ is the desired inter-vehicle distance between vehicles i and j. Several spacing policies are proposed in the literature [20], [21]. For high capacity, small vehicle to vehicle spacing, the constant spacing policy method is utilized.

D. Control Objectives

The platoon has to travel in urban environment (curvilinear path) and track a leader that is either autonomous or manually driven, and each follower vehicle, i, has the following information:

- position, velocity, and acceleration of the leader, s_0, q_0, η_0 via communication-based link;
- position of the front vehicle, i.e., its predecessor s_{i-1} via sensor-based link due to the limited communication capability of the on-board communication module.

Under these conditions, $\forall\ i = 1, \dots, N$, the following goals have to be achieved:

- Asymptotic stability of the tracking errors.
- String stability.

III. CONTROL DESIGN

The information flow structure among platoon can be modeled by a graph in which each vehicle is a node whose dynamics are described by (12). The information flow among followers can be modeled by a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{M})$ characterized by the set of nodes $\mathcal{V} = 1, \dots, N$ and set of edge $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Each edge represents a directional information exchange between two vehicles. The topology of the graph is described by an adjacency matrix $\mathcal{M} = [m_{ij}]_{N\times N}$, in which the element $m_{ij} = 1$ if vehicle i can obtain information from vehicle j; otherwise, $m_{ij} = 0$. Also, the self-edges are not allowed, i.e., $m_{ii} = 0$. To include the leader (with index 0) in the network, an augmented digraph $\bar{\mathcal{G}}$ is used.

Authors propose the following control law

$$u_i = \eta_0 + b(q_0 - q_i) + \sum_{j=0}^{N} k_{ij} m_{ij} (s_j - s_i - d_{r,ij}), \tag{13}$$

where, b and k_{ij} are tuning parameters, $d_{r,ii-1}$ is given by

$$d_{r,ii-1} = l_{f,i} + d_{bb,ii-1} + l_{r,i-1}, \tag{14}$$

where $l_{f,i}$ and $l_{r,i}$ is the distance from the front and rear bumpers to the rear axle center, O_{v_i}, of vehicle i, respectively, and $d_{bb,ii-1}$ is the desired constant space between vehicle i and its predecessor (bumper-to-bumper distance).

For the Predecessor - Leader Following (PLF) information flow topology, where the leader communicates with all the vehicles in broadcast, and every other vehicle also considers information from its predecessor, (13) can be recast as

$$u_i = \eta_0 + b e_{q,i0} + k_0 e_{s,i0}, \qquad \forall\ i = 1, \tag{15}$$

and

$$u_i = \eta_0 + b e_{q,i0} + k_0 e_{s,i0} + k_1 e_{s,i}, \qquad \forall\ i > 1, \tag{16}$$

where $e_{s,i0} = s_0 - s_i - d_{r,i0}$, $e_{q,i0} = q_0 - q_i$, $e_{s,i} = s_j - s_i - d_{r,i}$, for k_{i0}; assume $k_{10} = k_{20} = \dots = k_{N0} = k_0$, and for k_{ij}; assume $k_{21} = k_{32} = \dots = k_{NN-1} = k_1$.

Theorem 1. *Consider a N car-like vehicle platoon following a leader, the longitudinal model expressed as* (1), (2), *(4 - 10), and* (12), *with assumptions 1 - 3, and the control algorithm described in* (15) *and* (16) *is applied to it. Then, the system is asymptotically stable and the platoon is strong string stable as long as the following conditions are satisfied:* $\forall\ \gamma < 1$, $b > 0$, $c = 0.25b^2$, $k_1 = \gamma c$, *and* $k_0 = (1 - \gamma)c$.

Proof. Firstly, the internal stability analysis is done as follows.

Applying the control law (16) to the system (12), and if one define the inter-vehicle distance error, $e_{s,i}$, in terms of the errors with leader, $e_{s,i0}$, as $e_{s,i} = s_j - s_i - d_{r,i} = e_{s,i0} - e_{s,j0}, d_{r,i} = d_{r,i0} - d_{r,j0}, j = i - 1$, then the error dynamics in terms of the error with leader is given by

$$\begin{aligned} \dot{e}_{s,i0} &= e_{q,i0}, \\ \dot{e}_{q,10} &= -b e_{q,10} - k_0 e_{s,10}, \\ \dot{e}_{q,i0} &= -b e_{q,i0} - (k_0 + k_1)\, e_{s,i0} + k_1 e_{s,j0}, \forall\ i > 1. \end{aligned} \tag{17}$$

Let us define $e_s = [e_{s,10}, \dots, e_{s,i0}, \dots, e_{s,N0}]^T$ and $e_q = [e_{q,10}, \dots, e_{q,i0}, \dots, e_{q,N0}]^T$ as the position and speed error vectors, respectively, then the error state vector, $\mathcal{X}(t) = [e_s\ e_q]^T$, is given by

$$\dot{\mathcal{X}}(t) = A\mathcal{X}(t), \tag{18}$$

where A is described as follows

$$A = \begin{bmatrix} O_N & I_N \\ -P & -D \end{bmatrix}, \tag{19}$$

where I_n and O_n denote $(n \times n)$ identity and $(n \times n)$ null matrices, respectively, $D = diag\{b, \dots, b\} \in \mathbb{R}^{N\times N}$, and $P = [p_{ii}] \in \mathbb{R}^{N\times N}$ is represented by

$$p_{ii} = \begin{cases} k_0, & j = i, i = 1, \\ k_0 + k_1, & j = i, i > 1, \\ -k_1, & j = i - 1, i > 1, \\ 0, & otherwise. \end{cases} \tag{20}$$

The eigenvalues of P, $\lambda_{p,i}$, can be calculated as

$$\lambda_{p,i} = \begin{cases} k_0, & i = 1, \\ k_0 + k_1, & i > 1. \end{cases} \tag{21}$$

Thus, $\forall\ k_0 > 0$ and $k_1 > 0$, the matrix P is positive stable. In addition, $\forall\ b > 0$, the matrix D is positive stable.

Lemma 1. Schur's formula [22]. *Let matrices* $\mathcal{F}_{11}, \mathcal{F}_{12}, \mathcal{F}_{21}, \mathcal{F}_{22} \in \mathbb{R}^{N\times N}$ *and* $\mathcal{M} = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{21} & \mathcal{F}_{22} \end{bmatrix}$. *If* $\mathcal{F}_{11}, \mathcal{F}_{12}, \mathcal{F}_{21}, \mathcal{F}_{22}$ *commute pairwise, i.e.,* $\mathcal{F}_{rs}\mathcal{F}_{nm} = \mathcal{F}_{nm}\mathcal{F}_{rs}$ *for all possible pairs of indices* r, s *and* n, m*, then the determinate of* $\mathcal{M}$*,* $\det(M) = \det(\mathcal{F}_{11}\mathcal{F}_{22} - \mathcal{F}_{12}\mathcal{F}_{21})$

Lemma 2. [23]. *Let a complex coefficient polynomial,* $f(z) = z^2+(a+ib)z+(c+id)$*, where a, b, c, and* $d \in \mathbb{R}$. $f(z)$ *is Hurwtiz stable if and only if* $a > 0$ *and* $abd+a^2c-d^2 > 0$.

According to Lemma 1, the characteristics polynomial of A can be calculated as follows:

$$\begin{aligned} \det(SI_{2N} - A) &= \det\left(\begin{bmatrix} SI_N & -I_N \\ P & SI_N + D \end{bmatrix}\right) \\ &= \det\left(S^2 I_N + bI_N S + P\right) \\ &= \prod_{i=1}^{N}\left(S^2 + bS + \lambda_{p,i}\right). \end{aligned} \tag{22}$$

The polynomial $S^2 + bS + \lambda_{p,i}$ is Hurwitz stable, according to Lemma 2 and the conditions $b > 0$, $k_0 > 0$, $k_1 > 0$. This implies that all eigenvalues of A have negative real parts. Consequently, the error dynamics is asymptotically stable i.e., $\lim_{t\to\infty} \mathcal{X}(t) = 0$.

Secondly, the string stability analysis is done as follows. For simplicity, let us define the following errors $e_i = s_j - s_i - d_{r,i}$, $\dot{e}_i = q_j - q_i$, $\ddot{e}_i = u_j - u_i$, $e_{i0} = s_0 - s_i - d_{r,i0}$, and $\dot{e}_{i0} = q_0 - q_i$.

Form (16) one can write $\ddot{e}_i$ as

$$\begin{aligned} \ddot{e}_i &= u_j - u_i \\ &= b\dot{e}_{j0} + k_0 e_{j0} + k_1 e_j - b\dot{e}_{i0} - k_0 e_{i0} - k_1 e_i, \end{aligned} \tag{23}$$

Writing (23) in the S-domain results in

$$\begin{aligned} S^2 E_i(S) = bSE_{j0}(S) + k_0 E_{j0}(S) + k_1 E_j(S) \\ -bSE_{i0}(S) - k_0 E_{i0}(S) - k_1 E_i(S). \end{aligned}$$

Substituting with $e_{i0} = e_i + e_{j0}$ or $E_{i0}(S) = E_i(S) + E_{j0}(S)$ into (24), then the transfer function $H(S) = \frac{E_i(S)}{E_j(S)}$ can be defined as

$$H(S) = \frac{k_1}{S^2 + bS + c}, \tag{24}$$

where $c = k_0 + k_1$.

Conditions for string stability:

- In time domain, for *strong string stability*: $\left\|h(t)\right\|_1 < 1$
- In frequency domain
  - for *strong string stability*: $\left\|H(S)\right\|_\infty < 1$ and $h(t) > 0$

The poles of $H(S)$, second order system, are $p_{1,2} = -0.5b \pm \sqrt{(0.5b)^2 - c}$,

To achieve $h(t) > 0$, the response of $H(S)$ has to be over-damped, i.e., $(0.5b)^2 > c$, or critical-damped, i.e., $(0.5b)^2 = c$. However, for faster response the parameters of $H(S)$ are chosen for a critical-damped response, i.e., $(0.5b)^2 = c$. In this case, the impulse response is represented by

$$h(t) = k_1 t e^{-0.5bt}. \tag{25}$$

Thus, $\forall\ k_1 > 0$, one can obtain $h(t) > 0$.

Lemma 3. [24]. *If* $F(s)$ *is a stable, proper transfer function, with* $\left|F(0)\right| = \beta$*, and if* $f(t) = \mathcal{L}^{-1}\{F(s)\}$ *is its impulse response, then* $\left\|h(t)\right\|_1 = \beta$ *if and only if* $h(t)$ *does not change sign.*

According to Lemma 3,

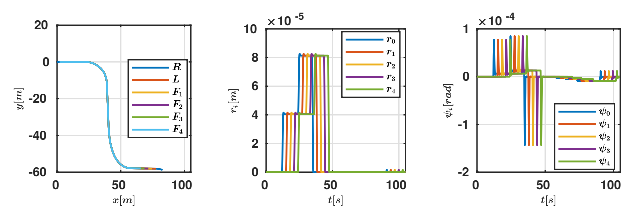
$$\begin{aligned} \left\|h(t)\right\|_1 &= \left|H(0)\right| \\ &= \frac{k_1}{c} = \gamma, \end{aligned} \tag{26}$$

Thus, by choosing $k_1 = \gamma c$ with $\gamma < 1$, then $\left\|h(t)\right\|_1 < 1$ and as a result the platoon is strong string stable. □

IV. RESULTS

A. Environment

The previously proposed control strategy is simulated in MATLAB/SIMULINK program for the considered Urban cars platooning. We consider a platoon composed of a homogeneous 4 vehicles plus a manually driven leader. For emulation of motion in urban environment, the trajectory shown in Fig. 3, is used as a reference trajectory to be followed by all the cars. The vehicles are initially parked near the reference path with different initial poses. The lateral controller (3) is in charge of producing steering angle of the vehicle. Gains of the lateral control law are tuned in order to get a satisfactory response of the lateral and angular deviations, see Fig. 3 for the results. The path tracking capability of the vehicle is enlightened in Fig. 3. The selected lateral control parameters are $k_p = 200$ and $k_d = 95$.



(a) Trajectory Tracking (b) r_i (c) ψ_i

Fig. 3: Lateral Control : R: Reference Trajectory, L: Leader Actual Trajectory, and F_i: Follower i Actual Trajectory.

The longitudinal controller is implemented at rate of 100 Hz assuming that the vehicle states are available at this rate. Control parameters are selected to guarantee the internal stability and string stability. The selected control parameters are reported in Table I.

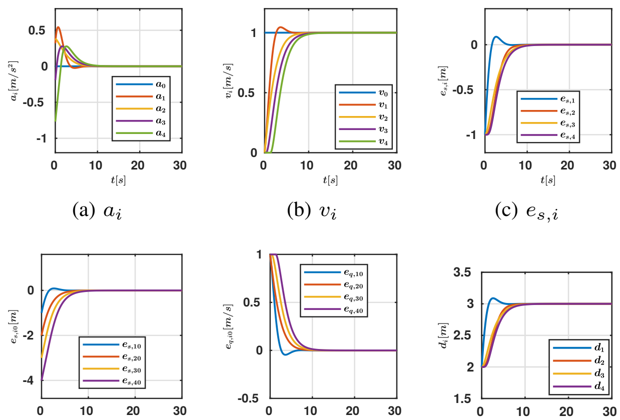
B. Normal Platooning

Fig. 4 shows the results for the normal consensus scenario. The leader travels with constant velocity. The results confirm the capability of the proposed approach of creating and maintaining the platoon in case of leader travel with constant velocity. The vehicles are starting from distances different from the one required by the spacing policy and reach

TABLE I: Simulation Parameters

Parameter	Value	Parameter	Value
L	$1\,\mathrm{m}$	a_{max}	$1\,\mathrm{m/s^2}$
a_{min}	$-3\,\mathrm{m/s^2}$	v_{max}	$8\,\mathrm{m/s}$
v_{min}	$0\,\mathrm{m/s}$	b	1.6
$d_{r,i}$	$3\,\mathrm{m}$	γ	0.1

the consensus and the curvilinear position and speed errors converge asymptotically. Moreover, the followers track the leader zero acceleration (i.e., the control effort reaches zero after consensus).



(a) a_i (b) v_i (c) $e_{s,i}$ (d) $e_{s,i0}$ (e) $e_{q,i0}$ (f) d_i

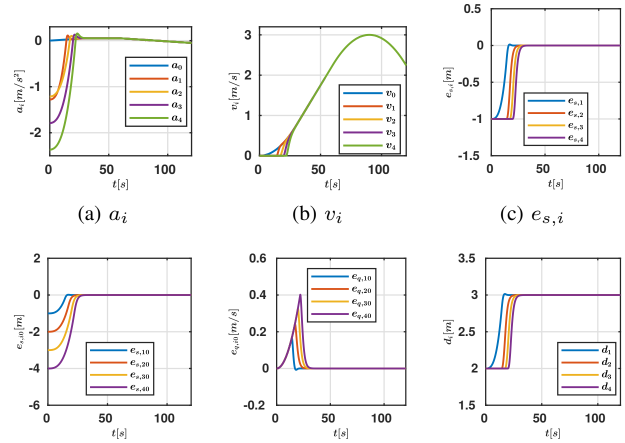
Fig. 4: Normal Platooning Longitudinal Control Results

C. Variable Speed Scenario

Fig. 5 shows the results for the leader tracking consensus scenario. The results confirm the capability of the proposed approach of creating and maintaining the platoon in case of a manually driven leader (i.e., the velocity is not constant). The vehicles are starting from distances different from the one required by the spacing policy and reach the consensus and the curvilinear position and speed errors converge asymptotically. Furthermore, the followers track the leader acceleration that has a trapezoidal shape.

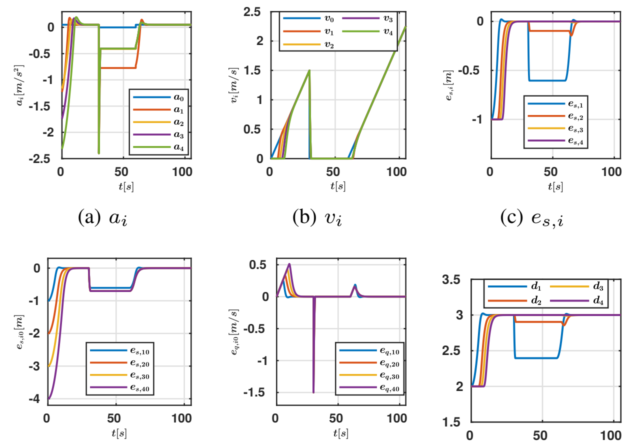
D. Braking Scenario

To investigate the controller robustness against different driving scenarios, we test the ability of the proposed strategy in case of the leader sudden braking. Results in Fig. 6 show how the platoon reacts in the case of a braking maneuver performed by the leader from $1.5\,\mathrm{m/s}$ to a full stop. The results show that the inter-vehicle distance between vehicle 0 and vehicle 1, d_1, suffers an approximately $0.6\,\mathrm{m}$ decrease, while d_2 suffers an approximately $0.09\,\mathrm{m}$ decrease, and d_3 and d_4 is further smaller. These results illustrate that the sudden disturbance on the platoon is attenuated along the rest of the platoon (i.e., string stable platoon).



(a) a_i (b) v_i (c) $e_{s,i}$ (d) $e_{s,i0}$ (e) $e_{q,i0}$ (f) d_i

Fig. 5: Variable Speed Platooning Longitudinal Control



(a) a_i (b) v_i (c) $e_{s,i}$ (d) $e_{s,i0}$ (e) $e_{q,i0}$ (f) d_i

Fig. 6: Platooning Longitudinal Control Results: Leader Sudden Brake

V. CONCLUSIONS

A consensus-based control technique for platooning in urban environment is proposed. A second-order longitudinal model of the platoon in the path coordinates is driven. General architecture is proposed to design the controller in the path coordinates in order to achieve some objectives defined the path coordinates with a relation that maps the control signals in the Cartesian and path coordinates. The proposed algorithm does't need for the predecessor's velocity, so one can depend only on the distance sensor without the need to communicate with the predecessor's such that lower bandwidth communication module can be utilized. A constant-spacing policy is used to get a high capacity flow of vehicles. The controller is design to achieve zero tracking errors in case the leader is manually driven (i.e., leader travels with variable velocity) to be suited to work in VALET project. Internal stability is analyzed and the asymptomatic stability of the tracking errors is proven.