

# Genetic Nonlinear Controller of a Skid Steering Mobile Robot with Slip Conditions

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**Abstract**—In this paper, the kinematics of a skid steering mobile robot is controlled using a nonlinear controller augmented with a genetic algorithm which is used to tune the controller parameters. The controller is simple in terms of design and implementation. Moreover, it maintains good performance and has stronger robustness. Furthermore, the proposed controller is used to control an extended kinematics model taking into account slip effects to test the validity and enhancement of the controller. Simulation results are included.

**Index Terms**—Skid Steering Mobile Robots, Genetic Algorithm, Nonlinear Controller tuning, Slip Conditions.

## I. INTRODUCTION

Skid steering mobile robots are widely used as outdoor mobile robots. They are suitable for terrain traversal such as loaders, farm machinery, mining and military applications, due to the simple and robust mechanical structure, faster response, high maneuverability, strong traction, and high mobility[1], [2]. Due to complex kinematic constraints and wheel/ground interactions, considering the kinematics model with the consideration of the slip conditions and designing a proper controller for skid-steering mobile robots (SSMR) are challenging tasks.

A number of research papers have been published on the topic of modeling and control of a skid steering mobile robots. Wheeled skid steering mobile robots stability has been studied by some authors using model based nonlinear control techniques by explicitly considering dynamics and drive models [3], [4], [5]. Furthermore, in some works the kinematics have been addressed as the relation of linear and angular velocities with the position of the vehicle [6], [7]. But, major skid effects have not been considered, which arise at a lower level, in the relation between drive velocities and vehicle velocities. An on-line adaptive control for wheeled skid steering mobile robot has been considered for estimating tire/ground friction of a simplified dynamic model [8]. Control methods of wheeled skid-steering mobile robot trajectory tracking on a rough terrain were presented in [9] including practical fuzzy lateral control, longitudinal control and the sensor pan-tilt control; the authors used ADAMS and MATLAB co-simulation platform to assess these control laws. In [10],[11], a thorough dynamic analysis of a skid-steered vehicle has been introduced; this

analysis considers steady-state (i.e., constant linear and angular velocities) dynamic models for circular motion of tracked vehicles.

Some of the previous works stated above designed a control system that based on the selection of its parameters using trial and error method which is immensely time-consuming and tedious process. while, others did not consider the slip conditions for 4-wheeled SSMR which plays a critical role in kinematics modeling of SSMR. Therefore, our main contribution in this research is to design a nonlinear controller augmented with an adaptation algorithm, based on genetic algorithm(GA), which tunes and optimizes the controller parameters, for controlling an SSMR extended kinematics model with slip conditions. The consideration of the slip conditions in the SSMR is essential as it plays an important role in robot control.

In this paper, an extended kinematics model of the kinematics of the SSMR based on [3],[12] is developed. This extended model takes into account the slip conditions which affect the performance of the motion of mobile robots. Then, a model based nonlinear controller augmented with a genetic algorithm to tune the nonlinear controller parameters is developed. Finally, a comparison between the classical and the extended kinematic models responses is performed to show the enhancement provided by the proposed controller. Design and experimental implementation simplicity, strong robustness, and good performance are the main advantages of the proposed controller.

The rest of the paper is organized as follows: In section II an SSMR extended kinematics model is presented in a systematic way. Section III focuses on the development of a model based genetic nonlinear controller to ensure robustness to the nonlinearities of system model. Section IV is dedicated for extensive simulation results considering trajectory tracking problem. Section V is devoted to conclusions and ideas for future work. Finally, acknowledgments and references complete the paper.

## II. SSMR MODEL

A mathematical description of the kinematics of an SSMR moving on a planar surface is reviewed in this section. The mathematical model of the vehicle [3] can be divided into three

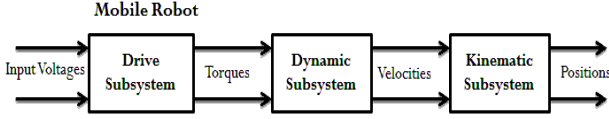


Fig. 1. An electrically driven mobile robot decomposition

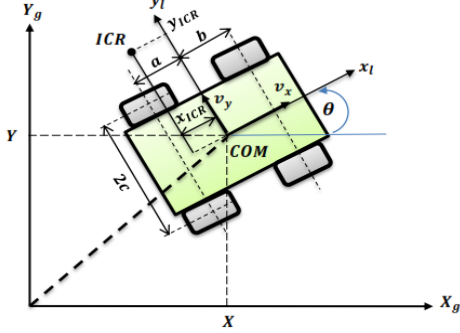


Fig. 2. Schematic diagram of SSMR

parts: kinematics, dynamics and drive subsystems, see Fig. 1. In this paper, we focus on the last block, i.e., the kinematic subsystem, and use it for reference tracking control of the desired trajectory.

#### A. Classical Kinematic Model

The main equation that describes the kinematic subsystem of the SSMR moving on a planar surface as shown in Fig. 2 is given by [3], [4]:

$$\dot{q} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & x_{ICR}\sin\theta \\ \sin\theta & -x_{ICR}\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ w \end{bmatrix} \quad (1)$$

where  $v_x$  is the longitudinal velocity,  $w$  is the angular velocity of the robot,  $q = [X \ Y \ \theta]^T$  represents the generalized coordinates of the center of mass (COM) of the robot, i.e., the COM position, with  $X$  and  $Y$ ; and  $\theta$  is the orientation of the local coordinate frame with respect to the inertial frame, and the coordinate of the instantaneous center of rotation (ICR) is defined as  $(x_{ICR}, y_{ICR})$ .

This can be rewritten as follows:

$$\dot{q} = S(q) \begin{bmatrix} v_x \\ w \end{bmatrix} = S(q)\eta \quad (2)$$

Equation (2) describes the kinematics of the robot, which can be considered as an underactuated system because  $\dim(\eta) = 2 < \dim(q) = 3$ .

#### B. Extended Kinematics Model

The classic kinematics model of an SSMR will be extended to include the slip effect and resulting into the extended kinematics model [12]. As known, The linear velocity of the

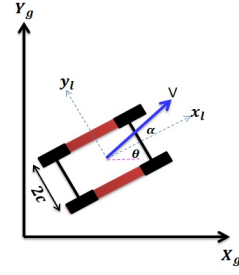


Fig. 3. Free body diagram of an SSMR during steering

wheels of a mobile robot in absence of wheel slip can be given by:

$$v_r = rw_r, \quad v_l = rw_l. \quad (3)$$

where:  $r$  denotes the so-called effective rolling radius of that wheel,  $v_r$  and  $v_l$  denote the linear velocities of the right and left wheels respectively,  $w_r$  and  $w_l$  are the angular velocities of the right and left wheels, respectively. In this way, the kinematics model for an SSMR is given by:

$$\begin{aligned} \dot{X} &= \frac{v_r + v_l}{2} \cos\theta + \frac{v_r - v_l}{2c} x_{ICR} \sin\theta, \\ \dot{Y} &= \frac{v_r + v_l}{2} \sin\theta - \frac{v_r - v_l}{2c} x_{ICR} \cos\theta, \\ \dot{\theta} &= \frac{v_r - v_l}{2c}. \end{aligned} \quad (4)$$

where  $2c$  is the distance between wheels centers.

But, the real velocity of the wheels will be lower than the theoretical velocity of the wheels when the robot moves under slip conditions according to the following formulas:

$$\begin{aligned} v_{rslip} &= v_r[1 - i_r] = rw_r[1 - i_r], \\ v_{lslip} &= v_l[1 - i_l] = rw_l[1 - i_l]. \end{aligned} \quad (5)$$

such that  $i_r$  and  $i_l$  are the right and left wheels slip, respectively,  $v_{rslip}$  and  $v_{lslip}$  are the linear velocities of the right and left wheels under slip conditions.

The kinematics equations for an SSMR during turning, as shown in Fig. 3, are given by

$$\begin{aligned} \dot{X}_{slip} &= \frac{v_{rslip} + v_{lslip}}{2} \cos\theta_{slip} + \frac{v_{rslip} - v_{lslip}}{2c} x_{ICR} \sin\theta_{slip} \\ &\quad - \frac{v_{rslip} + v_{lslip}}{2} \sin\theta_{slip} \tan\alpha, \\ \dot{Y}_{slip} &= \frac{v_{rslip} + v_{lslip}}{2} \sin\theta_{slip} - \frac{v_{rslip} - v_{lslip}}{2c} x_{ICR} \cos\theta_{slip} \\ &\quad + \frac{v_{rslip} + v_{lslip}}{2} \cos\theta_{slip} \tan\alpha, \\ \dot{\theta}_{slip} &= \frac{v_{rslip} - v_{lslip}}{2c}. \end{aligned} \quad (6)$$

where,  $\alpha$  is the slip angle,  $[X_{slip} \ Y_{slip} \ \theta_{slip}]$  is the location of the real mobile robot under slip conditions.

Therefore, to test the slip conditions effects on the SSMR motion, the extended kinematic equation (6) can be used.

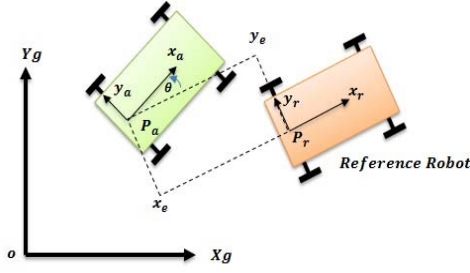


Fig. 4. Reference vehicle and error coordinates.

### III. CONTROLLER DESIGN

In this section, a model based nonlinear controller is introduced and an overview of genetic algorithm which is used in tuning the parameters of the nonlinear controller is given. This tuned controller is used in a reference tracking problem for both of the classical and extended kinematics models.

#### A. Model Based Nonlinear Controller Design

We will consider the problem of a tracking of a reference vehicle with the same kinematics in both position and orientation [13] as shown in Fig. 4. According to the control literature, the tracking problem is usually associated with the problem of asymptotically stabilizing the reference trajectory. So in such case, the feasibility of reference is a necessary condition for the existence of a control solution. For some specific control input  $t \mapsto (u_{1r}(t), u_{2r}(t))^T$ , called the reference control, a feasible trajectories  $t \mapsto (X_r(t), Y_r(t), \theta_r(t))$  which are smooth time functions which are solution to the robots kinematic model. For an SSMR for example, this means in view of (1) that:

$$\begin{aligned}\dot{X} &= v_x \cos \theta + w x_{ICR} \sin \theta, \\ \dot{Y} &= v_x \sin \theta - w x_{ICR} \cos \theta, \\ \dot{\theta} &= w.\end{aligned}\quad (7)$$

This equation can be written as a function of reference control inputs  $u_{1r}, u_{2r}$ :

$$\begin{aligned}\dot{X}_r &= u_{1r} \cos \theta_r + u_{2r} x_{ICR} \sin \theta_r, \\ \dot{Y}_r &= u_{1r} \sin \theta_r - u_{2r} x_{ICR} \cos \theta_r, \\ \dot{\theta}_r &= u_{2r}.\end{aligned}\quad (8)$$

The main concern here is to determine a feedback control which asymptotically stabilizes the tracking error  $(X - X_r, Y - Y_r, \theta - \theta_r)$  at zero, with  $(x_r, y_r)$  being the coordinates of  $P_r$  in the global frame  $OX_g Y_g$ , and  $\theta_r$  the oriented angle between  $x$  and  $x_r$ .

By derivation of the tracking error in position and orientation  $(X - X_r, Y - Y_r, \theta - \theta_r)$  with respect to the reference robot frame gives the vector:

$$\begin{bmatrix} X_e \\ Y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta_r & \sin \theta_r & 0 \\ -\sin \theta_r & \cos \theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X - X_r \\ Y - Y_r \\ \theta - \theta_r \end{bmatrix} \quad (9)$$

where  $X_e = X - X_r$ ,  $Y_e = Y - Y_r$ , and  $\theta_e = \theta - \theta_r$ .

By calculating the time derivative of this above equation, this will yield:

$$\begin{bmatrix} \dot{X}_e \\ \dot{Y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} u_{2r} Y_e + u_{1r} \cos \theta_e - u_{1r} \\ -u_{2r} X_e + u_{1r} \sin \theta_e \\ u_2 - u_{2r} \end{bmatrix} \quad (10)$$

Now by considering the following change of coordinates and control variables in order to determine a control  $(u_1, u_2)$  which asymptotically stabilizes the error  $(X_e, Y_e, \theta_e)$  at zero:

$$(X_e, Y_e, \theta_e, u_1, u_2) \mapsto (q_1, q_2, q_3, R_1, R_2) \quad (11)$$

which are defined by:

$$\begin{aligned}q_1 &= X_e, \quad q_2 = Y_e, \quad q_3 = \tan \theta_e, \\ R_1 &= u_{1r} \cos \theta_e - u_{1r}, \\ R_2 &= \frac{u_2 - u_{2r}}{\cos^2 \theta_e}.\end{aligned}\quad (12)$$

By assuming that we have the following control laws:

$$\begin{aligned}R_1 &= -K_1 |u_{1r}| (q_1 + q_2 q_3) \quad (K_1 > 0), \\ R_2 &= -K_2 u_{1r} q_2 - K_3 |u_{1r}| q_3 \quad (K_2, K_3 > 0).\end{aligned}\quad (13)$$

Finally, the control signals to the SSMR system  $(u_1, u_2)$  which represent the linear velocity  $v_x$  and angular velocity  $w$  defined in (7) are as follows:

$$u_1 = \frac{R_1 + u_{1r}}{\cos \theta_e}, \quad u_2 = R_2 \cos^2 \theta_e + u_{2r}. \quad (14)$$

#### B. Tuning of Nonlinear Controller Using Genetic Algorithm

1) *Genetic Algorithm Overview:* The Genetic Algorithm, GA, is considered as a powerful optimization searching technique based on the principles of natural genetics and natural selection [14]. Compared with other optimization techniques, GA is theoretically and empirically proven to provide robust search in complex spaces, offering a valid approach to problems requiring efficient and effective search. Moreover, GA is very easy to understand and it practically does not demand the knowledge of mathematics. Also, it is a suitable optimization technique especially in noisy environments. Furthermore, it provides an optimal solutions which get better with time. GA starts with an initial population containing a number of chromosomes. Each chromosome represents a solution of the problem which performance is evaluated by a fitness function. Basically, GA consists of three main elements used for the searching procedure: Selection, Crossover and Mutation. The creation of new individuals which may be better than their parents can be allowed by the application of these three basic operations. The procedure of a GA is given as follows:

- 1) *[Start]*: Generate randomly a population of chromosomes.
- 2) *[Fitness]*: Calculate the fitness for each chromosome in the population.
- 3) *[Newpopulation]*: Create offsprings by using genetic operators (selection, crossover and mutation).

- 4) [*Replace*]: Use the newly generated population for a further run of the algorithm.
- 5) [*Test & Loop*]: Stop if the search goal is achieved. Otherwise continue with step 2.

This above procedure is repeated for many generations and finally stops when reaching individuals that represent the optimum solution to the searching problem. A flow chart of the general scheme of the implementation of the GA is shown in the left side of Fig. 5. In this paper, the GA is implemented using MATLAB toolbox which allows the user to establish the GA parameters (the size of the population, the type of selection scheme, crossover and mutation and the probability of applying the genetic operators).

2) *Controller Parameters Tuning*: The main difficulty in the controller described by equation (13) is the selection and tuning of its parameters ( $K_1, K_2, K_3$ ) especially when the system has complex nonlinearities. So to overcome the difficulty of tuning of the controller parameters by trial and error method, we may use one of modern heuristic optimization technique such as GA. In GA, each chromosome comprises of three parameters, ( $K_1, K_2, K_3$ ) with value bounds varied depend on the objective functions used. The tuning procedure flowchart is depicted in Fig. 5. The GA parameters that used in this paper are as follows: maximum number of generations=200, population size=25, the fitness function is the mean of squared error (MSE). The main objective of GA is to seek for minimum fitness value.

$$MSE = \frac{1}{t} \int_0^t (e(t))^2 d\tau \quad (15)$$

The parameters bounds used in this study are selected to be:  $K_1 \in [1, 5]$ ,  $K_2 \in [1, 50]$ , and  $K_3 \in [1, 20]$ . Finally, after running the GA, the optimal parameters obtained are:  $K_1 = 4.0736$ ,  $K_2 = 32.5716$ , and  $K_3 = 9.0248$ .

#### IV. SIMULATION RESULTS

In this simulation, we considered the kinematics models which are described by (1) and (6) for the No-Slip and Slip cases, respectively. To demonstrate the effectiveness of the proposed controller (nonlinear controller tuned by genetic algorithm), we compared the results obtained from the proposed controller and those which obtained using the same nonlinear controller described by (13) but by selecting the controller parameters by trial and error. The system parameters applied for simulation are shown in Table I. In practice, it is difficult to measure  $x_{ICR}$  value. So it is assumed here to be [3], [4]:

$$x_{ICR} = \text{constant} = x_0 \quad x_0 \in (-a, b) \quad (16)$$

where  $a$  and  $b$  are positive kinematic parameters of the robot depicted in Fig. 2.

##### A. Genetic Nonlinear Controller Responses

To validate the performance of the proposed controller, simulation results using the Matlab/Simulink environment are

TABLE I  
PARAMETERS OF THE MODEL

Variable	Value	Unit
$a = b$	60	cm
$c$	30	cm
$r$	20	cm
$x_0$	-15	cm
$i_l$	18 %	-
$i_r$	23.3 %	-
$\alpha$	47	degree

presented for circular contour trajectory tracking, The reference trajectory is as follows:

$$\begin{aligned} x_r &= 7\cos(0.02\pi t), \\ y_r &= 7\sin(0.02\pi t), \\ \theta_r &= \frac{\pi}{2} + 0.02\pi t. \end{aligned} \quad (17)$$

In simulation, we test two cases for the system. First, we assume the system without slip conditions as described by (1). Second, we add slip conditions to the system as described by (6) to check its effect on the response and assess the controller performance. Fig. 6 shows the system response to circular contour described by (17). The system can track the desired inputs quickly and without any overshoot.

##### B. Trial and Error Nonlinear Controller Responses

For comparison purposes, the No-Slip and Slip cases of the system are tested but by selecting the controller parameters by trial and error. The gains found in (13) was selected to be:  $K_1 = 0.35$ ,  $K_2 = 25$ , and  $K_3 = 10$ . Fig. 7 shows the system response. It is clear that the system can track the desired inputs quickly and without any overshoot. But, it can be shown that there are some steady state errors in these responses.

For numerical comparison between the responses shown in Fig. 6 and Fig. 7, the Mean Squared Error (MSE) described by (15) is calculated for the two cases (No-Slip, Slip) using the two controllers and the MSE numerical values for  $X$ ,  $Y$ , and  $\theta$  references are shown in Table II. It is clear from this numerical comparison that the genetic nonlinear controller provides better responses than the controller based on trial and error selection of its parameters.

#### V. CONCLUSION

A nonlinear controller with genetic algorithm is presented by merging genetic procedure with a model based nonlinear controller to tune its parameters which improve the tracking accuracy and overcoming the effect of slip conditions added to the system. For comparison, a nonlinear controller is also designed by selecting its parameters by trial and error. The genetic nonlinear controller shows satisfactory results. In the future work, the proposed controller responses with another nonlinear controllers from the main literature will be investigated. Moreover, a development of a genetic nonlinear

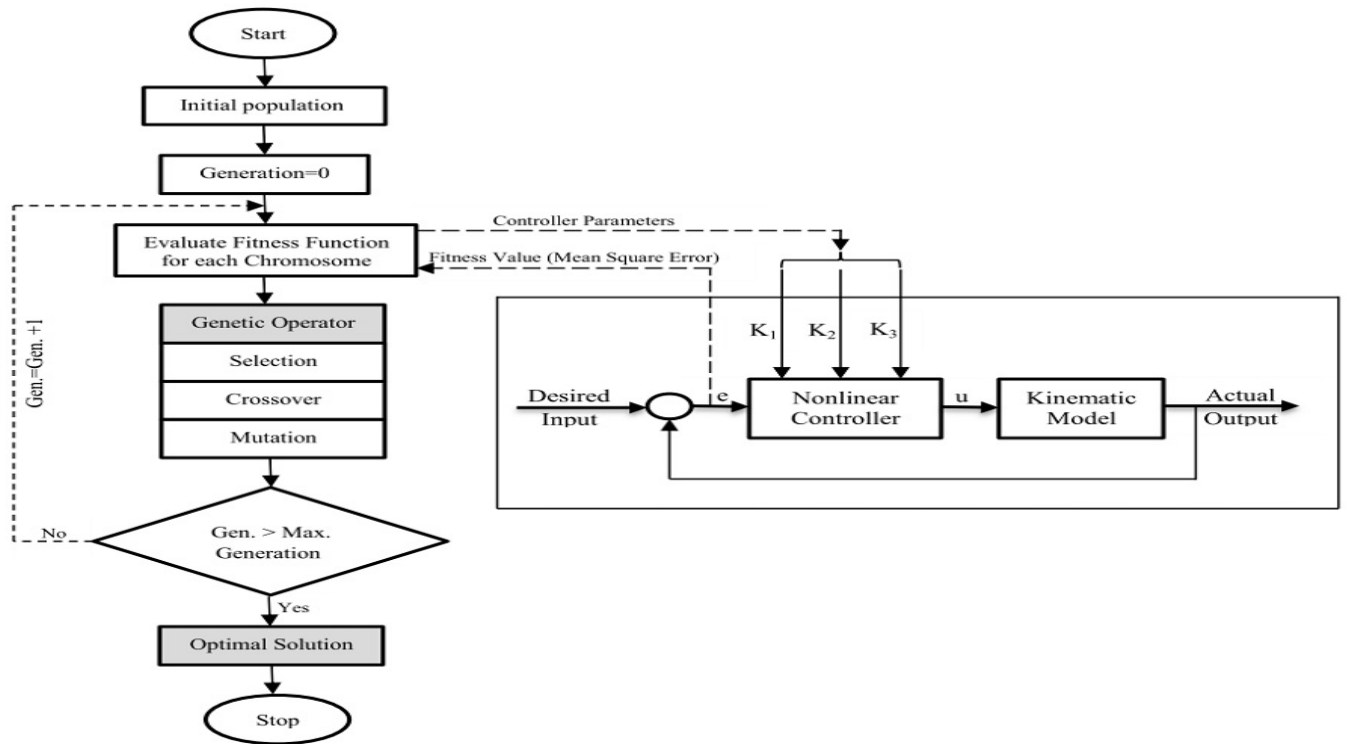


Fig. 5. Genetic Algorithm tuning procedure for the controller Parameters.

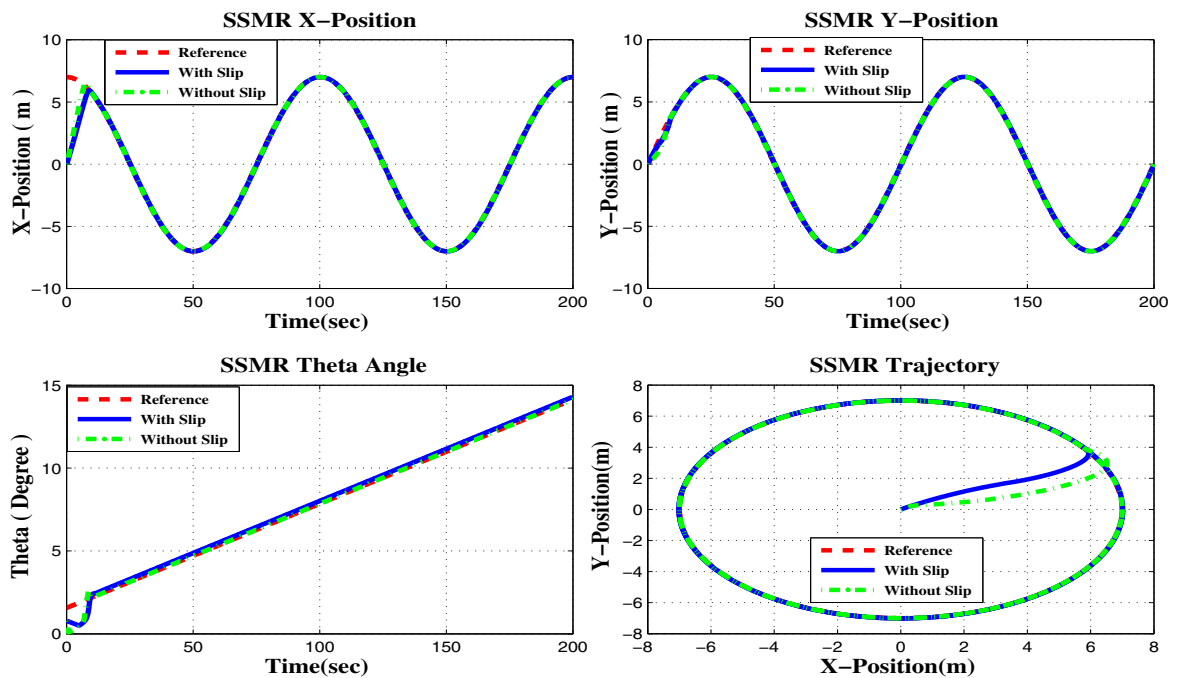


Fig. 6. Genetic nonlinear controller response for No-Slip and Slip cases

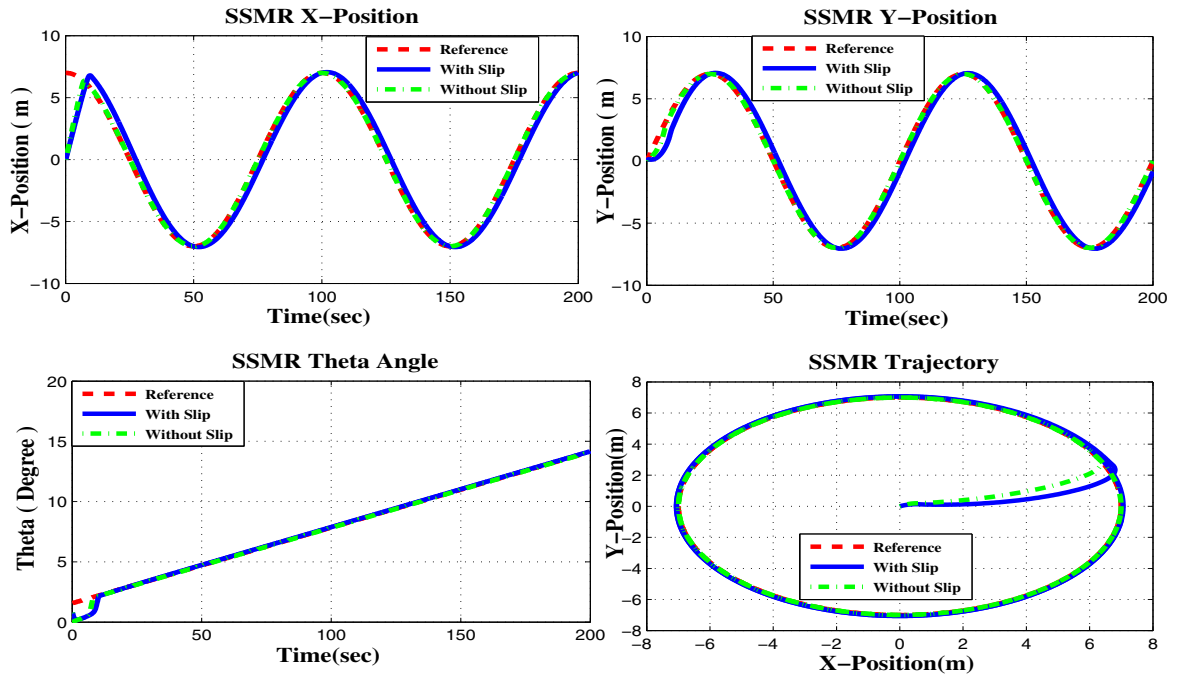


Fig. 7. Trial and error nonlinear controller response for No-Slip and Slip cases

TABLE II  
MEAN SQUARE ERROR COMPARISON

Case	Reference Input	Trial and Error Controller	Genetic Tuned Controller
No-Slip Case	$X - Position$	0.5774	0.5769
	$Y - Position$	0.0159	0.0154
	$\theta - Angle$	0.0735	0.0728
Slip Case	$X - Position$	1.0392	0.9428
	$Y - Position$	0.2623	0.1172
	$\theta - Angle$	0.3985	0.3205

controller to be used with the dynamical models of SSMR. Experimental implementation will be considered as well.

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