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A Novel Modification for a Quadrotor Design

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***Abstract*—Conventional quadrotor UAVs have crucial underactuation limitations. This appears in the coupling between the roll angle and the movement in y-direction, and between the pitch angle and the movement in x-direction. The quadrotor capability of hovering with either roll or pitch angle is restricted due to these limitations. In this paper, a novel design modification is proposed in order to increase the quadrotor degrees of freedom. Four additional rotations for the propellers grant overactuated system. It could be used to make either an oriented hovering without any translational movements or make horizontal movement with zero inclination angle, as followed in this work. A PID controller is used for the quadrotor control. Simulation results show the validity of the proposed control scheme.**

Keywords— quadrotor; quadcopter; quad-tilt-rotor; flying robot

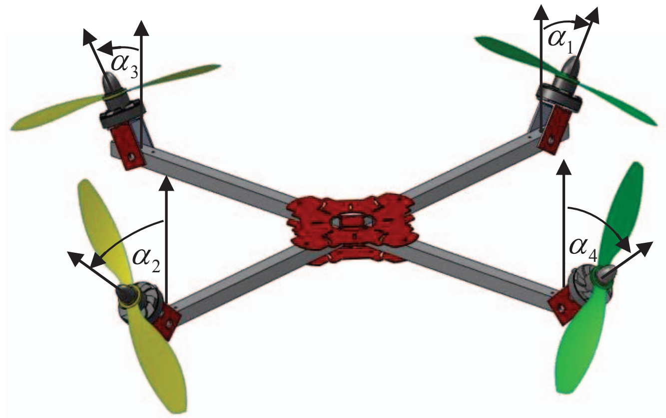
I. Introduction

The quadrotor or quadcopter is a unique type of Unmanned Aerial Vehicles (UAVs) which has Vertical Take Off and Landing (VTOL) ability. The quadrotor has an advantage of maneuverability due to its inherent dynamic nature. While the small size of such UAVs is especially suitable for some applications like surveillance tasks and military purposes. Quadrotors also have potential applications in other areas like earth sciences. Vision systems could be implemented in UAVs which permit covering areas such as object detection and object tracking. Study of quadrotors are interested by researchers due to their capability to perform tasks efficiently and accurately. Recently, focus has shifted to modify the quadrotor design to overcome its existence limitations.

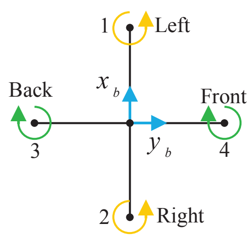
Conventionally, the quadrotor attitude is controlled by changing the rotational speed of each motor. Front and back rotors rotate in a clockwise direction, while right and left rotors rotate in a counter-clockwise direction as in Fig. (1-b). This configuration is devised in order to balance the moment created by each of the spinning rotor pairs, [1] while the yaw angle could be controlled by increasing or decreasing the rotational speed of any pair of rotors. By a similar way, the roll and pitch angles might be controlled. The roll angle (angle about x-axis) will take different values if the rotational speed of front and back propellers are different, associated with translational movement in y-direction. The same coupling between pitch angle (angle about y-axis) and translational movement in x-direction will occur under a speed difference between right and left propellers. This resultant coupling between roll angle and corresponding movement in y-direction, and pitch angle and corresponding movement in x-direction is due to the quadrotor underactuation.

Inherent underactuation of conventional quadrotor is due to the presence of only 4 independent control inputs (the 4 propeller spinning velocities). This arrangement does not allow to independently control the position and orientation of the quadrotor at the same time. For instance, horizontal translation necessarily implies a change in the attitude, symmetrically, a quadrotor can hover in place only when being horizontal w.r.t. the ground plane [2]. In addition, for trajectory tracking problems, only the Cartesian position and yaw angle (4 DOFs) can be independently controlled, while the behavior of the remaining roll and pitch angles (2 DOFs) is completely determined by the trajectory chosen for the former 4 DOFs . As quadrotors are being more and more exploited as autonomous flying service robots, it is important to explore different actuation strategies that can overcome the aforementioned underactuation problem and allow for full motion/force control in all directions in space [3].

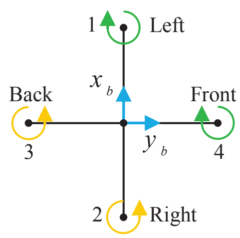
A concept for a quadrotor UAV with actuated tilting propellers, i.e., with propellers able to rotate around the axes connecting them to the main body frame is introduced in [1], [2], [3] and [4] as an approach to overcome the underactuation property. The approach proposed by Ryll et al. [4] is practically validated through making simple hovering on spot and also eight shape trajectory tracking problem. While a PD controller is used in [1] to track trajectories for the three Cartesian coordinates plus the pitch or roll angle one at a time. Both, the former mentioned tilting concept together with another tilting about the axes perpendicular to the arms represent an extension to the idea of overactuated quadrotors [5][6][7][8]. Simulations are performed in [8] for attitude control, using PID controller, and compared with the conventional model response showing that these modifications led to better performance. Gasco et al. in [5] and [6] implemented this concept for the four rotors and validated it through experiments on a rig using ball joint. These experiments were to examine performed was for horizontal stability, disturbance eliminating and reference angle track. A PD controller has been tuned and used for simple flight test which implies take off, hovering and landing.



(a)



(b)



(c)

Fig.1 The proposed modification on the quadrotor design; (a) the new tilt angles (b) the conventional directions of rotation (c) the recent directions of rotation

This paper proposes a novel actuation concept for a quadrotor UAV in which the (usually fixed) propellers are allowed to tilt about the axes perpendicular to the arms, see Fig. (1-a). This configuration involves four additional motors (one for each propeller). For the proposed novel design, it's needed to make a slight change in the conventional configuration which maintains the moment balanced. This different configuration is to make front and left propellers rotate clockwise while back and right ones rotate counterclockwise as in Fig. (1-c). The purpose of this change will be shown later.

The paper is organized as follows: section II will present the mathematical model of the quadrotor's conventional design and the modified one. Simplification of the modified quadrotor model and control system design is introduced in section III. Simulation results and discussion are presented in section IV. Conclusions and recommendations for future work are given in section V.

II. MATHEMATICAL MODEL

A. Conventional Model

In this section, the mathematical model of the conventional quadrotor without any tilting in its rotors will be introduced.

One of the effective parameters in the final form of the model is the choice of the rotational matrix between the body frame and the earth frame. The differences between the rotational matrices is the order of rotations considered to form the final matrix. Considering the rotation about the thrust axis as a first rotation, the final form of the model will be simple to some extent. Then the rotations considered will be $R_{x\to y\to z}$ to which is referred as roll-pitch-yaw rotation. This matrix is presented as following, noting that all the symbols used in this paper are found in Table 5 in the Appendix.

$$R_{body}^{earth} = R_{zyx} = R_{z,\psi}.R_{y,\theta}.R_{x,\varphi}$$
$$= \begin{bmatrix} c\psi c\theta & c\psi s\theta s\varphi - s\psi c\varphi & c\psi s\theta c\varphi + s\psi s\varphi \\ s\psi c\theta & s\psi s\theta s\varphi + c\psi c\varphi & s\psi s\theta c\varphi - c\psi s\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} \quad (1)$$

However the quadrotor has four inputs, namely, the four rotational speeds of the four rotors, alternative four virtual inputs are used to drive the system. These inputs are the total thrust force, the roll moment, the pitch moment and the yaw moment. These virtual inputs are described in the set of (2).

$$\left.\begin{aligned} u_1 &= b(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2) \\ u_2 &= b(\omega_3^2-\omega_4^2) \\ u_3 &= b(\omega_1^2-\omega_2^2) \\ u_4 &= d(-\omega_1^2+\omega_2^2+\omega_3^2-\omega_4^2) \end{aligned}\right\} \quad (2)$$

Where the rotational direction are defined in Fig. 1 as previously stated in the introduction.

Using Euler's method (represented in (3)) a set of differential equations describe the rotational motion could be obtained. The reason behind deriving the rotational equations of motion in the body frame and not in the inertial frame, is to have the inertia matrix independent on time [10].

$$I\ddot{\Omega}+\dot{\Omega}\times I\dot{\Omega} = M - M_G \quad (3)$$

$$\left.\begin{aligned} \ddot{\varphi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y-I_z}{I_x}\right) - \frac{I_R}{I_x}\dot{\theta}g_\omega + \frac{L}{I_x}u_2 \\ \ddot{\theta} &= \dot{\varphi}\dot{\psi}\left(\frac{I_z-I_x}{I_y}\right) + \frac{I_R}{I_y}\dot{\varphi}g_\omega + \frac{L}{I_y}u_3 \\ \ddot{\psi} &= \dot{\theta}\dot{\varphi}\left(\frac{I_x-I_y}{I_z}\right) + \frac{1}{I_z}u_4 \end{aligned}\right\} \quad (4)$$

Where inertia matrix I, the rotor inertia I_R, the vector M that describes the torque applied to the vehicle's body, the vector M_G of the gyroscopic torques.

Newton's second law of motion (represented in (5)) is used to derive the translational model of the quadrotor.

$$F_{ext.} = m\frac{dV_e}{dt} = m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} \quad (5)$$

$$\left.\begin{aligned}\ddot{x} &= -(c\psi s\theta c\varphi + s\psi s\varphi).\frac{u_1}{m}\\ \ddot{y} &= -(s\psi s\theta c\varphi - c\psi s\varphi).\frac{u_1}{m}\\ \ddot{z} &= g-(c\theta c\varphi).\frac{u_1}{m}\end{aligned}\right\} \quad (6)$$

Equations (4) and (6) represent the mathematical model of the conventional design of the quadrotor. More details about the conventional mathematical model derivation are given in [10], [11], [13] and [12]. In the next section, the modification supposed in this paper and the associated mathematical model will be described.

B. Modified Model

The main modification of the quadrotor/quadcopter model is to permit every rotor to make tilt angle about the axes perpendicular to its arm. Noting that the rotational speed directions taken as shown in Fig. 2. Additional frames should be considered together with the main two frames, namely, the body and the earth frame. As a certain result from these additional frames, each frame should has its own rotation matrix to the body frame and the earth frame.

As shown in Fig. 2 all the additional frames have the same orientation of the body frame. Thus the new rotation matrices will be standard rotation matrices about y-axis for the left and right propellers and about x-axis for the front and back propellers. The following equations represent the four additional rotation matrices as a function of the tilted angle (α_i).

$$R_{n1}^{b} = R_{y,\alpha_1} = R_{y_1} = \begin{bmatrix} c\alpha_1 & 0 & s\alpha_1 \\ 0 & 1 & 0 \\ -s\alpha_1 & 0 & c\alpha_1 \end{bmatrix}$$

$$R_{n2}^{b} = R_{y,\alpha_2} = R_{y_2} = \begin{bmatrix} c\alpha_2 & 0 & s\alpha_2 \\ 0 & 1 & 0 \\ -s\alpha_2 & 0 & c\alpha_2 \end{bmatrix}$$

$$R_{n3}^{b} = R_{y,\alpha_3} = R_{x_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_3 & -s\alpha_3 \\ 0 & s\alpha_3 & c\alpha_3 \end{bmatrix}$$

$$R_{n4}^{b} = R_{y,\alpha_4} = R_{y_4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_4 & -s\alpha_4 \\ 0 & s\alpha_4 & c\alpha_4 \end{bmatrix}$$

Using these matrices the thrust force and moment vectors can be calculated in the body fixed frame as following:

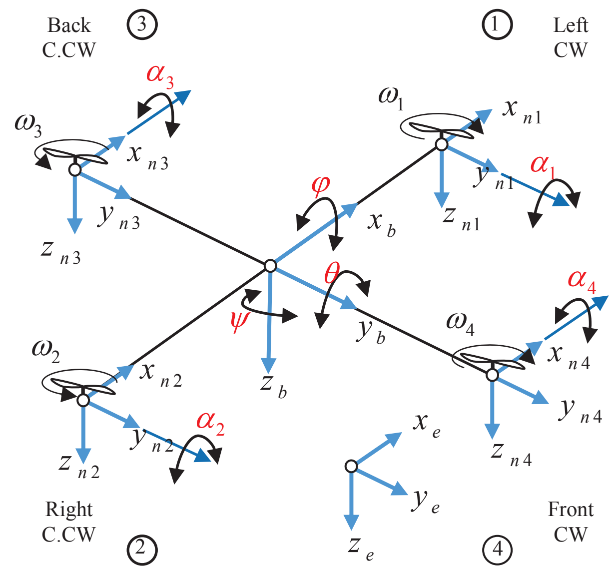


Fig. 2 Quad-Tilt-Rotor with earth, body and additional new frames

a) The thrust force vectors:

$$\left.\begin{aligned} F_{th_1}^{b} &= R_{y_1}.F_{th_1} = \begin{bmatrix} -b\,\omega_1^2 s\alpha_1 \\ 0 \\ -b\,\omega_1^2 c\alpha_1 \end{bmatrix}\\ F_{th_2}^{b} &= R_{y_2}.F_{th_2} = \begin{bmatrix} -b\,\omega_2^2 s\alpha_2 \\ 0 \\ -b\,\omega_2^2 c\alpha_2 \end{bmatrix}\\ F_{th_3}^{b} &= R_{x_3}.F_{th_3} = \begin{bmatrix} 0 \\ b\,\omega_3^2 s\alpha_3 \\ -b\,\omega_3^2 c\alpha_3 \end{bmatrix}\\ F_{th_4}^{b} &= R_{x_4}.F_{th_4} = \begin{bmatrix} 0 \\ b\,\omega_4^2 s\alpha_4 \\ -b\,\omega_4^2 c\alpha_4 \end{bmatrix} \end{aligned}\right\} \quad (7)$$

b) The moment vectors:

$$\left.\begin{aligned} M_1^{b} &= R_{y_1}.M_1 = \begin{bmatrix} -d\,\omega_1^2 s\alpha_1 \\ 0 \\ -d\,\omega_1^2 c\alpha_1 \end{bmatrix}\\ M_2^{b} &= R_{y_2}.M_2 = \begin{bmatrix} d\,\omega_2^2 s\alpha_2 \\ 0 \\ d\,\omega_2^2 c\alpha_2 \end{bmatrix}\\ M_3^{b} &= R_{x_3}.M_3 = \begin{bmatrix} 0 \\ -d\,\omega_3^2 s\alpha_3 \\ d\,\omega_3^2 c\alpha_3 \end{bmatrix}\\ M_4^{b} &= R_{x_4}.M_4 = \begin{bmatrix} 0 \\ d\,\omega_4^2 s\alpha_4 \\ -d\,\omega_4^2 c\alpha_4 \end{bmatrix} \end{aligned}\right\} \quad (8)$$

From the thrust force vectors in (7) and moment vectors (8), it can be observed that the virtual inputs to the system will take a different form as following:

$$\left.\begin{aligned} u_1 &= b\omega_1^2 c\alpha_1 + b\omega_2^2 c\alpha_2 \\ &\quad + b\omega_3^2 c\alpha_3 + b\omega_4^2 c\alpha_4 \\ u_2 &= b\omega_3^2 c\alpha_3 - b\omega_4^2 c\alpha_4 \\ &\quad - \frac{d}{L}\omega_1^2 s\alpha_1 + \frac{d}{L}\omega_2^2 s\alpha_2 \\ u_3 &= b\omega_1^2 c\alpha_1 - b\omega_2^2 c\alpha_2 \\ &\quad - \frac{d}{L}\omega_3^2 s\alpha_3 + \frac{d}{L}\omega_4^2 s\alpha_4 \\ u_4 &= -d\omega_1^2 c\alpha_1 + d\omega_2^2 c\alpha_2 \\ &\quad + d\omega_3^2 c\alpha_3 - d\omega_4^2 c\alpha_4 \end{aligned}\right\} \quad (9)$$

Again, by using the Euler and Newton equations the final form of the model will be given as:

$$\ddot{\varphi} = \dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} + \frac{I_R}{I_x}(-\dot{\theta}\, g_{\omega 3} + \dot{\psi}\, g_{\omega 2}) + \frac{L}{I_x}u_2 \quad (10)$$

$$\ddot{\theta} = \dot{\theta}\dot{\psi}\frac{(I_z - I_x)}{I_y} + \frac{I_R}{I_y}(-\dot{\varphi}\, g_{\omega 3} + \dot{\psi}\, g_{\omega 1}) + \frac{L}{I_y}u_3 \quad (11)$$

$$\ddot{\psi} = \dot{\varphi}\dot{\theta}\frac{(I_x - I_y)}{I_z} + \frac{I_R}{I_z}(-\dot{\varphi}\, g_{\omega 2} + \dot{\theta}\, g_{\omega 1}) + \frac{u_4}{I_z} \quad (12)$$

$$\begin{aligned} m\ddot{x} &= -b\omega_1^2(s\alpha_1 c\psi c\theta) - b\omega_2^2(s\alpha_2 c\psi c\theta) \\ &\quad + b\omega_3^2(s\alpha_3 c\psi s\theta s\varphi) - b\omega_3^2(s\alpha_3 s\psi c\varphi) \\ &\quad + b\omega_4^2(s\alpha_4 c\psi s\theta s\varphi) - b\omega_4^2(s\alpha_4 s\psi c\varphi) \\ &\quad + (-c\psi s\theta c\varphi - s\psi s\varphi)u_1 \end{aligned} \quad (13)$$

$$\begin{aligned} m\ddot{y} &= -b\omega_1^2(s\alpha_1 s\psi c\theta) - b\omega_2^2(s\alpha_2 s\psi c\theta) \\ &\quad + b\omega_3^2(s\alpha_3 s\psi s\theta s\varphi) + b\omega_3^2(s\alpha_3 c\psi c\varphi) \\ &\quad + b\omega_4^2(s\alpha_4 s\psi s\theta s\varphi) + b\omega_4^2(s\alpha_4 c\psi c\varphi) \\ &\quad + (-s\psi s\theta c\varphi + c\psi s\varphi)u_1 \end{aligned} \quad (14)$$

$$\begin{aligned} m\ddot{z} &= mg + b\omega_1^2(s\alpha_1 s\theta) + b\omega_2^2(s\alpha_2 s\theta) \\ &\quad + b\omega_3^2(s\alpha_3 c\theta s\varphi) + \mathrm{b}\,\omega_4^2(s\alpha_4 c\theta s\varphi) \\ &\quad + (-c\theta c\varphi)u_1 \end{aligned} \quad (15)$$

While:

$$g_{\omega 1} = (\omega_1 s\alpha_1 - \omega_2 s\alpha_2)$$

$$g_{\omega 2} = (\omega_3 s\alpha_3 - \omega_4 s\alpha_4)$$

$$g_{\omega 3} = (\omega_1 c\alpha_1 - \omega_2 c\alpha_2 - \omega_3 c\alpha_3 + \omega_4 c\alpha_4)$$

This design modification grants a total of 8 control inputs (4 propeller spinning velocities and 4 tilting angle), and makes it possible to obtain complete controllability over the main body 6 DOFs configuration thus rendering the quadrotor UAV an over actuated flying vehicle [3].

C. *Modified Model Validation*

To ensure that the novel mathematical model is valid, it's recommended to put the four new tilting angles, *i.e.* $\alpha_{1\to 4}$ with zero in (9) to (15). That will produce the same conventional model which described before in (2), (4) and (6).

III. CONTROL SYSTEM DESIGN

Tilt-design makes the dynamics of the quadcopter more complex, and introduces additional challenges in the control design. However, tilting rotor quadcopter, designed by using additional four servo motors that allow the rotors to tilt, is an overactuated system that potentially can track an arbitrary trajectory over time [1].

A. *Simplified Model*

One of the benefits of this modification is to increase the ability of the quadrotor to make roll or pitch angle while hovering in the air and also to move horizontally without inclination angle. These tasks will be discussed in the next sections. Noting that the conventional quadrotor was not able to achieve those tasks.

However, some simplifications are needed in order to transform it into a model suited for control design. First, as in many practical situations, we assume that the motors actuating the tilting/spinning axes are implementing a fast high-gain local controller able to impose desired values with negligible transient dynamics. This allows us to neglect the motor dynamics [2]. While other simplifications will be done in the mathematical model to get two cases from it. One case is to implement rolled hovering, i.e., make the craft hover while tilting its body by desired roll angle and with no any motion in y-axis. And the other case is to achieve horizontal motion in y-axis without any inclination in the quadrotor body about x-axis.

<u>*Case (1):*</u> This case discusses hovering with roll angle.
The following procedure explain how to implement this case:

- The rotors-1, 2 will be used to produce a torque that control the roll angle, their tilt angles will be equal in their values but have a different signs, and produced by the controller. According to the direction of the tilting, the direction of the moment will be determined as in Fig. 3.
- The other two rotors tilt angles should have the same value which is varying according to the relation in (18) to prevent the quadrotor moving in y-direction as the horizontal components of the forces will be balanced as shown in Fig. 4, $(\alpha_3 = \alpha_4 = \alpha_{34})$.
- To prevent the craft from making yaw angle about z-axis it's needed to maintain the rotational speeds of all rotors the same $(\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega)$.

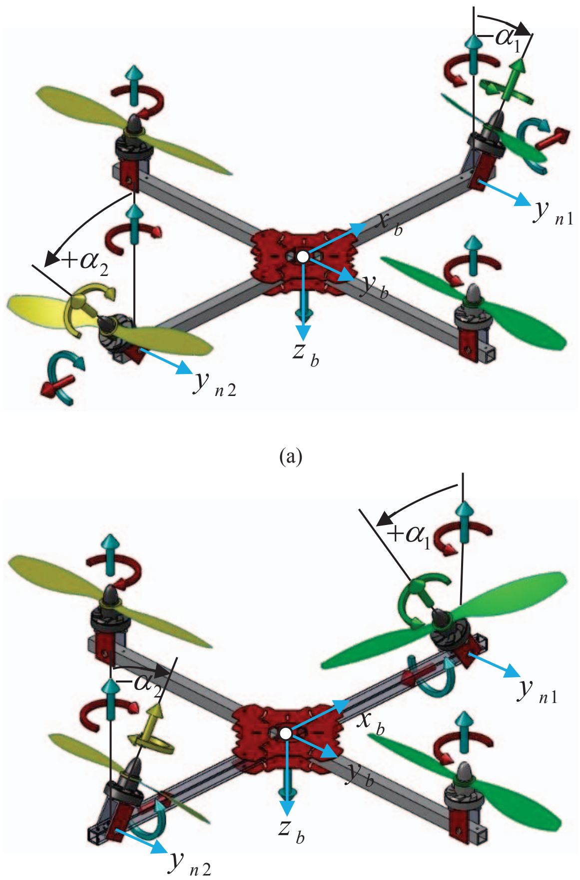


Fig. 3 Configuration shows how the motors produce the moments about x-axis in any direction

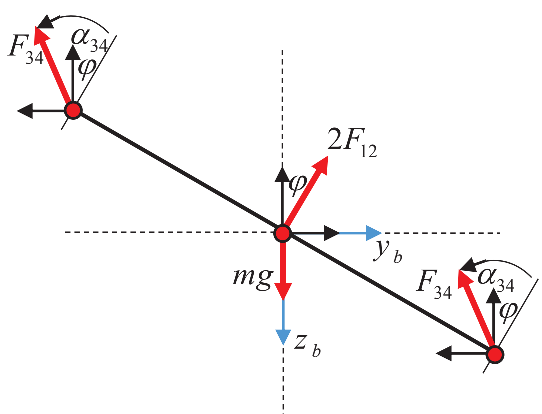


Fig. 4 Free body diagram for the tilted hovering mode with roll angle, x-axis is perpendicular to the paper and inward

Table 1 summarizes all the values used in the mathematical model to achieve the first case which is the rolled hovering.

Substituting with the values given in Table 1, the virtual inputs relations, given in (9), become:

$$\left.\begin{aligned} & u_1 = 2b\omega^2(c\alpha_{12} + c\alpha_{34}) \\ & u_2 = -\frac{2d}{L}\omega^2 s\alpha_{12} \\ & u_3 = 0 \qquad , \qquad u_4 = 0 \end{aligned}\right\} \quad (16)$$

Considering both the new relations of the virtual inputs and the values from Table 1, a simple form of the quadrotor model namely, (10) to (15), could be written in the form:

$$\left.\begin{aligned} & \ddot{\varphi} = \frac{L}{I_x} u_2 \qquad , \qquad \ddot{\theta} = 0 \\ & \ddot{\psi} = 0 \qquad , \qquad \ddot{x} = 0 \\ & \ddot{y} = \frac{2b\omega^2}{m}(c\alpha_{12} s\varphi + s(\alpha_{34} + \varphi)) \\ & \ddot{z} = g - \frac{2b}{m}\omega_{34}^2 \end{aligned}\right\} \quad (17)$$

The tilt angle constrain which obtained by equating the acceleration in y-axis , (17), to zero is:

$$\alpha_{34} = -\varphi + s^{-1}(-c\alpha_{12}.s\varphi) \quad (18)$$

<u>*Case (2):*</u> This case discusses the horizontal movement in y-direction without an inclination angle.

The procedure for controlling the y-position without any inclination angle are:

- Rotors-3, 4 will be used to produce forces along y-axis, see Fig. 5, their tilt angles are produced from the controller and have the same value $(\alpha_3 = \alpha_4 = \alpha_{34})$.
- The other two rotors should preserved vertical to avoid generating any moments or forces in x-direction, their tilt angles' values are equal to zero $(\alpha_1 = \alpha_2 = 0)$.
- To restrict the quadrotor from rotating about z-axis, *i.e.* making yaw angle, the rotational speeds of the rotors-1, 2 should have the same value $(\omega_1 = \omega_2 = \omega_{12})$ and the speeds of the rotors-3, 4 also should have the same value $(\omega_3 = \omega_4 = \omega_h)$; where ω_h denotes to the hovering speed : $\omega_h = \sqrt{\frac{mg}{4b}}$

Assumptions of this case are given in Table 2, while Fig. 6 illustrate this case.

TABLE 1. THE SIMPLIFICATIONS USED FOR THE ROLL CONTROL CASE

Variable	Constrained Value	Variable	Constrained Value
ω_1	ω	α_1	α_{12}
ω_2		α_2	$-\alpha_{12}$
ω_3		α_3	α_{34}
ω_4		α_4	

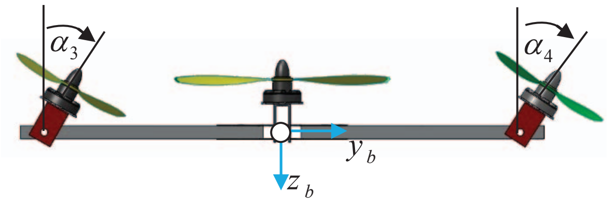


Fig. 5 Illustration of the horizontal moving mode

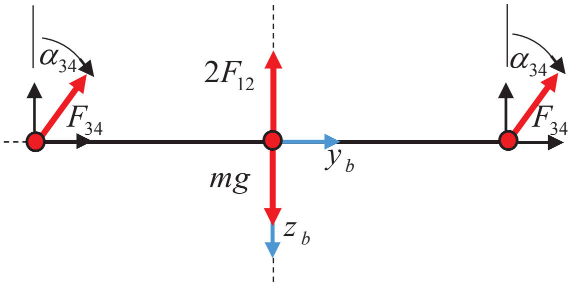


Fig. 6 the horizontal moving toward y-direction without inclination angle, x-axis is perpendicular to the paper and inward

TABLE 2. THE SIMPLIFICATIONS USED FOR THE Y-POSITION CONTROL CASE

Variable	Constrained Value	Variable	Constrained Value
ω_1	ω_{12}	α_1	0
ω_2		α_2	
ω_3	ω_h	α_3	α_{34}
ω_4		α_4	

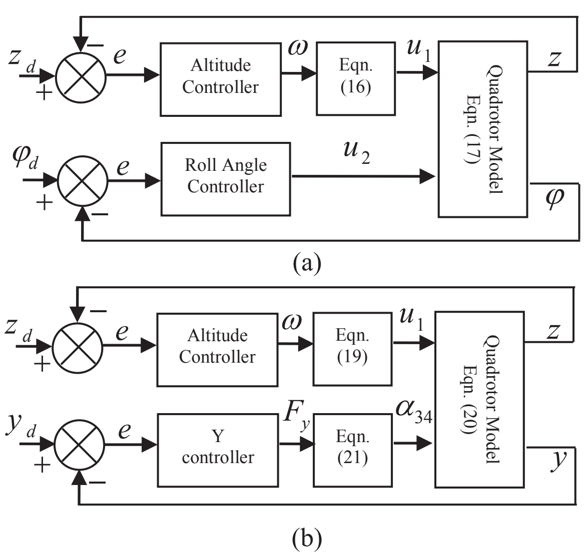


Fig. 7 The control scheme used for : (a) case 1, (b) case 2

The virtual inputs are given by:

$$\left.\begin{array}{lll} u_1 = 2b(\omega_{12}^2 + \omega_{34}^2 c\alpha_{34}) & , & u_2 = 0 \\ u_3 = 0 & , & u_4 = 0 \end{array}\right\} \quad (19)$$

The simplified model yields to:

$$\left.\begin{array}{lll} \ddot{\varphi} = 0 & , & \ddot{\theta} = 0 \\ \ddot{\psi} = 0 & , & \ddot{x} = 0 \\ \ddot{y} = \dfrac{2b}{m}\omega_{34}^2 s\alpha_{34} & & \\ \ddot{z} = g - \dfrac{2b}{m}(\omega_{12}^2 + \omega_{34}^2 c\alpha_{34}) & & \end{array}\right\} \quad (20)$$

B. Control System

The prototype is capable of two flight modalities: a fixed configuration in which it essentially behaves as a standard underactuated quadrotor, and a variable tilt angle configuration which guarantees some degree of full actuation. This control system is designed for the second mode. The control technique used in this paper is a PID control. The inputs to the controller will be the error in the roll angle and the error in the altitude in the first case, and the error in the y-position and the altitude in the second case. For every case there are two controllers, one for the altitude control and the other for the roll angle / y-position according to the relevant case.

In the first case, the roll angle controller, the output of the controller is the corresponding control action, i.e.,u_2 for the roll angle case (φ). While for the second controller the output is the rotational speed of the motors from which the control action u_1 can be calculated. Fig. (7-a) shows the control system for the roll case. Noting that in this case the tilt angle (α_{34}) should calculated from (18). In the second case, the y-position controller produces the force in y-direction (21) as a control action then this force used to calculate the tilt angle (α_{34}) as shown in Fig. (7-b).

From (7) the forces in y-direction is:

$$F_y = 2b\,\omega_h^2 s\alpha_{34} \quad (21)$$

IV. Results

The performance of the designed controller for the proposed modifications of the quadrotor is evaluated and discussed in this section. The reference altitude is one meter following the profile in Fig. (8-a) and the reference values of the roll angle are 10° and 30° and their profiles are shown in Fig. (8-b). For case (2), the same reference is used for the altitude and the reference values for the y-position are 1 and 3 meters as shown in Fig. (8-c). These profiles are 5^{th} order polynomials which satisfy the following conditions:

<u>*For the altitude:*</u>

$$t = 0 \rightarrow z = 0 \quad , \quad \dot{z} = \ddot{z} = 0$$
$$t = 3 \rightarrow z = 1 \quad , \quad \dot{z} = \ddot{z} = 0$$

<u>*For the roll angle, case (1):*</u>

$$t = 12 \rightarrow \varphi = 0 \quad , \quad \dot{\varphi} = \ddot{\varphi} = 0$$
$$t = 15 \rightarrow \varphi = 10°, 30° \quad , \quad \dot{\varphi} = \ddot{\varphi} = 0$$

TABLE 3. THE SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
m	$1.25kg$	L	$0.2m$
b	2.92×10^{-6}	d	1.12×10^{-7}
I_x	0.002353	I_y	0.002353
I_z	0.004706	I_R	2×10^{-5}

TABLE 4. THE GAINS OF THE CONTROLLERS USED IN THE SIMULATION

		K_p	K_i	K_d
Case(1)	**Altitude Controller**	2500	1300	500
	Angle Controller	20	2	0.7
Case(2)	**Altitude Controller**	1500	1300	300
	Y-position Controller	700	1300	500

For the y-position, case (2):

$$t=12\rightarrow y=0 \quad , \quad \dot{y}=\ddot{y}=0$$
$$t=15\rightarrow y=1,3 \quad , \quad \dot{y}=\ddot{y}=0$$

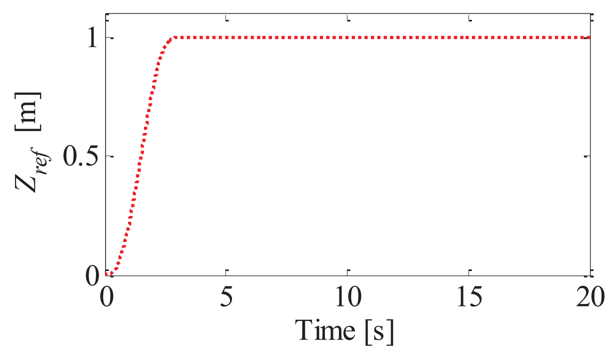
The parameters used in the simulation are given in Table 3. Also the parameters used in the PID controllers are found in Table 4.

A. *Simulation results of case (1):*

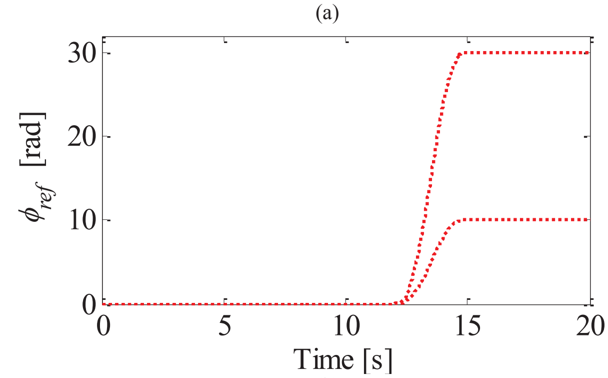
Figure 9 presents the simulation results for the first case. Figure (9-a) shows the roll angle response for both trajectories while Fig. (9-b) shows the altitude response with settling time 5.67 s calculated when the output reaches to $(\pm2\%)$ of the desired value. Figure (9-c) proves the decoupling between the roll angle and displacement in y-direction where there is no motion in y-direction, although the roll angle has a nonzero value. The other state variables of the quadrotor are maintained at zero value. Figure (10-a) shows the change of α_1 for both trajectories of the roll angle (ϕ). The trend of α_2 change will be the same as α_1 but in opposite direction, see Fig. 2. The tilt angle α_{34} is realized in Fig. (10-b) where it settled at the value $(-2\varphi_{des.})$, relation (21), just when the value of (α_{12}) went back to zero, the same result might be obtained from (21) when (α_{12}) equals to zero. Referring to Fig. 4, as the roll angle increases, the need for more thrust force increases to make its vertical component opposes the weight, this is obvious in Fig. (10-c). Because of using I_x and I_y with the same values, the performance with desired pitch angle instead of roll one will be the same and did not mentioned here to avoid redundancy.

B. *Simulation results of case (2):*

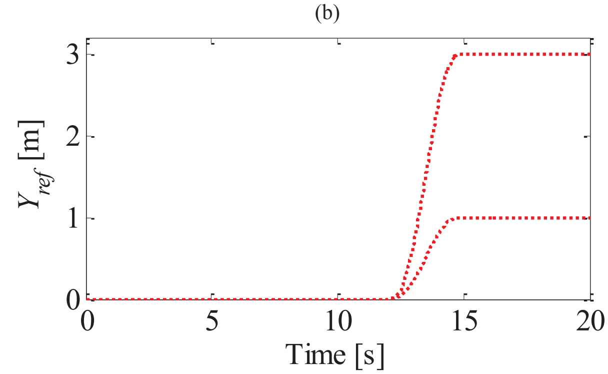
Figure 11 presents the simulation results for the second case. Figure (11-a) shows the y-position response for both trajectories while Fig. (11-b) shows the altitude response with settling time 4.12 s. Figure (11-c) proves again the decoupling between the roll angle and displacement in y-direction. The change of the tilt angles of rotors-3, 4 is given in Fig. (12-a), where both angles have the same value (α_{34}) and direction. As the desired profile for y-position starts at 12 s the tilt angles (α_{34}) also start changing at the same second and then the net vertical thrust forces' components reduced instantaneously which cause some oscillation in altitude at the same time, Fig (11-b). To restore the altitude again, the other two vertical rotors-1, 2 increase their rotational speed and hence their resultant thrust forces to oppose the weight of the quadrotor until the rotors-3, 4 went back vertical, i.e. $\alpha_{34}=0$, see Fig. (12-b).



(a)



(b)



(c)

Fig. 8 The reference profiles used in the simulation

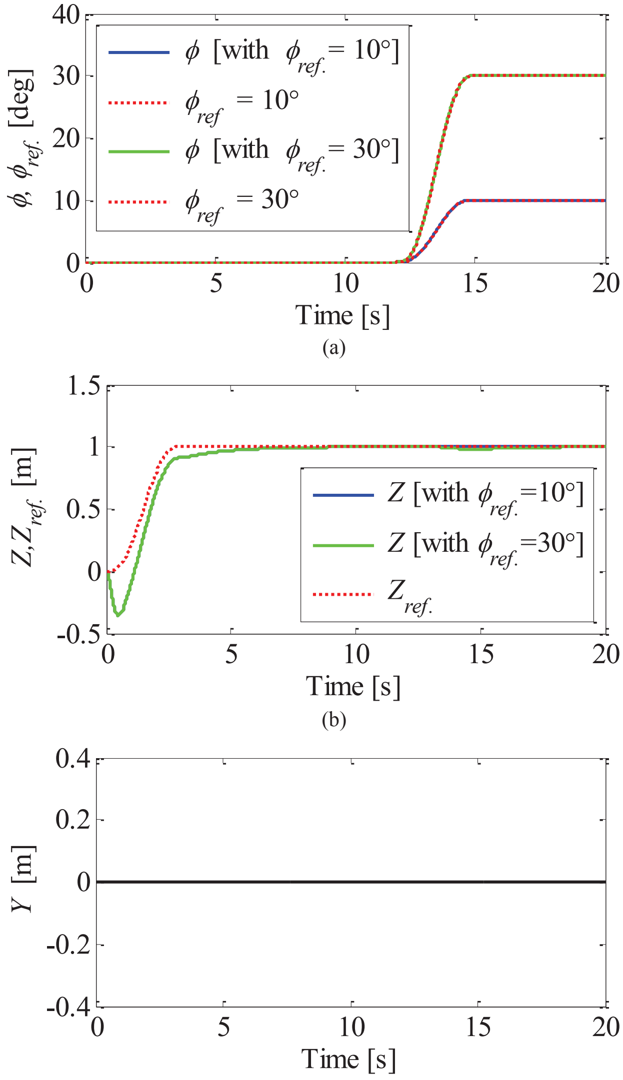


Fig. 9 The response of the controlled variables in *case (1)* " the rolled hovering "; (a) roll angle response, (b) altitude response (c) y-position response

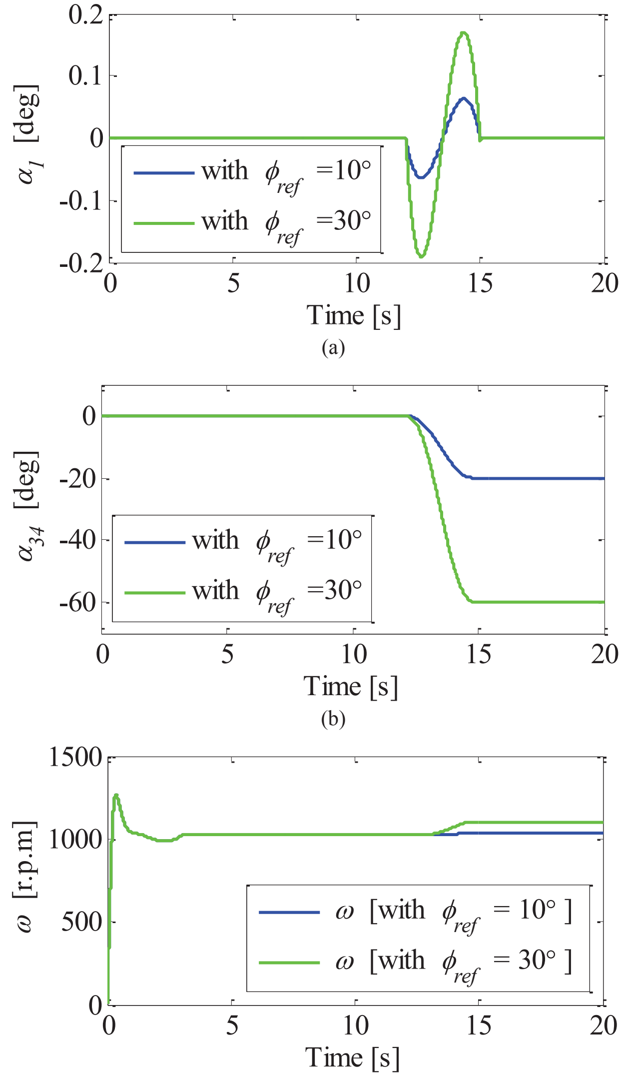


Fig. 10 The response of the other variables in the system in *case (1)*; (a) the tilt angle at rotor 1 and 2, (b) the tilt angle at rotors-3, 4, (c) the rotational speed at the four rotors

V. CONCLUSIONS

In this paper a novel design modification of the quadrotor is introduced to treat its underactuation property and decouple both the roll angle from the y-translation and the pitch angle from the x-translation. This is achieved by adding four additional rotations for the propellers. The designed controller for the proposed modification of the quadrotor achieved tilted hovering without any translational motion. It also succeeded in moving the quadrotor horizontally without any inclination. Simulation results of the designed controller showed satisfactory performance.

The research trend now is to realize these results practically and using this novel modification to track other arbitrary trajectory.

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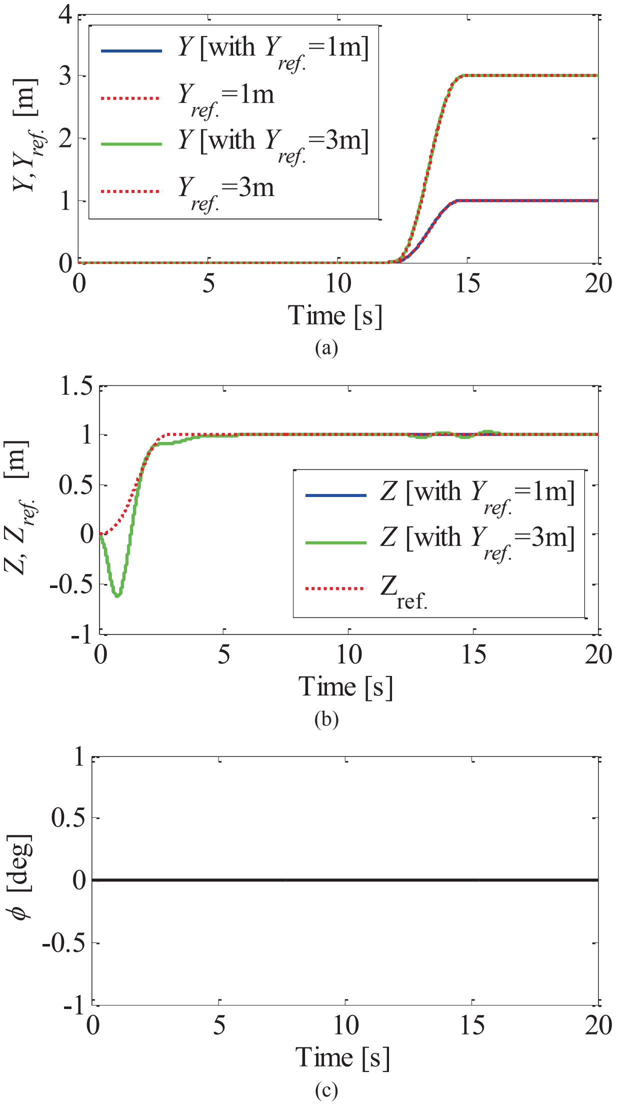

Fig. 11 The response of the controlled variables in *case (2)* " the horizontal moving in y-axis "; (a) y-position response, (b) altitude response (c) roll angle response

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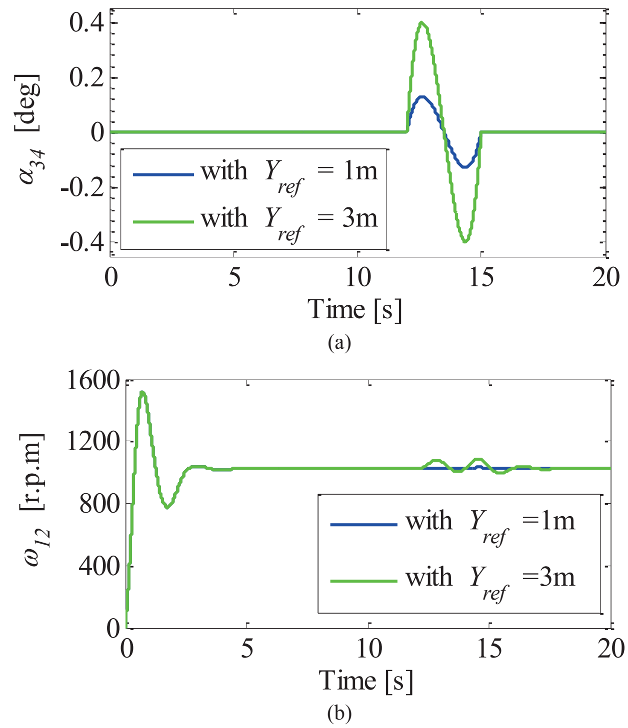

Fig. 12 The response of the other variables in the system in Case (2); (a) the tilt angle at rotors-3, 4 (b) the rotational speeds at the rotors-1, 2.

APPENDIX

TABLE 5. Nomenclature

α	Tilt angle
x_b	X-axis in the body frame
y_b	Y-axis in the body frame
z_b	Z-axis in the body frame
R	Rotation matrix
φ	Roll angle
θ	Pitch angle
ψ	Yaw angle
u	Virtual input
ω	Rotational speed
b	Thrust coefficient
d	Drag coefficient
F_{th}	Thrust force
$\ddot{x}$	Acceleration in x-direction
$\ddot{y}$	Acceleration in y-direction
$\ddot{z}$	Acceleration in z-direction
m	Mass
t	Time
x_e	X-axis in the earth frame
y_e	Y-axis in the earth frame
z_e	Z-axis in the earth frame
x_n	X-axis in the new frame
y_n	Y-axis in the new frame
z_n	Z-axis in the new frame
g	Gravitational acceleration (9.81 m/s2)
e	Error signal