Adaptive Fuzzy Iterative Learning Controller for X-Y Table Position Control

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Abstract—In this paper, an adaptive fuzzy iterative learning control algorithm is proposed for controlling one of the Mechatronics systems. The proposed control scheme is based upon a proportional-derivative-integral (PID) iterative learning control (ILC), for which a fuzzy control is added to tune the parameters of the PID-type ILC. Moreover, an adaptation law is added to the fuzzy control in order to automatically select the proper fuzzy membership functions. The performance of proposed algorithm was assessed in computer numerical controlled (CNC) machine X-Y table to illustrate the validation and the effectiveness of the proposed procedure. The simulation results show that the proposed algorithm can reduce the trajectory error in a far less number of iterations.

I. INTRODUCTION

In general, computer numerical controlled (CNC) machines are usually used for repetitive tasks, this repetitive nature makes it possible to apply iterative learning control (ILC) scheme to improve the tracking and control performance from iteration to iteration or improves the performance of the system in case of control task repetition. The research and application of ILC is receiving more and more attention with the wide application of repetitive/batch process [1], [2]. These control systems are required to carry out control tasks with high speeds and high precision subject to modeling uncertainty, repeatability imperfections and disturbances [3]. The higher quality of their performance in efficiency and accuracy can increase productivity, improve efficiency enhance product quality, and reduce costs. A strengthened learning control system is the control system which becomes efficient by learning how to fulfill the main central aims expressed as high-performance indices in desired or reference tracking and regulation with respect to load disturbance inputs [4]. During the last twenty years, a great number of ILC algorithms have been developed [5]. ILC has become a framework including many varieties of control approach. Each one has its merits in terms of performance such as convergence, robustness, learning speed, or suitability for special plants [6]–[8]. The main reason why ILC has attracted considerable research efforts is its simplicity, potential effectiveness and less information about the system to be controlled is needed [9]. The combination of fuzzy control and the ILC scheme is aimed to improve the performance of fuzzy control systems by alleviation of the disadvantages and make use of the advantages of both ILC and fuzzy controllers by using feedback control and feed-forward compensation benefits [10]. This research is based on the ideas presented in [11] and [12].

In this paper, an adaptive fuzzy iterative learning controller is proposed which makes use of fuzzy logic control to enhance the performance of the ILC scheme for controlling an X-Y table. In this scheme, a fuzzy system is used to update the ILC scheme parameters to be self-tuned and can be changed according to the changes in the system dynamics. Moreover, an adaptive algorithm is added to the fuzzy controller in order to make the fuzzy system automatically select the membership functions of the controller rather than selecting them by the operator because this selection depends upon his experience about the controlled process and may mismatch in case of dynamic changes that can occur in the controlled process.

The rest of this paper is organized as follows: Some notations and preliminaries on the structure of ILC scheme are presented in section II. In section III, CNC machine X-Y table model used for simulation is presented with highlighting of some nonlinearities that may occur in the system such as deadzone and saturation. In section IV, a focus on adaptive fuzzy control with ILC algorithm, adaptation and tuning of the ILC scheme parameters using adaptive fuzzy control are presented. Section V applies the proposed algorithm to an X-Y table and presents simulation results comparison between the proposed adaptive fuzzy iterative learning control (AFILC) algorithm and classical PID-type ILC to show the effectiveness of the proposed algorithm. The conclusion is highlighted in section VI.

II. ILC NOTATIONS AND PRELIMINARIES

Iterative learning control (ILC) can be defined as a control technique that improves the performance of dynamical systems which perform the same tasks repetitively over a finite time interval. ILC emulates the human capability of learning from practice. The basic idea behind ILC is that it uses previous trial information to update and improve the control signal for the next trial in order to improve the controlled dynamical system performance and make the tracking error to be small as possible [13], [14] . This idea can be illustrated in Fig. 1.
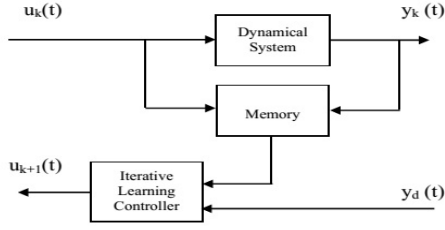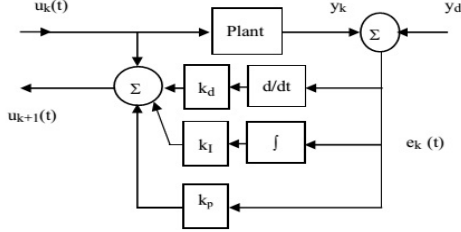
Fig. 1. Iterative learning controller configuration.

Fig. 2. Block of PID-type ILC algorithm.

A. Basic ILC algorithm

For a given dynamical system with input u and output y, there are some postulations for the formulation of ILC problem, that can be described as follows [14], [15] :

1) The process repeatedly performs the same task that ends in a fixed duration ($T > 0$), (T: sample period)
2) The desired output, $y_d(t)$ is pre-defined with $t \in [0, T]$.
3) For each trail (cycle, batch, iteration, repetition) the initial states are the same. In other words, the initial states of the objective process can be set to the same point at the beginning of each iteration.
4) The plant output, $y_k(t)$, is observable.

B. PID-type ILC algorithm

The goal of the ILC is to find an iterative form of learning control law such that the next iteration control signal is a function of the previous iteration control signal and the error signal such that the tracking objective $e_k(t) \to 0$ as $k \to \infty$ is achieved . Under the above assumptions, if the dynamical system relative degree is one, the PID-type ILC scheme derived from the original ILC scheme of the "Arimoto D- type" [7] is given by Eq. (1) and can be illustrated in the block diagram shown in Fig. 2.

$$u_{k+1} = u_k + K_p e_k(t) + K_I \int_0^t e_k(\tau)d\tau + K_d \frac{d}{dt} e_k(t) \quad (1)$$

where : $u_k(t)$ is the control signal at k^{th} iteration, $e_k(t)$ is the tracking error at k^{th} iteration that defined by $e_k(t) = y_d(t) - y_k(t)$ with $y_d(t)$ the desired output trajectory, $y_k(t)$ the actual system output at k^{th} iteration, and K_p, K_I and K_d are the PID learning gains to be designed. This ILC scheme described by Eq. (1) work properly for system with relative degree one

Fig. 3. X-Y feed system.

such that :

$$\lim_{k \to \infty} y_k(t) \to y_d(t) \quad (2)$$

for all $t \in [0, T]$, if

$$|1 - K_p h_1|_i < 1 \quad (3)$$

where: h_1 is the first Markov parameter. In the state space representation (A, B, C) , $h_1 = CB \neq 0$.

III. CNC MILLING MACHINE X-Y TABLE

In most advanced manufacturing processes, two and/or three dimensional motions are in high demand for industrial applications such as parts assembly, component insertion, machining, etc. A traditional X-Y table often utilizes a rotary motor and connect its output shaft to mechanical translators such as gears or bears to perform linear motion, by the vertical arrangement of two such linear motion implementations, two-dimensional movement is achieved. In a direct-drive system, the mechanical output is directly generated to the actuator and load and it has the characteristics of high force density, high precision and low production cost [16]. In this paper, a linear-drive based X-Y feed table of a high-speed milling machine is used and it is driven by two ETEL iron-core linear motors, one motor in the X-direction and the other motor in the Y-direction as shown in Fig. 3. The system dynamics can be described by two single-input single-output models. The models are obtained through the frequency domain identification based on the measured frequency response functions [17]. The models that relate the control input voltage (u) to the table position (z), with $z = X$ and $z = Y$ for the X and Y axes, respectively, are of second order with a time delay T_d.

$$G(s) = \frac{z(s)}{u(s)} = \frac{B}{s(s+A)} e^{-sT_d} \quad (4)$$

where s is the Laplace operator.

A. Basic Friction Model

Friction is responsible for several servomechanisms problems and their elimination is always a challenge of control engineers. Friction must be considered seriously because it is one of the greatest obstacles in high precision positioning systems. Static friction or kinetic friction models, which are also called steady state friction, compute friction torque as
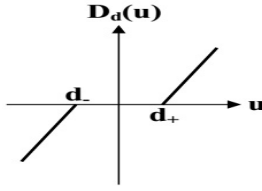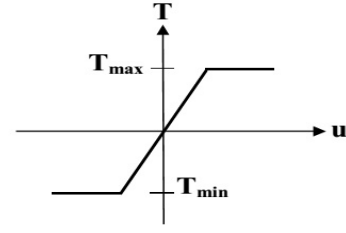
Fig. 4. Nonsymmetric Deadzone Nonlinearity.

Fig. 5. Saturation Nonlinearity.

a function of slowly varying velocity. The classical friction model is described by Eq. (5) [18]:

$$f = f_c sgn(\omega(s)) + b\omega(s) \tag{5}$$

Where: f_c is the coulomb friction level, b is the viscous friction coefficient, $\omega(s)$ is the velocity, and

$$sgn(\omega(s)) = \begin{cases} +1 & \text{if } \omega(s) > 0 \\ 0 & \text{if } \omega(s) = 0 \\ -1 & \text{if } \omega(s) < 0 \end{cases} \tag{6}$$

B. Dead Zone Nonlinearity in DC Motor

Dead zone nonlinearity is a common type of nonlinear characteristic that occurs in many mechanical systems such as DC motors. The ideal representation of dead zone nonlinearity is given in Fig. 4 [19]. For the DC motor, it exhibits this type of nonlinearity when the armature voltage is around zero. When a continuous signal applied to the armature of the driving DC motor goes through zero volts, the system stays motionless for some time. This is a result of the fact that the mechanical system cannot respond immediately to input signal commands when it is at rest. The coulomb friction causes mechanisms to be resistant to move from rest. Such characteristics of coulomb friction form a dead-zone nonlinearity in a DC motor. If u and τ are scalars, the non-symmetric dead-zone nonlinearity, shown in Fig. 4, is given by:

$$\tau = D_d(u) \begin{cases} u + d_-, & u < -d_- \\ 0, & -d_- \leq u < d_+ \\ u + d_+, & u \geq d_+ \end{cases} \tag{7}$$

The width of the motion dead-band can be characterized by the parameter vector $d = [d_+, d_-]^T$.

C. Saturation Nonlinearity in DC Motor

Saturation is one of the most common actuator nonlinearities in practical control systems. Saturation nonlinearity exists in almost real control systems. Actuator saturation not only deteriorates the control performance, causing large overshoots and large settling times, but also leads to instability since the feedback loop is broken in such situations [20]. If u and T are scalars, the saturation nonlinearity, shown in Fig. 5, is given by:

$$T = Sat(u), \begin{cases} T_{max} & u(t) \geq T_{max} \\ u(t), & T_{max} < u < T_{min} \\ T_{min}, & u(t) \leq T_{min} \end{cases} \tag{8}$$

where T_{max} is the chosen positive and T_{min} is the negative saturation limits. If $u(t)$ falls outside the range of the actuator, actuator saturation occurs.

IV. THE PROPOSED APPROACH (AFILC)

To derive an accurate tracking controller for a system, an adaptive fuzzy iterative learning control (AFILC) method is proposed based on the rules constructed from the experts' experiences. However, the ILC with fixed learning gains suffers from the problems of slow convergence in tracking error, and sensitivity to noise and modeling uncertainty. So a fuzzy mechanism is incorporated to properly tune the learning gains K_p, K_I, and K_d to achieve a fast convergence rate and enhanced robustness. Moreover, there are certain problems that are encountered in practical control situations during the design of the fuzzy controller, including choosing the the membership functions and/or rule-base and may later the designed controller perform inadequately if significant and unpredictable plant parameter variations occur, or if there is noise or some type of disturbance or some other environmental effects [21]. So it would be nice if there is a method that could automatically perform the whole design task for us initially so that it would also synthesize the fuzzy controller for the nominal condition. In this paper, an adaptive algorithm is used which depends on the least square method that can automatically synthesize and tune fuzzy controllers. The overall block diagram of adaptive fuzzy iterative learning control (AFILC) is shown in Fig. 6. The aim of the proposed AFILC is to remove uncertainty associated with linguistic variables, to select the learning gains according to the change in system dynamics due to unpredictable variations such that load disturbance inputs and measurement noise, and to converge with respect to given steady state error and percentage overshoot. In the proposed algorithm, PID-type ILC scheme described by Eq. (1) is used with a fuzzy controller to build up the fuzzy controller structure with ILC described in Fig. 6. AFILC controller means that the three parameters K_p, K_I and K_d of the ILC scheme are tuned by using an adaptive fuzzy tuner which select the output membership functions for the three parameters K_p, K_I and K_d of the ILC scheme to avoid their selection according to the experts' experiences. It is possible to use the least square method to tune fuzzy systems either in a batch or real-time mode. First, we consider a fuzzy system

output [22]–[25]:

$$U_{COA} = \frac{\sum_{i=1}^{n} \mu_i(c)c_i}{\sum \mu_i(c)} \qquad (9)$$

where : U_{COA} is the crisp control value of the defuzzification process, $\mu(c)$ is the membership function grade and c_i is the centers of the output membership functions.

$$U_{COA} = \frac{\mu_1(c)c_1}{\sum \mu_i(c)} + \frac{\mu_2(c)c_2}{\sum \mu_i(c)} + ... + \frac{\mu_n(c)c_n}{\sum \mu_i(c)} \qquad (10)$$

and that if we define:

$$\xi_i(c) = \frac{\mu_i(c)}{\sum \mu_i(c)} \qquad (11)$$

Then

$$U_{COA} = c_1\xi_1(c) + c_2\xi_2(c) + ... + c_n\xi_n(c) \qquad (12)$$

Hence, if we define

$$\xi(c) = [\xi_1, \xi_2, ..., \xi_n]^T, \quad \theta(c) = [c_1, c_2, ..., c_n]^T \qquad (13)$$

Then

$$U_{COA} = \xi^T \theta(c) \qquad (14)$$

Let $\phi = \xi^T$, So $U_{COA} = \phi\theta(c)$
Let the error is defined by:

$$e = U_{COA} - \phi\theta(c) \qquad (15)$$

Define

$$E(n) = [e_1, e_2, ..., e_n]^T \qquad (16)$$

Such that:

$$E(n) = U_{COA} - \phi\theta \qquad (17)$$

The best values of θ such that minimizing the squared error must be found:

$$E^T E = (U_{COA} - \phi\theta)^T((U_{COA} - \phi\theta)),$$
$$E^T E = (U_{COA}^T U_{COA} - 2\theta^T\phi^T U_{COA} + \theta^T\phi^T\phi\theta). \qquad (18)$$

In order to minimize the squared error, we find the partial differentiation of squared error and equalizing it by zero as follows:

$$\frac{\partial E^T E}{\partial \theta} = 0 \qquad (19)$$

By partial differentiation of Eq. (18) with respect to θ.

$$\frac{\partial E^T E}{\partial \theta} = 0 - 2\phi^T U_{COA} + \phi^T\phi\theta + \theta^T\phi^T\phi = 0,$$
$$(\phi^T\phi)^{-1}\phi^T\phi\theta = (\phi^T\phi)^{-1}\phi^T U_{COA}. \qquad (20)$$

From the above equation described by Eq.(20), the least squares estimate of θ can be found as :

$$\hat{\theta}_{LS} = (\phi^T\phi)^{-1}\phi^T U_{COA} \qquad (21)$$

The discrete form of the above algorithm could be found in [26]. Finally, the above least square algorithm produces and estimates the best centers for the output membership function centers c_i. In this paper, the above algorithm is applied to the three outputs of the fuzzy controller to estimate the centers

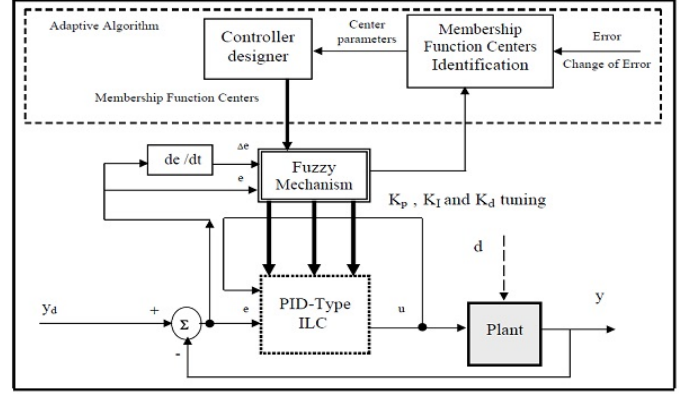| Ce/e	VS	S	M	L	VL
VS	VL	VS	M	L	L
S	VL	S	M	S	L
M	VL	S	L	L	L
L	VL	M	L	L	L
VL	VS	M	L	S	L

Fig. 6. Structure of adaptive fuzzy PID-type ILC controller.

of the proportional gain K_p, the integral gain K_I, and the derivative gain K_d of the ILC controller. Regarding to the fuzzy structure, there are two inputs to fuzzy inference: error $e(t)$ and change of error $ce(t)$, and three outputs for the ILC scheme parameters respectively K_p, K_I and K_d. Mamdani model is applied as the structure of fuzzy inference with some modification to obtain the best value for K_p, K_I and K_d. The rule base can be considered as the most important part of the fuzzy controller [21]. The full sets of rules are summarized in Table I. The linguistic varying levels used in inputs and output membership functions are assigned as (VS) Very Small, (S) Small, (M) Medium, (L) Large, and (VL) Very Large.

V. SIMULATION RESULTS

In this section, simulation trials are performed using MAT-LAB Software to demonstrate the effectiveness of the proposed algorithm (AFILC) and the improvements in the system performance by comparing its results with the system performance using classical PID-type ILC. Both of the proposed algorithm and classical PID-type ILC were applied to an X-Y feed table of a high-speed milling machine model described in section III. The two axes have two symmetrical motors and each axis is controlled independently using the same control structure for X-axis motor to Y-axis motor. The parameters of motor are described in Table II [27]. There are many cases that have been performed with the two controllers and the system is tested for step input reference.

1) Case(1): DC motors with constant friction.
   Fig. 7 shows the response of one motor of the X-Y table and the other one are the same because the same motor for X-axis and Y-axis was used for step input

TABLE II
THE MOTOR PARAMETERS

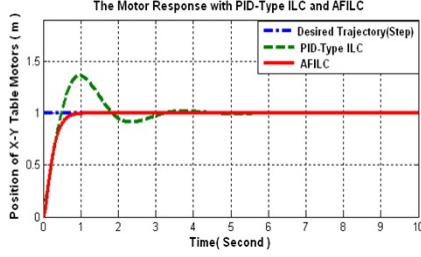| Parameter	Value	Unit(British System)
Motor Torque Constant(K)	$6x10^{-5}$	$lb.ft./Amp.$
Armature Resistance(R_a)	0.2	Ohm
Moment f Inertia(J)	$4.4x10^{-5}$	$lb.ft.Sec.^2$
Friction(f)	$4x10^{-4}$	$lb.ft./(Rad.Sec.)$
Back EMF Constant(K_b)	$5.5x10^{-2}$	$Volts.Sec./Rad.$
Delay Time(T_d)	0.00065	$Sec.$
Samling Time(T_s)	0.0026	$Sec.$

Fig. 7. Step change in the set point results.

using classical PID-type ILC and AFILC controllers. Where, the gains selected for the classical PID-type ILC controller are: $K_p = 0.35$, $K_I = 0.0019$, and $K_d = 0.4$. It is clear from the figure that the motor can track the reference input signal after some iterations using the two controllers but with better performance and no overshoot using AFILC compared to classical one that have large values of overshoot.

2) Case(2): DC motors with constant friction with added disturbance.
In order to demonstrate the effectiveness of both controllers against any load disturbance input that may affect performance of the process to be controlled, disturbance of the value $(d = 0.15 * Reference\ input)$ was added from iteration 500 to the end of iterations as shown in Fig. 8. It is clear that the AFILC controller forces the system to track the input again and there are less overshoot as compared with classical one.
3) Case(3): DC motors with variable friction.
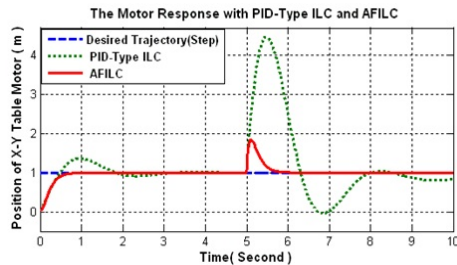To check the friction effect on the system responses, the

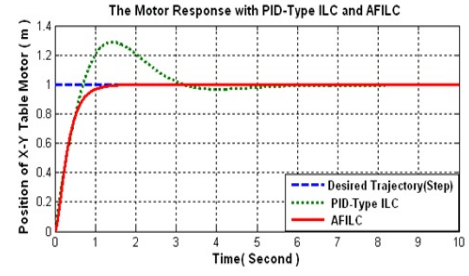Fig. 8. The disturbance effect on the two controllers.
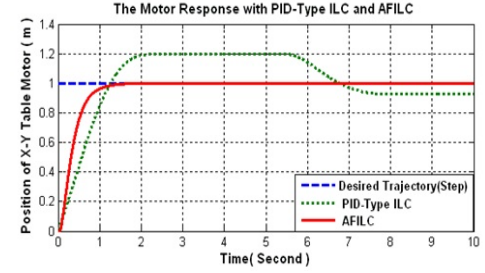

Fig. 9. Variable friction results.

Fig. 10. Variable friction with added deadzone nonlinearity results.

friction was made to be variable. It is clear from Fig. 9 that the friction causes reduction of the speed of the system response.

4) Case(4): DC motors with variable friction with the presence of dead-zone nonlinearity.
To assess the dead-zone nonlinearity effect on the system responses, a dead-zone was added to the system. It is clear from Fig. 10 that the dead-zone deteriorate the system response for the classical PID-type ILC but there is no effect on the proposed controller AFILC system response.

5) Case(5): DC motors with variable friction with the presence of saturation nonlinearity.
In order to test the saturation nonlinearity effect, a saturation was added to the system. It is clear from Fig. 11 that the saturation affects the two controllers but with less overshoot as compared with classical one that has large values of the overshoot.

VI. CONCLUSION

An AFILC algorithm is proposed by merging fuzzy control with ILC to improve its tracking accuracy in a closed-loop dynamical process and adding the adaptive algorithm based on batch least square method to the fuzzy controller to adapt the fuzzy membership functions. The algorithm achieves tracking accuracy with robustness against unpredictable disturbances and perturbations. Finally, an X-Y table system is used as a simulation example to demonstrate the proposed algorithm. Simulation results are reported to show the effectiveness and enhancement achieved by the proposed algorithm that finally
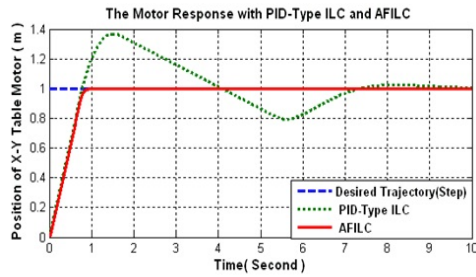
Fig. 11. Variable friction with added saturation nonlinearity results.

reflect the good performance of the proposed algorithm rather than the classical PID-type ILC. In the future work, a combination between a systematic method [28] and the proposed algorithm in this paper will be studied. Also, practically a comparison between the proposed method and other adaptive algorithms will be done.

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