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A Control Strategy for Designing an Intelligent Controller for Highly Dynamic/Perturbed Systems

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Abstract: In this paper a control strategy is proposed to deal with highly dynamic systems. It is based on designing an ANFIS based controller, with minimum number of inputs, by training it from the response of a conventional controller working on the same system. The performance of the resultant fuzzy controller is enhanced by optimizing its PID-like gains using Genetic Algorithms. The proposed control strategy is applied to a double inverted pendulum system, when a payload of significant mass is attached to the second link. This issue is important because of its relevance to some applications like humanoid robots. The added mass increases the system perturbation due to its inertial effect. The purpose of the controller is to balance the links around the equilibrium position, defined as the vertical upward one.

Keywords: Design of Fuzzy Logic Controller, Genetic Algorithms, Double Inverted Pendulum System

1. INTRODUCTION

The stabilization of inverted pendulum systems is an ongoing task in most areas of control theories. They are analogous to several real life systems like biped robots. These systems are highly perturbed and exhibits non negligible nonlinearities due to their dynamical structure. It is proposed here a cart type double inverted pendulum system when a significant payload is added to the second link. This issue is important since it is relevant to several important applications in different fields such as humanoid robots and mobile manipulators [1 - 4].

The inertial effect of the payload addition increases the system perturbations. This effect appears in the sort of reducing the working range that the conventional controller can stabilize the system within [1], compared to the same system without the payload. Thus the idea of this paper is to design an intelligent controller that has the same good performance of the conventional one, and at the same time is more capable of tolerating with highly perturbed systems, like the proposed one.

Fuzzy logic control is used to control inverted pendulum systems, as an alternative to conventional techniques. The double inverted pendulum system is considered as a multi-input multi-output system, since it has six state variables. This leads to the problem of rules number explosion if all the states are used as inputs to the fuzzy controller.

Many approaches have been introduced to overcome this problem. A dynamical hierarchical structure was designed to stabilize the system [5]. A multistage fuzzy controller was also proposed to stabilize the system [6]. Another technique based on combining the inputs into reducing the number of inputs was developed in [7,8].

Another problem in using fuzzy logic controller is the tuning of the controller parameters [9]. The hybrid control algorithms of fuzzy and neural network provide efficient solution. An ANFIS (Adaptive-Neuro Fuzzy Inference System) based controller was proposed to balance such systems [8,10]. It is based on optimizing the membership functions parameters according to training data sets from an optimal LQR controller that is working on balancing the same system.

Optimization techniques play an important role in the design of different controllers used to stabilize inverted pendulum systems. The performance of a human-simulated intelligent controller was enhanced using multi-objective genetic algorithms [11]. In another work, a neural controller based on chaos optimization method was designed [12].

In this work, a proposed control strategy is implemented to balance the suggested loaded double inverted pendulum system. The proposed control strategy depends on designing an ANFIS based controller with minimum number of rules based on training data sets taken from a conventional one, such as LQR controller, used to control the same system. Then, its performance is enhanced more by optimizing its PID-like gains using Genetic Algorithms (GAs). In this paper; firstly, the mathematical model of the system is briefly deduced. After that the ANFIS based controller design procedure is discussed. Finally, the controller performance is implemented in the simulation.

2. MATHEMATICAL MODEL OF DOUBLE INVERTED PENDULUM SYSTEM WITH PAYLOAD

2.1 Dynamic Analysis

The system consists of a cart that moves horizontally, two links with uniform mass distribution over the whole link length, and a payload, with significant inertia, that is attached to the second link, as shown in Fig. 1. The system dynamics is modeled using Lagrange method [1]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial P}{\partial q} + \frac{\partial S}{\partial \dot{q}} = F \quad (1)$$

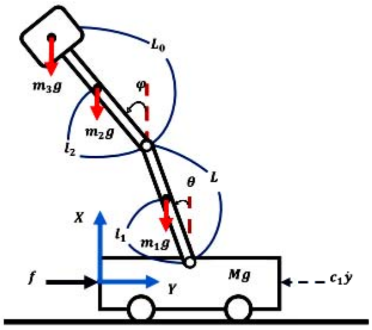

Fig.1 Physical Parameters of the Loaded Double Inverted Pendulum

Where T is the kinetic energy, P is the potential energy, and S is the energy loss. q is a variable of the generalized coordinates (y, θ, φ), and F is a vector of forces or moments acting in the direction of the generalized coordinates.

After determining the kinetic and potential energies of the system parts and the energy loss, and substituting in Eq. (1), the dynamics of the considered system are given as:

$$d_1\ddot{y} + d_4(\cos\theta)\ddot{\theta} + d_5(\cos\varphi)\ddot{\varphi} + c_1\dot{y}$$
$$-d_4(\sin\theta)\dot{\theta}^2 - d_5(\sin\varphi)\dot{\varphi}^2 = f \tag{2}$$

$$d_4(\cos\theta)\ddot{y} + d_2\ddot{\theta} + d_6(\cos(\theta-\varphi))\ddot{\varphi} + (c_2+c_3)\dot{\theta}$$
$$+\{d_6(\sin(\theta-\varphi))\,\dot{\varphi} - c_3\}\dot{\varphi} - d_4g(\sin\theta) = 0 \tag{3}$$

$$d_5(\cos\varphi)\ddot{y} + d_6(\cos(\theta-\varphi))\ddot{\theta} + d_3\ddot{\varphi} + c_3\dot{\varphi}$$
$$-\{d_6(\sin(\theta-\varphi))\,\dot{\theta} + c_3\}\dot{\theta} - d_5g(\sin\varphi) = 0 \tag{4}$$

The system equations can be written in a more compact matrix form:

$$N(q)\ddot{q} + H(q,\dot{q})\dot{q} + g(q) = F$$

Where:

$$N(q) = \begin{bmatrix} d_1 & d_4(\cos\theta) & d_5(\cos\varphi) \\ d_4(\cos\theta) & d_2 & d_6(\cos(\theta-\varphi)) \\ d_5(\cos\varphi) & d_6(\cos(\theta-\varphi)) & d_3 \end{bmatrix}$$

$$H(q,\dot{q}) = \begin{bmatrix} c_1 & -d_4(\sin\theta)\dot{\theta} & -d_5(\sin\varphi)\dot{\varphi} \\ 0 & c_2+c_3 & d_6(\sin(\theta-\varphi))\,\dot{\varphi} - c_3 \\ 0 & -d_6\,(\sin(\theta-\varphi))\,\dot{\theta} - c_3 & c_3 \end{bmatrix}$$

$$q = \begin{bmatrix} y \\ \theta \\ \varphi \end{bmatrix}, \quad g(q) = \begin{bmatrix} 0 \\ -d_4 g\sin\theta \\ -d_5 g\sin\varphi \end{bmatrix}, \quad F = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}$$

The meanings and values of the system parameters are given in Table 1 in the Appendix.

2.2 Model Linearization

The main goal of the controller is to balance the links in the vertical upward position. Thus this point is considered as the equilibrium point, and hence the nonlinear description of the system Eqns. (2), (3), and (4) are linearized around this point by assuming: $\cos\theta \cong 1, \cos\varphi \cong 1, \sin\theta \cong \theta, \sin\varphi \cong \varphi,$ $\cos(\theta-\varphi) \cong 1, \sin(\theta-\varphi) \cong \theta-\varphi, \dot{\theta}^2 = \dot{\varphi}^2 \cong 0.$

The linear state space and output equations of the system are described by:

$$\dot{x} = Ax + Bu$$
$$y = cx + Du \tag{5}$$

The force f acting on the cart is substituted by the symbol u, which denotes the control input signal. The state variables x and the output y are described as:

$$x = [y \quad \theta \quad \varphi \quad \dot{y} \quad \dot{\theta} \quad \dot{\varphi}]^T, \qquad y = [y \quad \theta \quad \varphi]$$
$$\dot{x} = [\dot{y} \quad \dot{\theta} \quad \dot{\varphi} \quad \ddot{y} \quad \ddot{\theta} \quad \ddot{\varphi}]^T = f(x,u) \tag{6}$$

The system matrix A, and the control input matrix B are determined by computing the Jacobean, such that:

$$A = \frac{\partial f(x,u)}{\partial x}, B = \frac{\partial f(x,u)}{\partial u} \tag{7}$$

Based on the values of the system parameters given in Table 1, the matrices A and B are calculated and then substituting in Eq. 5, the state space description is given as:

$$\begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \dot{\varphi} \\ \ddot{z} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -4.437 & 0.338 & -0.119 & 0.007 & 0.022 \\ 0 & 118.580 & -76.521 & 0.362 & -0.273 & 0.180 \\ 0 & -114.001 & 109.224 & -0.041 & 0.303 & -0.215 \end{bmatrix} \begin{bmatrix} z \\ \theta \\ \varphi \\ \dot{z} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.570 \\ -1.725 \\ 0.196 \end{bmatrix} u$$

$$\begin{bmatrix} z \\ \theta \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \theta \\ \varphi \\ \dot{z} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$

3. PROPOSED CONTROL STRATEGY

The proposed control strategy can be summarized in the following steps:

Step 1 Designing a controller based on conventional techniques to stabilize the linearized system which is selected as LQR controller.

Step 2 Designing an ANFIS based controller for the nonlinear system, based on training data sets acquired from the LQR controller.

Step 3 Enhancing the ANFIS based controller by optimizing its PID-like gains using Genetic Algorithms.

3.1 LQR Controller Design

For a linear quadratic regulator (LQR) controller, the control goal is to get the optimal control value $u^*(t)$ that minimizing the quadratic performance index function. The optimal control value equals:

$$u^*(t) = -R^{-1}B^TPx(t) = -Kx(t) \tag{8}$$

Where; K is the state feedback gain, and P is the unique positive definite solution of the Riccati equation:

$$PA + A^TP - PBR^{-1}B^TP + Q = 0 \tag{9}$$

The structure of the LQR controller is shown in Fig. 2. Putting $Q = \mathrm{diag}(7200, 36, 900, 252, 18, 36)$, and $R = 5.5$ [1], then the value of *K* can be computed as:

$$K = R^{-1}B^TP = [k_1\ k_2\ k_3\ k_4\ k_5\ k_6]$$
$$= [36.18\ -498.75\ 661.54\ 37.55\ \ -5.30\ 69.36] \tag{10}$$

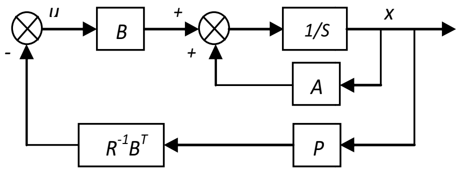

Fig.2 Optimal Control System Structure

3.2 ANFIS Based Controller Design

The Adaptive Neuro-Fuzzy Inference System (ANFIS) was early produced by Jang [9]. It is based on Takagi-Sugano model, where the premise part of the fuzzy rule is a fuzzy proposition, while the consequent part is a mathematical function, usually constant or first order polynomial.

The system has six state variables $y, \theta, \varphi, \dot{y}, \dot{\theta}, \dot{\varphi}$. Reducing the number of inputs to the fuzzy controller will overcome the problem of rule numbers explosion arises from using all the states as inputs. This will be achieved by combining the six inputs into only two inputs; the error and error variation, illustrated as:

Error: $$E = k_y e_y + k_\theta e_\theta + k_\varphi e_\varphi$$

Error variation: $$EC = k_{\dot{y}} \dot{e}_y + k_{\dot{\theta}} \dot{e}_\theta + k_{\dot{\varphi}} \dot{e}_\varphi \qquad (11)$$

Where; $e_y, e_\theta,$ and e_φ are the errors between the desired and the actual cart position and the two links' angles respectively. While $\dot{e}_y$, $\dot{e}_\theta$, and $\dot{e}_\varphi$ are the errors corresponding to their derivatives.

A weighted coefficient vector K_w is defined as:

$$K_w = \left[k_y \; k_\theta \; k_\varphi \; k_{\dot{y}} \; k_{\dot{\theta}} \; k_{\dot{\varphi}}\right]^T \qquad (12)$$

Its value is selected from the normalized values of state feedback gain K of the LQR controller, Eq. 10, such that:

$$K_x = \left[k_y \; k_\theta \; k_\varphi\right]^T = \left[\frac{k_1}{k_3} \; \frac{k_2}{k_3} \; 1\right]^T$$
$$K_{\dot{x}} = \left[k_{\dot{y}} \; k_{\dot{\theta}} \; k_{\dot{\varphi}}\right]^T = \left[\frac{k_4}{k_6} \; \frac{k_5}{k_6} \; 1\right]^T \qquad (13)$$

The inputs to the ANFIS based controller will be E and EC in this case. A schematic diagram illustrates the construction of the system with the fuzzy controller, is shown in Fig. 3.

The; $K_e, K_{ec},$ and K_u are the error, error variation, and the fuzzy controller output gains respectively. At the beginning, they are proposed to be equal to unity. Later on, they will be optimized using Genetic Algorithms.

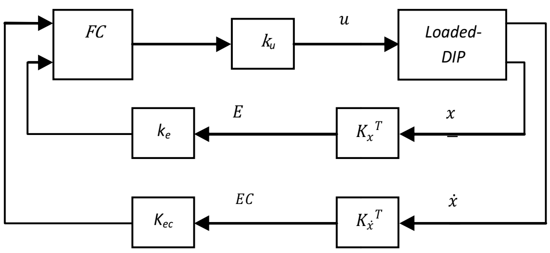

Fig.3 Loaded System with Fuzzy Controller

The design procedure of the ANFIS based controller to optimize its premise and consequent parameters is done as:

Step 1 Determining the working range of the initial states of y_{int}, θ_{int}, and φ_{int} that the LQR is capable of balancing the system within.

Step 2 Balancing the loaded system using the LQR controller for a specified initial state and calculating u from Eq. (8).

Step 3 Calculating the E and EC from Eq. (11).

Step 4 Acquiring the values of E, EC, and u; called "training data set" for this initial state.

Step 5 Returning to step2 for another initial state.

Step 6 Loading all the "training data sets" into the ANFIS graph editor.

Step 7 Optimizing the premise and consequent parameters of the ANFIS based controller.

Thus, the system has been tested with different initial states for which the LQR controller is able to work. The range of the initial states is selected such that: y_{int} changes from -0.1m to 0.1m, θ_{int} from -0.15rad to 0.15rad, and φ_{int} from -0.15rad to 0.15rad.

The ANFIS based controller is designed to have two inputs with three fuzzy sets for each of the Gaussian type, and one output. The training parameters are selected as hybrid learning algorithm, with number of epochs 10. The resultant surface of the fuzzy controller is shown in Fig. 4.

3.3 Optimization of the ANFIS Based Controller PID-like Gains

A typical Genetic Algorithm (GA) is an optimization technique that differs from traditional methods in many ways. It works with a coding of the parameters set, not the parameters themselves, searches within a population of points not a single point, and uses an Objective Function not derivative or other auxiliary information.

For the designed ANFIS based controller, GA has been applied to optimize its PID-like gains values. The *Objective Function* is assumed to be the summation of the squared error for the cart position and the two links' angles such that:

$$F = 1000 * \left(\sum e_y{}^2 + \sum e_\theta{}^2 + \sum e_\varphi{}^2\right) \qquad (14)$$

The maximum number of generations is selected as 1000 with 2 subpopulations for each. The number of individuals is taken as 25 for every subpopulation.

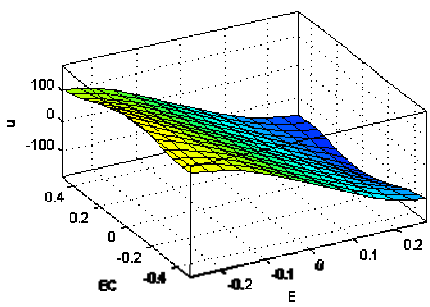

Fig.4 The Control Surface of the ANFIS Based Controller

The optimized gains have the values:

$$\mathrm{K_e} = 1.0228\,, \mathrm{K_{ec}} = 1.1048\,, \mathrm{K_u} = 1.1066 \qquad (15)$$

The nonlinear model of the system described by equations (2), (3), and (4) is represented in Simulink in Fig. 5.

4. SIMULATION RESULTS

The performance of both the LQR controller and the trained ANFIS based controller is compared together at the beginning. After that, the effect of the optimized PID-like gains of the ANFIS based controller is verified in terms of; increasing the initial states of the links' angles and the load variation.

4.1 LQR and ANFIS Based Controllers Comparison

The two controllers are compared at two points:

1) $y_{int} = 0.1m, \theta_{int} = 0.1rad\,,\ and\ \varphi_{int} = -0.1rad$, as shown in Fig. 6.

2) $y_{int} = 0.1m, \theta_{int} = 0.15rad\,,\ and\ \varphi_{int} = -0.15rad$, at the boundaries of the training data sets, as shown in Fig. 7.

For the first point, the two controllers are identical to each other. They have the same settling time of 2.5sec, and the same overshoot, for the cart position and the links' angles.

For the second point, the settling time is kept constant at 2.5sec and also the maximum overshoot for the cart movement and the links' angles. The difference here is that the two controllers are not identical to each other. The fuzzy controller has slightly higher oscillations rather than the LQR one.

Although the ANFIS based controller is trained from the LQR one, but their performances are not identical at all points. However, its performance satisfies the same settling time and maximum overshoot of the basic controller.

4.2 Effect of PID-like Gains Optimization

The effect of optimizing the PID-like gains appears in the sort of extending the working range of the initial conditions and the controller becomes more robust.

1) Extending the working range of the initial states, verified by testing a point of $y_{int} = 0.1m, \theta_{int} = 0.15rad\,,\ and\ \varphi_{int} = -0.19rad$. The settling time is kept constant at 3sec, as shown in Fig. 8.

2) The fuzzy controller is much more robust against load variation, verified by changing the load to the second link mass rartio $(R = m_3/m_2)$, given in Table 2 in the Appendix and shown in Fig. 9 and Fig. 10.

The state feedback gain value K, Eq. (10), is designed at mass ratio R (3). Its value is kept constant while the mass ratio R is changing. This is to verify the controllers' robustness. In addition the initial states are held constant at $y_{int} = 0.1m, \theta_{int} = 0.05rad$, $and\ \varphi_{int} = -0.08r$ with the load variations.

The two controllers show nearly the same settling time at each mass ratio R, but slightly lesser overshoot for the LQR controller. The advantage here is that the fuzzy controller is able to balance the system at mass ratio R (5), while the LQR one is not able to perform the task.

5. CONCLUSION

A proposed control strategy based on an intelligent controller is produced to balance the loaded double inverted pendulum system. This system is a highly perturbed one due to the existence of the mass attached to the second link. Firstly, an LQR controller is designed to stabilize the system, and then using its response to train an ANFIS based controller. Later on, the PID-like gains of the fuzzy controller are optimized using GA.

The fuzzy controller is designed with minimum number of rules by reducing the number of inputs and the assigned number of membership functions. The resultant fuzzy controller has the same performance of the LQR one in terms of the settling time (2.5sec) and maximum overshoot. In addition its PID-like gains optimization extends the working range of the links' initial angles by 26% (0.04rad). It is also much more robust against load variation.

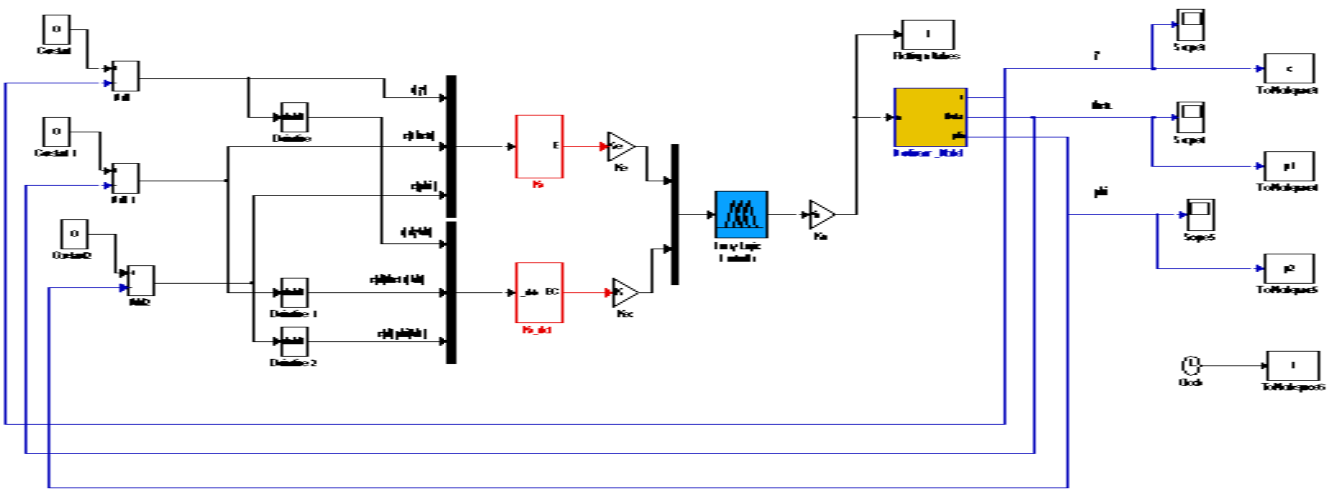

Fig. 5 Simulink Model of the System Utilizing ANFIS Based Controllers

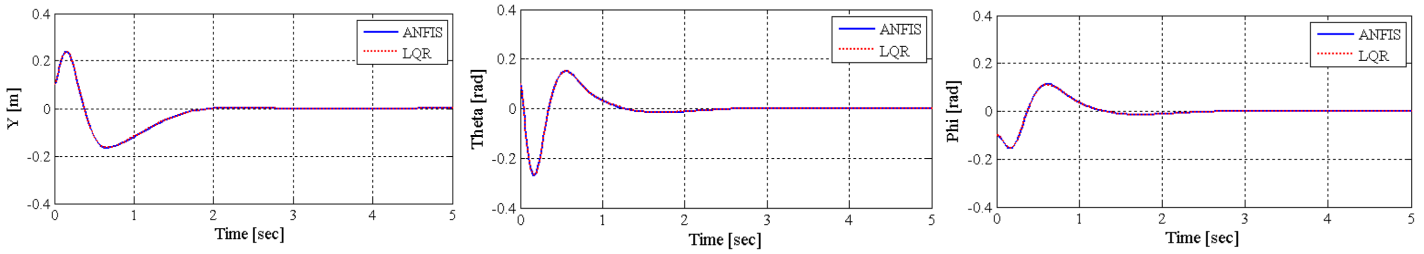

Fig. 6 Comparison of the LQR and ANFIS Based Controllers Performances: a) cart, b) first link, c) second link

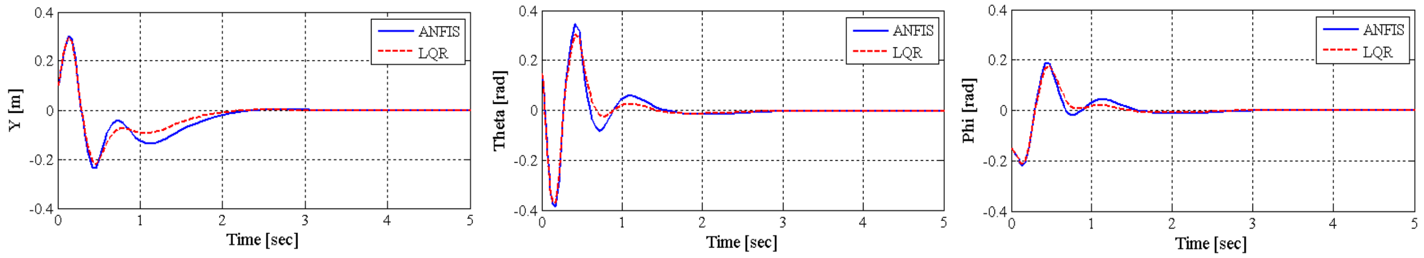

Fig. 7 Testing a Point at the Boundary of the Training Data Sets: a) cart, b) first link, c) second link

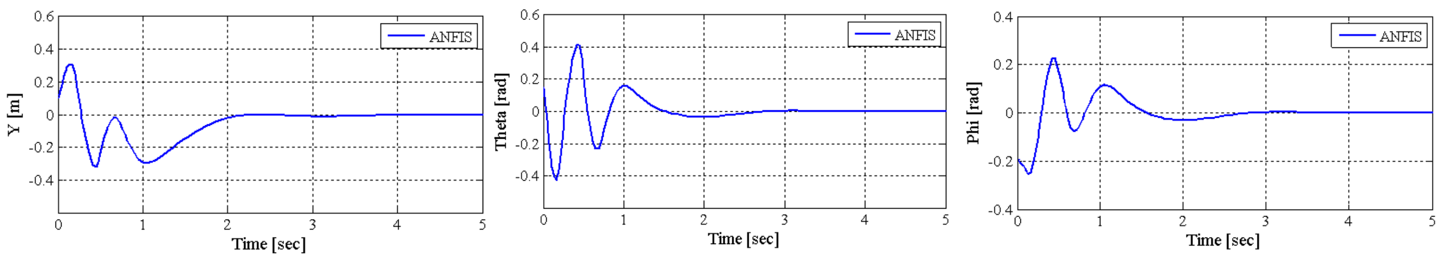

Fig. 8 Working Range Extension of the ANFIS Based Controller: a) cart, b) first link, c) second link

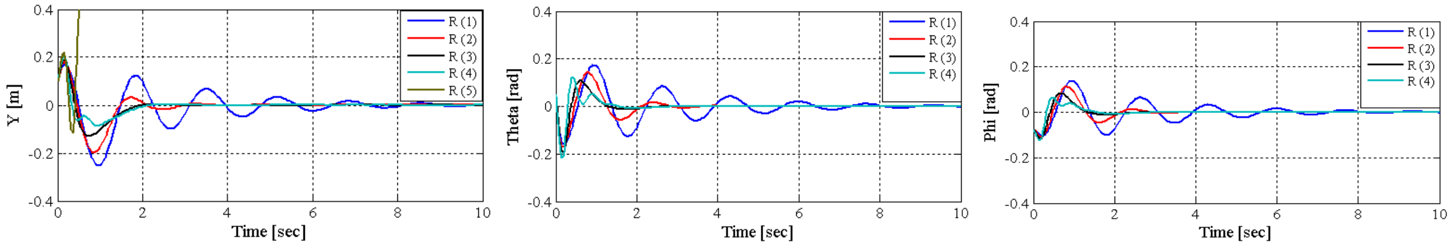

Fig. 9 Performance of the LQR Controller against Load Variation: a) cart, b) first link, c) second link

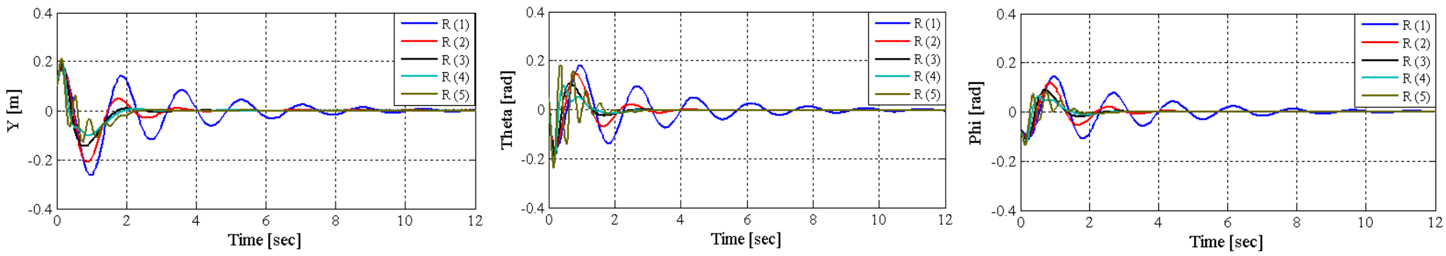

Fig. 10 Performance of the ANFIS Based Controller against Load Variation: a) cart, b) first link, c) second link

ACNOLDEGEMENT

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APPENDIX

Table 1System Parameters from SolidWorks Drawings

Symbol	**Description**
m_1	: mass of the first link assembly (0.205kg)
m_2	: mass of the second link assembly (0.178kg)
m_3	: mass of the payload (0.432kg)
M	: mass of the cart assembly (1.67Kg)
l_1	: length from the first joint to the center of mass of the first link (0.178m)
l_2	: length from the second joint to the center of mass of the second link(0.115m)
L	: length of the first link (0.36m)
L_0	: length of the second link(0.37m)
J_1	: Mass moment of inertia of the first link assembly around its center of gravity (0.0041kg.m^2)
J_2	: Mass moment of inertia of the second link assembly around its center of gravity (0.0024kg.m^2)
J_3	: Mass moment of inertia of the payload around its center of gravity (0.00014kg.m^2)
c_1	: Friction coefficient between the cart and the linear guide (0.21N.sec/m).
c_2, c_3	: Viscous coefficient for the two joints between the first link and the cart, and between the two links respectively (0.002Kg.m^2/sec).

The values of $d_1, d_2, \dots\dots, d_6$ are given s follows:

$$d_1 = M + m_1 + m_2 + m_3$$
$$d_2 = J_1 + m_1 {l_1}^2 + m_2 L^2 + m_3 L^2$$
$$d_3 = J_2 + m_2 {l_2}^2 + J_3 + m_3 {L_0}^2$$
$$d_4 = m_1 l_1 + m_2 L + m_3 L$$
$$d_5 = m_2 l_2 + m_3 L_0$$
$$d_6 = m_2 L l_2 + m_3 L L_0$$

Table 2 Load/Second Link Mass Ratio

Mass m_3 (Kg)	**Load/Second Link Mass Ratio ($R = m_3 / m_2$)**
$92.74 * 10^{-3}$	$R\,(1) = 0.52$
$234 * 10^{-3}$	$R\,(2) = 1.32$
$432 * 10^{-3}$	$R\,(3) = 2.42$
$686.5 * 10^{-3}$	$R\,(4) = 3.85$
$997.5 * 10^{-3}$	$R\,(5) = 5.6$