Thesis Proposal:

A Software Approach to Global Optimization

by

Marina Schmidt

Abstract

Still need to write.

Contents

List of Tables

List of Figures

List of Abbreviations

CCCZ	Controlled controlled controlled Z gate
qubit	Quantum bit

Chapter 1

Introduction

I introduce my thesis problem.

Chapter 2

Background

Toffolli single shot model

Toffoli and the Hadamard gate make up the universal gate set for quantum computing, which is used for information processing. Thus it needs to have fault tolerance incorporated in the circuit design of the error correction component. The component's fault tolerance is measured by fidelity, thus wanting a high fidelity it most desirable in a quantum circuit. The single-shot Toffoli circuit [**?**] has shown to be the superior design in obtaining higher fidelities by reducing the error correction processing time. When less time is required to correct the quantum bit (qubit) system, noise and decoherence is reduced allowing for a higher fidelity to be achieved.

The single-shot Toffoli gate achieves this by optimizing the gate constant parameters to describe an optimal pulse. It has shown to be superior for a three qubit system [**?**] and now the four qubit. The four qubit system is desirable because of the ability to encode code words (bit stings) as a future method to be used in encryption. Later five qubit system will be used for decryption creating an overall security system for quantum computers and networks.

A specific single-shot Toffoli gate is looked at in this paper is the controlled-controlled-controlled Z (CCCZ) gate, which is when the fourth qubit is flipped only if the first three qubits are in $|1\rangle$ state and not flipped otherwise. States represent the energy excitation state, which is the bit state value for a computer and is represented as:

$$|j\rangle_k$$

Where $j < 4$ and $j = 0$ is the ground state. k is the qubit location in system, starting at one and going to four in the four qubit system. The other main components of CCCNOT gate is the shift frequency ($\Delta_k(t)$) that describes the optimized pulse for each qubit and the anharmonicity (η_{jk}) for each energy state of each qubit. Therefore the energy of a qubit at the jth excitation level can be represented as:

$$E = h(j\Delta_k(t) - \eta_{jk}) \tag{2.1}$$

The qubits also interact with their closest neighbour, when the exicitation energy is above ground state, by factor of a coupling strength (g_k) between the X and Y axis. The X and Y coupling operators [?] are represented as:

$$X_k = \sum_{j=1}^{n} \sqrt{j} \Big|j-1\Big\rangle_k \Big\langle j|_k + hc$$
$$Y_k = -\sum_{j=1}^{n} \sqrt{-j} \Big|j-1\Big\rangle_k \Big\langle j|_k + hc$$

By using the above equation and 2.1, the general Hamiltonian of qubit system [?] can be obtained as:

$$\frac{\hat{H}(t)}{h} = \sum_{k=1}^{n}\sum_{j=0}^{n}(j\Delta_k(t) - \eta_{jk})|j\rangle_k\langle j|_k + \sum_{k=1}^{n-1}\frac{g_k}{2}(X_k X_{k+1} + Y_k Y_{k+1}) \tag{2.2}$$

The above Hamiltonian can be separated into two parts, the drift Hamiltonian:

$$\frac{\hat{H}^{dr}(t)}{h} = \sum_{k=1}^{n-1}\frac{g_k}{2}(X_k X_{k+1} + Y_k Y_{k+1})$$

that describes the uncontrolled part of the qubit system [?].

The other part of the equation 2.2 that is dependent on time, t, is the controlled Hamiltonian:

$$\frac{\hat{H}^{C}(t)}{h} = \sum_{k=1}^{n}\sum_{j=0}^{n}(j\Delta_k(t) - \eta_{jk})|j\rangle_k\langle j|_k \tag{2.3}$$

The controlled Hamiltonian describes the part of the qubit system that can be controlled and modified by an external pulse, $\Delta_k(t)$. The external pulse can change the energy states of the qubits to correct for dampening effects from environmental factors known as decoherence.

$\Delta_k(t)$ is a vector that represents different characteristics of the pulse for example amplitude and frequency. By applying the pulse vector, $\Delta_k(t)$, the Hamiltonian of the qubit system (2.2) then becomes:

$$\hat{H}\big(\Delta_k(t)\big) = \hat{H}^{dr} + \sum_{k=1}^{n} \Delta_k(t)\hat{H}^{C}(t) \tag{2.4}$$

By varying the above equation over a duration of time, Θ, evolution operator for time-ordered T:

$$U\big(\Delta_k(\Theta);\big) = Te^{\left\{-i\int_0^{\Theta} \hat{H}\left(\Delta_k(\tau)\right)d\tau\right\}} \tag{2.5}$$

The evolution operator does not take into account the decoherence effect from outside particles interacting with the qubits. Because decoherence is incorporated in open systems it is difficult to simulate, the intrinsic fidelity (with out decoherence) is first optimized to achieve a closer guess to optimize for a gate fidelity (with decoherence and system noise).

The performance of a closed system is further assessed by projecting the evolution operate 2.5 to computational subspace:

$$U_{\mathscr{P}}\big(\Delta_k(\Theta)\big) := \mathscr{P}U\big(\Delta_k(\Theta)\big)\mathscr{P} \tag{2.6}$$

is known as the "intrinsic fidelity" with respect to the ideal gate. The intrinsic fidelity for four qubit system using the CCCZ gate is:

$$\mathscr{F}\big(\Delta_k(\Theta)\big) = \frac{1}{N}\left|Tr\Big(CCCZ^{\dagger}U_{\mathscr{P}}\big(\Delta_k(\Theta)\big)\Big)\right| \tag{2.7}$$

with $\mathscr{F} = 1$ if $U_{\mathscr{P}}\big(\Delta_k(\Theta)\big) = CCCZ$ and $0 \leq \mathscr{F} < 1$ otherwise.

Once the intrinsic fidelity is optimized over $\Delta_k(t)$ to have intrinsic fidelity $\mathscr{F}\big(\Delta_k(\Theta)\big) >$ 99.99% [?] to obtain fault tolerance in the four qubit system. The pulse parameters from $\Delta_k(t)$ can then be experimentally applied to an open system to obtain the average gate fidelity for the overall circuit design.

Chapter 3

Method

Global Search

Chapter 4

RESULTS

My results on all my problems I have solved.

Qubit error control

Chapter 5

CONCLUSION

I conclude that I have solved the problem!