# Logic and Applications: Task 9

Mario Tsatsev
s1028415 Group 8

## Exercise 1

(i) Theorem: $P \vdash \neg\neg P$

Abbreviations:

Derivation tree:

$$\dfrac{\dfrac{}{P, \neg P \vdash \neg P}\,hyp \qquad \dfrac{}{P, \neg P \vdash P}\,hyp}{P \vdash \neg\neg P}\,\neg I$$

(ii) Theorem: $\neg\neg P \vdash P$

Abbreviations:

Derivation tree:

$$\dfrac{\dfrac{}{\neg\neg P, \neg P \vdash \neg\neg P}\,hyp \qquad \dfrac{}{\neg\neg P, \neg P \vdash \neg P}\,hyp}{\neg\neg P \vdash P}\,\neg E*$$

(iii) Theorem: P $\rightarrow Q \vdash \neg Q \rightarrow \neg P$

Abbreviations:

Derivation tree:

$$\dfrac{\dfrac{\dfrac{}{P \rightarrow Q, \neg P, P, \neg Q \vdash \neg Q}\,hyp \qquad \dfrac{\dfrac{}{P \rightarrow Q, \neg P, P, \neg Q \vdash P \rightarrow Q}\,hyp \qquad \dfrac{}{P \rightarrow Q, \neg P, P, \neg Q \vdash P}\,hyp}{P \rightarrow Q, \neg P, P, \neg Q \vdash Q}\rightarrow E}{P \rightarrow Q, \neg Q \vdash \neg P}\,\neg E}{P \rightarrow Q \vdash \neg Q \rightarrow \neg P}\rightarrow I$$

(iv) Theorem: $\neg P \wedge \neg Q \vdash \neg(P \vee Q)$
Abbreviations: let $\Gamma = \neg P \vee \neg Q$

$Let\ \alpha = \neg P \vee \neg Q, P \vee Q$

Derivation tree:

$$\dfrac{\dfrac{\dfrac{}{\alpha \vdash \neg P \wedge \neg Q}\,hyp}{\alpha \vdash \neg P}\,\wedge E2 \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\alpha, Q, P, Q \vdash \neg P \wedge \neg Q}\,hyp}{\alpha, Q, P \vdash \neg Q}\,\wedge E2 \quad \dfrac{}{\alpha, Q, P \vdash Q}\,hyp}{\alpha, Q \vdash P}\,\neg E* \quad \dfrac{}{\alpha, P \vdash P}\,hyp \quad \dfrac{}{\alpha \vdash P \vee Q}\,hyp}{\alpha \vdash P}\,\vee E}{\Gamma \vdash \neg(P \vee Q)}\,\neg I$$

(v) Theorem: $\vdash (P \to Q) \vee (Q \to P)$

Abbreviations: $\Gamma = (P \to Q) \vee (Q \to P) Derivationtree:$

$$\dfrac{\dfrac{\dfrac{}{\neg((P \to Q) \vee (Q \to P)), P \to Q \vdash \neg((P \to Q) \vee (Q \to P))}\,hyp \quad \dfrac{\dfrac{}{\neg((P \to Q) \vee (Q \to P)), P \to Q \vdash P \to Q}\,hype}{\neg((P \to Q) \vee (Q \to P)), P \to Q \vdash (P \to Q) \vee (Q \to P)}\,\vee I1}{\neg((P \to Q) \vee (Q \to P)) \vdash \neg(P \to Q)}\,\neg I \quad \dfrac{}{\neg((P \to Q) \vee (Q \to P)) \vdash P \to Q}\,\mathrm{Branch}_3}{\vdash (P \to Q) \vee (Q \to P)}\,\neg E*$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q, Q \to P \vdash Q \to P}\,hyp}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q, Q \to P \vdash (P \to Q) \vee (Q \to P)}\,\vee I2 \quad \dfrac{}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q, Q \to P \vdash \neg((P \to Q) \vee (Q \to P))}\,hyp}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q \vdash \neg(Q \to P)}\,\neg I \quad \dfrac{\dfrac{}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q, Q \vdash P}\,hyp}{\neg((P \to Q) \vee (Q \to P)), P, \neg Q \vdash Q \to P}\,\to I}{\neg((P \to Q) \vee (Q \to P)), P \vdash Q}\,\neg E*}{\neg((P \to Q) \vee (Q \to P)) \vdash P \to Q}\,\to I}{\vdash \neg(P \to Q) \vee Q \to P}$$

(vi) Theorem: $\vdash ((P \to Q) \to P) \to P$

Abbreviations: Let $\Delta = (P \to Q) \to P$
$let \Gamma = ((P \to Q) \to P) \to P$

Derivation tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Delta, \neg P, P \vdash \neg P}\,hyp \quad \dfrac{}{\Delta, \neg P, P \vdash P}\,hyp}{\Delta, \neg P, P \vdash Q}\,\neg E}{\Delta, \neg P \vdash P \to Q}\,\to I \quad \dfrac{\dfrac{}{\Delta, \neg P, P \to Q \vdash P \to Q}\,hyp \quad \dfrac{\dfrac{\dfrac{}{(P \to Q) \to P, \neg P, P \to Q \vdash P \to Q}\,hyp \quad \dfrac{}{(P \to Q) \to P, \neg P, P \to Q \vdash (P \to Q)P}\,hyp}{(P \to Q) \to P, \neg P, P \to Q \vdash P}\,\to E \quad \dfrac{}{\Delta, \neg P, P \to Q \vdash \neg P}\,hyp}{\Delta, \neg P, P \to Q \vdash \neg(P \to Q)}\,\neg I}{\Delta, \neg P \vdash \neg(P \to Q)}\,\neg I}{(P \to Q) \to P \vdash P}\,\neg E*}{\vdash ((P \to Q) \to P) \to P}\,\to I$$

I had to use DELTA alot but there was no way that i could fit it here :(
Also note that $\Delta$ was only used when i did not need to use that to prove the assumption

## Exercise 2

I did succeed in proving theorem `oldExamQuestion2`.

I succeded directly the first time.

## Exercise 3

a) I did succeed in proving the theorem in file `Task09_prop001.v`.

   I succeeded directly the first time

b) I did succeed in proving the theorem in file `Task09_prop016.v`.

   I succeeded in the end but it cost me some time (like 1 hour or so)

c) I did succeed in proving the theorem in file `Task09_prop020a.v`.

   I succeeded directly the first time

d) I did succeed in proving the theorem in file `Task09_prop030.v`.

   I succeded directly the first time (althogh it still took alot of time to write all the code still i did not delete and go back and such)

e) I did succeed in proving the theorem in file `Task09_prop107.v`.

   Initially i succeeded directly but Coq didnt save so i had to re-do it. In the re-doing i had about 15-20 code less so in the first attempt i have looped many times

## Exercise 4

a) I did not succeed in proving lemma `theGoldIsInSuitcase1`.

   ...

   The reason that I didn't succeed is that I got stuck at the proof obligation..... Its complicated. I managed to close 1 of the 3 branches but on the second one i felt as if i was just looping doing the same thing over and over :/

b) I did not succeed in proving lemma `theTrueStatementIsOnSuitcase4`.

   I did not even get there

c) I did not succeed in proving theorem `theSolutionIs`.

   could not even get there.