

Logic and Applications: Task 8

Mario Tsatsev
s1028415 Group 8

April 21, 2020

Exercise 1

a) My student number is 1028415 so I try to create the odd numbered derivation trees.

b) (i) Theorem: $P \rightarrow Q, Q \rightarrow R, P \vdash R$

Abbreviations: Let $\Gamma = P \rightarrow Q, P$.

Derivation tree:

$$\frac{\frac{\frac{}{\Gamma, Q \rightarrow R \vdash Q \rightarrow R} hyp \quad \frac{\frac{}{\Gamma, P \rightarrow Q \vdash P \rightarrow Q} hyp \quad \frac{}{\Gamma \vdash P} hyp}{\Gamma \vdash Q} \rightarrow E}{P \rightarrow Q, Q \rightarrow R, P \vdash R} \rightarrow E$$

Note that if it is too difficult to draw the trees using the package `prooftree.sty`, you may simply include *clear* pictures of your drawings as well!

(iii) Theorem: $P \wedge Q \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$

Abbreviations:

Derivation tree:

$$\frac{\frac{\frac{}{(P \wedge Q) \rightarrow R, P, Q \vdash P} hyp \quad \frac{}{(P \wedge Q) \rightarrow R, P, Q \vdash Q} hyp}{(P \wedge Q) \rightarrow R, P, Q \vdash P \wedge Q} \wedge I \quad \frac{}{(P \wedge Q) \rightarrow R, P, Q \vdash (P \wedge Q) \rightarrow R} hyp}{\frac{(P \wedge Q) \rightarrow R, P, Q \vdash R}{(P \wedge Q) \rightarrow R, P \vdash Q \rightarrow R} \rightarrow I} \rightarrow E$$

(v) Theorem: $P \vee Q \vdash Q \vee P$

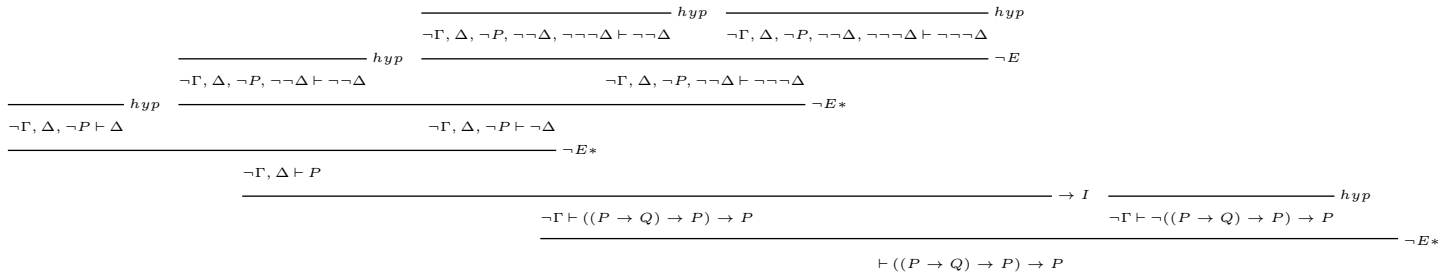
Abbreviations: Let $\Gamma = P \vee Q$

Derivation tree:

$$\frac{\frac{}{\Gamma \vdash P \vee Q} hyp \quad \frac{\frac{}{\Gamma, P \vdash P} hyp}{\Gamma, P \vdash Q \vee P} \vee I2 \quad \frac{\frac{}{\Gamma, Q \vdash Q} hyp}{\Gamma, Q \vdash Q \vee P} \vee I1}{P \vee Q \vdash Q \vee P} \vee E$$

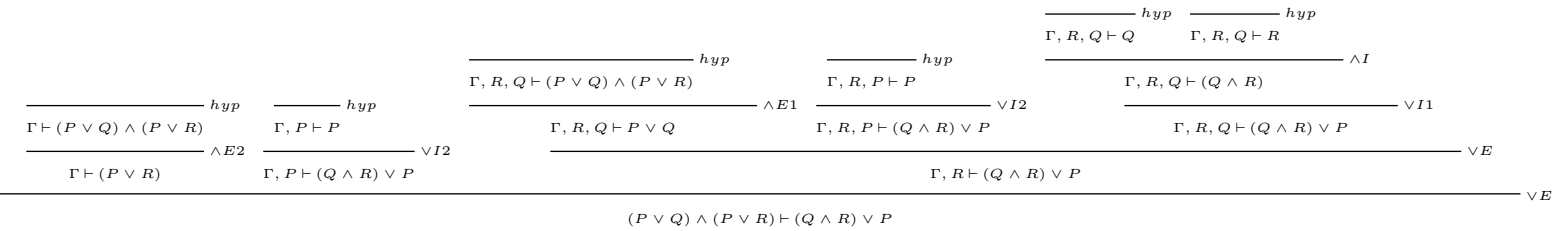
Theorem: $\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$

Derivation tree:

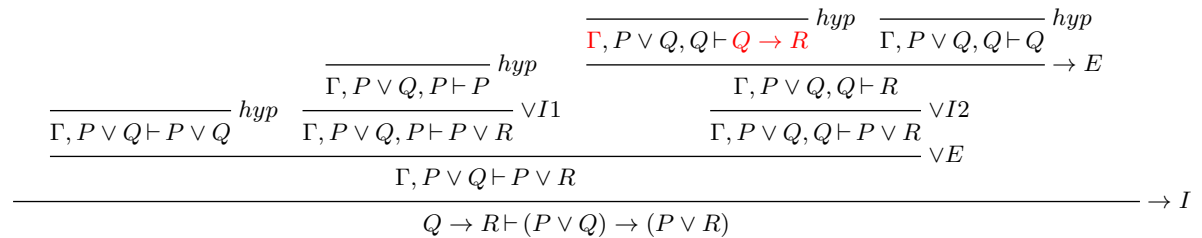


c) Theorem: $(P \vee Q) \wedge (P \vee R) \vdash (Q \wedge R) \vee P$

Derivation tree: sorry for the size :(



Derivation tree:



Theorem	#hyp	$\# \rightarrow E$	$\# \rightarrow I$	$\# \wedge E1$	$\# \wedge E2$	$\# \wedge I$	$\# \vee E$	$\# \vee I1$	$\# \vee I2$	#total
(i)	3	2								5

Theorem	#hyp	$\# \rightarrow E$	$\# \rightarrow I$	$\# \wedge E1$	$\# \wedge E2$	$\# \wedge I$	$\# \vee E$	$\# \vee I1$	$\# \vee I2$	#total
(iii)	3	1	2			1				7

Exercise 4

I managed to solve the theorems: one, two, three, four ,five ,six

```
Variables A B C P Q R: Prop.
```

```
Theorem one:
  (A ∧ B)
->
  (B ∧ A)
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

```
Theorem two:
  ((P -> Q) ∧ (Q -> R))
->
  (P -> R)
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

```
Theorem three:
  (¬A ∧ ¬B)
->
  (¬((A -> B) -> B))
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

```
Theorem four:
P ∨ ¬P
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

```
Theorem five:
¬((A ∧ (C -> ¬B)) ∧ (B ∧ (¬C -> ¬A)))
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

```
Theorem six:
((P -> Q) -> P) -> P
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

I failed to solve the theorems: none

Exercise 5

I managed to solve the lemmas: 1, 2, 3, 4, 5, 6, 7

```
Lemma reals1:
  ~ 37 < 37.
Proof.
apply Rlt_irrefl.
```

```

Qed.

Lemma reals2:
7 >= 3 -> ~ 3 > 7.
Proof.
apply Rge_not_gt.
(*! benb_proof *)
Qed.

Lemma reals3:
3 < 7 -> 7 > 3.
Proof.
apply Rlt_gt.
(*! benb_proof *)
Qed.

Lemma reals4:
~ 3 <= 7 -> 7 < 3.
Proof.
apply Rnot_le_lt.
(*! benb_proof *)
Qed.

Lemma reals5:
forall a:R,
  forall b:R,
    ~ a < b
  ->
    b <= a.
Proof.
apply Rnot_lt_le.
(*! benb_proof *)
Qed.

Lemma reals6:
forall a:R,
  forall b:R,
    a < b \/ a > b
  ->
    a <> b.
Proof.
apply Rlt_dichotomy_converse.
(*! benb_proof *)
Qed.

Lemma reals7:
forall a:R,
  forall b:R,
    a = b \/ a <> b.
Proof.
apply Req_dec.
(*! benb_proof *)
Qed.

```

I failed to solve the lemmas: none