Logic and Applications: Task 8

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Exercise 1

a) My student number is 1028415 so I try to create the odd numbered derivation trees.

b) (i) Theorem: $P \to Q\ , Q \to R, P \quad \vdash \quad R$

Abbreviations: Let $\Gamma = P \to Q, P$.

Derivation tree:

$$\dfrac{\dfrac{}{\Gamma, Q \to R \vdash Q \to R}\,\mathit{hyp} \qquad \dfrac{\dfrac{}{\Gamma, P \to Q \vdash P \to Q}\,\mathit{hyp} \quad \dfrac{}{\Gamma \vdash P}\,\mathit{hyp}}{\Gamma \vdash Q}\to E}{P \to Q, Q \to R, P \vdash R}\to E$$

Note that if it is too difficult to draw the trees using the package `prooftree.sty`, you may simply include *clear* pictures of your drawings as well!

(iii) Theorem: P $\wedge Q \to R \vdash P \to (Q \to R)$

Abbreviations:

Derivation tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(P \wedge Q) \to R, P, Q \vdash P}\,\mathit{hyp} \quad \dfrac{}{(P \wedge Q) \to R, P, Q \vdash Q}\,\mathit{hyp}}{(P \wedge Q) \to R, P, Q \vdash P \wedge Q}\wedge I \quad \dfrac{}{(P \wedge Q) \to R, P, Q \vdash (P \wedge Q) \to R}\,\mathit{hyp}}{(P \wedge Q) \to R, P, Q \vdash R}\to E}{(P \wedge Q) \to R, P \vdash Q \to R}\to I}{(P \wedge Q) \to R \vdash P \to (Q \to R)}\to I$$

(v) Theorem:P $\vee Q \vdash Q \vee P$

Abbreviations: Let $\Gamma = P \vee Q$

Derivation tree:

$$\dfrac{\dfrac{}{\Gamma \vdash P \vee Q}\,\mathit{hyp} \qquad \dfrac{\dfrac{}{\Gamma, P \vdash P}\,\mathit{hyp}}{\Gamma, P \vdash Q \vee P}\vee I2 \quad \dfrac{\dfrac{}{\Gamma, Q \vdash Q}\,\mathit{hyp}}{\Gamma, Q \vdash Q \vee P}\vee I1}{P \vee Q \vdash Q \vee P}\vee E$$

Theorem: $\vdash ((P \to Q) \to P) \to P$

Abbreviations: Let $\Delta = (P \to Q) \to P$
$let \Gamma = ((P \to Q) \to P) \to P$

Derivation tree:

$$\dfrac{\dfrac{\dfrac{}{\neg\Gamma, \Delta, \neg P \vdash \Delta}\,hyp \quad \dfrac{\dfrac{}{\neg\Gamma, \Delta, \neg P, \neg\neg\Delta \vdash \neg\neg\Delta}\,hyp \quad \dfrac{\dfrac{}{\neg\Gamma, \Delta, \neg P, \neg\neg\Delta, \neg\neg\neg\Delta \vdash \neg\neg\Delta}\,hyp \quad \dfrac{}{\neg\Gamma, \Delta, \neg P, \neg\neg\Delta, \neg\neg\neg\Delta \vdash \neg\neg\neg\Delta}\,hyp}{\neg\Gamma, \Delta, \neg P, \neg\neg\Delta \vdash \neg\neg\neg\Delta}\,\neg E}{\neg\Gamma, \Delta, \neg P \vdash \neg\Delta}\,\neg E*}{\dfrac{\neg\Gamma, \Delta \vdash P}{\neg\Gamma \vdash ((P \to Q) \to P) \to P}\,\to I}\,\neg E* \quad \dfrac{}{\neg\Gamma \vdash \neg((P \to Q) \to P) \to P}\,hyp}{\vdash ((P \to Q) \to P) \to P}\,\neg E*$$

I had to use DELTA alot but there was no way that i could fit it here :(

c) Theorem: (P $\vee Q) \wedge (P \vee R) \vdash (Q \wedge R) \vee P$

Abbreviations: Let $\Gamma = (P \vee Q) \wedge (P \vee R)$

Derivation tree: sorry for the size :(

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash (P \vee Q) \wedge (P \vee R)}\,hyp}{\Gamma \vdash (P \vee R)}\,\wedge E2 \quad \dfrac{\dfrac{}{\Gamma, P \vdash P}\,hyp}{\Gamma, P \vdash (Q \wedge R) \vee P}\,\vee I2 \quad \dfrac{\dfrac{\dfrac{}{\Gamma, R, Q \vdash (P \vee Q) \wedge (P \vee R)}\,hyp}{\Gamma, R, Q \vdash P \vee Q}\,\wedge E1 \quad \dfrac{\dfrac{}{\Gamma, R, P \vdash P}\,hyp}{\Gamma, R, P \vdash (Q \wedge R) \vee P}\,\vee I2 \quad \dfrac{\dfrac{\dfrac{}{\Gamma, R, Q \vdash Q}\,hyp \quad \dfrac{}{\Gamma, R, Q \vdash R}\,hyp}{\Gamma, R, Q \vdash (Q \wedge R)}\,\wedge I}{\Gamma, R, Q \vdash (Q \wedge R) \vee P}\,\vee I1}{\Gamma, R \vdash (Q \wedge R) \vee P}\,\vee E}{(P \vee Q) \wedge (P \vee R) \vdash (Q \wedge R) \vee P}\,\vee E$$

d) Theorem: Q $\to R \vdash (P \vee Q) \to (P \vee R)$

Abbreviations: Let $\Gamma = Q \to R$

Derivation tree:

$$\dfrac{\dfrac{\dfrac{}{\Gamma, P \vee Q \vdash P \vee Q}\,hyp \quad \dfrac{\dfrac{}{\Gamma, P \vee Q, P \vdash P}\,hyp}{\Gamma, P \vee Q, P \vdash P \vee R}\,\vee I1 \quad \dfrac{\dfrac{\dfrac{}{\Gamma, P \vee Q, Q \vdash {\color{red}Q \to R}}\,hyp \quad \dfrac{}{\Gamma, P \vee Q, Q \vdash Q}\,hyp}{\Gamma, P \vee Q, Q \vdash R}\,\to E}{\Gamma, P \vee Q, Q \vdash P \vee R}\,\vee I2}{\Gamma, P \vee Q \vdash P \vee R}\,\vee E}{Q \to R \vdash (P \vee Q) \to (P \vee R)}\,\to I$$

Theorem	#hyp	$\#{\to}E$	$\#{\to}I$	$\#{\wedge}E1$	$\#{\wedge}E2$	$\#{\wedge}I$	$\#{\vee}E$	$\#{\vee}I1$	$\#{\vee}I2$	#total
(i)	3	2								5

Theorem	#hyp	$\#{\to}E$	$\#{\to}I$	$\#{\wedge}E1$	$\#{\wedge}E2$	$\#{\wedge}I$	$\#{\vee}E$	$\#{\vee}I1$	$\#{\vee}I2$	#total
(iii)	3	1	2			1				7

Theorem	#hyp	$\#\to E$	$\#\to I$	$\#\land E1$	$\#\land E2$	$\#\land I$	$\#\lor E$	$\#\lor I1$	$\#\lor I2$	#total
(v)	3						1	1	1	6

Theorem	#hyp	$\#\to E$	$\#\to I$	$\#\land E1$	$\#\land E2$	$\#\land I$	$\#\lor E$	$\#\lor I1$	$\#\lor I2$	#total
(vii)	6			1	1	1	2	1	2	14

Theorem	#hyp	$\#\to E$	$\#\to I$	$\#\land E1$	$\#\land E2$	$\#\land I$	$\#\lor E$	$\#\lor I1$	$\#\lor I2$	#total
(ix)	4	1	1				1	1	1	9

Exercise 2

This is my alternative proof for showing that 'Ken is happy' follows from the assumptions, even if we omit $PN \to WT$ and $WT \to \neg KHo$.

Theorem: $\Gamma \quad \vdash \quad KHa$

Abbreviations: Γ is already defined in the assignment.

Derivation tree:

$$\dfrac{\dfrac{}{\Gamma \vdash KHo \lor \neg KHo}\,hyp \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, KHo \vdash JS \to ML}\,hyp \quad \dfrac{\dfrac{}{\Gamma, KHo \vdash JS \land PN}\,hyp}{\Gamma, KHo \vdash JS}\,\land E1}{\Gamma, KHo \vdash ML}\,\to E \quad \dfrac{}{\Gamma, KHo \vdash KHo}\,hyp}{\Gamma, KHo \vdash KHo \land ML}\,\land I \quad \dfrac{}{\Gamma, KHo \vdash KHo \land ML \to KHa}\,hyp}{\Gamma, KHo \vdash KHa}\,\to E \qquad \dfrac{}{\Gamma, \neg KHo \vdash KHa}\,B3}{\Gamma \vdash KHa}\,\lor E$$

B3: $$\dfrac{\dfrac{}{\Gamma, \neg KHo \vdash \neg KHo}\,hyp \quad \dfrac{}{\Gamma \vdash \neg KHo \to KHa}\,hyp}{\underline{\Gamma, \neg KHo \vdash KHa}}\,\to E$$

Exercise 3

a) This is my definition for `Button`:

$$\ldots$$

b) This is my derivation tree.

Theorem: $\Gamma \quad \vdash \quad \text{ButtonIn} \leftrightarrow \text{Ring}$

Abbreviations: Γ is already defined in the assignment.

Derivation tree:

$$\dfrac{}{\Gamma \quad \vdash \quad \text{ButtonIn} \leftrightarrow \text{Ring}}\,\ldots$$

Exercise 4

I managed to solve the theorems: one, two, three, four ,five ,six

```
Variables A B C P Q R: Prop.


Theorem one:
  (A /\ B)
->
  (B /\ A)
.
Proof.
tauto.
(*! benbta_proof *)
Qed.


Theorem two:
  ((P -> Q) /\ (Q -> R))
->
  (P -> R)
.
Proof.
tauto.
(*! benbta_proof *)
Qed.


Theorem three:
  (~A /\ ~B)
->
  (~((A -> B) -> B))
.
Proof.
tauto.
(*! benbta_proof *)
Qed.


Theorem four:
P \/ ~P
.
Proof.
tauto.
(*! benbta_proof *)
Qed.


Theorem five:
~((A /\ (C -> ~B)) /\ (B /\ (~C -> ~A)))
.
Proof.
tauto.
(*! benbta_proof *)
Qed.

Theorem six:
((P -> Q) -> P) -> P
.
Proof.
tauto.
(*! benbta_proof *)
Qed.
```

I failed to solve the theorems: none

Exercise 5

I managed to solve the lemmas: 1, 2, 3, 4, 5, 6, 7

```
Lemma reals1:
~ 37 < 37.
Proof.
apply Rlt_irrefl.
```

```
Qed.

Lemma reals2:
7 >= 3 -> ~ 3 > 7.
Proof.
apply Rge_not_gt.
(*! benb_proof *)
Qed.

Lemma reals3:
3 < 7 -> 7> 3.
Proof.
apply Rlt_gt.
(*! benb_proof *)
Qed.

Lemma reals4:
~ 3 <= 7 -> 7 < 3.
Proof.
apply Rnot_le_lt.
(*! benb_proof *)
Qed.

Lemma reals5:
forall a:R,
  forall b:R,
      ~ a < b
    ->
      b <= a.
Proof.
apply Rnot_lt_le.
(*! benb_proof *)
Qed.

Lemma reals6:
forall a:R,
  forall b:R,
      a<b \/ a>b
    ->
      a<>b.
Proof.
apply Rlt_dichotomy_converse.
(*! benb_proof *)
Qed.

Lemma reals7:
forall a:R,
  forall b:R,
    a=b \/ a<>b.
Proof.
apply Req_dec.
(*! benb_proof *)
Qed.
```

I failed to solve the lemmas: none