Logic and Applications: Task 10

Mario Tsatsev
s1028415 Group 8

April 21, 2020

Exercise 1

(i) Theorem: $\forall x : D, \forall y : D, Rxy \vdash \forall z : D, Rzz$

Abbreviations (if any used):
Let $\Gamma = \dots$.

Let $\Sigma = \dots$.

Derivation tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x : D, \forall y : D,\ \mathrm{R\ x\ y}\ , x : D \vdash \forall x : D, \forall y : D\ ,\mathrm{R\ x\ y}}\ hyp}{\forall x : D, \forall y : D,\ \mathrm{R\ x\ y}\ , x : D \vdash \forall y : D\ ,\mathrm{R\ x\ y}}\ \forall Ex}{\forall x : D, \forall y : D,\ \mathrm{R\ x\ y}\ , x : D \vdash \mathrm{R\ x\ x}}\ \forall Ey}{\forall x : D, \forall y : D, \mathrm{R\ x\ y} \vdash \forall z : D, \mathrm{R\ z\ z}}\ \forall I$$

(ii) Theorem: $\forall x : D, \mathrm{P\ x}\ \rightarrow\ \mathrm{Q\ x}\ , \exists y : D,\ \mathrm{P\ y}\ \vdash \exists x : D\ ,\mathrm{P\ x}$

Abbreviations: Let $\Gamma = \forall x : D\ \mathrm{P\ x}\ \rightarrow\ \mathrm{Q\ x}\ Let \Sigma = x$
Derivation tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma, y, Qy \vdash \mathrm{P\ x}}\ hyp}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma, y, Qy \vdash \exists x : D\ ,\mathrm{P\ x}}\ \exists I \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma \vdash Px}\ hyp \quad \dfrac{\dfrac{}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma \vdash \forall x : D\ ,\ \mathrm{P\ x} \rightarrow \mathrm{Q\ x}}\ hyp}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma \vdash Px \rightarrow Qx}\ \forall E}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma \vdash Qx}\ \rightarrow E}{\Gamma, \exists y : D\ \mathrm{P\ y}\ , Px, \Sigma \vdash \exists y : D\ ,\mathrm{Q\ y}}\ \exists Ix}{\Gamma, \exists y : D\ \mathrm{P\ y}\ \vdash \exists x : D\ ,\mathrm{P\ x}}\ \exists E \qquad \dfrac{}{\Gamma, \exists y : D\ ,\mathrm{P\ y}\ \vdash \exists y : D\ ,\ \mathrm{P\ y}}\ hyp}{\Gamma, \exists y : D\ ,\mathrm{P\ y}\ \vdash \exists x : D\ ,\ \mathrm{P\ x}}\ \exists E$$

(iii) Theorem: $\exists x : D\ \mathrm{P\ x}\ \vee\ \mathrm{Q\ x}\ \vdash \exists x : D\ ,\ \mathrm{P\ x}\ \vee \exists x : D\ ,\ \mathrm{Q\ x}$

Abbreviations: Let $\Gamma = \exists x : D\ \mathrm{Q\ x}\ \vee\ \mathrm{P\ x}$

Derivation tree:

$$\dfrac{\dfrac{}{\Gamma, x,\ \text{P x} \vee \text{Q x} \vdash \text{P x} \vee \text{Q x}}\,hyp \quad \dfrac{\dfrac{\dfrac{}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{P x} \vdash \text{P x}}\,hyp}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{P x} \vdash \exists x : D,\ \text{P x}}\,\exists Ex}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{P x} \vdash \exists x : D,\ \text{Q x} \vee \exists x : D,\ \text{P x}}\,\vee I2 \quad \dfrac{\dfrac{\dfrac{}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{Q x} \vdash \text{Q x}}\,hyp}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{Q x} \vdash \exists x : D,\ \text{Q x}}\,\exists Ex}{\Gamma, x,\ \text{P x} \vee \text{Q x},\ \text{Q x} \vdash \exists x : D,\ \text{Q x} \vee \exists x : D,\ \text{P x}}\,\vee I1}{\Gamma, x,\ \text{P x} \vee \text{Q x} \vdash \exists x : D\ ,\text{P x}}\,\vee E \quad \dfrac{}{\Gamma \vdash \exists y : D,\ \text{P x} \vee \text{Q x}}\,hyp}{\Gamma, \vdash \exists x : D,\ \text{Q x} \vee \exists x : D,\ \text{P x}}\,\exists E$$

(iv) Theorem: $\exists x : D$, R x x $\wedge$ P x $\vdash \neg(\forall x : D$, P x $\rightarrow \neg(\exists y : D$ R x y))

Abbreviations:

Derivation tree:

(v) Theorem:

Abbreviations:

Derivation tree:

(vi) Theorem: $\forall x : D$, P X$\vee$ R x x ,$\forall x : D$, P x $\rightarrow$ ($\exists y : D$, R x y$\wedge$ R y x) $\vdash$ $\forall x : D$,$\exists y : D$ R x y

Abbreviations: Let $\Gamma = \forall x : D$, P X$\vee$ R x x and Let $\Delta = \forall x : D$, P x $\rightarrow$ ($\exists y : D$, R x y$\wedge$ R y x)
Let $\Phi =$ R x y $\wedge$ R y x

Derivation tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \Delta, x : D \vdash \forall x : D,\text{P x} \vee \text{R x x}}\,hyp}{\Gamma, \Delta, x : D \vdash \text{P x} \vee \text{R x x}}\,\forall E \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \Delta, x : D, Px \vdash \forall x : D, \exists y : D\ \text{R x y} \wedge \text{R y x}}\,hyp}{\Gamma, \Delta, x : D, Px \vdash \exists y : D\ \text{R x y} \wedge \text{R y x}}\,\forall E \quad \dfrac{}{\Gamma, \Delta, x : D, Px \vdash \text{P x}}\,hyp}{\Gamma, \Delta, x : D, Px \vdash \exists y : D,\text{R x y} \wedge \text{R y x}}\,\rightarrow E \quad \dfrac{\dfrac{\dfrac{}{\Gamma, \Delta, x, Px, y, \Phi \vdash \text{R x y} \wedge \text{R y x}}\,hyp}{\Gamma, \Delta, x : D, Px, y : D, \Phi \vdash \text{R x y}}\,\wedge E1}{\Gamma, \Delta, x : D, Px, y : D, \Phi \vdash \exists y : D\ \text{R x y}}\,\exists I}{\Gamma, \Delta, x : D, Px, \vdash \exists y : D,\text{R x y}}\,\exists E \quad \dfrac{\dfrac{}{\Gamma, \Delta, x, Rxx \vdash \exists y : D,\text{R x x}}\,hyp}{\Gamma, \Delta, x, Rxx \vdash \exists y : D,\text{R x y}}\,\exists I}{\Gamma, \Delta, x : D, \vdash \exists y : D,\text{R x y}}\,\vee E}{\Gamma, \Delta \vdash \forall x : D, \exists y : D,\text{R x y}}\,\forall I$$

In some places i had x:D or y:D but i had to remove it to fit it on the page, also again sorry for the size :/

Exercise 2

I did `Theorem aMarriedPersonLooksAtAnUnmarriedPerson`.

Well not necessarily start over but i was not sure if i could use LEM so i tried without it and then failed so i did it with LEM at the end

Exercise 3

I did manage to prove `Theorem theResponseLecturesOfLogicAndApplicationsPayOff`.

directly, no start overs

Exercise 4

a) This theorem basically holds because of the strange combination of quantifiers and operators. To be more precise, ...

   Note that it is crucial that the set of students is not empty, because it is not empty then there is no student for who anything could hold

b) I did not manage to prove `Theorem drinkers_paradox`.

   I REALLY TRIED

   ‘

Exercise 5

a) I did manage to prove the theorem from `Task10_pred001.v`.

   directly

b) I did not manage to prove the theorem from `Task10_pred009.v`.

   started over

c) I did manage to prove the theorem from `Task10_pred020.v`.

   don’t remember honestly :/

d) I did not manage to prove the theorem from `Task10_pred031.v`.

   directly

e) I did manage to prove the theorem from `Task10_pred042.v`.

   took some time but it worked and i did not have to start over