

Semantics and Rewriting lambda

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Exercise 2

a)

$$\begin{aligned}\mathbf{SKK} &\rightarrow_{\beta} \mathbf{I} \\ \mathbf{SKK} &\equiv (\lambda xyz.xz(yz))\mathbf{KK} \\ &\rightarrow (\lambda yz.\mathbf{K}z(yz))\mathbf{K} \\ &\rightarrow (\lambda z.\mathbf{K}z(Kz)) \\ &\rightarrow (\lambda z.(\lambda xy.x)z(\mathbf{K}z)) \\ &\rightarrow (\lambda z.(\lambda y.z)(\mathbf{K}z)) \\ &\rightarrow (\lambda z.(\lambda.z)) \\ &\rightarrow (\lambda z.z) \\ &\equiv \mathbf{I}\end{aligned}$$

b)

$$\begin{aligned}\mathbf{KI} &\rightarrow_{\beta} K_* \\ \mathbf{KI} &\equiv (\lambda xy.x)\mathbf{I} \\ &\rightarrow (\lambda y.\mathbf{I}) \\ &\rightarrow y \\ &\equiv \mathbf{K}_*\end{aligned}$$

Exercise 3

To determine that we must reduce until possible.

$$\mathbf{SKIKISS} \rightarrow_{\beta} (\lambda x.x)(\mathbf{KIKISS}) \quad (1)$$

$$\rightarrow_{\beta} (\lambda x.x)(\lambda xy.x)\mathbf{IKISS} \quad (2)$$

$$\rightarrow_{\beta} (\lambda xy.x)\mathbf{IKISS} \quad (3)$$

$$\rightarrow_{\beta} (\lambda xy.x)(\lambda x.x)(\lambda xy.x)\mathbf{ISS} \quad (4)$$

$$\rightarrow_{\beta} (\lambda y.(\lambda x.x))(\lambda xy.x)\mathbf{ISS} \quad (5)$$

$$\rightarrow_{\beta} (\lambda x.x)\mathbf{ISS} \quad (6)$$

$$\rightarrow_{\beta} (\lambda x.x)(\lambda x.x)\mathbf{SS} \quad (7)$$

$$\rightarrow_{\beta} (\lambda x.x)\mathbf{SS} \quad (8)$$

$$\rightarrow_{\beta} (\lambda x.x)(\lambda xyz.xz(yz))\mathbf{S} \quad (9)$$

$$\rightarrow_{\beta} (\lambda xyz.xz(yz))(\lambda xyz.xz(yz)) \quad (10)$$

$$\rightarrow_{\beta} (\lambda yz.(\lambda xyz.xz(yz))z(yz)) \quad (11)$$

$$\rightarrow_{\beta} (\lambda yz.(\lambda yz.zz(yz))(yz)) \quad (12)$$

$$\rightarrow_{\beta} (\lambda yz.(\lambda z.zz(yzz))) \quad (13)$$