CS 584: Spatial Algorithms
Emory University
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Homework # 8

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1. **Compute the probability that at least three wolves are close to the group! You may use the Generating Function based approach covered in class**.

Solution. We start with the Generating Function based method: For the wolves $w_i, i \in \{1, \cdots, 5\}$, they have $\mathbb{P}(w_i) = \{0.8, 0.5, 0.2, 0.1, 0.3\}$ of being "close to the group of tourists" . This means $1 - \mathbb{P}(w_i) = \{0.2, 0.5, 0.8, 0.9, 0.7\}$ of the wolves being "far from the group of tourists". Then the Generating Function Method becomes:

$$\begin{aligned}\prod_{i=1}^{5} \mathbb{P}(w_i)w_i + (1 - \mathbb{P}(w_i)) &= (0.8x + 0.2)(0.5x + 0.5)(0.2x + 0.8)(0.1x + 0.9)(0.3x + 0.7) \\ &= (0.4x^2 + 0.5x + 0.1)(0.2x + 0.8)(0.1x + 0.9)(0.3x + 0.7) \\ &= (0.08x^3 + 0.42x^2 + 0.42x + 0.08)(0.1x + 0.9)(0.3x + 0.7) \\ &= (0.008x^4 + 0.114x^2 + 0.42x^2 + 0.386x + 0.072)(0.3x + 0.7) \\ &= 0.0024x^5 + 0.0398x^4 + 0.2058x^3 + 0.4098x^2 + 0.2918x + 0.0504\end{aligned}$$

This means that $\mathbb{P}(w = 5) = 0.00024, \mathbb{P}(w = 4) = 0.0398, \mathbb{P}(w = 3) = 0.2058, \mathbb{P}(w = 2) = 0.4098, \mathbb{P}(w = 1) = 0.2918, \mathbb{P}(w = 0) = 0.0504$. Since we are asked about the $\mathbb{P}(w \geqslant 3)$, then we compute

$$\begin{aligned}\mathbb{P}(w \geqslant 3) &= \mathbb{P}(w = 5) + \mathbb{P}(w = 4) + \mathbb{P}(w = 3) \\ &= 0.0024 + 0.398 + 0.2058 \\ &= 0.248\end{aligned}$$

2. **Use a polynomial-time algorithm for this purpose.**

Solution. While these computations were done by Mathematica, we briefly discuss how to do these computations in polynomial time. First, observe that multiplying a polynomial of degree m with a polynomial of degree n, we have $(m + 1)(n + 1)$ multiplies and mn additions, which is $\mathcal{O}(2mn + m + n + 1) = \mathcal{O}(mn)$. If we do divide and conquer, then we will only have $\lg w$ levels of this algorithm, where w is the number of binomials we are multiplying (number of wolves for the above problem). This yields a polynomial time algorithm for computing what we want.

If we want lower, we can recognized that we only have to do 3 multiplications for two binomials, so using divide and conquer algorithms, we can get $\mathcal{O}\left(n^{\lg 3}\right)$. If we desire faster, we can use the FFT and iFFT (both done in $\mathcal{O}(n \lg n)$ with n evalutations for the interpolation, so doing this process $\lg w$, we end up with an $\mathcal{O}\left(n \lg^2 n\right)$ algorithm.

3. **Given your result above. Should you go out there to protect the tourist groups? Or is the probability low enough that you can safely take a nap**

   Solution. I don't know how to answer this, as it appears to be more of an ethics question then anything else. The probability is low (≈ 0.25), but that is still too high for me to risk the loss of human life.

Acknowledgements

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