CS 584: Spatial Algorithms
Emory University
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Homework # 6

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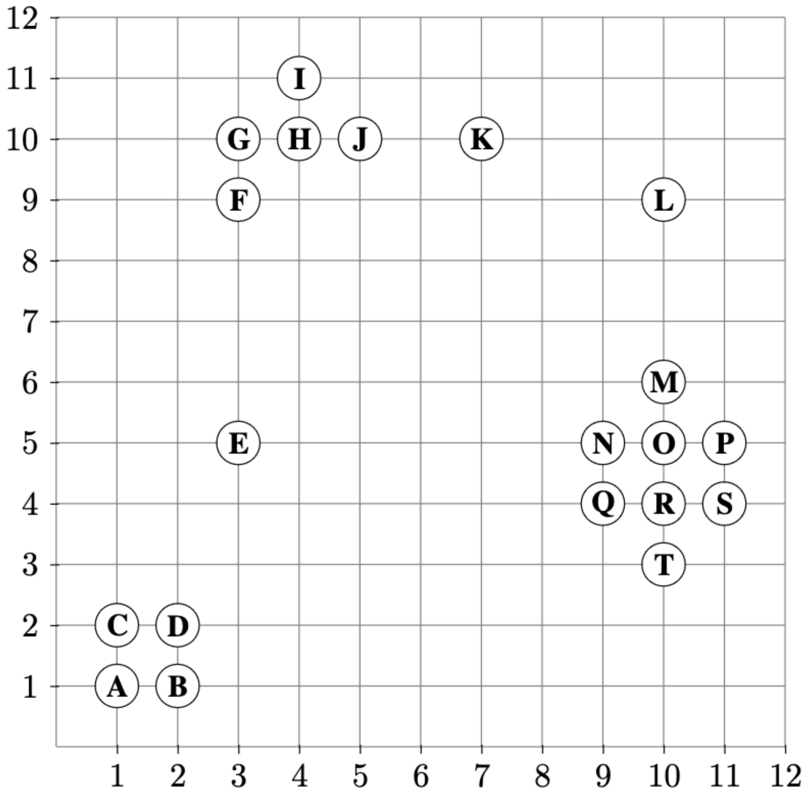

Figure 1

General Formulae

First recall that there are three main formulas for computing the local outlier factor (LOF):

$$\text{r-dist}_k(p, q) := \max(\text{k-dist}(q), \text{dist}(p, q)) \tag{1}$$

$$\text{lrd}_k(o) := \left(\frac{\sum_{o \in N_k(o)} \text{r-dist}_k(o, o')}{\|N_k(o)\|} \right)^{-1} \tag{2}$$

$$\text{LOF}(o) = \frac{\sum_{o' \in N_k(o)} \frac{\text{lrd}_k(o')}{\text{lrd}_k(o)}}{\|N_k(0)\|} \tag{3}$$

Applied To Point L with $k = 3$

Since our problem is concerned with $k = 3$ looking at point L, we can rewrite Eqn.3 in the following way:

$$\mathrm{LOF}_3(L) := \frac{\sum_{o' \in N_3(L)} \frac{\mathrm{lrd}_3(o')}{\mathrm{lrd}_3(L)}}{\|N_3(L)\|}$$

First, we need to ascertain what $N_3(L)$ is. Intuitively, $N_k(O)$ is the set of all points that are k-nearest neighbors with point O. However, this set may not be just k points, especially if there is a tie then we might have to include more points. But for this case, $N_3(L) = \{K, M, O\}$. Since there is only one points 3 away and exactly two points 4 away from L, then $\|N_3(L)\| = 3$. Combining all of this information, we arrive at:

$$\mathrm{LOF}_3(L) = \frac{\frac{\mathrm{lrd}_3(K)}{\mathrm{lrd}_3(L)} + \frac{\mathrm{lrd}_3(M)}{\mathrm{lrd}_3(L)} + \frac{\mathrm{lrd}_3(O)}{\mathrm{lrd}_3(L)}}{3}$$

Computing $\mathbf{lrd}_3(K)$

The first part of Eqn. **??** is computing $lrb_3(K)$. First, we observe that $N_3(K) = \{J, H, I, G, L\}$, as there is a 3-way tie for the third nearest neighbor, so we have to enclude G,I, and L, which means that $\|N_3(K)\| = 5$. Now using eqn. 2, modified for our problem, we have

$$\begin{aligned}
\mathrm{lrd}_3(K) &= \left(\frac{\text{r-dist}_3(K,J) + \text{r-dist}_3(K,H) + \text{r-dist}_3(K,I) + \text{r-dist}_3(K,G) + \text{r-dist}_3(K,L)}{5}\right)^{-1} \\
&= (\max(2,2) + \max(2,3) + \max(3,4) + \max(2,4) + \max(4,4)) \\
&= \left(\frac{2+3+4+4+4}{5}\right)^{-1} \\
&= \left(\frac{17}{5}\right)^{-1} \\
&= \frac{5}{17}
\end{aligned}$$

Computing $\mathbf{lrd}_3(M)$

Next, observe that $N_3(M) = \{O, P, R, N\}$. This means that $\|N_3(M)\| = 4$. Also

$$\begin{aligned}
\mathrm{lrd}_3(M) &= \left(\frac{\text{r-dist}_3(M,O) + \text{r-dist}_3(M,P) + \text{r-dist}_3(M,R) + \text{r-dist}_3(M,N)}{4}\right)^{-1} \\
&= \left(\frac{\max(1,1) + \max(2,2) + \max(1,2) + \max(2,2)}{4}\right)^{-1} \\
&= \left(\frac{1+2+2+2}{4}\right)^{-1} \\
&= \left(\frac{7}{4}\right)^{-1} \\
&= \frac{4}{7}
\end{aligned}$$

Computing $\mathbf{lrd}_3(O)$

Now observe that $N_3(O) = \{M, N, R, P\}$. This means that $\|N_3(O)\| = 4$. Also,

$$\begin{aligned}
\mathrm{lrd}_3(O) &= \left(\frac{\text{r-dist}_3(O,M) + \text{r-dist}_3(O,N) + \text{r-dist}_3(O,R) + \text{r-dist}_3(O,P)}{4}\right)^{-1} \\
&= \left(\frac{\max(2,1) + \max(2,2) + \max(1,1) + \max(2,1)}{4}\right)^{-1} \\
&= \left(\frac{2+2+1+2}{4}\right)^{-1} \\
&= \left(\frac{7}{4}\right)^{-1} \\
&= \frac{4}{7}
\end{aligned}$$

Computing $\mathbf{lrd}_3(L)$

Now observe that $N_3(L) = \{M, O, K\}$. This means that $\|N_3(L)\| = 3$. Also,

$$\begin{aligned}
\mathrm{lrd}_3(L) &= \left(\frac{\text{r-dist}_3(L,M) + \text{r-dist}_3(L,O) + \text{r-dist}_3(L,K)}{3}\right)^{-1} \\
&= \left(\frac{\max(3,2) + \max(1,4) + \max(3,4)}{3}\right)^{-1} \\
&= \left(\frac{3+4+4}{3}\right)^{-1} \\
&= \left(\frac{11}{3}\right)^{-1} \\
&= \frac{3}{11}
\end{aligned}$$

Final Computation

Now that we have computed all of the preliminary steps, we can compute the local outlier factor, in eqn **??**. We see that

$$\begin{aligned}
\mathrm{LOF}_3(L) &= \frac{\frac{\mathrm{lrd}_3(K)}{\mathrm{lrd}_3(L)} + \frac{\mathrm{lrd}_3(M)}{\mathrm{lrd}_3(L)} + \frac{\mathrm{lrd}_3(O)}{\mathrm{lrd}_3(L)}}{3} \\
&= \frac{\frac{\frac{5}{17}}{\frac{3}{11}} + \frac{\frac{5}{17}}{\frac{3}{11}} + \frac{\frac{5}{17}}{\frac{3}{11}}}{3} \\
&= \frac{\frac{55}{51} + \frac{44}{21} + \frac{44}{21}}{3} \\
&= \frac{385}{357} + \frac{748}{357} + \frac{748}{357} \\
&= \frac{1881}{357 \cdot 3} \\
&= \frac{627}{357} \approx 1.75630252
\end{aligned}$$

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