By symmetry, there are only slight differences from dielectric in terms of calculation.

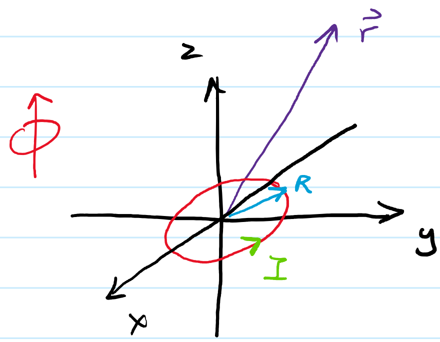
- Derivation of magnetic dipole potential / field
- Magnetic material
  - Magnetization, bound current
  - Auxillary field, free current
  - Special case: Linear magnetic material

Magnetic Dipole's field / potential

Recall that potential is easier to calculate than field because field is a vector and is annoying to solve each component

$\Rightarrow$ Usual approach: Find $V / \vec{A} \xrightarrow{\text{div / curl}} \vec{E} / \vec{B}$

Simple magnetic dipole



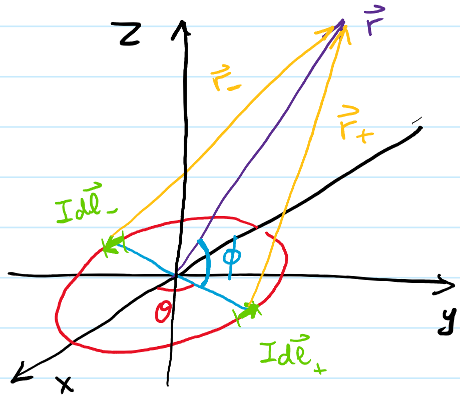
Model:

- Ampere's Loop with current I
- Loop's radius $= R$
- Only consider the potential very far away, $|\vec{r}| \gg R$

(So the loop looks like a point when zoom out)

Standard derivation

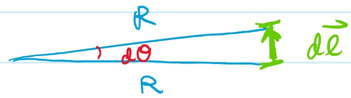
By symmetry, we can first look at the potential contributed by 2 segments opposite in the loop :



1. Write their contributions of magnetic vector potential $\vec{A}$ like in electric dipole

$$d\vec{A} = \frac{\mu_0}{4\pi}\left[\frac{Id\vec{l}_+}{|\vec{r}_+|} - \frac{Id\vec{l}_-}{|\vec{r}_-|}\right]$$

By geometry



$$= \frac{\mu_0 I}{4\pi}\left[\frac{\boxed{Rd\theta}}{|\vec{r}_+|}\cdot(+\hat{\theta}) + \frac{\boxed{Rd\theta}}{|\vec{r}_-|}\cdot(-\hat{\theta})\right]$$

$d\vec{l}_+$, $d\vec{l}_-$ are in opposite direction

$$= \frac{\mu_0 IRd\theta}{4\pi}\left[\frac{1}{\sqrt{r^2+R^2-2rR\cos\phi}} - \frac{1}{\sqrt{r^2+R^2+2rR\cos\phi}}\right]\cdot(\hat{\theta})$$

$$= \frac{\mu_0 IR\,d\theta}{4\pi}\cdot\frac{1}{r}\left[\left(1+\frac{R}{r}\cos\phi\right) - \left(1-\frac{R}{r}\cos\phi\right)\right]\cdot(\hat{\theta})$$

$$= \frac{\mu_0 IRd\theta}{4\pi}\,\frac{2R}{r^2}\cos\phi\cdot(\hat{\theta})$$

2. To integrate all θ, first need to turn parameters that depend on θ to expression of θ

- Recall in rotation, angular unit vector $\hat{\theta}_{(r,\theta)}$ depends on θ

$$\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

- ϕ is geometrically depending on θ. First express the position vectors by $\{\hat{x}/\hat{y}/\hat{z}\}$

   - Position of the measurement point $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$
   - Position of $Id\vec{l}_+$ $\vec{R} = R\cos\theta\,\hat{x} + R\sin\theta\,\hat{y}$

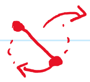
$\Rightarrow$ By dot product, can express ϕ as

$$\cos\phi = \frac{\vec{r}\cdot\vec{R}}{|\vec{r}|\,|\vec{R}|} = \frac{xR\cos\theta + yR\sin\theta}{rR}$$

[3] Subst. back these expressions, do the integration.

$$\int d\vec{A} = \int_0^{\pi} \frac{\mu_0 I R^2}{2\pi r^2}\left[\frac{xR\cos\theta + yR\sin\theta}{rR}\right]\left[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}\right] d\theta$$

Only need to sweep for 180°



$$= \frac{\mu_0 I R^2}{2\pi r^2}\int_0^{\pi} \frac{\underbrace{(-x\hat{x} + y\hat{y})\sin\theta\cos\theta}_{0} + \underbrace{(x\hat{y})\cos^2\theta}_{\frac{\pi}{2}} - \underbrace{(y\hat{x})\sin^2\theta}_{\frac{\pi}{2}}}{r}\, d\theta$$

☆ By integration result.

$$= \frac{\mu_0 I R^2}{2\pi r^2}\cdot\frac{\pi}{2}\,\boxed{\frac{(x\hat{y} - y\hat{x})}{r}}$$

$$= \frac{\mu_0}{4\pi r^2}\cdot I\pi R^2\cdot\boxed{(\hat{z}\times\hat{r})}$$

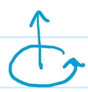
Cross product

$$\hat{z}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix}$$

Define magnetic dipole moment as $\vec{m} = I\vec{a} = I\cdot(\pi R^2\hat{z})$

area in the loop

$$\Rightarrow \boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}}$$



$\vec{m}$

(direction by right hand rule)

From then we can find $\vec{B}$ by $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\Rightarrow \boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3}}$$

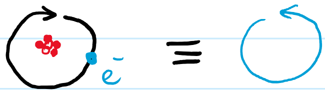
(Skip the steps here)

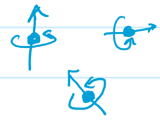
Source of magnetic field in matter

Individual atom has magnetic response due to its electrons

An electron behaves as a magnetic dipole in 2 ways.

[1] Electron rotate about nucleus (orbital motion)

$\Rightarrow$ Equivalent to a current loop 

[2] Electron carry intrinsic magnetic moment (spin) 

Spin = a quantum mechanical property carried

by point particles, describing how it reacts with B field

There is no classical correspondance. The closest math. description is rotation. This is why they are usually depicted as spinning arrows.

The magnetic moment carried by a charged particle is generally descibed as

$$\vec{m} = g \frac{q}{2m} \cdot \vec{L}$$

q, m = charge & mass of the particle

g = g-factor (a proportionality constant). $\vec{L}$ = some rotation vector

For orbital motion : $\begin{cases} g = -1 \\ \vec{L} = \text{angular momentum vector} \end{cases}$

$\sim m\vec{r} \times \vec{v}$ in classical pict. (from definition $I \cdot A = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{q}{2m} \cdot mrv$)

For Spin : $\begin{cases} g \simeq 2.002 \\ \vec{L} = \text{the spin vector} \end{cases}$ } Require quantum mechanics to explain

Note : Proton & Neutron also carries magnetic moment but because $\vec{m} \propto \frac{1}{mass}$, they are too small to compare with electron

($m_e \sim 10^{-31}$ kg. , $m_p, m_n \sim 10^{-27}$ kg)

Microscopic response to magnetic field

Orbital motion & spin of electron react oppositely to B field

- Effect from orbital motion always exist.
- Effect from spin is a lot stronger, but may or may not exist.

↳ If no spin → diamagnetism (orbital effect dominate)
  If have spin → paramagnetism (spin effect dominate)

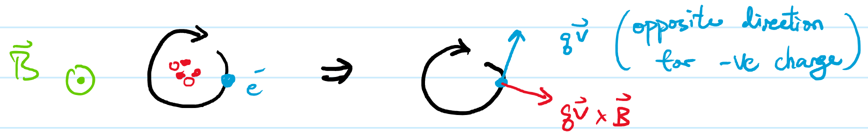
① Diamagnetism = Dipole opposite to applied $\vec{B}$

☆ Diagmatism effect exist in all material due to electron orbital motion

Half Classical Picture : External B field change e^-'s rotation speed

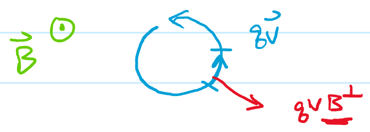
1. Applying a B field

⇒ Electron feels Lorentz force $q\vec{v} \times \vec{B}$



(Here choose $\vec{B}$ to be out of paper. You can also check the case of $\vec{B}$ into paper)

2. Write Newton 2nd Law along the radial direction



only B normal to rotation plane affect the radial force

For rotation, require $F = ma = \frac{mv^2}{r}$

$$\frac{mv^2}{r} = \frac{Zq^2}{r^2} - qvB^{\perp}$$

(Z = no. of nucleons)

$$\Rightarrow \quad v = \frac{-qB^{\perp} + \sqrt{q^2 {B^{\perp}}^2 + \frac{4mZq^2}{r^3}}}{2\frac{m}{r}}$$

require $v = \sqrt{\frac{Ze^2}{mr}}$ when $B = 0$. So take +ve

$$= \sqrt{\frac{Zq^2}{mr} + \left(\frac{qBr}{2m}\right)^2} - \frac{qBr}{2m}$$

$$< \sqrt{\frac{Zq^2}{mr}}$$

$\sqrt{1+x^2} - x < 1$ for any $x > 0$

So rotation speed $\downarrow$. $\Rightarrow$ Angular momentum $|\vec{L}| \downarrow$

$\Rightarrow$ Magnetic moment $|\vec{m}| = g \cdot \frac{q}{2m} |\vec{L}| \downarrow$

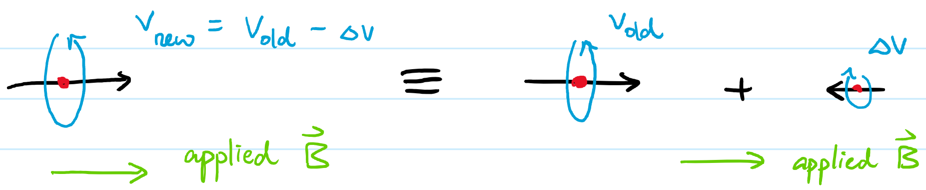
$\Rightarrow$ $\vec{B}$ in the same direction as $\vec{m}$ decreases $|\vec{m}|$

$\equiv$ Diamagnetism.

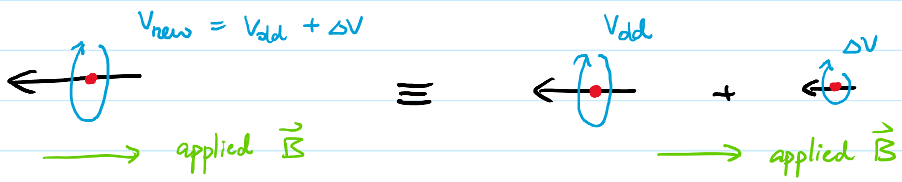
☆ Note that this is a half classical model because it requires radius of rotation to be unchanged, which is a statement from the Bohr model

$\boxed{3}$ Equivently, a small magnetic dipole is created to oppose the applied $\vec{B}$

$\Rightarrow$ Produce a small replusion against the applied $\vec{B}$.



prove this case by yourself



4 Because statiscally, the plane of rotation can be pointing in any direction. Therefore on average, magnetic moment due to V_{old} must be 0.

⇒ Magnetization must be coming from magnetic moment due to ΔV

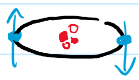
⇒ Diamagnetic material repels applied B field.

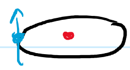
② Paramagnetism = Dipole same direction as applied $\vec{B}$

☆ Paramagnetism is only found in material with unpaired / free electron because Pauli Exclusion Principle require paired electrons to be in opposite spin.

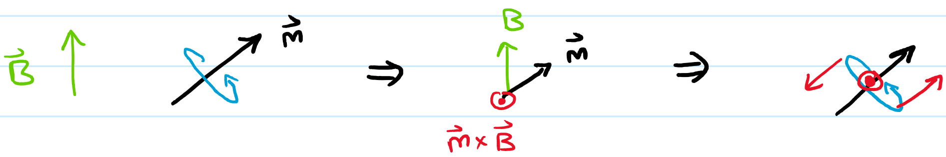
Half Classical Picture : External $\vec{B}$ field flip electron's spin

1 By Pauli exclusion principle, each orbit in an atom can contain at most 1 "up" spin and 1 "down" spin electron.

– Some orbits are fully occupied 

– Some orbits contain only 1 electron 

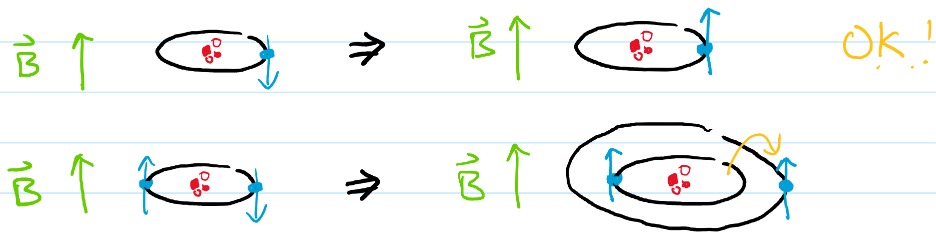
2 Spin is essentially magnetic moment. When B is applied, magnetic moment change direction due to torque $\vec{m} \times \vec{B}$



i.e. Torque will flip magnetic moment to align with B field

(In quantum mechanics, it simply describes as spin aligning to the direction of $\vec{B}$ gives the lowest energy.)

$\boxed{3}$ However not all electron can be flipped because of Pauli exclusion principle. Only unpair electron can be flipped



The only way to flip is by changing orbit which require energy input

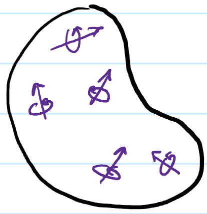
$\boxed{4}$ So the unpaired electron contribute a magnetization the same direction as the applied B field.

$\Rightarrow$ Paramagnetic material attract the applied B field

☆☆☆ Remind that these are just "half-classical" explanation. The real calculation / explanation of experimental relation must rely on calculation from quantum mechanics framework.

Magnetization (field) / Dipole density

Consider an object containing many dipoles. The potential distribution is



$$\vec{A}_{dip}(\vec{r}) = \sum (\text{each dipole contribution})$$

$$= \sum_i \frac{\mu_0}{4\pi} \frac{\vec{m}_i \times \hat{r}_i}{|\vec{r}_i|^2}$$

Extending to continuous distribution, like in Biot-Savat's Law

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{M}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2} \times \left[\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\right] d^3\vec{r}\,'$$

Define : $\vec{M}(\vec{r}) \equiv$ Magnetic dipole density at position $\vec{r}$

This formula is very similar to Biot-Savat's Law except it is $\sim \frac{1}{r^2}$

$$\frac{\mu_0}{4\pi} \iiint \frac{\boxed{\vec{J}(\vec{r}\,')}}{|\vec{r}\,'-\vec{r}|} \times \left[\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\right] d^3\vec{r}\,' \quad \text{V.S.} \quad \frac{\mu_0}{4\pi} \iiint \frac{\boxed{\vec{M}(\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|^2} \times \left[\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\right] d^3\vec{r}\,'$$

Source = current — $V \sim \frac{1}{r}$

Source = dipole — $V \sim \frac{1}{r^2}$

☆☆ $\vec{M}(\vec{r})$ has several names, eg.

- Magnetic dipole density ← Most accurate
- Magnetization field
- Magnetization ← Most common name in book

We can perform the same treatment like electric dipole density $\vec{P}$ to separate $\vec{M}$ into bound current densities.

(However it does not simplify the calculation much)

Bound currents

(After skipping a lot of annoying vector calculus simplification)

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \boxed{\oiint} \frac{\vec{M}(\vec{r}\,') \times \boxed{d^2\vec{r}\,'}}{|\vec{r}-\vec{r}\,'|} + \frac{\mu_0}{4\pi} \boxed{\iiint} \frac{\vec{\nabla}\times\vec{M}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} \boxed{d^3\vec{r}\,'}$$

Integral of some cross product on the material surface

Volume integral in the whole material

How to give these 2 terms physical meaning?

Recall Biot-Savart's Law of potential

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}}{r} \sim \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}}{r}\, d^3\vec{r} \sim \frac{\text{current}}{\text{distance}}$$

The numerator in the above 2 terms can be interpreted as some kind of current distribution

$$\frac{\mu_0}{4\pi}\oiint \frac{\boxed{\vec{M}(\vec{r}\,')\times} d^2\vec{r}\,'}{\boxed{|\vec{r}-\vec{r}\,'|}} \sim \frac{\text{current}}{\text{distance}} \sim \frac{\mu_0}{4\pi}\iiint \frac{\boxed{\vec{\nabla}\times\vec{M}(\vec{r}\,')}}{\boxed{|\vec{r}-\vec{r}\,'|}}\, d^3\vec{r}\,'$$

Defining the bound current distribution

normal vector on the surface

$$\vec{K}_b(\vec{r}\,') \equiv \vec{M}(\vec{r}\,') \times \hat{n} = \text{Surface bound current density}$$

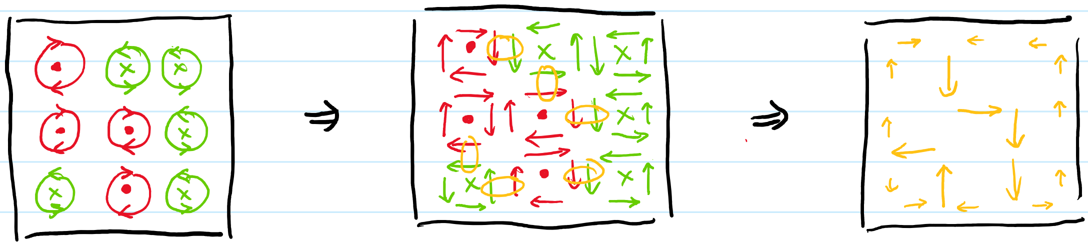
$$\vec{J}_b(\vec{r}\,') \equiv \vec{\nabla}\times\vec{M}(\vec{r}\,') = \text{Volume bound current density}$$

So we can interpret the dipole potential as a result of contribution of 2 kinds of sources

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\oiint \frac{\vec{K}_b(\vec{r}\,')\times d^2\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} + \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}_b(\vec{r}\,')\, d^3\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

$$= \left[\begin{array}{c}\text{Contribution by the}\\ \text{currents that are}\\ \text{on the material surface}\end{array}\right] + \left[\begin{array}{c}\text{Contribution by the}\\ \text{currents that are}\\ \text{inside the material}\end{array}\right]$$

Geometric interpretation of bound currents



If the dipole arrange like this ⇒ Visualize the loops as currents ⇒ Equivalent to have excess current at certain location = "bound current"

Short Summary ①

Given a magnetized object, if we know its dipole density distribution $\vec{M}$, we can compute the following directly:

- The (bound) current distribution on the object

$$\vec{K}_b = \vec{M} \times \hat{n} \quad , \quad \vec{J}_b = \vec{\nabla} \times \vec{M}$$

- The magnetic vector potential & B field it creates

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \times \left[\frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|} \right] d^3\vec{r}\,'$$

$$= \frac{\mu_0}{4\pi} \oint\!\!\oint \frac{\vec{K}_b(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^2\vec{r}\,' + \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}_b(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3\vec{r}\,'$$

and then $\vec{B} = \vec{\nabla} \times \vec{A}$

(However these calculation are very annoying)

☆☆☆ Note that $\vec{M}$ here is the magnetic dipole density naturally carried by the object. To find $\vec{M}$, we need to derive from the material's property (e.g. atomic arrangement)

Auxillary field / H field

$\vec{H}$ field in magnetization is the equivalent to $\vec{D}$ field in polarization

Problem: To learn the magnetic response of a material, one needs to apply external magnetic field to the material

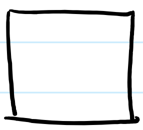
↪ Rearrangement of dipoles form bound current distribution

↪ Bound current distributions emit their own B field

⇒ Total B field = Superposition of the 2

= (Contribution by external B field) + (Contribution by bound current i.e. dipoles in the material)

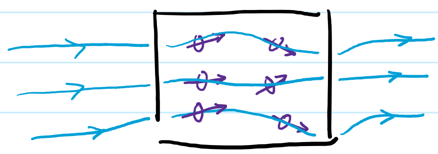
Depending on the material structure, the result B field can be very irregular



No external B field
material shows no magnetization

External B field is
originally very regular

But when put together, the B field distribution may be distorted



Free current distribution

Note that to create the external B field, we also need certain current distribution on our equipment.

⇒ These current distribution is completely controllable

↳ We have full control on current.

⇒ Call these currents "free currents"

$$\begin{cases} \vec{K}_f(\vec{r}) = \text{Surface free current density} \\ \vec{J}_f(\vec{r}) = \text{Volume free current density} \end{cases}$$

(But since we usually create the external B field by solenoid $\vec{K}_f$ is almost all you see. $\vec{J}_f$ is rare)

Then by Ampere's Law,

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B}_{tot}(\vec{r}) = (\text{all current distribution})$$

$$= \begin{pmatrix} \text{bound current} \\ \text{distribute on} \\ \text{material} \end{pmatrix} + \begin{pmatrix} \text{free current} \\ \text{distribute on} \\ \text{setup} \end{pmatrix}$$

$$= \vec{J}_b(\vec{r}) + \vec{J}_f(\vec{r})$$

By definition, $\vec{J}_b = \vec{\nabla} \times \vec{M}(\vec{r})$, so

$$\vec{J}_f(\vec{r}) = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B}_{tot}(\vec{r}) - \vec{\nabla} \times \vec{M}(\vec{r})$$

$$= \nabla \times \left(\frac{1}{\mu_0} B_{tot}(\vec{r}) - \vec{M}(\vec{r}) \right)$$

We can define a new vector field $\vec{H}$

$$\boxed{\vec{H}(\vec{r}) = \frac{1}{\mu_0} \vec{B}_{tot}(\vec{r}) - \vec{M}(\vec{r})}$$

So that $$\boxed{\begin{aligned} \vec{\nabla} \times \vec{H}(\vec{r}) &= \vec{J}_f(\vec{r}) \\ \vec{H}(\vec{r}) \times \hat{n} &= \vec{K}_f(\vec{r}) \end{aligned}}$$ looks like Ampere's Law

Finding $\vec{B}_{tot}$

So far we have 2 approaches to find $\vec{B}_{tot}$

$\boxed{1}$ If we know both $\vec{J}_b / \vec{K}_b$ and $\vec{J}_f / \vec{K}_f$, we can solve the PDE $\vec{\nabla} \times \vec{B}_{tot} = -\nabla^2 \vec{A} = \mu_0 (\vec{J}_b + \vec{J}_f)$

$\boxed{2}$ If we know both $\vec{M}$ and $\vec{H}$, simply use $\vec{B}_{tot} = \mu_0 (\vec{H} + \vec{M})$

Method $\boxed{1}$ requires solving PDE but $\boxed{2}$ does not.

So we would always want to use $\boxed{2}$ if possible.

- $\vec{M}$ can be derived by knowing the material structure (Basically all models require quantum mechanics)
- $\vec{H}$ may be derived from $\vec{J}_f / \vec{K}_f$, while they can be completely controlled by our experimental setup.

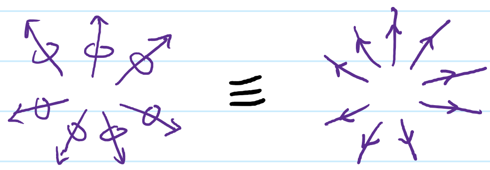
Bad news We cannot fully determine $\vec{H}$ by knowing where the free currents are if the config. is not symmetric

Although we have $\vec{\nabla} \times \vec{H} = \vec{J}_f$ which is similar to $\vec{\nabla} \times \vec{B}_{tot} = \mu_0 \vec{J}_{tot}$

We cannot solve for $\vec{H}$ by this because $\vec{H}$ may not be divergentless

Recall the PDE way to find $\vec{B}_{tot}$:

Given $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and $\vec{B}$ must be divergentless ($\vec{\nabla} \cdot \vec{B} = 0$) as always. We can define $\vec{B} = \vec{\nabla} \times \vec{A}$ and solve $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ The $\vec{B}$ we get is unique	Given $\vec{\nabla} \times \vec{H} = \vec{J}_f$ but $\vec{H}$ may not be divergentless ($\vec{\nabla} \cdot \vec{H}$ = something we don't know) $\Rightarrow$ Cannot continue.

Example of $\vec{H}$ having divergence $\neq 0$: 

Such magnetic dipole gives diverging $\vec{M}$ field ($\vec{\nabla}\cdot\vec{M} \neq 0$)

In order to achieve $\vec{\nabla}\cdot\vec{B}_{tot} = 0$, we require

$$\vec{\nabla}\cdot\vec{H} = \vec{\nabla}\cdot\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{\nabla}\cdot\vec{M} \neq 0$$

Good news : As free current comes from our setup,
We can choose a good layout to make $\vec{\nabla}\cdot\vec{H} = 0$

In symmetric config. we can even solve by Ampere's Law integral form

i.e. $\oint \vec{H}\cdot d\vec{\ell} = I_f \Rightarrow |\vec{H}| = \dfrac{(\text{total free current})}{(\text{total loop perimeter})\cdot \cos\theta}$

Short Summary ②

When applying external B field on a material,

the overall $\vec{B}_{tot}$ can be found by $\frac{1}{\mu_0}B_{tot} = \vec{H} + \vec{M}$

- Finding $\vec{M}$ require knowing the material's property i.e. how the dipole rearrange under a B field

  ↪ Need a model / description of $\vec{M}$ vs $\vec{B}$

- Finding $\vec{H}$ is possible only if the free currents are distributed in a symmetric layout (so that we have 100% certainty that $\vec{\nabla}\cdot\vec{H} = 0$).

  Then we can solve $\vec{\nabla}\times\vec{H} = \vec{J}_f$ just like normal Ampere's Law

Special Model : Linear magnetic material

There exists material with strange magnetization response.

In general, the magnetic dipole density is some function of overall $\vec{B}$

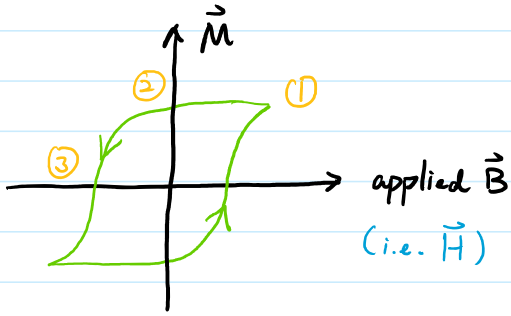
$$\vec{M} = f(\vec{B})$$

E.g. Natural magnet has complicate response

Apply B field → Magnetize → Remove B field → Still magnetize
(1) (2)

→ Apply reverse and stronger field → Demagnetize
(3)

⇒ So called "Hysterisis" behavior

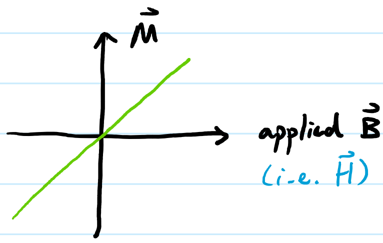


For simplicity, we will only discuss linear magnetic material, i.e.

$$|\vec{M}| \propto |\vec{B}| \propto |\vec{H}|$$

This only applies to simple material that are diamagnetic or paramagnetic

Not ferromagnetic material.



Mathematically we can write

$$\vec{M}(\vec{r}) = \chi_m \vec{H}(\vec{r})$$

Defining the proportionality "constant" $\boxed{\chi_m = \text{magnetic susceptibility}}$

In fact, χ_m can be a matrix (i.e. magnetization is directional)
But for simplicity, you will not see it in textbook questions

With $\begin{cases} \text{Diamagnetic} \Leftrightarrow \chi_m < 0 & \text{(magnetization opposite to applied } \vec{B}) \\ \text{Paramagnetic} \Leftrightarrow \chi_m > 0 & \text{(magnetization along applied } \vec{B}) \end{cases}$

★☆ Note: In electrostatic, χ_e is defined by $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

In magnetostatic, χ_m is defined by $\vec{M} = \chi_m \vec{H}$

This difference is mainly due to convenience in experiments:

- In E field experiment, voltage is the thing we control / measure
  - $\Rightarrow$ $\vec{E}$ is one of the first thing we can calculate ($E = -\vec{\nabla} V$)
- In B field experiment, free current is the thing we control / measure
  - $\Rightarrow$ $\vec{H}$ is one of the first thing we can calculate ($\vec{\nabla} \times \vec{H} = J_f$)

Also by definition of $\vec{H}$

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0} \vec{B}_{tot}(\vec{r}) - \vec{M}(\vec{r})$$

$$= \frac{1}{\mu_0} \vec{B}_{tot}(\vec{r}) - \chi_m \vec{H}(\vec{r})$$

$$(1+\chi_m)\vec{H}(\vec{r}) = \frac{1}{\mu_0} \vec{B}_{tot}(\vec{r})$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0 \boxed{(1+\chi_m)}} \vec{B}_{tot}(\vec{r})$$

$$= \frac{1}{\mu_0 \boxed{\mu_r}} \vec{B}_{tot}(\vec{r})$$

μ_r = relative permeability (no unit)

$$= \frac{1}{\boxed{\mu}} \vec{B}_{tot}(\vec{r})$$

$\mu = \mu_0 \mu_r$ = permeability (same unit as μ_0)

Short Summary ③

Knowing the material being linear makes calculation easy

1. Given we know the free current distribution is symmetric in our setup, we can find $\vec{H}$ from $\vec{J}_f$
2. Knowing the material is linear, $\vec{B}_{tot} = \mu \vec{H}$ inside the material, and $\vec{B}_{tot} = \mu_0 \vec{H}$ in vacuum.
3. Dipole density can also be found by $\vec{M} = \chi_m \vec{H}$ and thus the bound currents.