

- State : State Function vs Process
  - Reviewing 4 most common processes
    - { Formula      True for everything
    - vs
    - True for ideal gas only
  - Example calculation on engine (efficiency & COP)
- 

### Thermodynamics 1<sup>st</sup> Law

$$\delta Q = dU + \delta W$$

- $\delta Q$  = Heat input to the system
- $dU$  = Change in the system's internal energy
- $\delta W$  = W.D. by the system (Chemistry prefers W.D. to system)

Chemistry book prefer  
 $dU = \delta Q + \delta W$

Physics book prefer  
 $\delta Q = dU + \delta W$

This is basically Energy Conservation

### Terminologies in Thermodynamics

#### ① State

A system in a "specific state"

= The system can be "well-distinguished" by a set of parameters

↳ Different numbers  $\Rightarrow$  Different states

#### E.g Ideal Gas System

Every state can be purely described by 3 parameters (P,V,T)

But only 2 of them are independent, because  $PV = nRT$

## E.g. Mechanical System

No. of parameters =  $3+3+3+\dots$

↓ ↓ ↓

We can describe the state of 1 particle by  $(\vec{r}, \vec{v}, \vec{a}, \dots)$

But only  $\vec{r}$  &  $\vec{v}$  are independent, because mechanical system

are constrained by  $\vec{F} = m\vec{a}$ , a 2<sup>nd</sup> order ODE. We only need

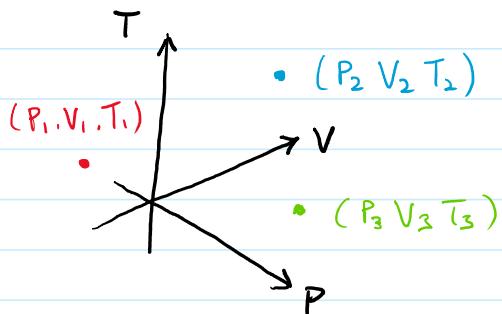
the initial position & velocity to determine subsequent motions

⇒ When there are  $N$  particles, each state is described by

$N$  sets of  $(\vec{r}, \vec{v})$  =  $6N$  parameters.

## (2) State Space

~ Graphical representation of every possible state



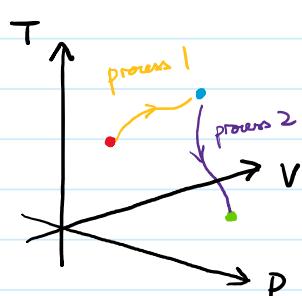
E.g. Every point in the PVT space

= A possible state of an ideal gas

## (3) Process

= The transition from 1 state to another

= Paths in the state space



★ Because only 2 of  $(P, V, T)$  are independent

All ideal gas processes can be projected onto

a 2D plane  $\Rightarrow$  PV diagram

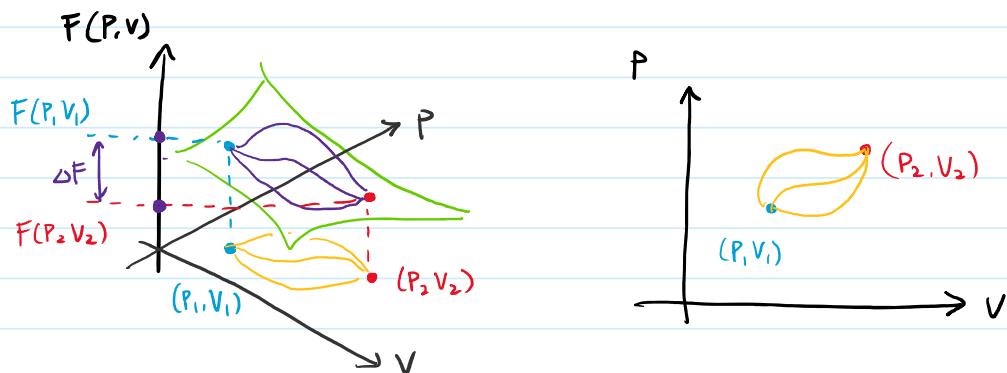
(or you can choose PT / VT plane)

#### ④ State Function

= Any function that depends on the state parameters only

= Function's value is well-defined for every state

E.g. For a scalar state function, we can draw it as a smooth height map



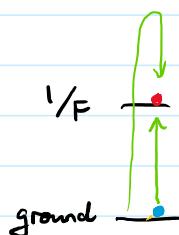
★ You can describe a state function like a conservative field

$$\text{Total change} = \int_{\text{g } F(P, V)}^{\text{any process}} dF = F(P_2, V_2) - F(P_1, V_1)$$

$(P_1, V_1) \rightarrow (P_2, V_2)$

#### Example of State Function

- Internal Energy, E.g. PE, KE



state parameter = height

The net change in PE is independent of the path

The thermodynamics internal energy  $U$  is similar

- All state parameters themselves are state function

E.g. You can always write sth. like  $F(P, V, T) = P$

- Entropy (only in some situations)

## Example of NOT a state function

### ① Work Done

$$\Delta W.D. = P \cdot \Delta V$$

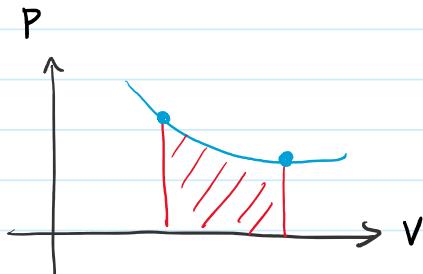
P is a state parameter  
no problem

$\Delta V$  = Change of state parameter  
NOT well-defined on a given state

You cannot define W.D. with one state

W.D. is a property of a process.

From its definition



W.D. is a weighted sum along a process

$$\sum_{\text{segment}} P_i \Delta V_i \sim \int_{\text{process}} P dV$$

= area under curve

### ② Heat

From 1<sup>st</sup> Law  $\delta Q = dU + \delta W$

$dU$  state function  $\delta W$  process dependent

$\therefore \delta Q$  must also be process dependent

As a comparison :

- $dU$  along a close path always = 0
- $\delta W, \delta Q$  along a close path may not be 0

## ★ Notation :

- Change in state function  $\Rightarrow$  use  $d$  e.g.  $dU$
- Change in path dependent function  $\Rightarrow$  use  $\delta$  e.g.  $\delta W, \delta Q$

When appearing in an integral =

$d$   $\Rightarrow$  Independent of path, simply substitute initial & final value

$\delta$   $\Rightarrow$  Have to do line integral explicitly

- Without the given path, we write  $\int \delta W, \int \delta Q$

- After given the path  $C$ , we can write  $\int_C dW, \int_C dQ$

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## Reviewing the 4 main processes

|   |                           |                    |
|---|---------------------------|--------------------|
| { | Isovolumetric (Isochoric) | $V = \text{const}$ |
|   | Isobaric                  | $P = \text{const}$ |
|   | Isothermal                | $T = \text{const}$ |
|   | Adiabatic                 | $\delta Q = 0$     |

We are interested in finding  $dU, \delta W, \delta Q$  for all 4 processes.

(In general, we can promote to any arbitrary process)

Here I split the formula into 2 kinds

Red Bounding box  $\Rightarrow$  True for all materials

Blue font  $\Rightarrow$  For ideal gas only  
∴ Require kinetic theory / Ideal gas law

## ① Internal Energy

$U = \text{State function} = \text{Path independent}$

$$\Delta U = U(P_2, V_2, T_2) - U(P_1, V_1, T_1) \quad \text{for any process}$$

For ideal gas only:

Deriving from kinetic theory of a box of free particle

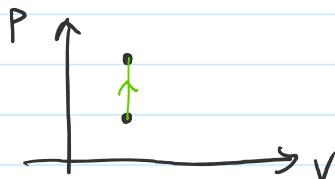
$$U(P, V) = \frac{i}{2} PV = \frac{i}{2} NkT$$

$i = \text{degree of freedom.}$

$$\therefore \Delta U = \frac{i}{2} (P_2 V_2 - P_1 V_1) = \frac{i}{2} Nk (T_2 - T_1)$$

Therefore  $U$  of ideal gas is a state function of  $T$  only

## ② Isovolumetric Process



W.D.

By definition

$$\Delta W = P \underline{dV} = 0$$

$V$  cannot change

### ③ Heat

By 1<sup>st</sup> Law,  $\Delta Q = dU + \Delta W = dU$

So we can write  $\int \Delta Q = \int_{\text{iso } V \text{ path}} dQ = \int dU = U_2 - U_1$

The heat capacity under constant  $V$  can be defined as

$$\int_{\text{iso. } V \text{ path}} dQ = \int C_V(T) dV \Leftrightarrow C_V(T) = \left( \frac{dQ}{dT} \right)_{\text{for } Q \text{ along iso } V \text{ path only}}$$

i.e. Rate of heat input per temperature change

(given that volume cannot change during heat transfer)

For ideal gas only

$$\text{Because } \Delta U = \frac{i}{2} (P_2 V_2 - P_1 V_1) = \frac{i}{2} V (P_2 - P_1)$$

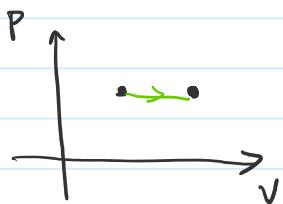
$$= \frac{i}{2} N k (T_2 - T_1)$$

$V = \text{const.}$

$$\Rightarrow \left[ \int_{\text{iso. } V} dQ = \Delta U = \frac{i}{2} V (P_2 - P_1) = \frac{i}{2} N k (T_2 - T_1) \right]$$

$$\Rightarrow \left[ C_v(T) = \left( \frac{dQ}{dT} \right)_{\text{iso. } V} = \frac{dU}{dT} = \frac{i}{2} N k = \text{const.} \right]$$

## ② Isobaric Process (const. P)



W.D.

$$\text{By } \int_{\text{iso. } P} P dV = P \int dV$$

$$= P(V_2 - V_1)$$

$\downarrow P = \text{const.}$

$= Nk(T_2 - T_1)$  for ideal gas

## ③ Heat

$$\text{By 1st Law: } \delta Q = dU + \delta W$$

$$\left[ \int_{\text{iso. } P} dQ = (U_2 - U_1) + P(V_2 - V_1) \right]$$

The heat capacity under constant P can be defined as

$$\left[ \int_{\text{iso. } P} dQ = \int C_p(T) dT \Leftrightarrow C_p(T) = \left( \frac{dQ}{dT} \right)_{\text{iso. } P} = \frac{dU}{dT} + P \frac{dV}{dT} \right]$$

For ideal gas only:

$$\left[ \int_{\text{iso. } P} dQ \right] = \frac{i}{2} (P_2 V_2 - P_1 V_1) + P(V_2 - V_1)$$

$P_2 = P_1$

$$= \frac{i+2}{2} P(V_2 - V_1) = \frac{i+2}{2} Nk(T_2 - T_1)$$

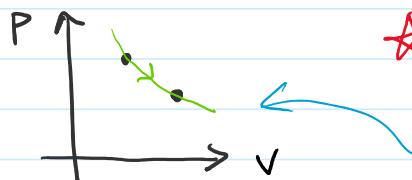
$$\left[ C_p(T) \right] = \frac{d}{dT} \left( \frac{i}{2} NkT \right) + P \cdot \frac{dV}{dT}$$

$$= \frac{i}{2} Nk + P \cdot \frac{d}{dT} \left( \frac{NkT}{P} \right)$$

$$= \frac{i}{2} Nk + NK$$

$$= \frac{i+2}{2} Nk = \text{const.}$$

### ③ Isothermal Process (const. T)



\* Iso. T curve on PV diagram depends on the material

This shape of curve is only true for ideal gas

W.D.

The form  $\left[ \int P dV \right]$  cannot be further simplified unless you know the relation between (i.e. knowing the material's property)

For ideal gas only:

The relation of  $P, V, T$  is well known :  $P = \frac{NkT}{V}$

Under constant  $T$ , this is enough for computing the integral

$$\left[ \int P dV = \int \frac{NkT}{V} dV = NkT \ln \left( \frac{V_2}{V_1} \right) \right]$$

## 2] Heat

Again, if relation of P, V, T is unknown

We cannot do any simplification.  $\boxed{\int \delta Q = U_2 - U_1 + \int pdV}$

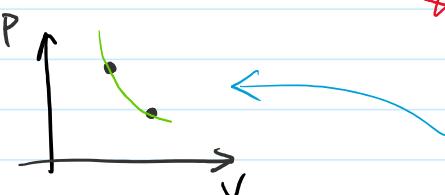
For ideal gas only:

Because ideal gas's internal energy solely depends on T

$$\Delta U = \frac{1}{2} Nk(T_2 - T_1) = 0 \text{ if } T \text{ is fixed}$$

$$\Rightarrow \boxed{\int_{\text{iso-T}} \delta Q = \int PdV = NkT \ln\left(\frac{V_2}{V_1}\right)}$$

## ④ Adiabatic ( $\delta Q = 0$ )



\* Again, the shape of the curve depends on the material

This shape is for ideal gas only

1] Heat :  $\boxed{\delta Q = 0}$  By definition

2] W.D. : By 1<sup>st</sup> Law  $\delta Q = dU + dW$

$$\boxed{dW = -dU}$$

**★★★** If we know the relation of U with P, V,

we can derive the adiabatic relation

For ideal gas only:

The adiabatic relation is derived as follows :

$\therefore$  We know  $U = \frac{1}{2}PV$  for ideal gas  $\Rightarrow dU = \frac{1}{2}(pdV + VdP)$

Substitute into  $-dU = \delta W$

$$-\frac{i}{2}(pdV + Vdp) = pdV$$

$$\frac{-i}{2} Vdp = \frac{i+2}{2} pdV$$

$$\frac{1}{p} dp = -\left(\frac{i+2}{2}\right) \frac{1}{V} dV$$

$$\int \frac{1}{p} dp = -\left(\frac{i+2}{2}\right) \int \frac{1}{V} dV$$

$$\ln P = -\left(\frac{i+2}{2}\right) \ln V + \text{const.}$$

$$\ln(PV^{\frac{i+2}{2}}) = \text{const.}$$

$$\boxed{PV^{\frac{i+2}{2}} = \text{const.}}$$

Adiabatic relation  
of ideal gas

★ In general, the adiabatic relation of most material

look like  $\boxed{PV^\gamma = \text{const.}}$

The index  $\gamma$  is called adiabatic constant

For ideal gas,  $\gamma = \frac{i+2}{i} = \frac{C_p}{C_v} = \text{const.}$

which is a numerical coincidence

However this is NOT correct for other materials

(The adiabatic "constant" may not even be a constant)

## Summary in a table

red underline = always True

blue = ideal gas only

|           | $dU$<br>is a state function   | $\delta W$<br>by definition: $\int pdV$  | $\delta Q$<br>by 1 <sup>st</sup> law: $\delta Q = dU + \delta W$   |
|-----------|---|--|--|
| Iso. V    | $\underline{U(P_2V_2T_2) - U(P_1V_1T_1)}$<br>$= \frac{i}{2}(P_2V_2 - P_1V_1)$<br>$= \frac{i}{2}NK(T_2 - T_1)$ | 0  | $\underline{\Delta U + 0}$<br>$= \underline{\int C_V(T) dT}$<br>$= \frac{i}{2} \underline{NK(T_2 - T_1)}_{C_V}$              |
| Iso. P    | (Same as iso. V)  | $P(V_2 - V_1)$<br>$= NK(T_2 - T_1)$  | $\underline{\Delta U + P(V_2 - V_1)}$<br>$= \underline{\int C_P(T) dT}$<br>$= \frac{i+2}{2} \underline{NK(T_2 - T_1)}_{C_P}$ |
| Iso. T    | (Same as iso. V)<br>$= 0$ for ideal gas   | $\int pdV$ ( <u>no further reduction</u> )<br>$= NKT \ln\left(\frac{V_2}{V_1}\right)$              | $\underline{\Delta U + \int pdV}$<br>$= NKT \ln\left(\frac{V_2}{V_1}\right)$   |
| Adiabatic | (Same as iso. V)  | $\underline{\delta W = -dU}$<br>$= -\frac{i}{2}(P_2V_2 - P_1V_1)$<br>$= -\frac{i}{2}NK(T_2 - T_1)$ | $\underline{0}$<br>(By definition)   |

## Standard thermal cycle practice problem



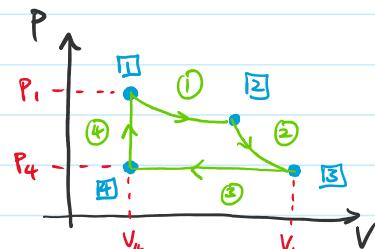
The most common question to ask :  
What is the engine efficiency? COP?

Some general steps to follow :

- ① Write down the relation of parameters at different states  
(Depends on the process that connects them)
- ② Write down the  $dU$ ,  $dW$ ,  $dQ$  of each process
- ③ Identify if the  $dQ$  of each process is an input or output
- ④ Calculate the efficiency / COP by formula

Example 1 : A cycle with all 4 kinds of process

1. Iso T (expand)



2. Adiabatic (expand)

3. Iso P (contract)

4. Iso V

(Assume running on ideal gas)

Step 1 : Write down the relation of parameters at different states

①

$$P_1, V_1, T_1$$

②

$$P_2, V_2, T_2$$

③

$$P_3, V_3, T_3$$

④

$$P_4, V_4, T_4$$

$$\boxed{1} \rightarrow \boxed{2} = \text{iso } T \Rightarrow T_1 = T_2$$

You can also use ideal gas law to get more relation

$$\boxed{2} \rightarrow \boxed{3} = \text{adiabatic} \Rightarrow P_2 V_2^\gamma = P_3 V_3^\gamma$$

$$\boxed{3} \rightarrow \boxed{4} = \text{iso } P \Rightarrow P_3 = P_4$$

But do it only if you really need (Don't waste time)

$$\boxed{4} \rightarrow \boxed{1} = \text{iso } V \Rightarrow V_4 = V_1$$

Step 2 : Write down the  $dU$ ,  $SW$ ,  $SO$  of each process

$$\boxed{1} \rightarrow \boxed{2} = \text{iso. T} \quad \left\{ \begin{array}{l} \Delta U = 0 \\ \int SW = NkT_1 \ln\left(\frac{V_2}{V_1}\right) \\ \int SO = NkT_1 \ln\left(\frac{V_2}{V_1}\right) > 0 \end{array} \right.$$

$$\boxed{2} \rightarrow \boxed{3} = \text{adiabatic} \quad \left\{ \begin{array}{l} \Delta U = \frac{i}{2}(P_3V_3 - P_2V_2) = \frac{i}{2}Nk(T_3 - T_2) \\ \int SW = \frac{i}{2}(P_3V_3 - P_2V_2) = \frac{i}{2}Nk(T_3 - T_2) \\ \int SO = 0 \end{array} \right.$$

$$\boxed{3} \rightarrow \boxed{4} = \text{iso. P} \quad \left\{ \begin{array}{l} \Delta U = \frac{i}{2}P(V_4 - V_3) = \frac{i}{2}Nk(T_4 - T_3) \\ \int SW = P(V_4 - V_3) = Nk(T_4 - T_3) \\ \int SO = \frac{i+2}{2}P(V_4 - V_3) = \frac{i+2}{2}Nk(T_4 - T_3) < 0 \end{array} \right.$$

$$\boxed{4} \rightarrow \boxed{1} = \text{iso. V} \quad \left\{ \begin{array}{l} \Delta U = \frac{i}{2}V_4(P_1 - P_4) = \frac{i}{2}Nk(T_1 - T_4) \\ \int SW = 0 \\ \int SO = \frac{i}{2}V_4(P_1 - P_4) = \frac{i}{2}Nk(T_1 - T_4) > 0 \end{array} \right.$$

Step 3 : Identify whether each  $SO$  is an input or output

★★ Recall in the Physics convention of 1<sup>st</sup> law

$$SO = dU + SW$$

↑ heat input to the system      ↑ change in U of the system      ↑ W.D. by the system

⇒ Any  $SO > 0$  is input  
 $SO < 0$  is output

⇒ Path with heat input :  $\boxed{1} \rightarrow \boxed{2}$ ,  $\boxed{4} \rightarrow \boxed{1}$

heat output :  $\boxed{3} \rightarrow \boxed{4}$

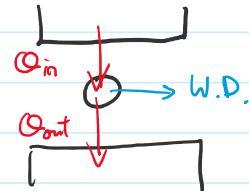
Step 4 : Calculate efficiency / COP by formulae

By definition :

$$\text{Efficiency} = \eta = \frac{\text{W.D.}}{\text{Heat input}} = 1 - \left| \frac{\text{Heat input}}{\text{Heat output}} \right|$$

i.e. What is the portion of heat input

can the engine convert into W.D.?



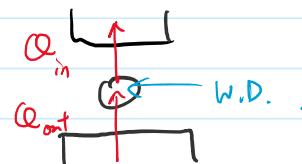
Eg. Efficiency of the above engine , by formulae

$$= 1 - \left| \frac{\frac{i+2}{2} Nk(T_4 - T_3)}{\frac{i}{2} Nk(T_1 - T_4) + NkT_1 \ln\left(\frac{V_2}{V_1}\right)} \right|$$

\* You may further simplify the expression by the state parameter relations from step 1 .

Also , if we operate the engine reversely , it becomes a refrigerator

i.e. Remove heat by applying work



$$\begin{aligned} \text{COP} \\ (\text{Coefficient of Performance}) &= \frac{\text{Heat remove}}{\text{W.D.}} = \left| \frac{\text{Heat output}}{\text{Heat input} - \text{Heat output}} \right| \end{aligned}$$

$$= \frac{1-\eta}{\eta} \quad (\text{After some algebra derivation})$$

## Example 2 : System of Photon gas

Photon's internal energy & pressure are given by

$$\left\{ \begin{array}{l} U = 3PV = aVT^4 \\ P = \frac{1}{3}aT^4 \end{array} \right. \quad \text{with } a = \text{some const.}$$

↑ substitute to get

For comparison,  
the formulae for ideal gas are

$$\left\{ \begin{array}{l} U = \frac{i}{2}PV = \frac{i}{2}NkT \\ P = \frac{NkT}{V} \end{array} \right. \quad \text{which we cannot use.}$$

\* Photon is NOT ideal gas. We cannot use any formulae that are derived from kinetic theory or ideal gas law.

(Any blue formulae on the table)

We must re-derive the  $dU$ ,  $dW$ ,  $dQ$  of 4 processes.

\* Remember  $U$  is always a state function.  $\Delta U$  is independent of process

$$\Delta U = \underline{U(P_2, V_2, T_2) - U(P_1, V_1, T_1)} \quad \text{true for all system.}$$

$$\begin{aligned} &= \underline{3(P_2V_2 - P_1V_1)} \\ &= \underline{a(V_2T_2^4 - V_1T_1^4)} \end{aligned} \quad \begin{aligned} &\text{Using photon gas's formulae} \\ &U = 3PV = aVT^4 \end{aligned}$$

### II Iso V.

$$-\Delta U = 3V(P_2 - P_1) = aV(T_2^4 - T_1^4)$$

$$-\int dW = \int pdV = \underline{0} \quad (\text{always true, by definition})$$

$$-\int dQ = \underline{\Delta U + 0} = 3V(P_2 - P_1) = \boxed{aV(T_2^4 - T_1^4)}$$

$$= \int C_V(T) dT$$

$$\Rightarrow C_V(T) = \left( \frac{dQ}{dT} \right)_{\text{Iso } V} = \frac{dU}{dT} = \boxed{4aVT^3}$$

## 2 Iso P (Note that for photon gas, $\text{iso P} = \text{iso T}$ )

\* Note : For photon gas,  $\underline{\text{iso P}} = \underline{\text{iso T}}$ .

$\because$  By  $P = \frac{1}{3}aT^4$ , if  $P$  is fixed, then  $T$  is fixed!

$$-\Delta U = 3P(V_2 - V_1) = aT^4(V_2 - V_1)$$

$$-\int \delta W = \underline{\int P dV} = P(V_2 - V_1) = \boxed{\underline{\frac{1}{3}aT^4(V_2 - V_1)}}$$

$$-\int \delta Q = \underline{\Delta U + \int \delta W} = 4P(V_2 - V_1) = \boxed{\underline{\frac{4}{3}aT^4(V_2 - V_1)}}$$

$$\text{BUT } C_p(T) = \left( \frac{dQ}{dT} \right)_{\text{iso P}} = \boxed{\underline{\text{undefined.}}}$$

$\therefore$  Under constant pressure, no matter how much  $Q$  is input to the system the temperature cannot change!

$$\int_{\text{iso P}} dQ = \int C_p(T) dT = \int (\text{undefined}) \cdot (0)$$

## 3 Adiabatic

$$-\int \delta Q = 0 \quad (\text{always true, by definition})$$

$$-\underline{\delta W} = -dU$$

$$P dV = -d(3PV)$$

$$= -3(PdV + VdP)$$

$$4PdV = 3VdP$$

$$\frac{4}{3} \int \frac{1}{V} dV = - \int \frac{1}{P} dP$$

$$\ln(V^{\frac{4}{3}}) = \ln(P^{-1}) + \text{const.}$$

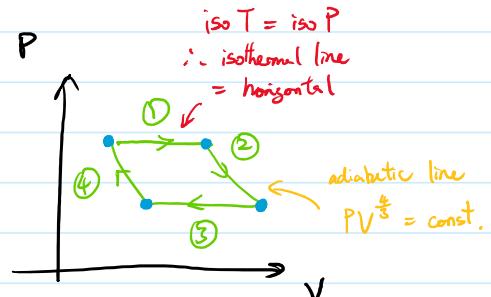
$$\boxed{PV^{\frac{4}{3}} = \text{const}}$$

$\therefore$  Adiabatic const. for photon gas =  $\frac{4}{3}$  and obviously  $\neq \frac{C_V}{C_P}$

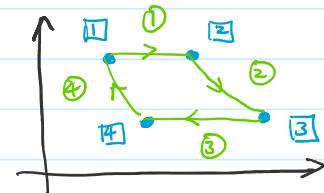
### Example 3    Carnot cycle by photon gas.

By definition, Carnot cycle is made of :

- ① Iso T. expansion
- ② Adiabatic expansion
- ③ Iso. T. contraction
- ④ Adiabatic contraction



Step 1 : Write out the relation between state parameter of different states.



$$\boxed{1} \rightarrow \boxed{2} = \text{iso. P/T} \Rightarrow P_1 = P_2, T_1 = T_2$$

$$\boxed{2} \rightarrow \boxed{3} = \text{adiabatic} \Rightarrow P_2 V_2^{\frac{4}{3}} = P_3 V_3^{\frac{4}{3}}$$

$$\boxed{3} \rightarrow \boxed{4} = \text{iso P/T} \Rightarrow P_3 = P_4, T_3 = T_4$$

$$\boxed{4} \rightarrow \boxed{1} = \text{adiabatic} \Rightarrow P_4 V_4^{\frac{4}{3}} = P_1 V_1^{\frac{4}{3}}$$

Step 2 : Write down  $\Delta U$ ,  $\delta W$ ,  $\delta Q$  of each process

and Step 3 : Identify whether each  $\delta Q$  is an input or output

$$\boxed{1} \rightarrow \boxed{2} = \text{iso P/T} \quad \left\{ \begin{array}{l} \Delta U = a T_1^4 (V_2 - V_1) \\ \int \delta W = \frac{1}{3} a T_1^4 (V_2 - V_1) \\ \int \delta Q = \frac{4}{3} a T_1^4 (V_2 - V_1) > 0 \end{array} \right.$$

$$\boxed{2} \rightarrow \boxed{3} = \text{adiabatic} \quad \left\{ \begin{array}{l} \Delta U = a (V_3 T_3^4 - V_2 T_2^4) \\ \int \delta W = -a (V_3 T_3^4 - V_2 T_2^4) \\ \int \delta Q = 0 \end{array} \right.$$

$$[3] \rightarrow [4] = \text{iso P/T}$$

$$\left\{ \begin{array}{l} \Delta U = a T_3^4 (V_4 - V_3) \\ \int \delta Q = \frac{1}{3} a T_3^4 (V_4 - V_3) \\ \int \delta Q = \frac{4}{3} a T_3^4 (V_4 - V_3) < 0 \end{array} \right.$$

$$[4] \rightarrow [1] = \text{adiabatic}$$

$$\left\{ \begin{array}{l} \Delta U = a (V_1 T_1^4 - V_4 T_4^4) \\ \int \delta Q = -a (V_1 T_1^4 - V_4 T_4^4) \\ \int \delta Q = 0 \end{array} \right.$$

Step 4 : Compute engine efficiency

$$\eta = 1 - \left| \frac{\text{Heat output}}{\text{Heat input}} \right| = 1 - \left| \frac{\frac{4}{3} a T_3^4 (V_4 - V_3)}{\frac{4}{3} a T_1^4 (V_2 - V_1)} \right|$$

The expression can be simplified using relations in step 1

By adiabatic relation

$$PV^{\frac{4}{3}} = \text{const}$$

$$\frac{1}{3} a T^4 V^{\frac{4}{3}} = \text{const}$$

$$T V^{\frac{1}{3}} = \text{const} \Rightarrow \underline{T^3 V = \text{const}}$$

From the relations between state parameters :

$$\left\{ \begin{array}{l} T_1 = T_2 , T_3 = T_4 \\ T_2^3 V_2 = T_3^3 V_3 , T_4^3 V_4 = T_1^3 V_1 \end{array} \right.$$

$$\begin{aligned} \Rightarrow \eta &= 1 - \left| \frac{\frac{T_3^4 (V_4 - V_3)}{T_1^4 (V_2 - V_1)}}{\frac{T_3^3 V_4}{T_2^3 V_2} - \frac{T_3^3 V_3}{T_1^3 V_1}} \right| \\ &= 1 - \frac{T_3}{T_1} \left| \frac{\frac{T_4^3 V_4}{T_2^3 V_2} - \frac{T_3^3 V_3}{T_1^3 V_1}}{\frac{T_4^3 V_4}{T_2^3 V_2} - \frac{T_3^3 V_3}{T_1^3 V_1}} \right| \\ &= 1 - \frac{T_3}{T_1} \end{aligned}$$

$\frac{T_4^3 V_4}{T_2^3 V_2} = T_1^3 V_1$   
 $\frac{T_3^3 V_3}{T_1^3 V_1} = T_2^3 V_2$

which is the same as Carnot cycle of ideal gas !