Magnetic Induction

by Tony Shing

Overview:

- Lorentz force

- Faraday's law, motional/transformer EMF & Lenz's law

In electromagnetism, theoretically every problem can be solved through a set of PDEs called the **Maxwell Equations**.

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad\qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\longrightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad\qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

However, a *system of PDEs* is too complicated to be solved. So we need to learn different "tricks" to avoid them, which are enough for some simple scenarios.

Electrostatics only concerns the 3rd equation of the set - Faraday's law.

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0   A Brief History of Electromagnetism

Electromagnetic induction is likely the most confusing topic in beginner E&M. In my opinion, knowing a bit of the history is helpful to unify the concepts you have learnt.

Year	Who	Advancement
<1500s		Different electrostatics phenomena were known. But they were not unified or explained at all.
1600	William Gilbert	The first person to use the word "electrical" to describe electrostatics phenomena. Also the first to propose that electrical effect is due to flows of particles.
1750	Benjamin Franklin	Developed a one "fluid" theory of electricity, and called this fluid "charge".
1784	Charles-Augustin de Coulomb	Experimentally showed that force between charged objects $\propto \frac{1}{r^2}$. (Coulomb's law $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$)
1800	Alessandro Volta	Made the first battery from electro-chemistry. (First time to have steady current.)

1820	Hans Christian Ørsted	Discovered that current wire can deflect compress. (First time to relate electric and magnetic phenomena.)
1820	André-Marie Ampère	Formulated and verified the force between current wires. ($F = I\vec{l}_1 \times \frac{\mu_0 I}{2\pi r}\vec{l}_2$)
1831	Michael Faraday	Discovered and experimented on magnetic induction, include - Across iron core: Reading appears at the instant when switch is on/off. - Moving frame: Reading appears when wireframe moves, changes shape or when magnetic field change. insertFig
1834	Emil Lenz	Explained direction of induced current by energy conservation. (Lenz's Law)
1860	James Clerk Maxwell	- Unified past discoveries into 20 equations. - Introduced field description. This is the first time we write **E** and **B**. Before Maxwell, everything was described in terms of force.
1893	Oliver Heaviside	Unified the 20 equations from Maxwell to 4, by vector calculus. (This is the version of Maxwell's equation we now know.)
1895	Hendrik Lorentz	Derive the correct force on charges under both $\vec{E}$ and $\vec{B}$. (Lorentz force law)

1   Motion under Lorentz Force

Nowadays we know that Lorentz force is the fundamental explanation to magnetic induction. But interestingly, it was formulated correctly only in the very late history of classical E&M.

$$\boxed{m\vec{a} = \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}}$$

In general, $\vec{E}$ and $\vec{B}$ may be function of time and position. Below we will only discuss the special case when the fields are constant. By symmetry, we can assume that $\vec{B}$ is only in $\hat{z}$ direction, i.e. $\vec{B} = B_z\hat{z}$. The Newton's 2$^{\text{nd}}$ law then writes as

$$m\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = q\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} + q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B_z \end{vmatrix}$$

which turns into a system of 3 1$^{\text{st}}$ order linear ODEs.

$$\begin{cases} m\dfrac{\mathrm{d}v_x}{\mathrm{d}t} = qE_x + qv_yB_z \\[2mm] m\dfrac{\mathrm{d}v_y}{\mathrm{d}t} = qE_y - qv_xB_z \\[2mm] m\dfrac{\mathrm{d}v_z}{\mathrm{d}t} = qE_z \end{cases}$$

2 Magnetic Induction

2.1 EMF - Force or Voltage?

The term **electromotive force** (EMF) was invented by Alessandro Volta in 1801, for explaining observations in electrochemistry.

Metal electrodes in electrolyte

   $\Rightarrow$ Current generates spontaneously.

   $\Rightarrow$ There must be some kind of "force" pushing the current!

(add figure here: electrolyte)

In early 1800s, scientists tended to describe things like mechanical system, i.e. any motions of objects must be driven by some kind of "force". So when there is a steady current, there must be a "force" that keep pushing the charges forward. namely the "electromotive force".

   – However, EMF took the unit of volt because voltage measurement was the only way to quantify the magnitude of EMF.

   – Later when Michael Faraday discovered the phenomena of magnetic induction, he used the same word to refer to the source of the induced current.

Today we still keep the name as a "force", even though we already know much better how current is generated in different sources. Maybe because people are already used to refer the same word "EMF" as the general name for ANY kind of current source, rather than discriminating them by their origin of energy.

(add figure here: i dun care)

Here we only focus on the two kinds of EMF that relate to magnetic induction phenomena.

   – **Motional EMF** : Source of energy = Motion of the charges' "container". Can be explained via Lorentz force.

   – **Transformer EMF** : Source of energy = Change in magnetic field. Explanation requires relativity.

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3   Basic Problems in Magnetic Induction

— The End —