

Relativistic Energy & Momentum

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Overview:

- 4-velocity and 4-momentum
- Common applications of 4-momentum
- Spacetime interval

1 The 4-vector Family

4-vectors are "vector-like objects" that

- Are constructed by physical quantities (the "observables").
- Respect the rule of Lorentz transformation for vectors.

Plainly speaking, we can write the values of some physical quantities into a combination of 4 items, such that by multiplying with Lorentz transformation matrix, it yields the values of the quantities that are observed by another observer.

$$\begin{array}{ccc}
 \begin{array}{l} \text{A 4x1 vector} \\ \text{made of } \lambda, \mu, \nu, \xi \\ \text{Substitute the values} \\ \text{that are observed by B} \end{array} & \longrightarrow & \begin{array}{c} \left(\begin{array}{c} \lambda' \\ \mu' \\ \nu' \\ \xi' \end{array} \right) \\ \hline \end{array} = \begin{array}{c} \left(\begin{array}{cccc} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \left(\begin{array}{c} \lambda \\ \mu \\ \nu \\ \xi \end{array} \right) \\ \hline \end{array} \longleftarrow \begin{array}{l} \text{A 4x1 vector} \\ \text{made of } \lambda, \mu, \nu, \xi \\ \text{Substitute the values} \\ \text{that are observed by A} \end{array} \\
 & & \begin{array}{c} \uparrow \\ \text{The Lorentz Transformation} \\ \text{A 4x4 matrix} \end{array}
 \end{array}$$

We have already seen the example of **4-position vector** $\vec{X} = \begin{pmatrix} ct & x & y & z \end{pmatrix}$, which is constructed by the time and position coordinate of an event, and can be used to show the values of the event's coordinate according to different observers.

$$\begin{array}{ccc}
 \begin{array}{l} \text{A 4x1 vector} \\ \text{made of } t, x, y, z \\ \text{Substitute the values} \\ \text{that are observed by B} \end{array} & \longrightarrow & \begin{array}{c} \left(\begin{array}{c} ct' \\ x' \\ y' \\ z' \end{array} \right) \\ \hline \end{array} = \begin{array}{c} \left(\begin{array}{cccc} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \left(\begin{array}{c} ct \\ x \\ y \\ z \end{array} \right) \\ \hline \end{array} \longleftarrow \begin{array}{l} \text{A 4x1 vector} \\ \text{made of } t, x, y, z \\ \text{Substitute the values} \\ \text{that are observed by A} \end{array}
 \end{array}$$

Starting from 4-position vector, we can construct other types of 4-vectors used in relativistics mechanics.

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