

Uncertainty, Evidence, and Truth in Non Axiomatic Logic

To analyze sentences like “Penguins are birds” or “Birds fly,” we first need to understand what truth actually means in this framework. In everyday thinking or classical logic, truth is often treated as absolute. A statement is either right or wrong. But in Non Axiomatic Logic (NAL), truth is not meant to perfectly describe an objective world, because a system never has complete knowledge. Instead, truth is a summary of the system’s experience.

Because of this, we cannot simply label statements as true or false. Every conclusion must be evaluated under the Assumption of Insufficient Knowledge and Resources (AIKR). The system never knows everything, and it does not have unlimited time. So any judgment is essentially a prediction based on past evidence. Uncertainty is not an error here. It is a natural consequence of reasoning with incomplete experience.

To evaluate statements, NAL uses evidence accumulated from observation. This evidence is summarized numerically using frequency and confidence, forming a two dimensional truth representation.

I. Evidence: The Basis of Reasoning

Reasoning in NAL is driven by experience based evidence. Evidence is not formal proof. It is accumulated support for or against a statement of the form:

$S \rightarrow P$

(Subject is a Predicate)

The system evaluates how observed instances of S relate to P.

Positive evidence (w^+) supports the statement.

Example: observing a robin with feathers supports “Robins are birds.”

Negative evidence (w^-) contradicts the statement.

Example: an observation that conflicts with bird characteristics.

Total evidence

$$w = w^+ + w^-$$

This total matters because it reflects how experienced the system is. A conclusion based on many observations is more reliable than one based on a few.

II. Measuring Uncertainty: Frequency and Confidence

NAL avoids collapsing uncertainty into a single probability number. A single value mixes randomness with ignorance. Instead, truth is expressed with two components:

Frequency (f)

Measures how often the statement is supported:

$$f = w^+ / w$$

It answers: *how often does this appear true?*

Confidence (c)

Measures reliability:

$$c = w / (w + k)$$

The parameter k represents system cautiousness. Even perfect frequency never guarantees perfect confidence when evidence is limited.

Truth value representation:

$$\langle f, c \rangle$$

This separation allows the system to distinguish strong evidence from weak guesses.

III. Representing Uncertainty: Three Equivalent Forms

NAL represents uncertainty in three interchangeable formats.

1. Evidence Form (w^+ , w^-)

This preserves raw observational history.

2. Truth Value $\langle f, c \rangle$

This is the operational form used during reasoning.

3. Truth Interval $[L, U]$

This expresses uncertainty as a range:

$$L = f \cdot c$$

$$U = 1 - c(1 - f)$$

As confidence increases, the interval narrows.

These forms describe the same knowledge from different perspectives and prevent hasty generalization.

IV. Concrete Analysis and Representation

To understand uncertainty fully, we estimate evidence values for specific statements. For these examples, $k = 1$.

(a) “Penguins are birds”

Context: A taxonomic truth strongly supported by biological observation.

Estimated evidence:

$$w^+ = 50$$

$$w^- = 0$$

$$w = 50$$

Form	Representation	Calculation / Note
Evidence	$w^+ = 50, w^- = 0$	Strong recurring support
Truth Value	$\langle 1.0, 0.98 \rangle$	$f = 1.0, c \approx 50/51$
Interval	$[0.98, 1.0]$	Very high certainty

Interpretation: Perfect observed consistency with high experiential confidence.

(b) “Birds fly”

Context: A general rule with exceptions.

Estimated evidence:

$$w^+ = 40$$

$$w^- = 10$$

$$w = 50$$

Form	Representation	Calculation / Note
Evidence	$w^+ = 40, w^- = 10$	Exceptions acknowledged
Truth Value	$\langle 0.80, 0.98 \rangle$	Majority support
Interval	$[0.78, 0.80]$	Narrow due to high experience

Interpretation: The system confidently recognizes a strong trend with known exceptions.

(c) “Painting is drawing”

Context: Conceptual overlap with ambiguity.

Estimated evidence:

$$w^+ = 2$$

$$w^- = 8$$

$$w = 10$$

Form	Representation	Calculation / Note
Evidence	$w^+ = 2, w^- = 8$	Mostly distinction
Truth Value	$\langle 0.20, 0.91 \rangle$	Low frequency
Interval	$[0.18, 0.27]$	Wider uncertainty

Interpretation: Limited experience and conflicting evidence produce broader uncertainty.

V. Language, Heuristics, and “Enough Thinking”

Although NAL represents belief using precise numerical structures, human reasoning rarely operates in explicit numbers. Instead,

people communicate uncertainty through linguistic approximations. We do not normally say, “This belief has frequency 0.8 and confidence 0.9.” We say, “Birds usually fly.” These expressions are not imprecise mistakes. They are practical containers that encode probabilistic meaning in natural language.

From the perspective of NAL, such phrases correspond to structured truth values. Everyday language implicitly separates two dimensions of belief: how often something appears true (frequency) and how strongly we trust that estimate (confidence). This mapping shows that natural language is not opposed to formal reasoning. It compresses numerical uncertainty into intuitive categories that allow fast communication and decision-making.

For example:

- “Always” or “Everyone” approximates a frequency near 1.0, reflecting overwhelming supporting evidence.
- “Usually” or “Mostly” suggests a strong but non-absolute trend.
- “Rarely” or “Hardly ever” indicates low observed frequency.
- “I guess” or “Presumably” signals low confidence due to limited experience.
- “Absolutely” or “Certainly” expresses high confidence based on extensive evidence.

These linguistic forms allow humans to communicate uncertainty efficiently while still preserving meaningful distinctions about reliability.

The Principle of “Enough Thinking”

Intelligent agents operate with limited time, memory, and computational resources, so seeking absolute certainty is often impractical. In real environments, decisions must be made with incomplete information. The principle of “Enough Thinking” captures this reality: instead of waiting for perfect certainty, a system acts once available evidence reaches a sufficient confidence threshold. Remaining uncertainty is accepted as unavoidable.

A simple survival analogy makes this clear. Delaying action until certainty is complete risks failure, while acting on strong evidence allows timely response. Real intelligence is therefore not about perfect accuracy, but about balancing correctness with speed.

Why This Matters for General Intelligence

General intelligence depends on functioning under uncertainty. Environments are dynamic, and waiting for perfect information can be worse than acting on reliable estimates. NAL formalizes this by representing beliefs as graded, enabling decisions based on confidence thresholds, and allowing beliefs to evolve with new evidence. Human reasoning already follows this pattern through expressions like “probably” or “almost certain.” NAL turns that intuition into a structured reasoning framework.

VI. Probability in Modern AI

The final question asks whether modern AI, especially deep learning and large language models, has actually solved the uncertainty problem discussed in the lecture.

On the surface, it looks like it has. Almost every modern AI system is built on probability: softmax outputs, Bayesian priors, loss functions, likelihood estimates. The math is everywhere.

But the real question is not whether AI uses probability. It is **what kind of uncertainty** that probability actually represents.

From the perspective of NAL, modern AI is still missing a crucial distinction. It treats uncertainty as a single optimized number, while intelligent reasoning requires separating statistical tendency from epistemic confidence. This difference creates what can be called an optimization trap: systems become extremely good at fitting past data, but fragile when facing the unknown.

1. The “Global” Illusion: Probability as a Single Number

In mainstream machine learning, probability is represented as a single scalar output. When an image classifier says:

Cat: 0.99

the implication is that the system is highly certain. But that number actually compresses multiple meanings into one value. It mixes how often something appeared in training data with how reliable that knowledge really is.

The system cannot express ignorance.

If the classifier is shown adversarial noise or an unfamiliar pattern that vaguely resembles a cat, it may still output a confident

label. Not because it understands the image, but because its architecture forces a decision among known categories. It must distribute probability somewhere, even when the input lies outside its experience.

This exposes a structural limitation. The model reports high probability without any measure of conceptual confidence. It has no internal representation for:

“I do not know what this is.”

NAL explicitly separates these dimensions. Frequency reflects statistical pattern. Confidence reflects the depth of supporting evidence. A strange or novel input would produce low confidence, signaling uncertainty and preventing overcommitment. This distinction is not cosmetic. It is what allows reasoning systems to behave cautiously instead of hallucinating certainty.

2. Hume’s Problem and the i.i.d. Assumption

At a deeper level, modern AI inherits a philosophical shortcut. Hume’s problem of induction asks how we justify expecting the future to resemble the past. Machine learning avoids this question by assuming that training and deployment data are independent and identically distributed.

In practice, this means the system is designed under the belief that tomorrow will statistically look like yesterday.

This assumption creates a sharp split between learning and action:

Training phase: the model performs extreme optimization. It repeatedly adjusts parameters to minimize error across massive datasets, effectively trying to approximate a global optimum. The goal is near-perfect statistical alignment with past observations.

Inference phase: once deployed, the model becomes largely static. It no longer revises its internal beliefs in response to new evidence. When the environment shifts, performance degrades because the system lacks mechanisms for ongoing reasoning or adaptive revision.

This pipeline resembles a system that thinks too much upfront and not at all afterward. It optimizes for a frozen snapshot of reality rather than treating knowledge as continuously evolving. NAL, by contrast, assumes that uncertainty persists and that reasoning must remain active under resource constraints. Learning and acting are intertwined, not separated.

3. “Enough Thinking” Versus Optimization

This leads to the core conceptual difference. Modern AI prioritizes optimization: finding the best parameters relative to historical data. The implicit assumption is that better optimization equals better intelligence.

NAL approaches intelligence differently. It emphasizes “enough thinking”: producing decisions that are sufficient given current evidence and limited resources.

Consider a driving scenario. A conventional ML system is trained on

enormous datasets to recognize predefined situations. The training process aims to cover as many cases as possible in advance. However, when an unexpected event occurs, the system has no principled way to represent novelty. It either misclassifies the situation or produces unstable behavior.

An NAL-style system would treat unfamiliar input as a signal of low confidence. Instead of forcing a precise classification, it evaluates what decision is necessary under uncertainty. The system does not need perfect semantic understanding to act safely. It only needs enough evidence to cross a decision threshold.

This distinction is critical. Optimization tries to eliminate uncertainty during training. “Enough thinking” manages uncertainty during operation. One assumes the world can be pre-solved. The other assumes surprise is inevitable and builds reasoning mechanisms around that reality.

VII. Conclusion

This paper shows that in Non Axiomatic Logic, truth is not absolute but a structured summary of experience under limited knowledge and resources. Evidence, frequency, and confidence allow uncertainty to be represented explicitly, making reasoning adaptive instead of rigid. The principle of “enough thinking” highlights that intelligence is not about perfect certainty, but about making practical decisions with incomplete information.

Compared to modern AI, which often compresses uncertainty into single optimized probabilities, NAL separates statistical

tendency from confidence, allowing systems to recognize ignorance and revise beliefs over time. This distinction matters because real intelligence operates in changing environments where certainty is never guaranteed.

NAL frames uncertainty as a normal and necessary part of reasoning. Intelligence is not defined by eliminating uncertainty, but by managing it effectively to support learning, adaptation, and action.