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Artificial General Intelligence

Homework 4

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## **NAL-1 Truth-Value Calculations and Properties**

### **Introduction**

This paper walks through a NAL-1 (Non-Axiomatic Logic Level 1) inference exercise. The goal is to understand how truth values move through a chain of deductions, how the system reacts when contradictions appear through revision, and how it generalizes from specific observations using induction and abduction.

NAL is built to operate under AIKR, the Assumption of Insufficient Knowledge and Resources. In other words, the system is never assumed to know everything or have unlimited time to compute. Because of that, the logic has to be practical. It must reason using incomplete evidence and stay flexible enough to update its conclusions when new information shows up.

The exercise provides six initial judgments (I1 to I6) and asks us to derive twenty conclusions (D1 to D20) using deduction, revision, induction, and abduction. After that, we modify the initial judgments to observe how the system behaves and what reasoning properties naturally emerge.

## Part 1: Turning Abstract Symbols into a Real Example

The abstract problem uses placeholder symbols: a, b, c, d, e, f. To make this meaningful, I replaced these with a coffee shop scenario.

Symbol	Concrete Interpretation
a	Espresso (specific subject)
b	Coffee (category)
c	Caffeinated Drink (higher category)
d	Stimulant (property/goal)
e	Bitter (observed attribute)
f	Energy Drink (new subject for comparison)

This mapping creates a realistic scenario. You start with general knowledge: Espresso is Coffee, Coffee is Caffeinated, Caffeinated Drinks are Stimulants. Then you get a contradiction: you drink an espresso and it does not stimulate you. Finally, you observe a new property (bitterness) and try to compare espresso with something else (energy drinks).

## Part 2: Excel Implementation and Verification

I implemented the NAL-1 truth-value calculation formulas in Excel. The formulas for each inference type are:

### Deduction

$$f = f_1 \times f_2$$

$$c = f_1 \times f_2 \times c_1 \times c_2$$

### Revision

$$w = c / (1 - c)$$

$$w_{\text{total}} = w_1 + w_2$$

$$c_{\text{new}} = w_{\text{total}} / (w_{\text{total}} + 1)$$

$$f_{\text{new}} = (w_1 \times f_1 + w_2 \times f_2) / w_{\text{total}}$$

## Induction

$$f = f_1$$

$$c = (f_2 \times c_1 \times c_2) / (f_2 \times c_1 \times c_2 + k)$$

where  $k = 1$  (evidence horizon)

## Abduction

$$f = f_2$$

$$c = (f_1 \times c_1 \times c_2) / (f_1 \times c_1 \times c_2 + k)$$

The results from the Excel sheet matched the expected values from Appendix F:

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
I1	1.00	0.90	0.00	0.00	-	1.00	0.90	
I2	1.00	0.90	0.00	0.00	-	1.00	0.90	
I3	1.00	0.90	0.00	0.00	-	1.00	0.90	
I4	1.00	0.90	0.00	0.00	-	0.00	0.90	
I5	1.00	0.90	0.00	0.00	-	1.00	0.90	
I6	1.00	0.90	0.00	0.00	-	1.00	0.90	
D1	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I1 + I2)
D2	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I2 + I3)
D3	1.00	0.90	1.00	0.81	Ded	1.00	0.73	(I1 + D2)
D4	0.00	0.90	1.00	0.73	Rev	0.23	0.92	(I4 + D3)
D5	0.00	0.90	1.00	0.90	Abd	0.00	0.45	(I4 + I3)
D6	0.00	0.45	1.00	0.81	Rev	0.84	0.84	(D5 + D1)
D7	0.00	0.90	1.00	0.90	Ind	0.00	0.45	(I4 + I1)

<b>D8</b>	0.00	0.45	1.00	0.81	<b>Rev</b>	0.84	0.84	(D7 + D2)
<b>D9</b>	0.00	0.90	1.00	0.81	<b>Abd</b>	0.00	0.42	(I4 + D2)
<b>D10</b>	0.00	0.42	1.00	0.90	<b>Rev</b>	0.93	0.91	(D9 + I1)
<b>D11</b>	0.00	0.90	1.00	0.81	<b>Ind</b>	0.00	0.42	(I4 + D1)
<b>D12</b>	0.00	0.42	1.00	0.90	<b>Rev</b>	0.93	0.91	(D11 + I3)
<b>D13</b>	0.00	0.45	1.00	0.90	<b>Ind</b>	0.00	0.29	(D5 + I1)
<b>D14</b>	0.00	0.29	1.00	0.90	<b>Rev</b>	0.96	0.90	(D13 + I2)
<b>D15</b>	1.00	0.90	1.00	0.90	<b>Ind</b>	1.00	0.45	(I5 + I1)
<b>D16</b>	1.00	0.90	1.00	0.81	<b>Ind</b>	1.00	0.42	(I5 + D1)
<b>D17</b>	1.00	0.90	1.00	0.73	<b>Ind</b>	1.00	0.40	(I5 + D3)
<b>D18</b>	1.00	0.90	1.00	0.90	<b>Abd</b>	1.00	0.45	(I6 + I3)
<b>D19</b>	1.00	0.90	1.00	0.81	<b>Abd</b>	1.00	0.42	(I6 + D2)
<b>D20</b>	1.00	0.90	1.00	0.73	<b>Abd</b>	1.00	0.40	(I6 + D3)

These calculations show the core behavior of NAL-1. Deductions chain together but lose confidence at each step (D1, D2, D3). Revisions blend conflicting evidence based on their confidence weights (D4 through D14). Inductions and abductions produce high frequency but deliberately low confidence, signaling that the system is making a generalization from limited data (D15 through D20).

### Part 3: Modifying Truth-Values and Observing Properties

To understand how NAL-1 behaves, I modified each initial statement one at a time and observed what happened to the derived conclusions. This reveals six key properties of the system.

## 1. Blame Attribution

Change: I1 (Espresso → Coffee) from  $\langle 1, 0.9 \rangle$  to  $\langle 1, 0.4 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
I1	1.00	0.90	0.00	0.00	-	1.00	0.40	
I2	1.00	0.90	0.00	0.00	-	1.00	0.90	
I3	1.00	0.90	0.00	0.00	-	1.00	0.90	
I4	1.00	0.90	0.00	0.00	-	0.00	0.90	
I5	1.00	0.90	0.00	0.00	-	1.00	0.90	
I6	1.00	0.90	0.00	0.00	-	1.00	0.90	
D1	1.00	0.40	1.00	0.90	Ded	1.00	0.36	(I1 + I2)
D2	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I2 + I3)
D3	1.00	0.40	1.00	0.81	Ded	1.00	0.32	(I1 + D2)
D4	0.00	0.90	1.00	0.32	Rev	0.05	0.90	(I4 + D3)
D5	0.00	0.90	1.00	0.90	Abd	0.00	0.45	(I4 + I3)
D6	0.00	0.45	1.00	0.36	Rev	0.41	0.58	(D5 + D1)
D7	0.00	0.90	1.00	0.40	Ind	0.00	0.26	(I4 + I1)
D8	0.00	0.26	1.00	0.81	Rev	0.92	0.82	(D7 + D2)
D9	0.00	0.90	1.00	0.81	Abd	0.00	0.42	(I4 + D2)
D10	0.00	0.42	1.00	0.40	Rev	0.48	0.58	(D9 + I1)
D11	0.00	0.90	1.00	0.36	Ind	0.00	0.24	(I4 + D1)
D12	0.00	0.24	1.00	0.90	Rev	0.97	0.90	(D11 + I3)
D13	0.00	0.45	1.00	0.40	Ind	0.00	0.15	(D5 + I1)
D14	0.00	0.15	1.00	0.90	Rev	0.98	0.90	(D13 + I2)
D15	1.00	0.90	1.00	0.40	Ind	1.00	0.26	(I5 + I1)
D16	1.00	0.90	1.00	0.36	Ind	1.00	0.24	(I5 + D1)
D17	1.00	0.90	1.00	0.32	Ind	1.00	0.23	(I5 + D3)

<b>D18</b>	1.00	0.90	1.00	0.90	<b>Abd</b>	1.00	0.45	(I6 + I3)
<b>D19</b>	1.00	0.90	1.00	0.81	<b>Abd</b>	1.00	0.42	(I6 + D2)
<b>D20</b>	1.00	0.90	1.00	0.32	<b>Abd</b>	1.00	0.23	(I6 + D3)

When I lowered the confidence of I1 to 0.4, the system became uncertain whether the drink was actually espresso. Later, when the contradiction arrived (I4: this espresso is not a stimulant), the system assigned the blame to the identity rather than to the general rule.

In the original case with  $c=0.9$ , D10 (Is this drink actually Espresso?) ended up at  $f=0.93$ , meaning the system was still fairly sure it was espresso. The system blamed the general rule instead. But with  $c=0.4$ , the system decided it probably was not espresso after all.

This is blame attribution. When something goes wrong, the system looks for which link in the chain was the weakest and assigns responsibility there. This is similar to credit assignment in reinforcement learning, where an agent has to figure out which past action caused a current reward or penalty.

## 2. Frequency Propagation

Change: I2 (Coffee  $\rightarrow$  Caffeinated) from  $\langle 1.0, 0.9 \rangle$  to  $\langle 0.8, 0.9 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
<b>I1</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I2</b>	1.00	0.90	0.00	0.00	-	0.80	0.90	
<b>I3</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I4</b>	1.00	0.90	0.00	0.00	-	0.00	0.90	
<b>I5</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I6</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>D1</b>	1.00	0.90	0.80	0.90	<b>Ded</b>	0.80	0.65	(I1 + I2)
<b>D2</b>	0.80	0.90	1.00	0.90	<b>Ded</b>	0.80	0.65	(I2 + I3)

<b>D3</b>	1.00	0.90	0.80	0.65	<b>Ded</b>	0.80	0.47	(I1 + D2)
<b>D4</b>	0.00	0.90	0.80	0.47	<b>Rev</b>	0.07	0.91	(I4 + D3)
<b>D5</b>	0.00	0.90	1.00	0.90	<b>Abd</b>	0.00	0.45	(I4 + I3)
<b>D6</b>	0.00	0.45	0.80	0.65	<b>Rev</b>	0.56	0.73	(D5 + D1)
<b>D7</b>	0.00	0.90	1.00	0.90	<b>Ind</b>	0.00	0.45	(I4 + I1)
<b>D8</b>	0.00	0.45	0.80	0.65	<b>Rev</b>	0.56	0.73	(D7 + D2)
<b>D9</b>	0.00	0.90	0.80	0.65	<b>Abd</b>	0.00	0.32	(I4 + D2)
<b>D10</b>	0.00	0.32	1.00	0.90	<b>Rev</b>	0.95	0.90	(D9 + I1)
<b>D11</b>	0.00	0.90	0.80	0.65	<b>Ind</b>	0.00	0.32	(I4 + D1)
<b>D12</b>	0.00	0.32	1.00	0.90	<b>Rev</b>	0.95	0.90	(D11 + I3)
<b>D13</b>	0.00	0.45	1.00	0.90	<b>Ind</b>	0.00	0.29	(D5 + I1)
<b>D14</b>	0.00	0.29	0.80	0.90	<b>Rev</b>	0.77	0.90	(D13 + I2)
<b>D15</b>	1.00	0.90	1.00	0.90	<b>Ind</b>	1.00	0.45	(I5 + I1)
<b>D16</b>	1.00	0.90	0.80	0.65	<b>Ind</b>	1.00	0.32	(I5 + D1)
<b>D17</b>	1.00	0.90	0.80	0.47	<b>Ind</b>	1.00	0.25	(I5 + D3)
<b>D18</b>	1.00	0.90	1.00	0.90	<b>Abd</b>	1.00	0.45	(I6 + I3)
<b>D19</b>	1.00	0.90	0.80	0.65	<b>Abd</b>	1.00	0.32	(I6 + D2)
<b>D20</b>	1.00	0.90	0.80	0.47	<b>Abd</b>	1.00	0.25	(I6 + D3)

I modified I2 to acknowledge that 20% of coffee is decaf. The system automatically propagated this frequency down the chain. D3 (Espresso is a Stimulant) dropped to  $f=0.8$  instead of 1.0. Unlike binary logic, where a single exception breaks a rule, NAL-1 maintains a statistical heuristic. The system accepts that rules can be mostly true and still useful. This is how humans actually reason. We know that coffee is usually caffeinated, and we use that knowledge even though we also know decaf exists.

### 3. Logical Stability

Change: I3 (Caffeinated  $\rightarrow$  Stimulant) from  $\langle 1, 0.9 \rangle$  to  $\langle 0, 0.9 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
I1	1.00	0.90	0.00	0.00	-	1.00	0.90	
I2	1.00	0.90	0.00	0.00	-	1.00	0.90	
I3	1.00	0.90	0.00	0.00	-	0.00	0.90	
I4	1.00	0.90	0.00	0.00	-	0.00	0.90	
I5	1.00	0.90	0.00	0.00	-	1.00	0.90	
I6	1.00	0.90	0.00	0.00	-	1.00	0.90	
D1	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I1 + I2)
D2	1.00	0.90	0.00	0.90	Ded	0.00	0.00	(I2 + I3)
D3	1.00	0.90	0.00	0.00	Ded	0.00	0.00	(I1 + D2)
D4	0.00	0.90	0.00	0.00	Rev	0.00	0.90	(I4 + D3)
D5	0.00	0.90	0.00	0.90	Abd	0.00	0.00	(I4 + I3)
D6	0.00	0.00	1.00	0.81	Rev	1.00	0.81	(D5 + D1)
D7	0.00	0.90	1.00	0.90	Ind	0.00	0.45	(I4 + I1)
D8	0.00	0.45	0.00	0.00	Rev	0.00	0.45	(D7 + D2)
D9	0.00	0.90	0.00	0.00	Abd	0.00	0.00	(I4 + D2)
D10	0.00	0.00	1.00	0.90	Rev	1.00	0.90	(D9 + I1)
D11	0.00	0.90	1.00	0.81	Ind	0.00	0.42	(I4 + D1)
D12	0.00	0.42	0.00	0.90	Rev	0.00	0.91	(D11 + I3)
D13	0.00	0.00	1.00	0.90	Ind	0.00	0.00	(D5 + I1)
D14	0.00	0.00	1.00	0.90	Rev	1.00	0.90	(D13 + I2)
D15	1.00	0.90	1.00	0.90	Ind	1.00	0.45	(I5 + I1)
D16	1.00	0.90	1.00	0.81	Ind	1.00	0.42	(I5 + D1)
D17	1.00	0.90	0.00	0.00	Ind	1.00	0.00	(I5 + D3)



<b>D18</b>	1.00	0.90	0.00	0.90	<b>Abd</b>	1.00	0.00	(I6 + I3)
<b>D19</b>	1.00	0.90	0.00	0.00	<b>Abd</b>	1.00	0.00	(I6 + D2)
<b>D20</b>	1.00	0.90	0.00	0.00	<b>Abd</b>	1.00	0.00	(I6 + D3)

I flipped I3 to represent a world where caffeine does not stimulate people. This is a biological fact reversal. Because I3 sits at the top of the inheritance chain, changing it flipped the entire deductive tree. Every conclusion about espresso being a stimulant became strongly negative ( $f=0$ ).

This demonstrates logical stability in the structural sense. The system's conclusions are stable relative to the premises. If you change a premise, the conclusions change in a predictable, traceable way. The problem here is not instability but sensitivity. Top-level premises have outsized influence on the entire belief structure.

#### 4. Resistance to Noise

Change: I4 (Espresso  $\rightarrow$  NOT Stimulant) from  $\langle 0, 0.9 \rangle$  to  $\langle 0, 0.1 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
<b>I1</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I2</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I3</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I4</b>	1.00	0.90	0.00	0.00	-	0.00	0.10	
<b>I5</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I6</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>D1</b>	1.00	0.90	1.00	0.90	<b>Ded</b>	1.00	0.81	(I1 + I2)
<b>D2</b>	1.00	0.90	1.00	0.90	<b>Ded</b>	1.00	0.81	(I2 + I3)
<b>D3</b>	1.00	0.90	1.00	0.81	<b>Ded</b>	1.00	0.73	(I1 + D2)
<b>D4</b>	0.00	0.10	1.00	0.73	<b>Rev</b>	0.96	0.74	(I4 + D3)

<b>D5</b>	0.00	0.10	1.00	0.90	<b>Abd</b>	0.00	0.08	(I4 + I3)
<b>D6</b>	0.00	0.08	1.00	0.81	<b>Rev</b>	0.98	0.81	(D5 + D1)
<b>D7</b>	0.00	0.10	1.00	0.90	<b>Ind</b>	0.00	0.08	(I4 + I1)
<b>D8</b>	0.00	0.08	1.00	0.81	<b>Rev</b>	0.98	0.81	(D7 + D2)
<b>D9</b>	0.00	0.10	1.00	0.81	<b>Abd</b>	0.00	0.07	(I4 + D2)
<b>D10</b>	0.00	0.07	1.00	0.90	<b>Rev</b>	0.99	0.90	(D9 + I1)
<b>D11</b>	0.00	0.10	1.00	0.81	<b>Ind</b>	0.00	0.07	(I4 + D1)
<b>D12</b>	0.00	0.07	1.00	0.90	<b>Rev</b>	0.99	0.90	(D11 + I3)
<b>D13</b>	0.00	0.08	1.00	0.90	<b>Ind</b>	0.00	0.07	(D5 + I1)
<b>D14</b>	0.00	0.07	1.00	0.90	<b>Rev</b>	0.99	0.90	(D13 + I2)
<b>D15</b>	1.00	0.90	1.00	0.90	<b>Ind</b>	1.00	0.45	(I5 + I1)
<b>D16</b>	1.00	0.90	1.00	0.81	<b>Ind</b>	1.00	0.42	(I5 + D1)
<b>D17</b>	1.00	0.90	1.00	0.73	<b>Ind</b>	1.00	0.40	(I5 + D3)
<b>D18</b>	1.00	0.90	1.00	0.90	<b>Abd</b>	1.00	0.45	(I6 + I3)
<b>D19</b>	1.00	0.90	1.00	0.81	<b>Abd</b>	1.00	0.42	(I6 + D2)
<b>D20</b>	1.00	0.90	1.00	0.73	<b>Abd</b>	1.00	0.40	(I6 + D3)

I treated the negative evidence as a weak rumor instead of a strong fact. With  $c=0.1$ , the revision rule effectively ignored the contradiction. The system kept its original deductive belief (D3) nearly intact.

This shows resistance to noise. The system does not overreact to low-quality data. It weighs evidence by confidence. A vague rumor does not overturn a well-established belief. This is important for any system operating in a noisy environment.

## 5. Negative Induction

Change: I5 (Espresso  $\rightarrow$  Bitter) from  $\langle 1, 0.9 \rangle$  to  $\langle 0, 0.9 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
I1	1.00	0.90	0.00	0.00	-	1.00	0.90	
I2	1.00	0.90	0.00	0.00	-	1.00	0.90	
I3	1.00	0.90	0.00	0.00	-	1.00	0.90	
I4	1.00	0.90	0.00	0.00	-	0.00	0.90	
I5	1.00	0.90	0.00	0.00	-	0.00	0.90	
I6	1.00	0.90	0.00	0.00	-	1.00	0.90	
D1	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I1 + I2)
D2	1.00	0.90	1.00	0.90	Ded	1.00	0.81	(I2 + I3)
D3	1.00	0.90	1.00	0.81	Ded	1.00	0.73	(I1 + D2)
D4	0.00	0.90	1.00	0.73	Rev	0.23	0.92	(I4 + D3)
D5	0.00	0.90	1.00	0.90	Abd	0.00	0.45	(I4 + I3)
D6	0.00	0.45	1.00	0.81	Rev	0.84	0.84	(D5 + D1)
D7	0.00	0.90	1.00	0.90	Ind	0.00	0.45	(I4 + I1)
D8	0.00	0.45	1.00	0.81	Rev	0.84	0.84	(D7 + D2)
D9	0.00	0.90	1.00	0.81	Abd	0.00	0.42	(I4 + D2)
D10	0.00	0.42	1.00	0.90	Rev	0.93	0.91	(D9 + I1)
D11	0.00	0.90	1.00	0.81	Ind	0.00	0.42	(I4 + D1)
D12	0.00	0.42	1.00	0.90	Rev	0.93	0.91	(D11 + I3)
D13	0.00	0.45	1.00	0.90	Ind	0.00	0.29	(D5 + I1)
D14	0.00	0.29	1.00	0.90	Rev	0.96	0.90	(D13 + I2)
D15	0.00	0.90	1.00	0.90	Ind	0.00	0.45	(I5 + I1)
D16	0.00	0.90	1.00	0.81	Ind	0.00	0.42	(I5 + D1)
D17	0.00	0.90	1.00	0.73	Ind	0.00	0.40	(I5 + D3)

<b>D18</b>	1.00	0.90	1.00	0.90	<b>Abd</b>	1.00	0.45	(I6 + I3)
<b>D19</b>	1.00	0.90	1.00	0.81	<b>Abd</b>	1.00	0.42	(I6 + D2)
<b>D20</b>	1.00	0.90	1.00	0.73	<b>Abd</b>	1.00	0.40	(I6 + D3)

I observed that a specific espresso was sweet instead of bitter. The system induced that coffee is NOT bitter (D15 became  $f=0$ ). This demonstrates that NAL-1 generalizes the absence of traits just as easily as their presence.

This is negative induction. The system can learn what categories lack based on specific examples. This is important because knowing what something is not can be as informative as knowing what it is.

## 6. Abductive Weakness

Change: I6 (Energy Drink  $\rightarrow$  Stimulant) from  $\langle 1, 0.9 \rangle$  to  $\langle 1, 0.2 \rangle$

ID	F1	C1	F2	C2	Operation	Result F	Result C	Notes
<b>I1</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I2</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I3</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I4</b>	1.00	0.90	0.00	0.00	-	0.00	0.90	
<b>I5</b>	1.00	0.90	0.00	0.00	-	1.00	0.90	
<b>I6</b>	1.00	0.90	0.00	0.00	-	1.00	0.20	
<b>D1</b>	1.00	0.90	1.00	0.90	<b>Ded</b>	1.00	0.81	(I1 + I2)
<b>D2</b>	1.00	0.90	1.00	0.90	<b>Ded</b>	1.00	0.81	(I2 + I3)
<b>D3</b>	1.00	0.90	1.00	0.81	<b>Ded</b>	1.00	0.73	(I1 + D2)
<b>D4</b>	0.00	0.90	1.00	0.73	<b>Rev</b>	0.23	0.92	(I4 + D3)
<b>D5</b>	0.00	0.90	1.00	0.90	<b>Abd</b>	0.00	0.45	(I4 + I3)
<b>D6</b>	0.00	0.45	1.00	0.81	<b>Rev</b>	0.84	0.84	(D5 + D1)

<b>D7</b>	0.00	0.90	1.00	0.90	<b>Ind</b>	0.00	0.45	(I4 + I1)
<b>D8</b>	0.00	0.45	1.00	0.81	<b>Rev</b>	0.84	0.84	(D7 + D2)
<b>D9</b>	0.00	0.90	1.00	0.81	<b>Abd</b>	0.00	0.42	(I4 + D2)
<b>D10</b>	0.00	0.42	1.00	0.90	<b>Rev</b>	0.93	0.91	(D9 + I1)
<b>D11</b>	0.00	0.90	1.00	0.81	<b>Ind</b>	0.00	0.42	(I4 + D1)
<b>D12</b>	0.00	0.42	1.00	0.90	<b>Rev</b>	0.93	0.91	(D11 + I3)
<b>D13</b>	0.00	0.45	1.00	0.90	<b>Ind</b>	0.00	0.29	(D5 + I1)
<b>D14</b>	0.00	0.29	1.00	0.90	<b>Rev</b>	0.96	0.90	(D13 + I2)
<b>D15</b>	1.00	0.90	1.00	0.90	<b>Ind</b>	1.00	0.45	(I5 + I1)
<b>D16</b>	1.00	0.90	1.00	0.81	<b>Ind</b>	1.00	0.42	(I5 + D1)
<b>D17</b>	1.00	0.90	1.00	0.73	<b>Ind</b>	1.00	0.40	(I5 + D3)
<b>D18</b>	1.00	0.20	1.00	0.90	<b>Abd</b>	1.00	0.15	(I6 + I3)
<b>D19</b>	1.00	0.20	1.00	0.81	<b>Abd</b>	1.00	0.14	(I6 + D2)
<b>D20</b>	1.00	0.20	1.00	0.73	<b>Abd</b>	1.00	0.13	(I6 + D3)

I reduced the confidence of I6 to represent only a vague suspicion that energy drinks are stimulants. The abductive guess that energy drinks are caffeinated (D18) dropped to extremely low confidence (0.15).

This reveals abductive weakness, which prevents overfitting. The system refuses to form a strong hypothesis between two terms unless they share a high-confidence common trait. It will not confidently conclude that energy drinks are caffeinated just because both categories vaguely relate to stimulation.

### Summary of Properties

These six modifications reveal how NAL-1 handles uncertainty:

- It attributes blame to the weakest link when contradictions occur
- It propagates frequencies through deduction chains, allowing for mostly-true rules
- It responds predictably when premises change, showing structural stability

- It resists noise by weighing evidence by confidence, not just frequency
- It generalizes both presence and absence of properties
- It prevents overfitting by keeping weak inferences at low confidence

#### **Part 4: Comparing NAL Induction with Other Theories**

NAL treats induction as a basic weak inference rule. This is very different from how classical logic, probability theory, and machine learning approach generalization.

##### **NAL vs. Classical Logic**

In classical binary logic, induction is considered invalid. Just because some A are B does not logically prove that all A are B. This is essentially Hume's problem. Because of this, classical logic avoids treating induction as a proper logical rule and instead pushes it into heuristics or abduction.

NAL takes a different position by redefining what "valid" means. An inference is valid if the conclusion is supported by the evidence in the premises. Induction in NAL is defeasible, meaning it can later be overturned, but it is still logically acceptable because it summarizes what the system currently knows.

This shows a deeper philosophical split. Classical logic demands strict truth preservation. NAL prioritizes experience summarization. Under AIKR, where knowledge and resources are limited, summarizing experience is more useful than demanding certainty that may never be achievable.

##### **NAL vs. Probability and Statistics**

Traditional probability and statistics assume there is some stable underlying distribution or that samples are random. They work best in a closed world where possibilities are predefined, or where the future statistically resembles the past.

NAL assumes an open world. It does not rely on random sampling or fixed distributions. Confidence represents how much evidence exists, not just how often something happens. In this framework, induction acts like the inverse of deduction. If Espresso is Coffee and Espresso is Bitter, the system induces that Coffee is Bitter. The strength of that conclusion depends on accumulated evidence rather than sampling assumptions.

Bayesian conditioning updates beliefs with new evidence but requires an initial prior distribution. NAL instead uses revision to merge evidence from different sources without needing predefined priors for every concept. This makes it more adaptable when the environment keeps changing.

### **NAL vs. Machine Learning**

Modern machine learning depends heavily on inductive generalization during training. A common problem is overfitting, where a model memorizes a small dataset too well and fails when facing new situations.

Machine learning addresses this with techniques like regularization, cross-validation, or built-in inductive biases. NAL approaches the same issue differently. Its confidence metric and the parameter  $k$  naturally limit how strongly a conclusion can be trusted when evidence is scarce. A generalization from a single example may have frequency 1.0, but its confidence stays low.

This weak inference signals that the rule is tentative. The system recognizes the pattern but avoids committing too strongly before gathering more evidence. In that sense, overfitting control is built directly into the reasoning framework rather than added as an external training step.

### **NAL vs. Fuzzy Logic**

Fuzzy logic focuses on vagueness. It measures partial truth, such as how tall someone is or how hot water feels, using membership functions and numeric combinations.

NAL focuses on uncertainty instead of vagueness. Its induction rule calculates how much evidence supports a general statement based on specific observations. Frequency captures how strong the pattern appears, while confidence captures how much evidence backs it. Both systems use numeric truth-values, but they represent different ideas. Fuzzy logic expresses degrees of truth. NAL expresses degrees of evidential support.

### Summary Table

Feature	Classical Logic	Probability	NAL
Justification	Truth-preservation	Frequency/Distribution	Experience-summary
Scope	Closed (Fixed Axioms)	Closed (Defined Space)	Open (New terms allowed)
Handling Inconsistency	System crashes	Dutch Book violation	Revision merges evidence
Hume's Problem	Unsolvable	Assumes stationarity	Relative rationality (AIKR)
New Evidence	Retract (Non-monotonic)	Bayesian Update	Revision (Pools evidence)

The main difference is that NAL is designed for open environments where knowledge and resources are limited. It assumes incomplete information and bounded processing. This makes it practical for systems operating in real situations where conditions change and new concepts constantly appear.



## Conclusion

This exercise demonstrates how NAL-1 performs reasoning under uncertainty. The system chains deductions, reconciles contradictions through revision, and generalizes from individual cases. It uses a two-part truth-value that tracks both pattern strength (frequency) and evidential support (confidence).

The experiments in Part 3 revealed several emergent properties, including blame attribution, frequency propagation, logical stability, resistance to noise, negative induction, and abductive weakness. These behaviors are not manually programmed. They arise naturally from the truth-value equations and how evidence is combined.

Comparing NAL with other reasoning frameworks shows that it occupies a distinct role. It does not aim to replace classical logic or probability theory. Instead, it targets scenarios where decisions must be made with incomplete information and limited computation. Induction is treated as experience summarization rather than guaranteed truth or probability estimation.

Whether this approach is ideal depends on the task. Systems that require continuous learning in changing environments benefit from NAL's flexibility. Systems demanding strict correctness or precise probabilistic guarantees may prefer other frameworks. But for an agent building knowledge incrementally under AIKR, NAL provides a consistent and practical reasoning structure.

## Reference

Wang, P. (2013). *Non-Axiomatic Logic: A Model of Intelligent Reasoning* (2nd ed.).