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Homework #4

7 a)

Show $5n^3 + 2n^2 + 3n = \Theta(n^3)$

Definition $f(n) = \Theta(g(n))$

if $f(n) \leq C \cdot g(n)$ for all $n \geq n_0$ where C & n_0 are positive constants.

Therefore $\frac{f(n)}{g(n)} \leq C$ for all $n \geq n_0$

$f(n) = 5n^3 + 2n^2 + 3n$

$g(n) = n^3$

$$\frac{5n^3 + 2n^2 + 3n}{n^3} \leq C \text{ for all } n \geq n_0$$

choose $n_0 = 1$

$$\frac{5n^3 + 2n^2 + 3n}{n^3} \leq C \text{ for all } n \geq 1$$

$$\frac{5n^3 + 2n^3 + 3n^3}{n^3} > \frac{5n^3 + 2n^2 + 3n}{n^3} \text{ for all } n \geq 1$$

$$= 5 + 2 + 3 > \frac{5n^3 + 2n^2 + 3n}{n^3} \text{ for all } n \geq 1$$

$$10 > \frac{5n^3 + 2n^2 + 3n}{n^3} \text{ for all } n \geq 1$$

$$\frac{5n^3 + 2n^2 + 3n}{n^3} \leq 10 \text{ for all } n \geq 1$$

Thus $f(n) = \Theta(g(n))$

a) cont.

If $5n^3 + 2n^2 + 3n \leq 10 \cdot g(n)$ for all $n \geq 1$

then $f(n) = O(g(n))$

$5n^3 + 2n^2 + 3n \leq 10n^3$ for all $n \geq 1$

$2n^2 + 3n \leq 5n^3$ for all $n \geq 1$

$0 \leq 5n^3 - 2n^2 - 3n$ for all $n \geq 1$

when $n=1$

$0 \leq 5(1)^3 - 2(1)^2 - 3(1)$

$0 \leq 0$

when $n=2$

$0 \leq 5(2)^3 - 2(2)^2 - 3(2)$

$0 \leq 40 - 8 - 6$

$0 \leq 28$

$\therefore \quad 5n^3 + 2n^2 + 3n = \Theta(n^3)$

7 b) Show $\sqrt{7n^2+2n-8} = \Theta(n)$

Definition $f(n) = \Theta\, g(n)$

if $f(n) \leq C \cdot g(n)$ for all $n \geq n_0$

where C & n_0 are positive constants

Therefore $\frac{f(n)}{g(n)} \leq C$ for all $n \geq n_0$

$f(n) = \sqrt{7n^2+2n-8}$

$g(n) = n$

$$\frac{\sqrt{7n^2+2n-8}}{n} \leq C \quad \text{for all } n \geq n_0$$

Choose $n = 2$

$$\frac{\sqrt{7n^2+2n-8}}{n} \leq C \quad \text{for all } n \geq 2$$

$$\frac{7n^2+2n-8}{n^2} > \frac{\sqrt{7n^2+2n-8}}{n} \quad \text{for all } n \geq 2$$

$$7 + \frac{2n-8}{n^2} > \frac{\sqrt{7n^2+2n-8}}{n} \quad \text{for all } n \geq 2$$

$$7 + \frac{2(2)-8}{(2)^2}$$

$$7 + \frac{-4}{4} = 6$$

b) Cont.

$$\frac{\sqrt{7n^2+2n-8}}{n} \leq 6 \quad \text{for all } n \geq 2$$

Thus $f(n) = \Theta g(n)$

If $\sqrt{7n^2+2n-8} \leq 6 \cdot n$ for all $n \geq 2$

then $f(n) = \Theta g(n)$

$$\sqrt{7n^2+2n-8} \leq 6n \quad \text{for all } n \geq 2$$

$$7n^2+2n-8 \leq 36n^2 \quad \text{for all } n \geq 2$$

$$0 \leq 29n^2-2n+8 \quad \text{for all } n \geq 2$$

when $n=2$

$$0 \leq 116$$

when $n=3$

$$0 \leq 262$$

$\therefore \quad \sqrt{7n^2+2n-8} = \Theta(n)$