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Homework #2

Question 8

If i) $A \rightarrow (B \wedge C)$

ii) $(B \vee D) \rightarrow E$

iii) $A \vee D$

iv) $\neg (B \wedge C)$

thus E is true

Proof

	Proposition	Reason
①	$\neg (B \wedge C)$	Premise iv
②	$A \rightarrow (B \wedge C)$	premise i
③	$\neg A$	steps ① and ② and modos tollens
④	$A \vee D$	premise iii
⑤	D	steps ③ and ④ and the Rule of Disjunctive Syllogism
⑥	$(B \vee D)$	step ⑤ and the Rule of Disjunctive Amplification
⑦	$(B \vee D) \rightarrow E$	premise ii
⑧	E	step ⑦ and Modos Ponens

Question 9

Let: $S(x)$ be the predicate "x is a student"

$F(x)$ be the predicate "x is a faculty member"

$A(x,y)$ be the predicate "x has asked y a question"

where the domain consists of all people associated with the school

a) Lois has asked Professor Michaels a question.

$A(\text{Lois}, \text{Professor Michaels})$

b) Every student has asked professor Gross a question.

↳ For every person at school x, if person x is a student, then x has asked Professor Gross a question.

$$\forall_x (S(x) \rightarrow A(x, \text{Professor Gross}))]$$

c) Some student has not asked any faculty member a question.

$$\exists_x [S(x) \wedge \forall_y (F(y) \rightarrow \neg A(x,y))]$$

d) There is a faculty member who has never been asked a question. by a student.

$$\exists_y [F(y) \wedge \forall_x (S(x) \rightarrow \neg A(x,y))]$$

e) Some student has asked every faculty member a question

$$\exists_x [S(x) \wedge \forall_y (F(y) \rightarrow A(x,y))]$$

f) Some student has never been asked a question by a faculty member.

$$\exists_x [S(x) \wedge \forall_y (F(y) \rightarrow \neg A(y,x))]$$

Question 10

a) $\forall_x \exists_y (x^2 = y)$ True

b) $\forall_x \exists_y (x = y^2)$ False

c) $\exists_x \forall_y (xy = 0)$ True

d) $\forall_x (x \neq 0 \rightarrow \exists_y (xy = 1))$ True

e) $\exists_x \forall_y (y \neq 0 \rightarrow xy = 1)$ False

f) $\exists_x \exists_y (x + 2y = 2 \wedge 2x + 4y = 5)$ False

g) $\forall_x \exists_y (x + y = 2 \wedge 2x - y = 1)$ False