Conor McLaughlin, Kristian Olsson, Michael Vaden
Professor Don Brown
DS 6040: Bayesian Machine Learning
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A Bayesian Approach to Predicting Algerian Forest Fires

Problem Description

In July of 2023, northern Algeria was ravaged by a series of wildfires that killed more than thirty people and displaced over fifteen hundred households (Sekkai). Forest fires are a recurring and brutal problem worldwide, and these natural disasters have very real consequences and victims. Algeria in particular is susceptible to these fires because many of its regions are often stricken by intense heat waves that have only grown stronger over recent years. Less than one percent of the country's landmass is covered by forest, and it is important to preserve this nature and best protect it from natural disasters (*Algeria*). In this project, we aim to understand how certain climatic measures contribute to the start of wildfires, investigate if there are regional differences in modeling the likelihood of a fire in Algeria, and determine the best way to predict fire likelihood given the provided measurements and the uncertainty surrounding our predictions.

Data Description

Our dataset for this project includes daily observations from June to September 2012 detailing forest fire occurrences and climatic measurements taking place in the northeastern Algeria region Bejaia, as well as the northwestern region Sidi Bel-abbes. The UCI Machine Learning Repository offers the data which contains 244 observations evenly divided between the two regions (Abid). In our initial preprocessing of the data, we had to address some irregularities in the CSV file to make the data appropriate for exploratory data analysis. Next, we considered which predictors to include in our predictive models. The initial dataset includes base climatic measurements of temperature, relative humidity, wind speed, and rain, as well as time measurements for the four months of 2012 data with day, month, and year features.

Additionally, the dataset offers six components shown in Figure 0.1 that are calculated by the Meteorological Service of Canada and are combinations of the four base climatic measurements mentioned above (*Canadian*). The Fire Weather Index (FWI) is the final combined index of all of these influencing components and acts as a unitless measurement of fire intensity. This index is universally standard, although the range of fire weather is not the same for different regions across the globe (*Fire*). This means the FWI and calculated components do not necessarily scale to local weather patterns, and we want to use only the original climatic measurements as predictors initially before comparing them to an FWI-based model. Our exploratory data analysis further shows that these components are all very highly correlated with each other (Fig 0.2-0.4). As a result, we drop the calculated indices from our dataset to avoid multicollinearity and limit our predictors to temperature, relative humidity, wind speed, and rain. Finally, we standard scale our variables to be appropriate for our model approaches and allow us to understand the relative importance of predictors in predicting the response. We also transform the rain predictor by taking its square root to try to model non-linear relationships with expected wildfires for the Bejaia and Sidi Bel-abbes regions.

Probability Models

Our model-building process hinges on a Bayesian logistic regression with No U-Turn Sampling to understand important predictors of fires in the two regions of Algeria. Our pooled models use Bayesian logistic regression with Gaussian distributions to represent the regression coefficients for each of our predictors. We built one model using the climate measurement predictors of temperature, rain, relative humidity, and wind speed (Model 1) and another using FWI as the only predictor (Model 4). The resulting values are then converted to probabilities between 0 and 1 using the inverse logit function. The converted probabilities are then used as the probability value in a Bernoulli distribution to determine the likelihood of a fire occurring on each day.

For our unpooled model (Model 2) and hierarchical models (Models 3, 5, and 6), we used the same structure as our pooled models. We utilized normal priors for the coefficients to develop a GLM that estimated the likelihood of a fire. However, since the models were unpooled and hierarchical, we estimated the parameters for each region separately. Our hierarchical model also utilized normal distributions as the hyperpriors for the mean of each coefficient and half-normal distributions as hyperpriors for the variance. We built the single unpooled model using the measurement predictors temperature, rain, relative humidity, and wind speed (Model 2). Then, we built three hierarchical models. The first hierarchical model is a linear model using the climate measurement predictors (Model 3). The next hierarchical model is a linear model using only FWI as the predictor (Model 5). The last hierarchical model is a non-linear model using the climate measurement predictors of temperature, rain, relative humidity, wind speed, and the square root of rain (Model 6).

Approaches

Before we built our models, we split our dataset into a training and testing dataset, so we could assess the out-of-sample performance of our models. We used an 80/20 train-test ratio, so our training set had 195 observations and our test set had 49 observations. We also ensured the test and training set remained balanced between the two regions to ensure the unpooled and hierarchical models had an equal amount of data to train and test on. We built a pooled model (Model 1) using each of the climatic measures to understand how certain climatic measures contribute to the likelihood of wildfires in Algeria. We used the estimated coefficients and their associated highest density intervals (HDI) to determine the effect each predictor has on the likelihood of a fire and the degree of confidence we have in that effect. We know that predictors with positive coefficients in our model with HDIs that do not include 0 will indicate a high degree of confidence that an increase in that predictor leads to an increase in the likelihood of a fire, while predictors with negative coefficients indicate a decrease in the likelihood of a fire.

We built unpooled (Model 2) and hierarchical (Model 3) models using each of the climatic measures to determine if there are regional differences between Sidi Bel-abbes and Bejaia. We compared both the direction and degree of the estimated coefficients for each predictor in each of the two regions Sidi Bel-abbes and Bejaia. If the HDI of a predictor for one region did not include 0 and the other did include 0, then we determined that the predictor was significant for one region and not the other. If the HDIs of a single predictor for each region were in the same direction but did not overlap, then we determined there was a difference in the degree of importance of that feature between the two regions.

In order to determine the best way to predict fire likelihood given the provided measurements, we built additional models and compared them using WAIC to determine the most successful models. We used our best FWI model and our best climatic measurements model to make predictions on our test set to assess the out-of-sample performance of our models. We used accuracy, area under the ROC curve (AUC), and visualized the predictions and each prediction's associated degree of uncertainty to evaluate the predictive performance of our model and the uncertainty of these predictions.

Results

The pooled model using base climatic predictors (Model 1) performs well in our sampling and posterior evaluations: the trace plot shows our chains are converging to a similar distribution, and our scale reduction factor values of 1.0 for all of the parameters confirm this conclusion (Fig 1.1-1.2). In addition, our Bayesian p-value plot and posterior predictive check show that our posterior predictive mean follows the observed values (Fig 1.3-1.4). Although our posterior predictive has high variance, our Bayesian p-value plot shows that we are within the expected deviations from uniformity for a dataset of the same sample size so we believe the model is a good fit. The predictor coefficients shown through the posterior coefficient plots suggest that temperature, rain, and relative humidity proved to be significant while wind speed was insignificant in predicting the likelihood of fires (Fig 1.5). As we would expect, an increase in temperature increases the likelihood of a forest fire while a decrease in rain and relative humidity increases the likelihood of a forest fire.

The unpooled and hierarchical models (Models 2 and 3) perform similarly to the pooled model in terms of the sampling and posterior evaluations (Fig 2.1-2.4 and 3.1-3.4), although the hierarchical model had slightly worse sampling with some slight deviations between the chains' traces. We further analyze the predictor coefficients for both models. Using our posterior coefficient plots (Fig 2.5 and 3.5) with 94% HDI, we were able to measure the uncertainty in the impacts of our regression coefficients on the fire response. For both the unpooled and hierarchical models we see that the intercept was significant (non-zero) and negative for the Bejaia region, indicating that there is less likelihood of forest fires compared to the Sidi-Bel Abbes region holding all other variables constant. For both regions, in both models, temperature was a significant predictor with a very similar positive coefficient estimate, indicating that as temperature increases, the likelihood of forest fires increases, as expected. Additionally, for both regions, in both models, rain was a significant predictor with varying negative estimates. In the Bejaia region, rain decreases the odds of a fire occurring much more compared to the Sidi-Bel Abbes region. In both models, humidity proved to be significant for only the Sidi-Bel Abbes region, whereas a decrease in humidity increases the odds of a fire.

We also found that both models outperformed the pooled model by a significant margin when comparing and weighing the models using the Widely Applicable Information Criterion (WAIC). More specifically, the unpooled model had a model weight of 0.819, while the pooled model had a weight of 0.181 (MC1). Similarly, the hierarchical model had a model weight of 0.797, compared to the pooled model's 0.203 (MC2). In addition, the hierarchical model outperformed the unpooled model as well, proving to be the best model of the three with a model weight of 1.0 versus 0.0 (MC3). This shows that the group differences between regions that are incorporated into both the hierarchical and unpooled models are impactful, and the hierarchical model has a better expected fit in terms of predictive accuracy.

In summary, the pooled model showed that temperature, rain, and relative humidity were important predictors of forest fires in both regions. However, the hierarchical model and unpooled model outperformed this model and showed that there are differences in the importance of the four base predictors between the two regions of Algeria. Thus, it is important to consider the differences in the climatic conditions of the two regions when predicting fire likelihood.

The pooled and hierarchical FWI models (Models 4 and 5) showed good sampling with convergence as well as stronger posterior predictions with lower variance (Fig 4.1-4.4 and 5.1-5.4). After building the two models, we found that the parameter estimates for the intercept and FWI predictor were similar between the two regions in the hierarchical model. We compared the two models using WAIC and found that the hierarchical model slightly outperforms the pooled model with a model weight of 0.545 versus 0.454 (MC4). Thus, although FWI's estimates aren't significantly different between the two regions, the hierarchical model with slight differences performs better. This implies that the interpretation of FWI between the two regions in northern Algeria could vary slightly.

In our aim to find the best model to accurately predict wildfires, we begin by comparing our hierarchical model with base measurements (Model 3) to our hierarchical FWI model (Model 5) using WAIC. We find that the hierarchical FWI model completely outperforms the base hierarchical model with a model weight of 1.0 (MC5). We tried to improve upon our predictive power by using the same hierarchical model by adding a non-linear term of rain with a square root transformation (Model 6). The non-linear model showed acceptable sampling with convergence and posterior predictions with higher variance than the FWI models (Fig 6.1-6.4). When comparing this nonlinear hierarchical model to the linear hierarchical model (Model 3), it is given a weight of 1.0 (MC6). However, when comparing it to the hierarchical FWI model, it was given a weight of 0, similar to the hierarchical measurement model (MC7).

Finally, to test the true predictive power of the non-linear hierarchical model and pooled FWI model, we made predictions onto the test set. The non-linear hierarchical model performed well with an accuracy of 0.857 and an area under the ROC curve (AUC) of 0.92 (Fig 6.6-6.7). In comparison, the hierarchical FWI model performed strongly with an accuracy of 0.939 and an AUC of 0.98 (Fig 5.6-5.7). Thus, both models are good predictors of forest fires, however, the FWI model is stronger and should be used over the hierarchical model, especially when so much is at stake from the detrimental impacts of forest fires. When FWI may not be available, our nonlinear hierarchical model could be used with caution.

Conclusion

The FWI does a great job of predicting the occurrence of fires, but our model also shows that the impact of FWI could vary slightly by region and may not consider different climatic conditions around the world or give insight into what climatic conditions are important to local areas. Our project helps us decipher how the original climatic measures can help impact estimated wildfires in two different regions of Algeria. We were also able to incorporate uncertainty into our approach through the use of priors and evaluate the confidence intervals of our predictors accordingly with posterior predictive results. Although we learned that we cannot beat the predictive power of the FWI, we found that the importance of climatic measures can vary by region, so it could be beneficial for fire prevention experts to develop regional-level FWIs.

A few limitations that we faced in our project include both a small sample size and a lack of seasonality due to our limited time frame of a summer of a single year. With more fire observations, climate measurements, and years of data, we would likely be able to produce better and more confident results. While we find that climatic measurements could generate regional specific insights into the likelihood of a fire that FWI does not capture, this is only based on our analysis of the Bejaia and Sidi Bel-abbes regions. We would need data from more regions around the world to generalize this conclusion.

Overall, this project and its outcomes were a success because we dug deeper and got significant results beyond the traditional estimate of wildfire danger.

References

Abid, Faroudja. (2019). *Algerian Forest Fires Dataset.* UCI Machine Learning Repository. https://doi.org/10.24432/C5KW4N.

Algeria - Forest Area (% Of Land Area) - 2023 Data 2024 Forecast 1990-2021 Historical. (2023). TRADING ECONOMICS. https://tradingeconomics.com/algeria/forest-area-percent-of-land-area-wb-data.html

Canadian Wildland Fire Information System | Canadian Forest Fire Weather Index (FWI) System. (n.d.). Natural Resources Canada. https://cwfis.cfs.nrcan.gc.ca/background/summary/fwi

Fire Weather Indices Wiki | Fire weather index. (n.d.). Swiss Federal Institute for Forest, Snow and Landscape Research WSL. https://wikifire.wsl.ch/tiki-index259b.html?page=Fire+weather+index

Sekkai, R., & Yee, V. (2023, July 25). *34 Killed in Algerian Wildfires as Heat Waves Parch Mediterranean Region*. The New York Times. https://www.nytimes.com/2023/07/25/world/middleeast/algeria-wildfires-heat-wave.html

Appendix

Exploratory Data Analysis

Figure 0.1 - Fire Weather Index Components

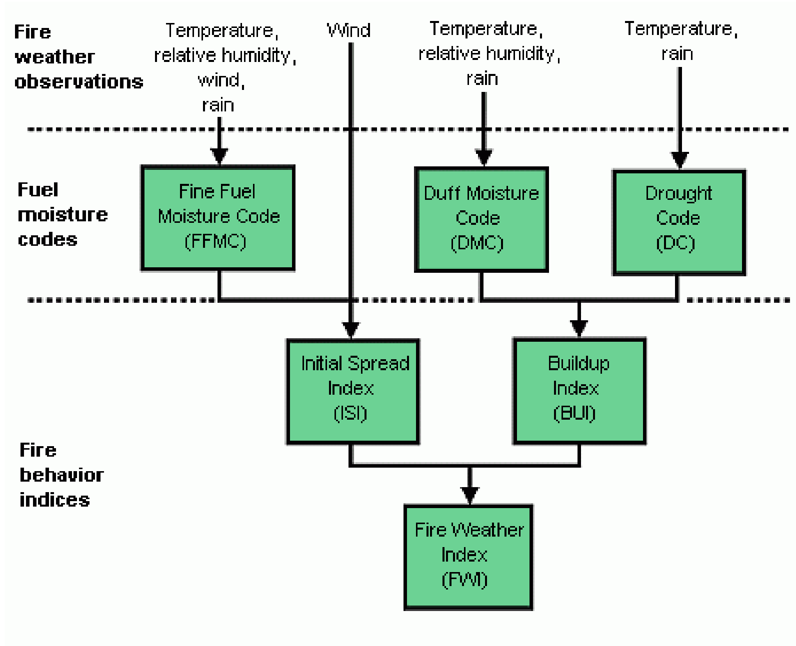


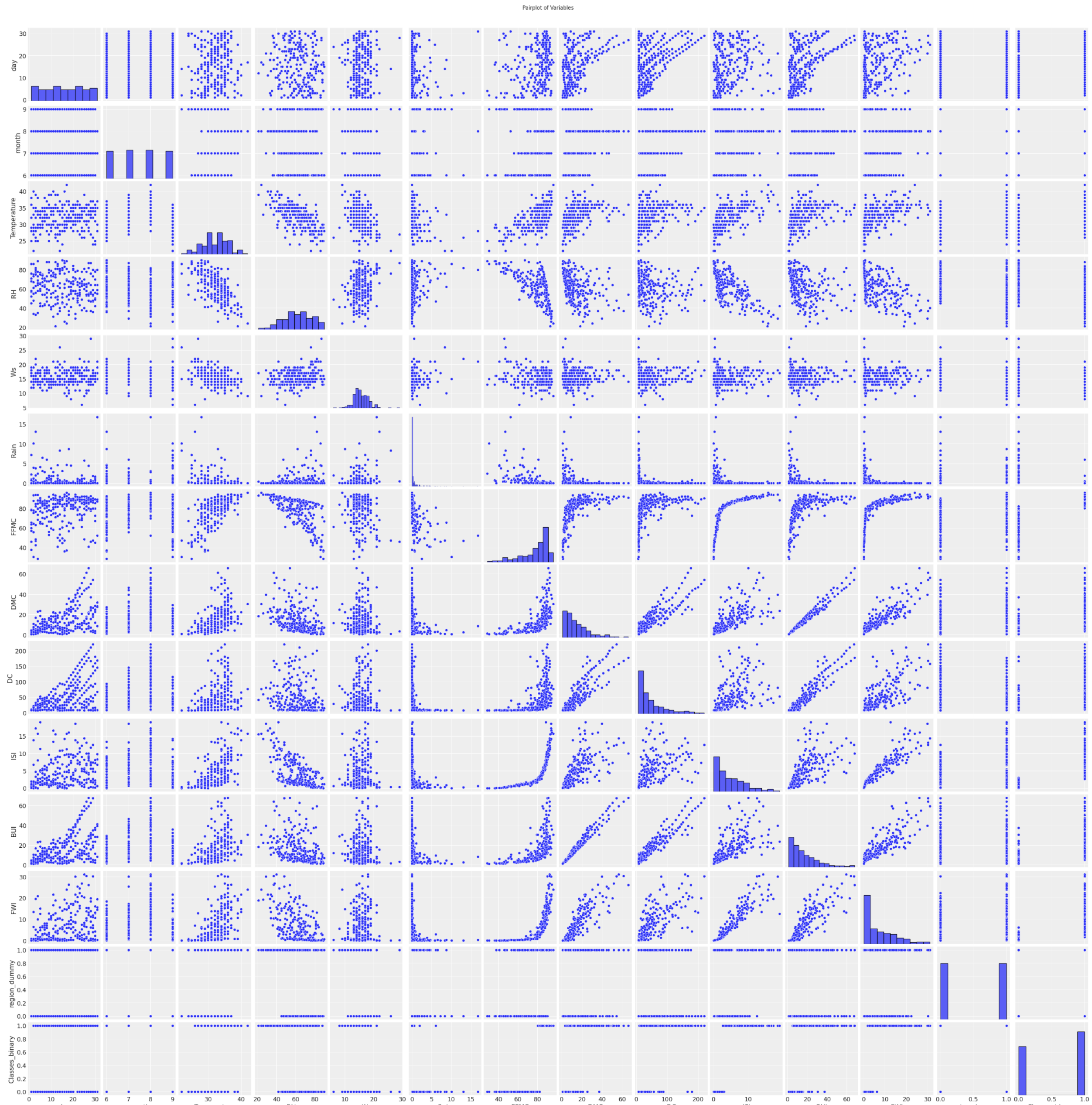
Figure 0.2 - Descriptive Statistics

	day	month	Temperature	RH	Ws	Rain	FFMC	DMC	DC	ISI	BUI	FWI	Classes	region	region_dummy	Classes_binary
count	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244.000000	244	244	244.000000	244.000000
unique	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	2	2	NaN	NaN
top	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	fire	Bejaia	NaN	NaN
freq	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	138	122	NaN	NaN
mean	15.754098	7.500000	32.172131	61.938525	15.504098	0.760656	77.887705	14.673361	49.288115	4.759836	16.673361	7.049180	NaN	NaN	0.500000	0.565574
std	8.825059	1.112961	3.633843	14.884200	2.810178	1.999406	14.337571	12.368039	47.619662	4.154628	14.201648	7.428366	NaN	NaN	0.501028	0.496700
min	1.000000	6.000000	22.000000	21.000000	6.000000	0.000000	28.600000	0.700000	6.900000	0.000000	1.100000	0.000000	NaN	NaN	0.000000	0.000000
25%	8.000000	7.000000	30.000000	52.000000	14.000000	0.000000	72.075000	5.800000	13.275000	1.400000	6.000000	0.700000	NaN	NaN	0.000000	0.000000
50%	16.000000	7.500000	32.000000	63.000000	15.000000	0.000000	83.500000	11.300000	33.100000	3.500000	12.450000	4.450000	NaN	NaN	0.500000	1.000000
75%	23.000000	8.000000	35.000000	73.250000	17.000000	0.500000	88.300000	20.750000	68.150000	7.300000	22.525000	11.375000	NaN	NaN	1.000000	1.000000
max	31.000000	9.000000	42.000000	90.000000	29.000000	16.800000	96.000000	65.900000	220.400000	19.000000	68.000000	31.100000	NaN	NaN	1.000000	1.000000

Figure 0.3 - Correlation Matrix

	day	month	Temperature	RH	Ws	Rain	FFMC	DMC	DC	ISI	BUI	FWI	region_dummy	Classes_binary
day	1.000	0.000	0.096	-0.074	0.047	-0.112	0.224	0.492	0.528	0.179	0.517	0.350	0.000	0.202
month	0.000	1.000	-0.059	-0.038	-0.041	0.035	0.016	0.068	0.128	0.064	0.086	0.082	-0.000	0.022
Temperature	0.096	-0.059	1.000	-0.654	-0.278	-0.327	0.677	0.483	0.370	0.606	0.456	0.567	0.273	0.518
RH	-0.074	-0.038	-0.654	1.000	0.236	0.223	-0.646	-0.405	-0.220	-0.688	-0.350	-0.580	-0.406	-0.435
Ws	0.047	-0.041	-0.278	0.236	1.000	0.170	-0.163	-0.001	0.076	0.012	0.030	0.034	-0.177	-0.067
Rain	-0.112	0.035	-0.327	0.223	0.170	1.000	-0.544	-0.289	-0.297	-0.348	-0.299	-0.325	-0.041	-0.379
FFMC	0.224	0.016	0.677	-0.646	-0.163	-0.544	1.000	0.602	0.504	0.741	0.590	0.691	0.225	0.770
DMC	0.492	0.068	0.483	-0.405	-0.001	-0.289	0.602	1.000	0.875	0.678	0.982	0.875	0.191	0.584
DC	0.528	0.128	0.370	-0.220	0.076	-0.297	0.504	0.875	1.000	0.504	0.942	0.737	-0.081	0.507
ISI	0.179	0.064	0.606	-0.688	0.012	-0.348	0.741	0.678	0.504	1.000	0.641	0.922	0.266	0.736
BUI	0.517	0.086	0.456	-0.350	0.030	-0.299	0.590	0.982	0.942	0.641	1.000	0.857	0.088	0.585
FWI	0.350	0.082	0.567	-0.580	0.034	-0.325	0.691	0.875	0.737	0.922	0.857	1.000	0.198	0.719
region_dummy	0.000	-0.000	0.273	-0.406	-0.177	-0.041	0.225	0.191	-0.081	0.266	0.088	0.198	1.000	0.165
Classes_binary	0.202	0.022	0.518	-0.435	-0.067	-0.379	0.770	0.584	0.507	0.736	0.585	0.719	0.165	1.000

Figure 0.4 - Pairplot Matrix



Model 1: Pooled Linear Measurement Model

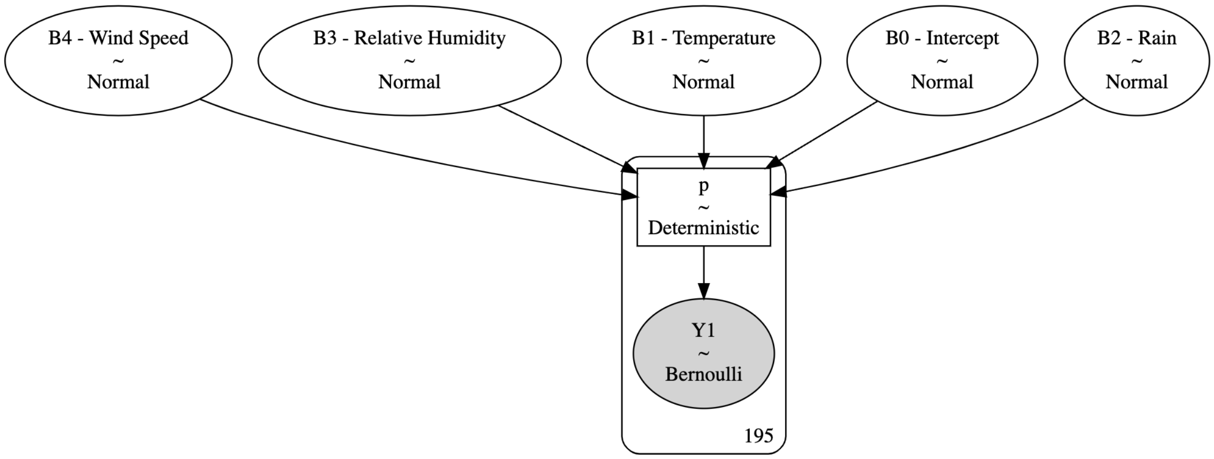


Figure 1.1 - Pooled Linear Measurement Model - Trace Plot

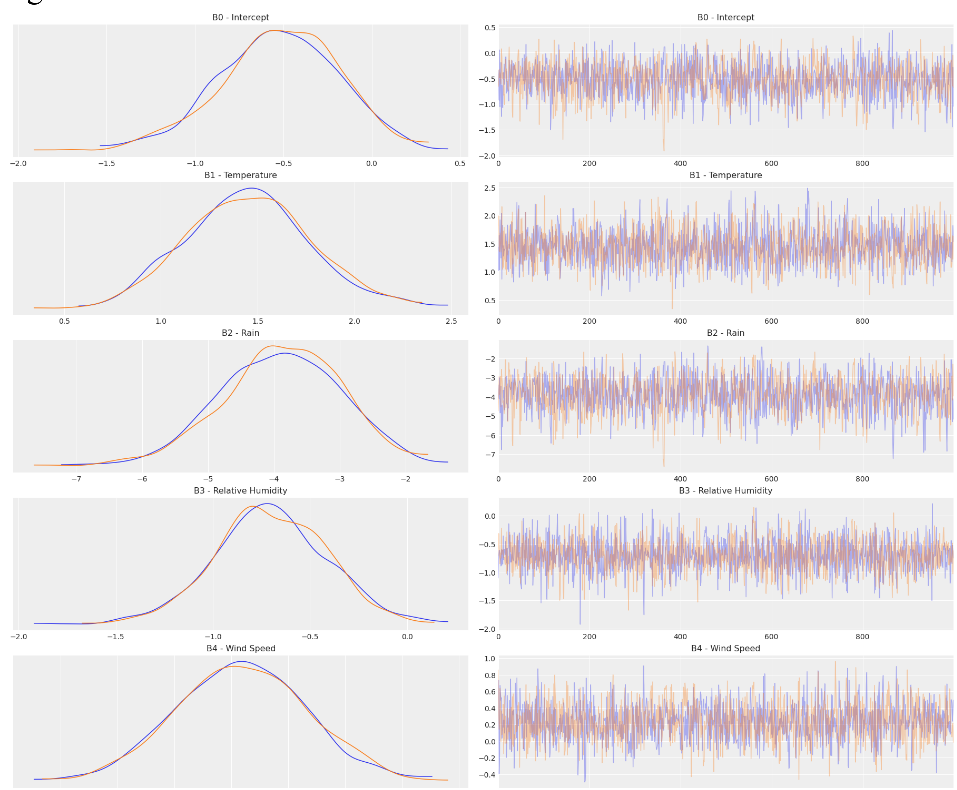


Figure 1.2 - Pooled Linear Measurement Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept	-0.527	0.337	-1.186	0.089	0.009	0.007	1433.0	1322.0	1.0
B1 - Temperature	1.453	0.321	0.860	2.058	0.008	0.006	1805.0	1441.0	1.0
B2 - Rain	-3.895	0.927	-5.548	-2.121	0.025	0.018	1406.0	1404.0	1.0
B3 - Relative Humidity	-0.703	0.289	-1.245	-0.173	0.006	0.004	2180.0	1540.0	1.0
B4 - Wind Speed	0.234	0.230	-0.174	0.691	0.005	0.004	1936.0	1491.0	1.0

Figure 1.3 - Pooled Linear Measurement Model - Bayesian p-Value Plot

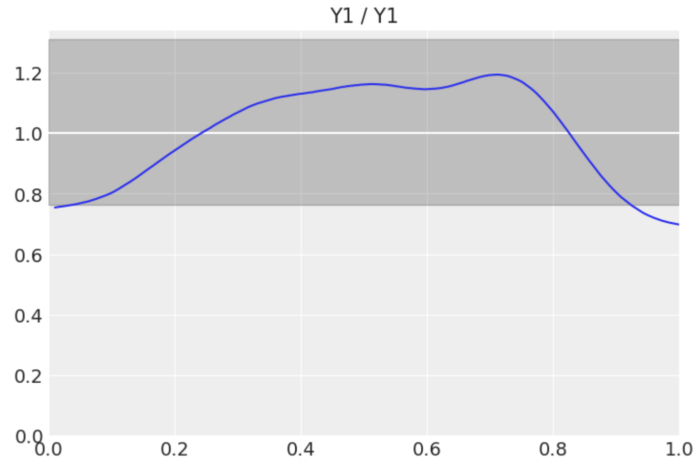


Figure 1.4 - Pooled Linear Measurement Model - Posterior Predictive Check Plot

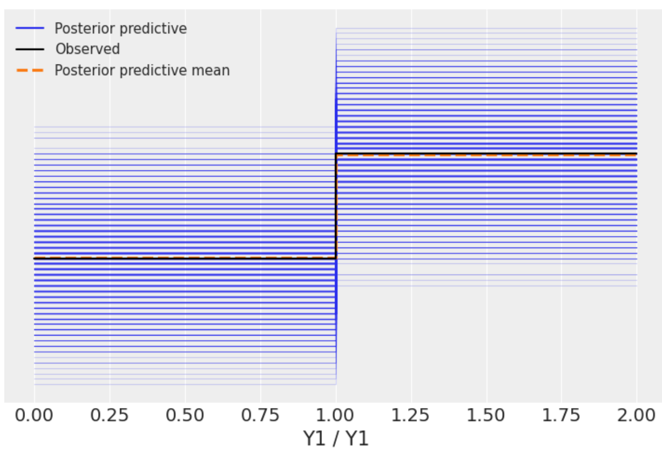
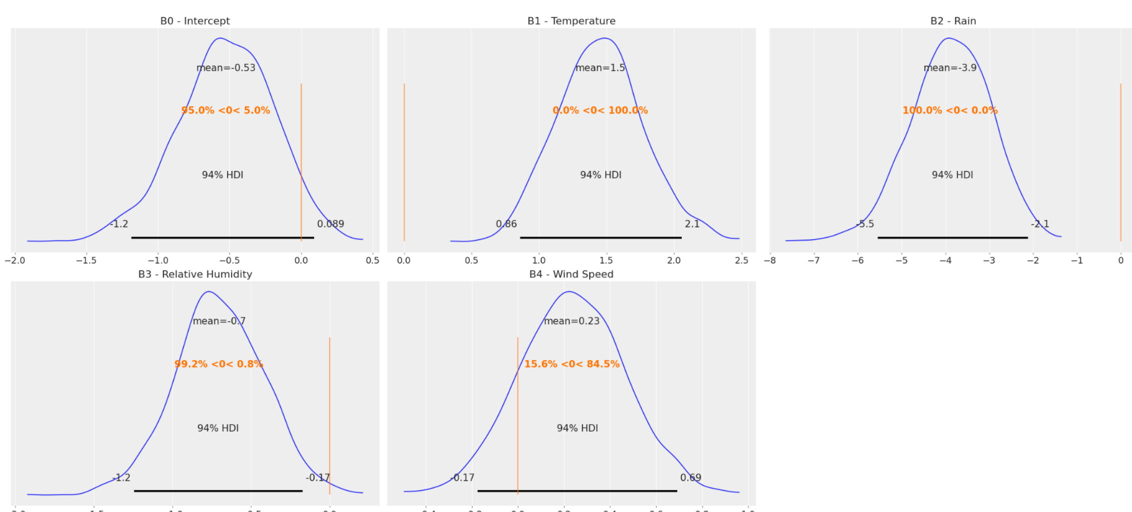


Figure 1.5 - Pooled Linear Measurement Model - Posterior Coefficient Plot



Model 2: Unpooled Linear Measurement Model

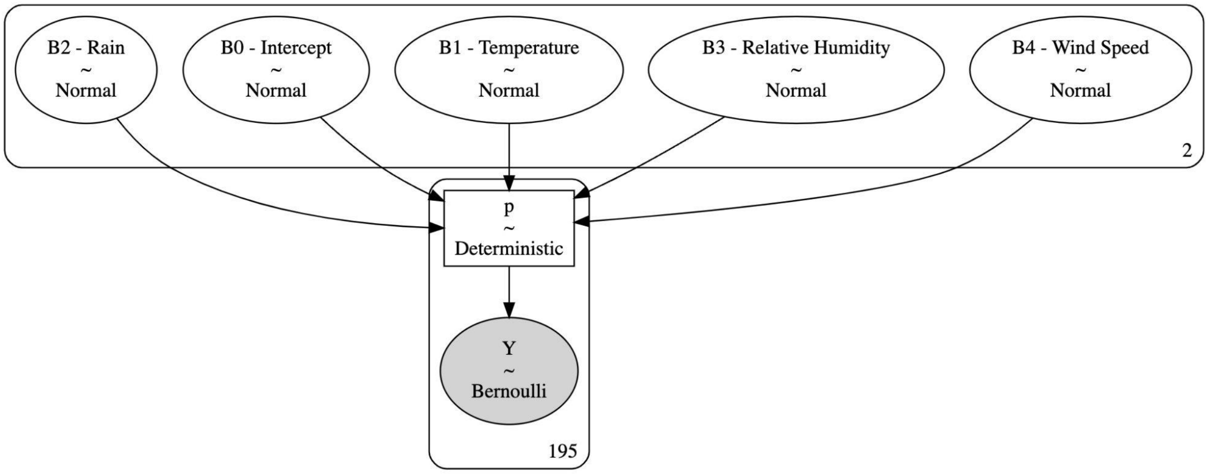


Figure 2.1 - Unpooled Linear Measurement Model - Trace Plot

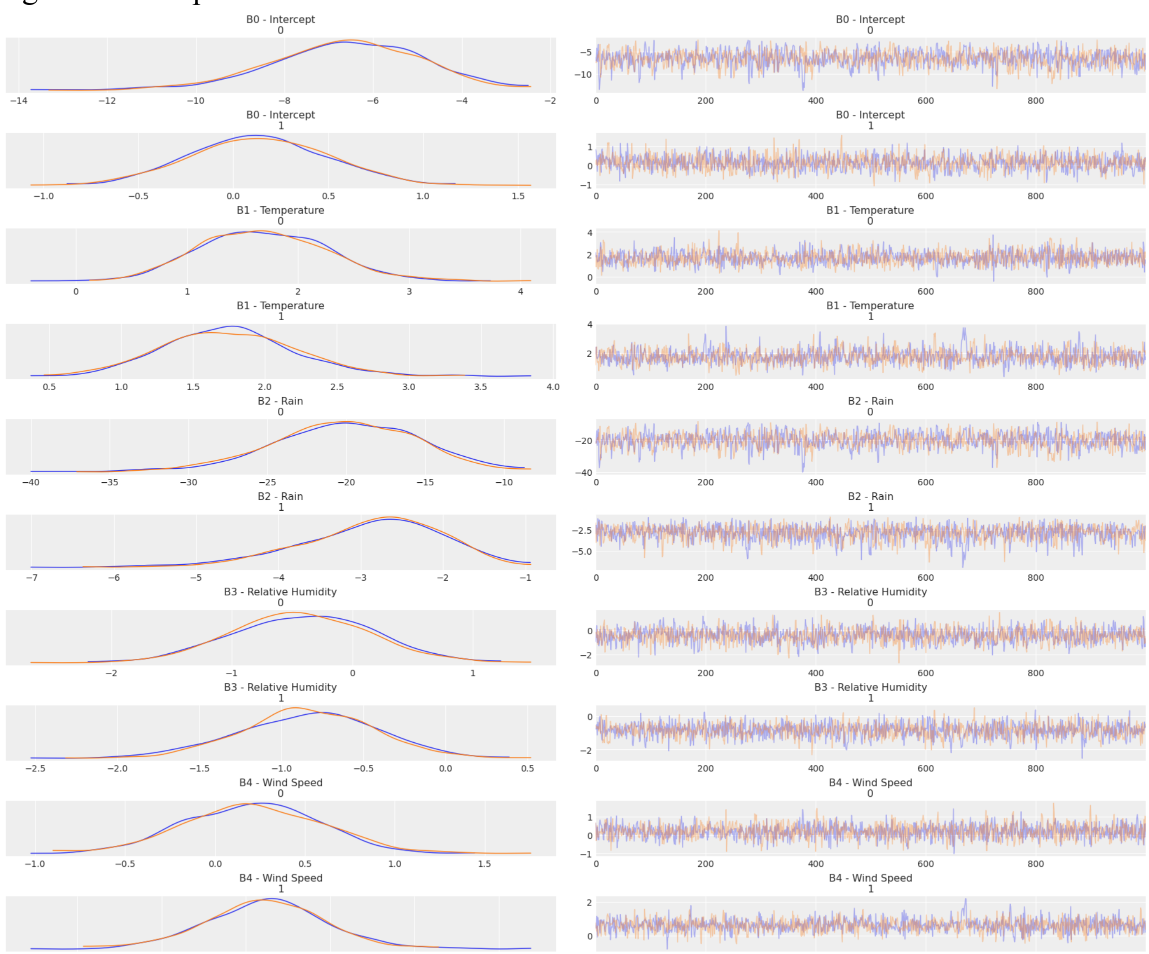


Figure 2.2 - Unpooled Linear Measurement Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept[0]	-6.624	1.728	-9.799	-3.416	0.054	0.040	1113.0	916.0	1.00
B0 - Intercept[1]	0.140	0.369	-0.550	0.809	0.009	0.007	1600.0	1553.0	1.00
B1 - Temperature[0]	1.691	0.575	0.647	2.745	0.014	0.011	1621.0	1491.0	1.00
B1 - Temperature[1]	1.757	0.481	0.815	2.601	0.015	0.011	1152.0	961.0	1.00
B2 - Rain[0]	-20.105	4.675	-28.864	-11.420	0.150	0.112	1031.0	687.0	1.00
B2 - Rain[1]	-2.874	0.885	-4.589	-1.378	0.026	0.021	1383.0	1020.0	1.00
B3 - Relative Humidity[0]	-0.418	0.578	-1.523	0.626	0.014	0.011	1602.0	1298.0	1.00
B3 - Relative Humidity[1]	-0.841	0.413	-1.682	-0.147	0.011	0.008	1574.0	1154.0	1.00
B4 - Wind Speed[0]	0.212	0.383	-0.466	0.953	0.009	0.008	1963.0	1348.0	1.01
B4 - Wind Speed[1]	0.617	0.362	-0.095	1.270	0.010	0.008	1423.0	1228.0	1.00

Figure 2.3 - Unpooled Linear Measurement Model - Bayesian p-Value Plot

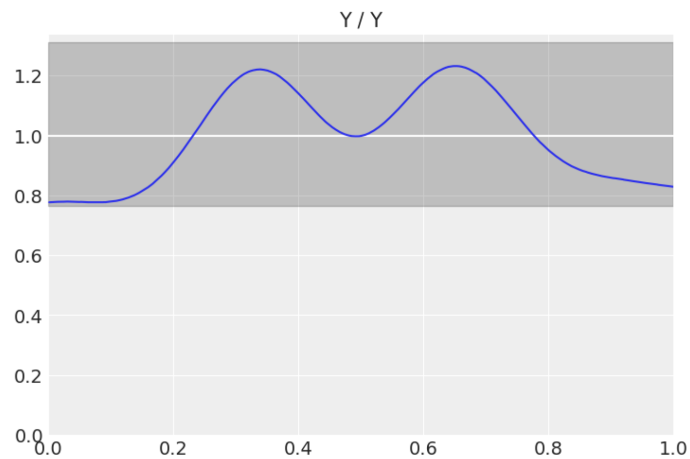


Figure 2.4 - Unpooled Linear Measurement Model - Posterior Predictive Check Plot

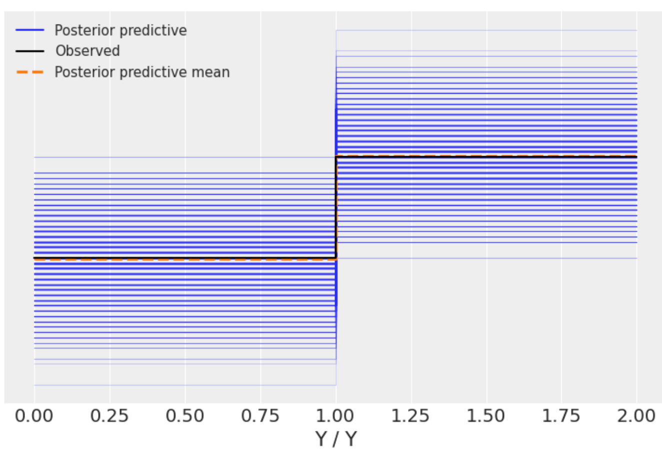
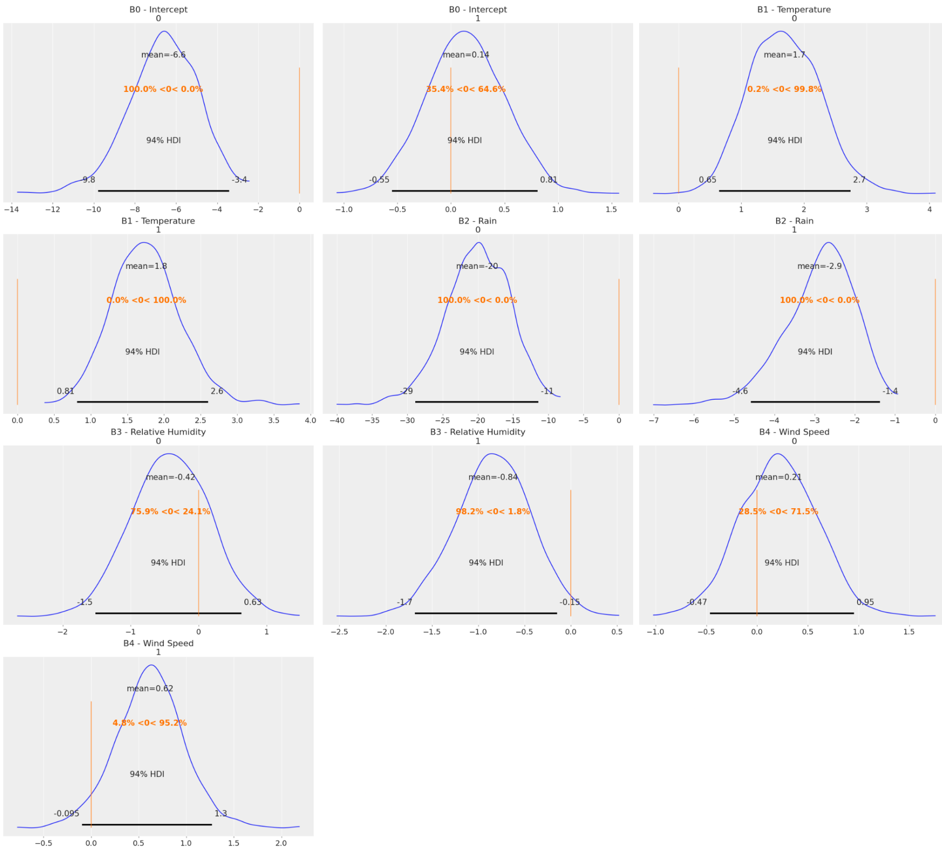


Figure 2.5 - Unpooled Linear Measurement Model - Posterior Coefficient Plot



Model 3: Hierarchical Linear Measurement Model

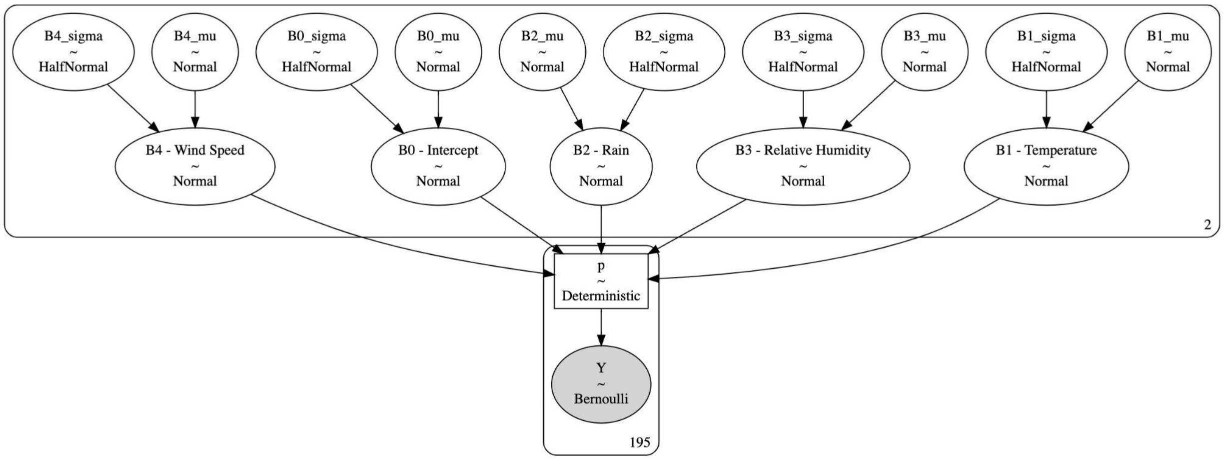


Figure 3.1 - Hierarchical Linear Measurement Model - Trace Plot

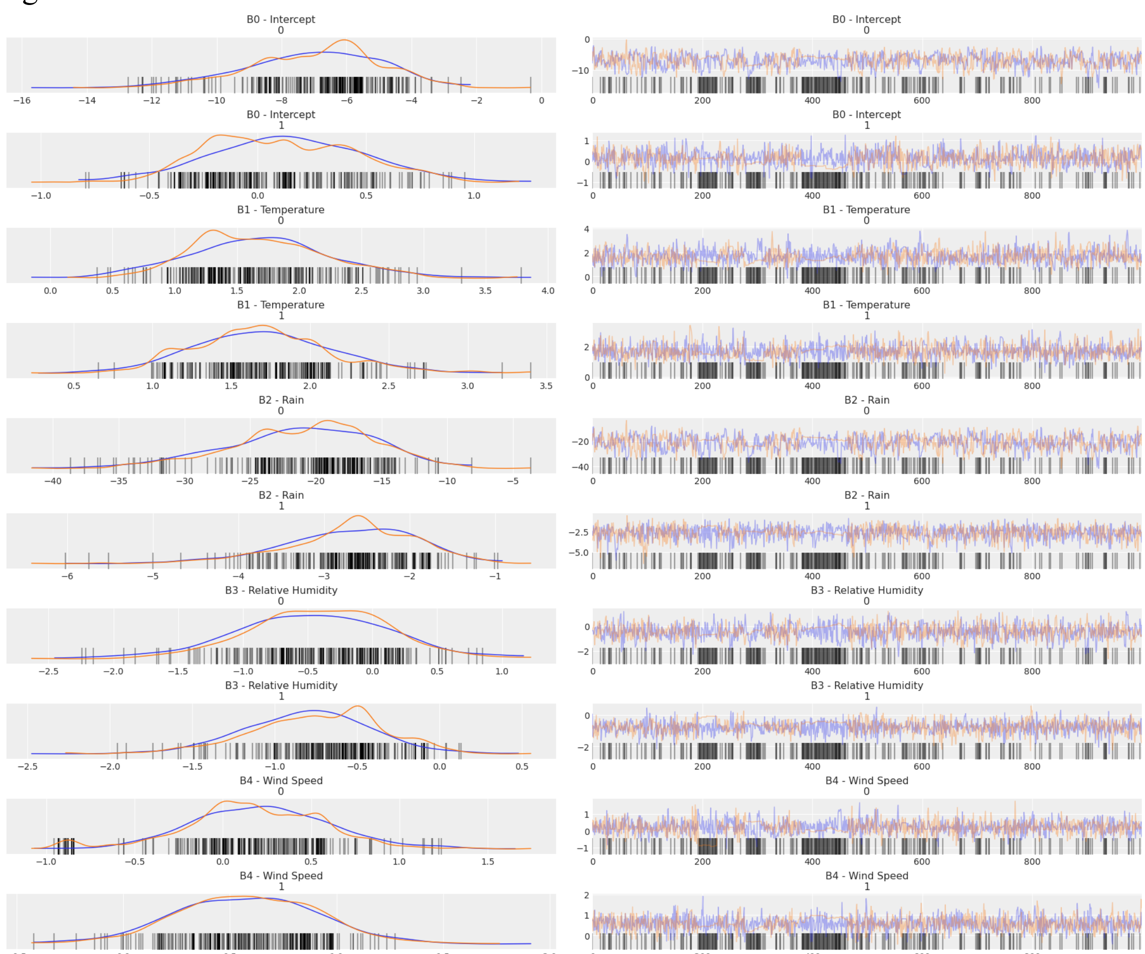


Figure 3.2 - Hierarchical Linear Measurement Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept[0]	-7.001	2.066	-11.266	-3.645	0.081	0.059	669.0	899.0	1.00
B0 - Intercept[1]	0.129	0.364	-0.541	0.799	0.016	0.011	504.0	985.0	1.01
B1 - Temperature[0]	1.685	0.570	0.690	2.818	0.020	0.015	853.0	999.0	1.00
B1 - Temperature[1]	1.702	0.450	0.931	2.613	0.014	0.010	855.0	1197.0	1.00
B2 - Rain[0]	-21.114	5.570	-31.914	-11.306	0.220	0.159	666.0	799.0	1.00
B2 - Rain[1]	-2.688	0.783	-4.210	-1.288	0.024	0.018	1138.0	963.0	1.01
B3 - Relative Humidity[0]	-0.435	0.569	-1.579	0.569	0.016	0.013	1226.0	1168.0	1.01
B3 - Relative Humidity[1]	-0.774	0.398	-1.510	-0.024	0.018	0.013	475.0	431.0	1.01
B4 - Wind Speed[0]	0.194	0.406	-0.642	0.924	0.021	0.015	445.0	140.0	1.01
B4 - Wind Speed[1]	0.582	0.339	-0.066	1.218	0.012	0.008	832.0	1329.0	1.00

Figure 3.3 - Hierarchical Linear Measurement Model - Bayesian p-Value Plot

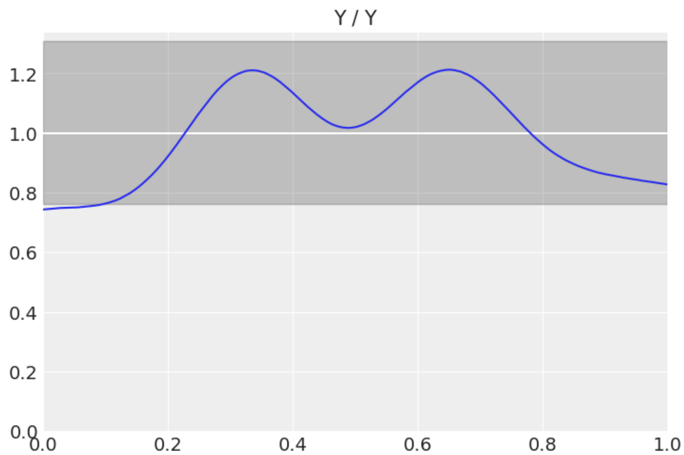


Figure 3.4 - Hierarchical Linear Measurement Model - Posterior Predictive Check Plot

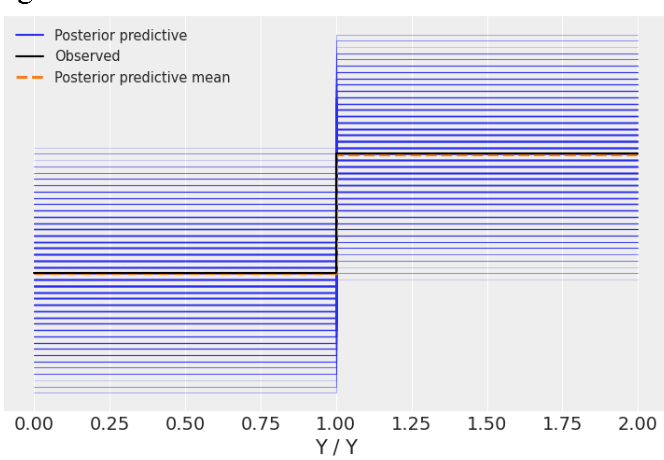
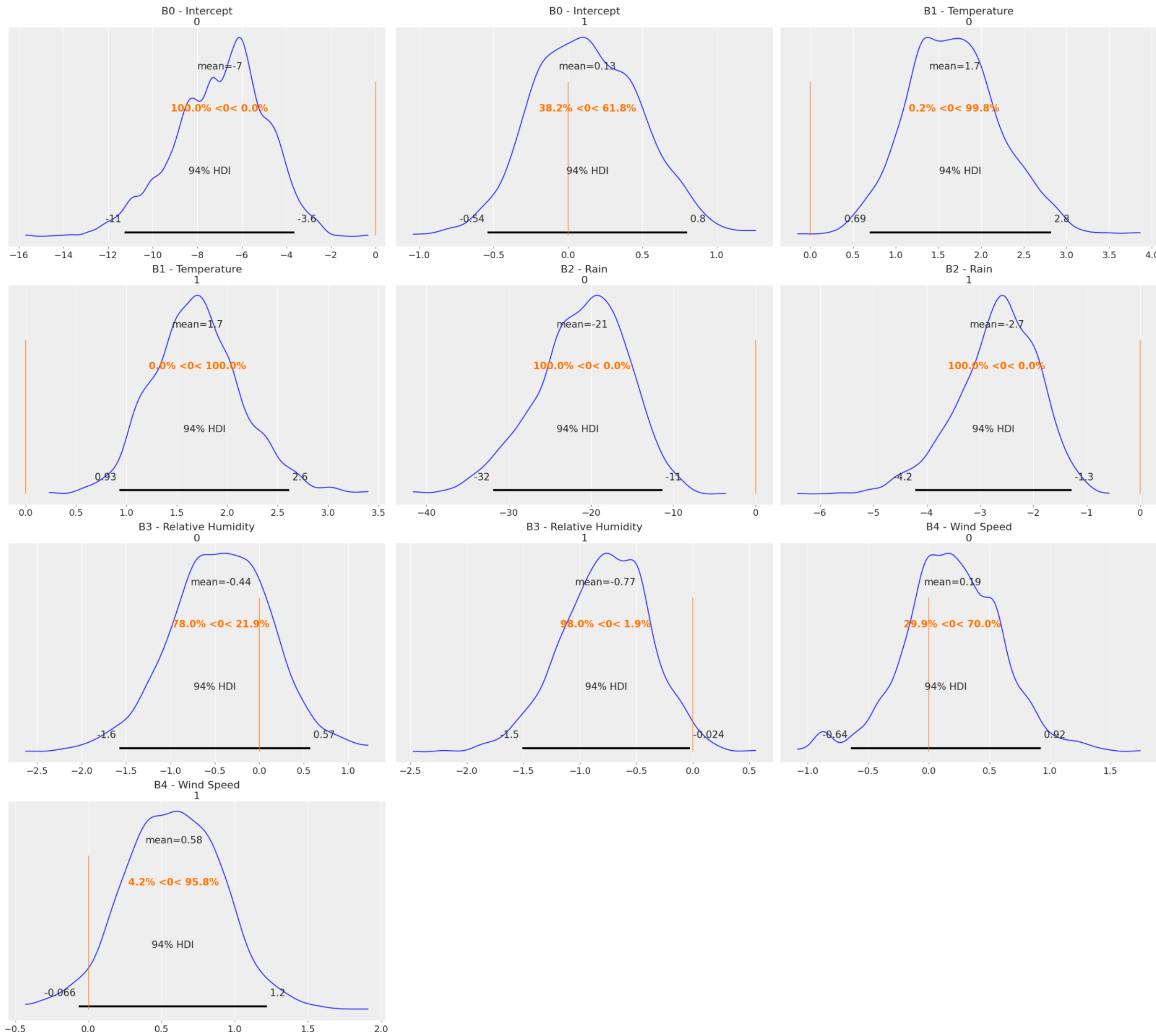


Figure 3.5 - Hierarchical Linear Measurement Model - Posterior Coefficient Plot



Model 4: Pooled FWI Model

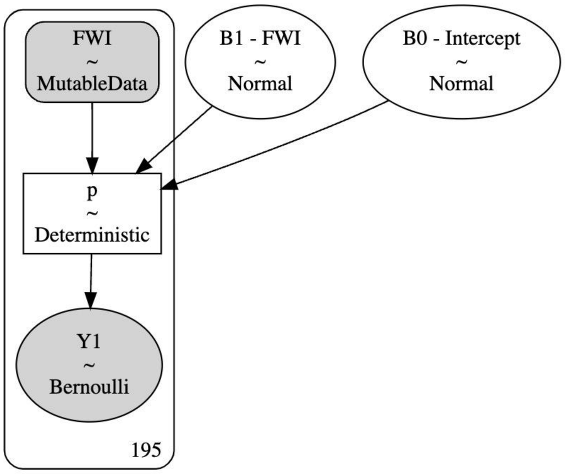


Figure 4.1 - Pooled FWI Model - Trace Plot

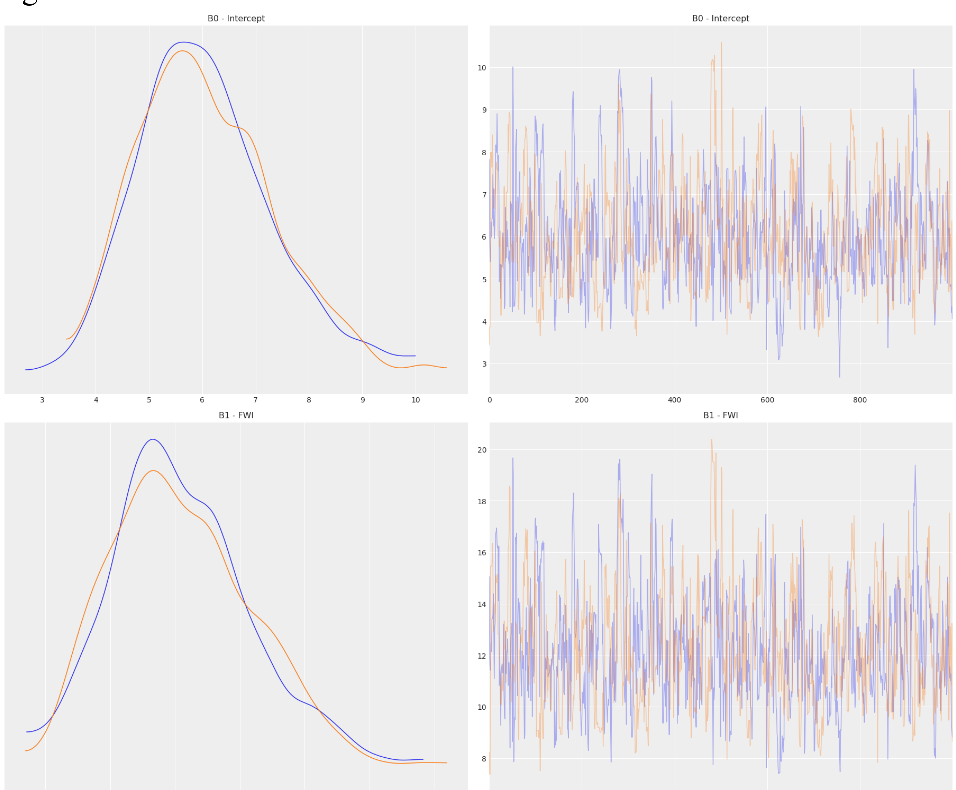


Figure 4.2 - Pooled FWI Linear Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept	6.062	1.268	3.80	8.427	0.072	0.051	308.0	409.0	1.01
B1 - FWI	12.283	2.271	8.23	16.478	0.129	0.092	318.0	407.0	1.01

Figure 4.3 - Pooled FWI Linear Model - Bayesian p-Value table

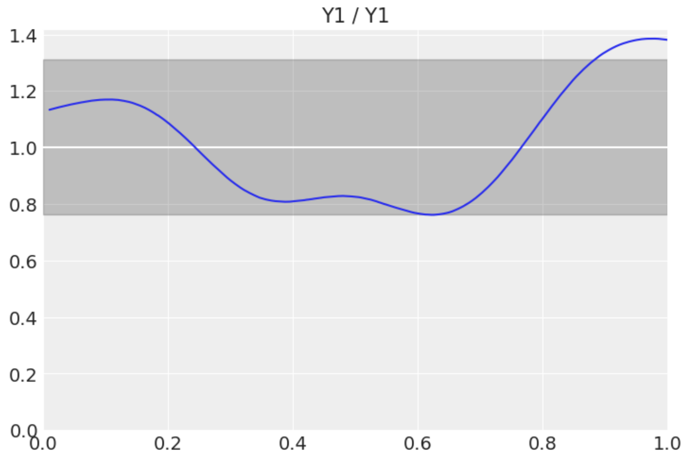


Figure 4.4 - Pooled FWI Linear Model - Posterior Predictive Check Plot

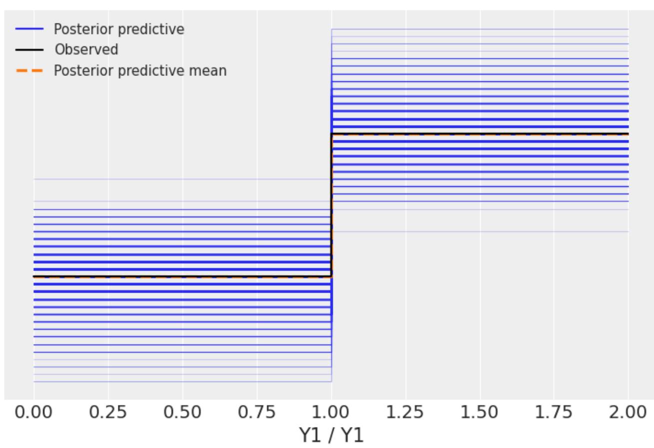
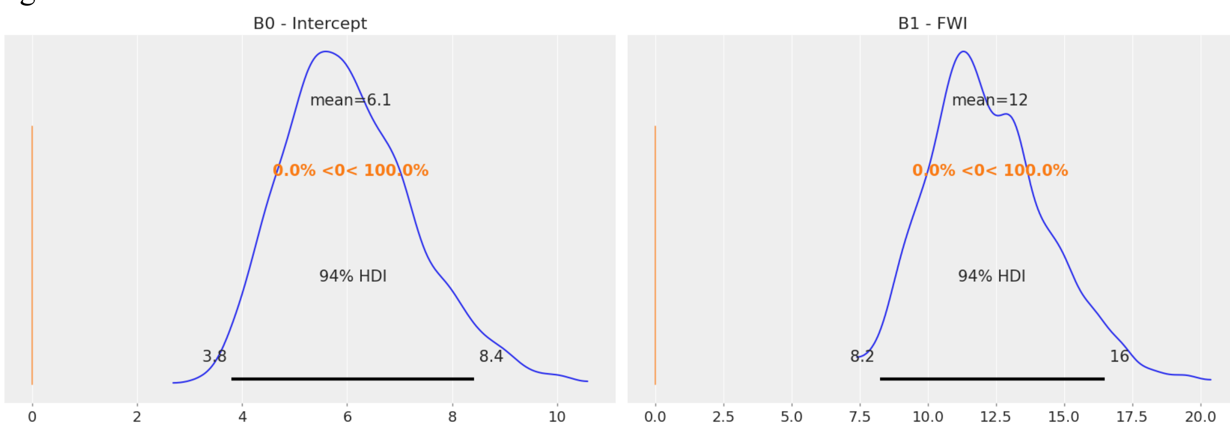


Figure 4.5 - Pooled FWI Linear Model - Posterior Coefficient Plot



Model 5: Hierarchical FWI Model

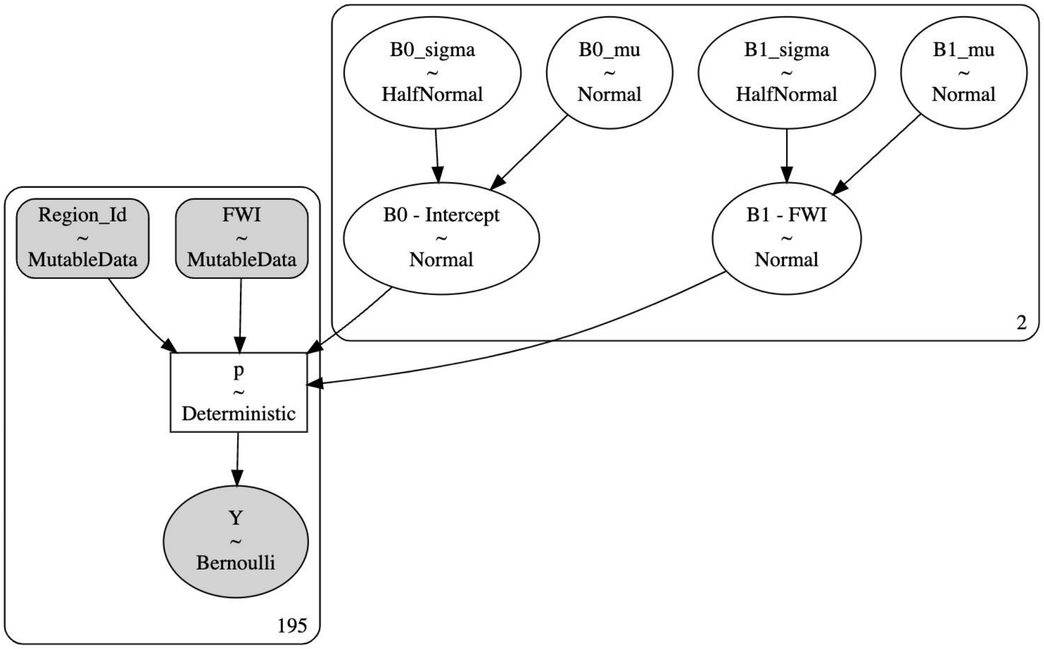


Figure 5.1 - Hierarchical FWI Model - Trace Plot

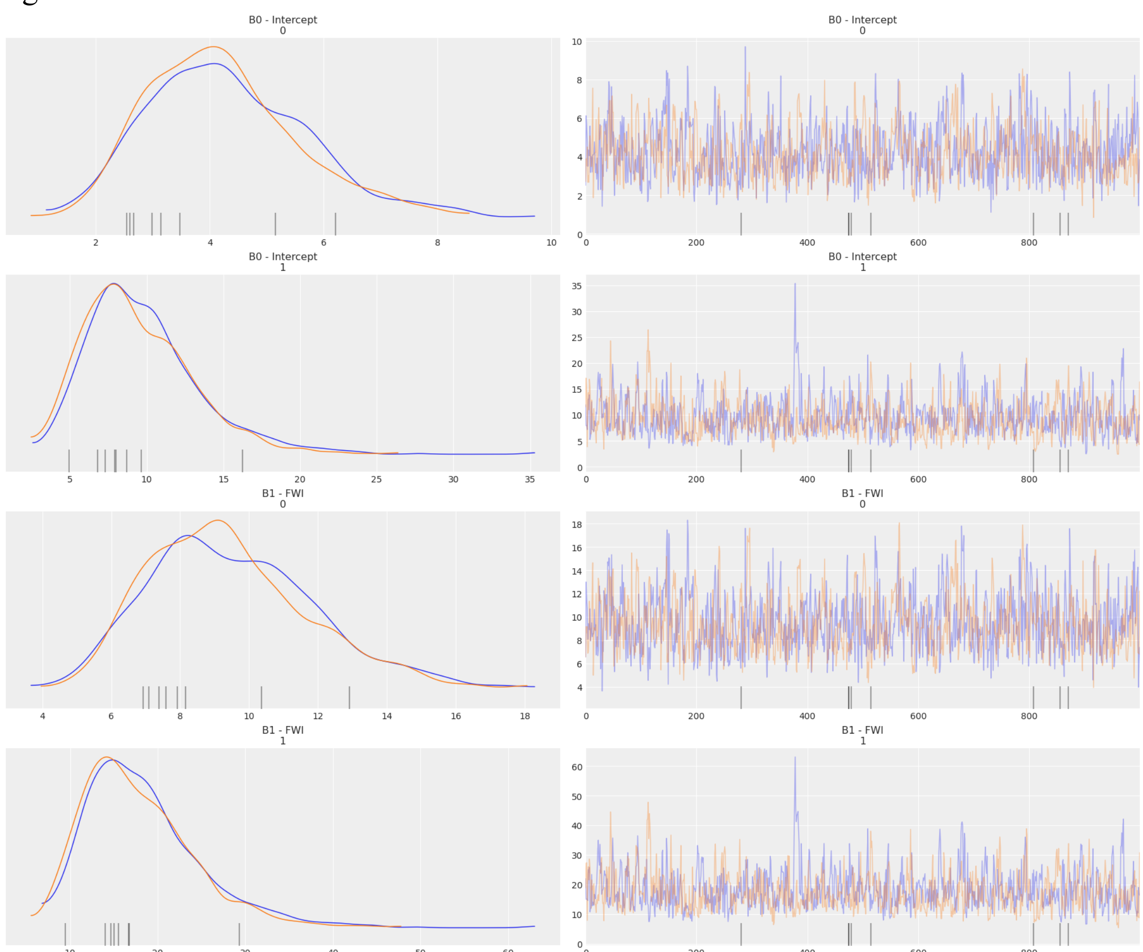


Figure 5.2 - Hierarchical FWI Linear Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept[0]	4.256	1.344	1.859	6.788	0.055	0.040	661.0	778.0	1.0
B0 - Intercept[1]	9.582	3.631	4.092	16.806	0.163	0.120	534.0	623.0	1.0
B1 - FWI[0]	9.506	2.444	5.232	14.168	0.097	0.071	707.0	710.0	1.0
B1 - FWI[1]	18.085	6.518	7.580	30.103	0.295	0.219	526.0	575.0	1.0

Figure 5.3 - Hierarchical FWI Linear Model - Bayesian p-Value Table

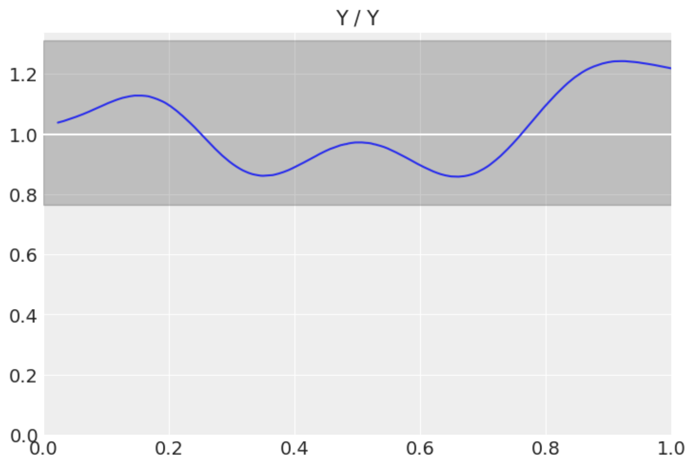


Figure 5.4 - Hierarchical FWI Linear Model - Posterior Predictive Check Plot

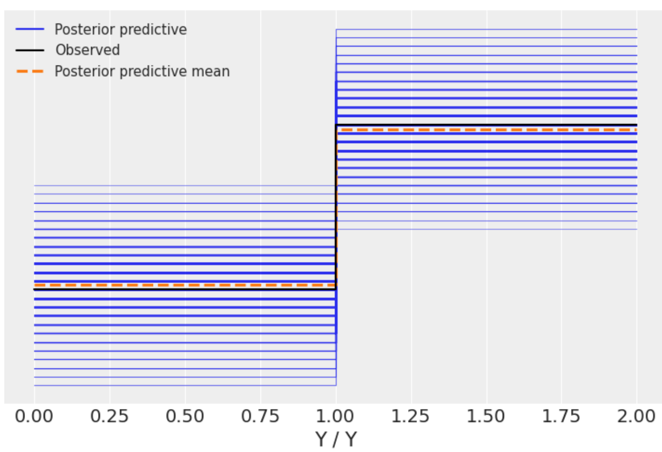


Figure 5.5 - Hierarchical FWI Linear Model - Posterior Coefficient Plot

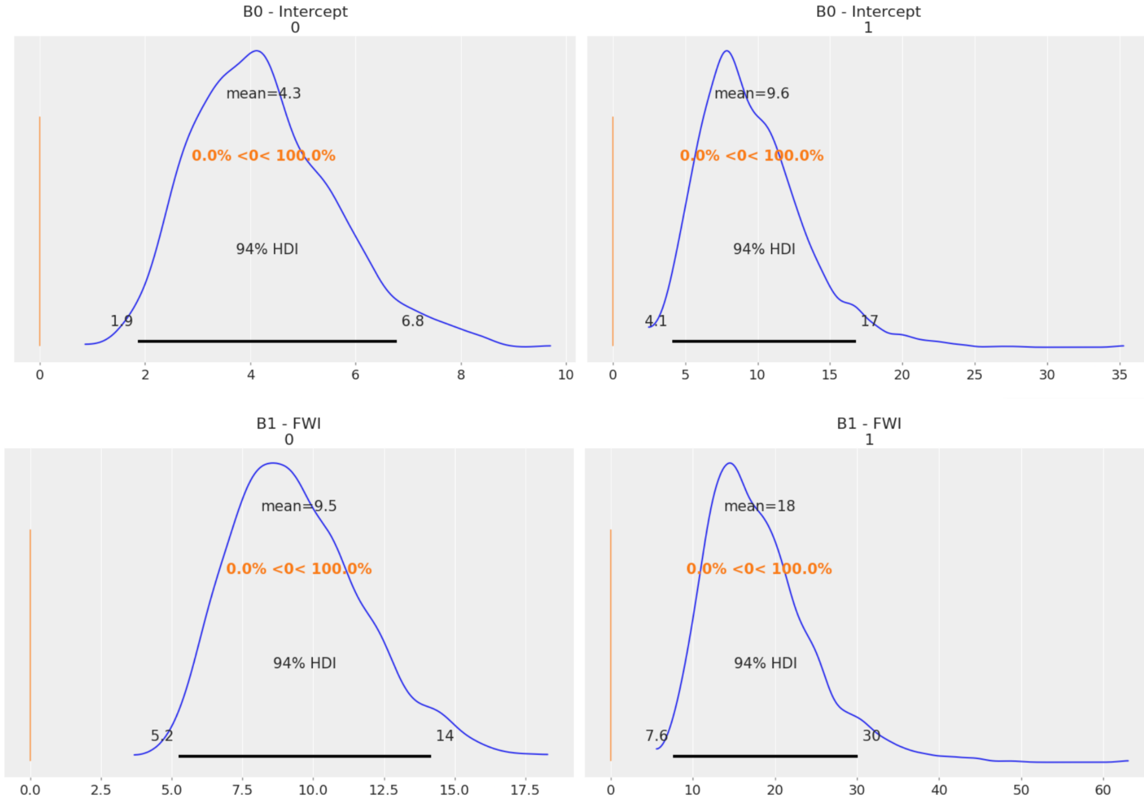


Figure 5.6 - Hierarchical FWI Linear Model - ROC Plot

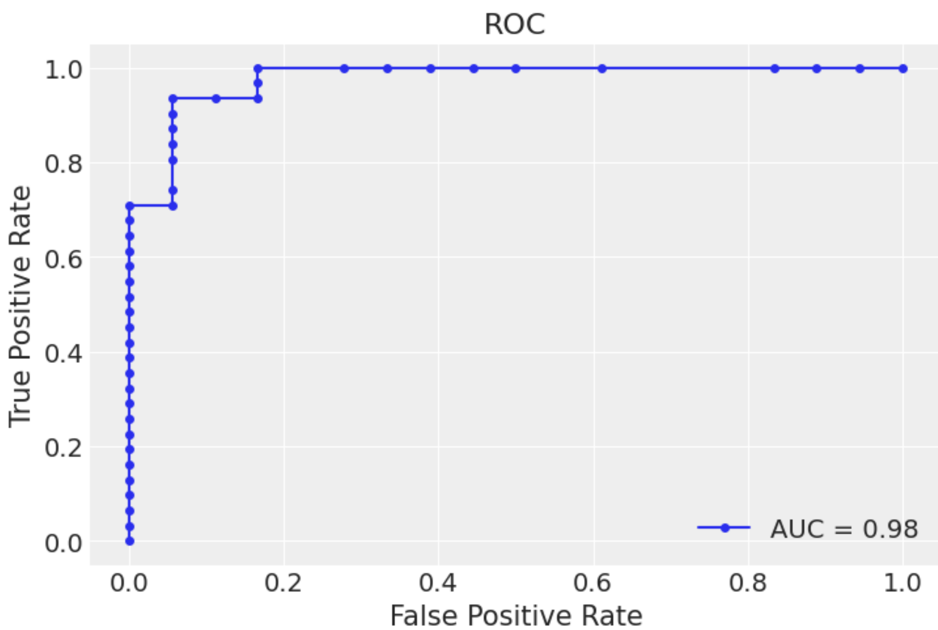
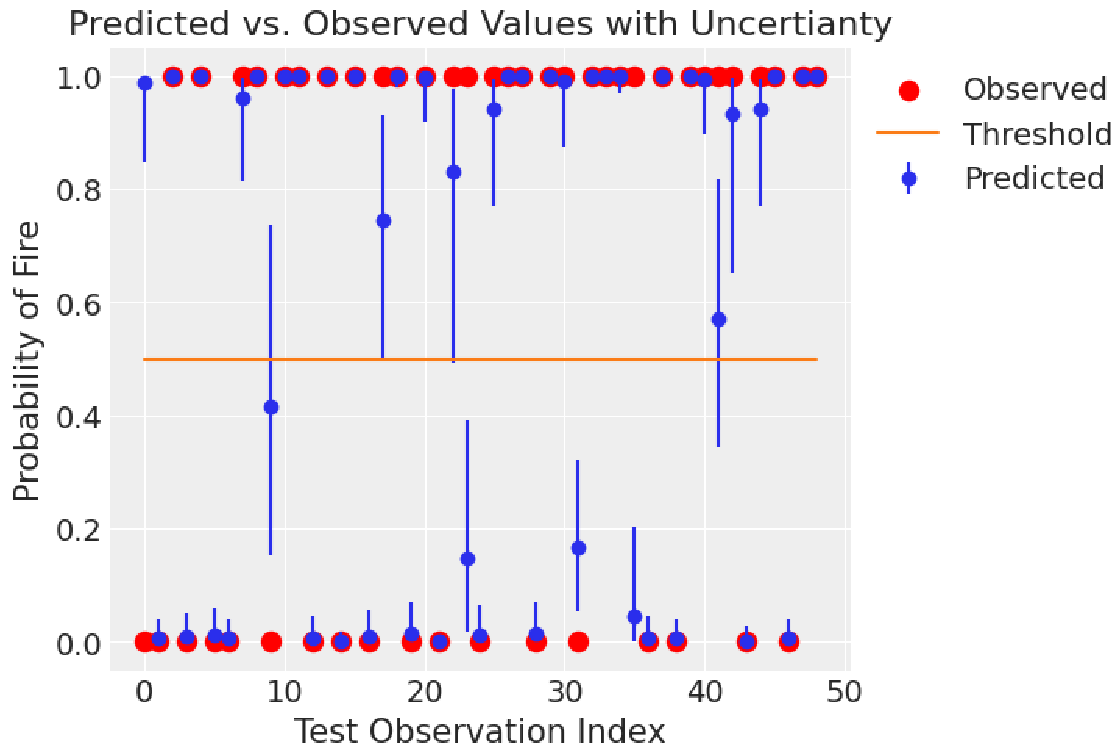


Figure 5.7 - Hierarchical FWI Linear Model - Predicted vs Observed Plot



Model 6: Hierarchical Non-Linear Measurement Model

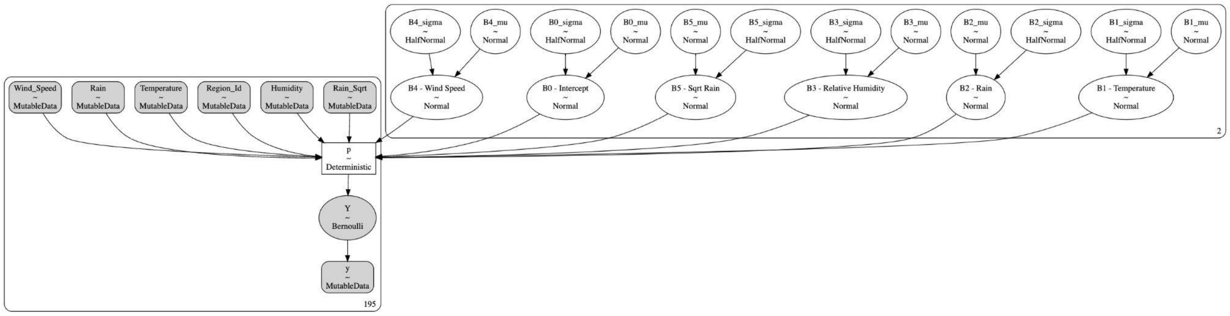


Figure 6.1 - Hierarchical Non-Linear Measurement Model - Trace Plot

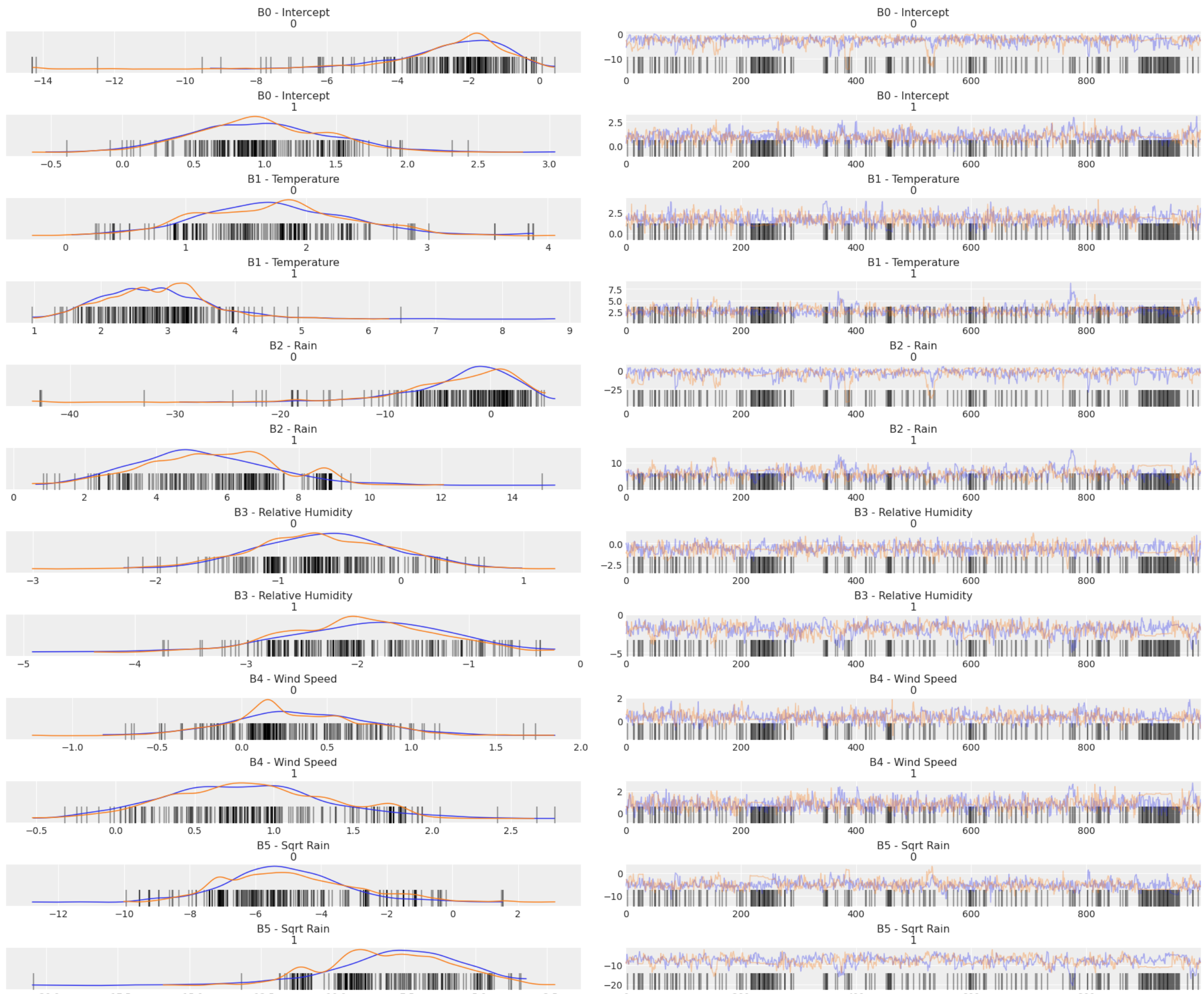


Figure 6.2 - Hierarchical Non-Linear Measurement Model - Summary Table

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
B0 - Intercept[0]	-2.583	1.839	-5.745	0.110	0.111	0.086	375.0	333.0	1.00
B0 - Intercept[1]	0.972	0.494	0.084	1.907	0.023	0.017	496.0	649.0	1.01
B1 - Temperature[0]	1.740	0.643	0.530	2.902	0.023	0.016	763.0	706.0	1.00
B1 - Temperature[1]	2.843	0.839	1.303	4.289	0.046	0.033	365.0	446.0	1.00
B2 - Rain[0]	-3.146	6.243	-13.943	4.979	0.391	0.293	348.0	315.0	1.01
B2 - Rain[1]	5.515	2.034	1.745	8.928	0.140	0.102	223.0	394.0	1.00
B3 - Relative Humidity[0]	-0.631	0.573	-1.657	0.456	0.021	0.015	728.0	955.0	1.00
B3 - Relative Humidity[1]	-1.947	0.718	-3.178	-0.543	0.040	0.029	341.0	500.0	1.00
B4 - Wind Speed[0]	0.334	0.406	-0.382	1.141	0.015	0.011	735.0	1026.0	1.00
B4 - Wind Speed[1]	0.848	0.523	-0.090	1.833	0.028	0.024	389.0	357.0	1.00
B5 - Sqrt Rain[0]	-5.025	1.939	-8.464	-1.095	0.131	0.093	225.0	325.0	1.02
B5 - Sqrt Rain[1]	-7.947	2.336	-11.918	-3.694	0.159	0.114	225.0	337.0	1.00

Figure 6.3 - Hierarchical Non-Linear Measurement Model - Bayesian p-Value Plot

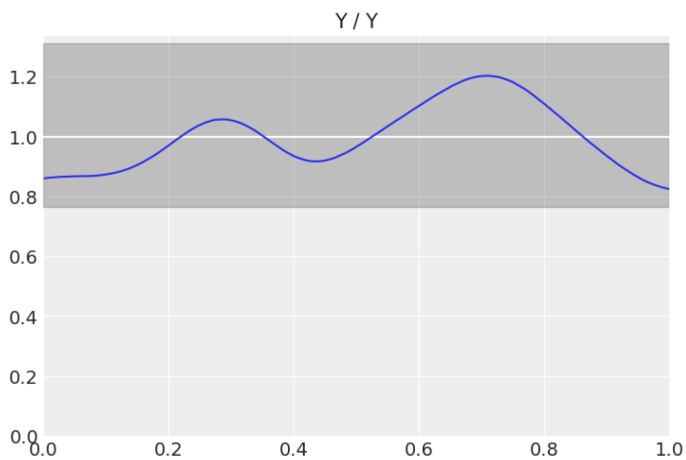


Figure 6.4 - Hierarchical Non-Linear Measurement Model - Posterior Predictive Check Plot

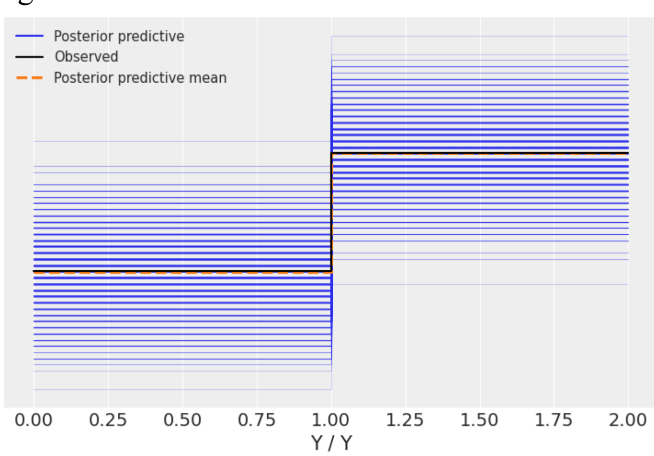


Figure 6.5 - Hierarchical Non-Linear Measurement Model - Posterior Coefficient Plot

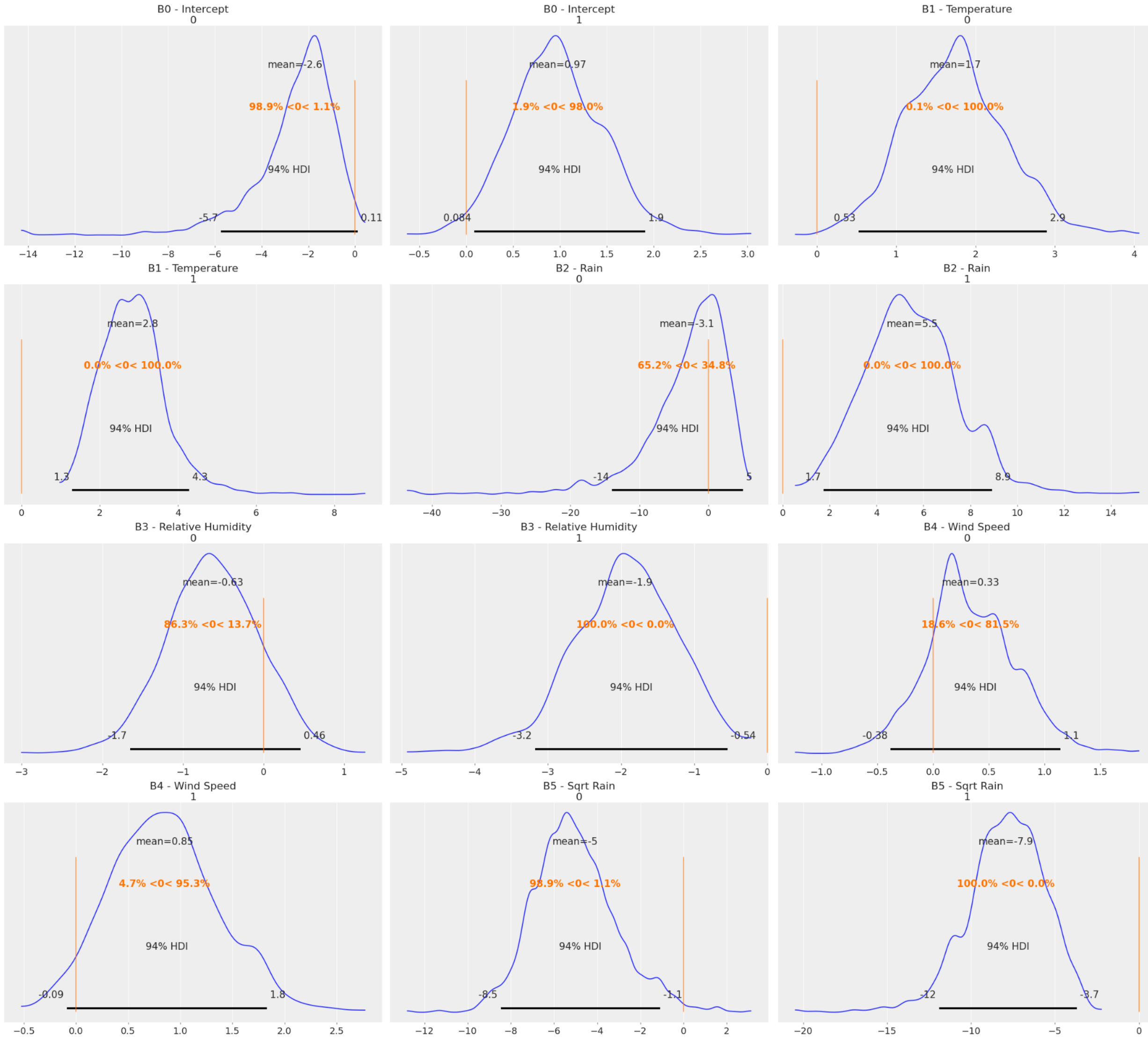


Figure 6.6 - Hierarchical Non-Linear Measurement Model - ROC Plot

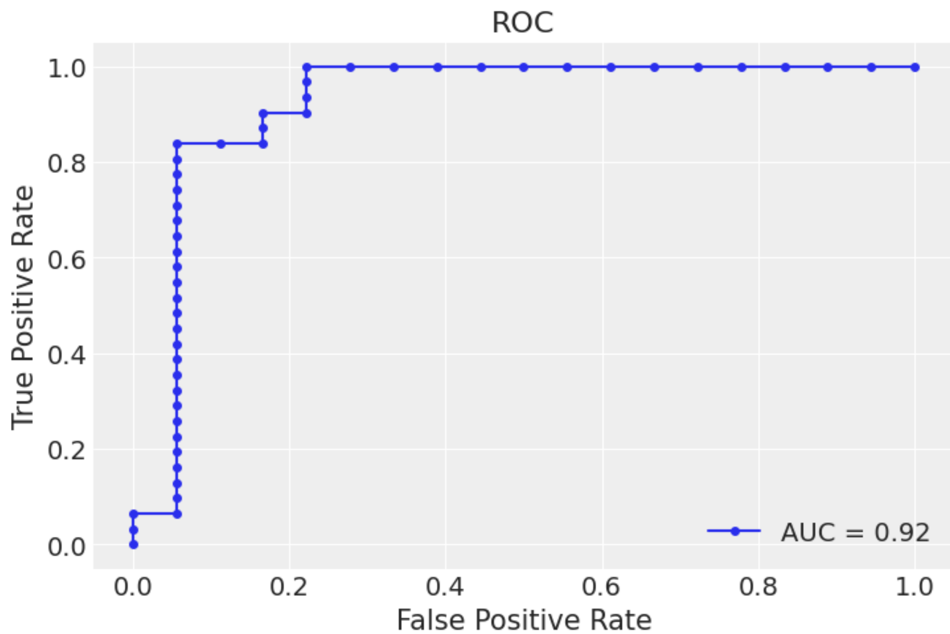
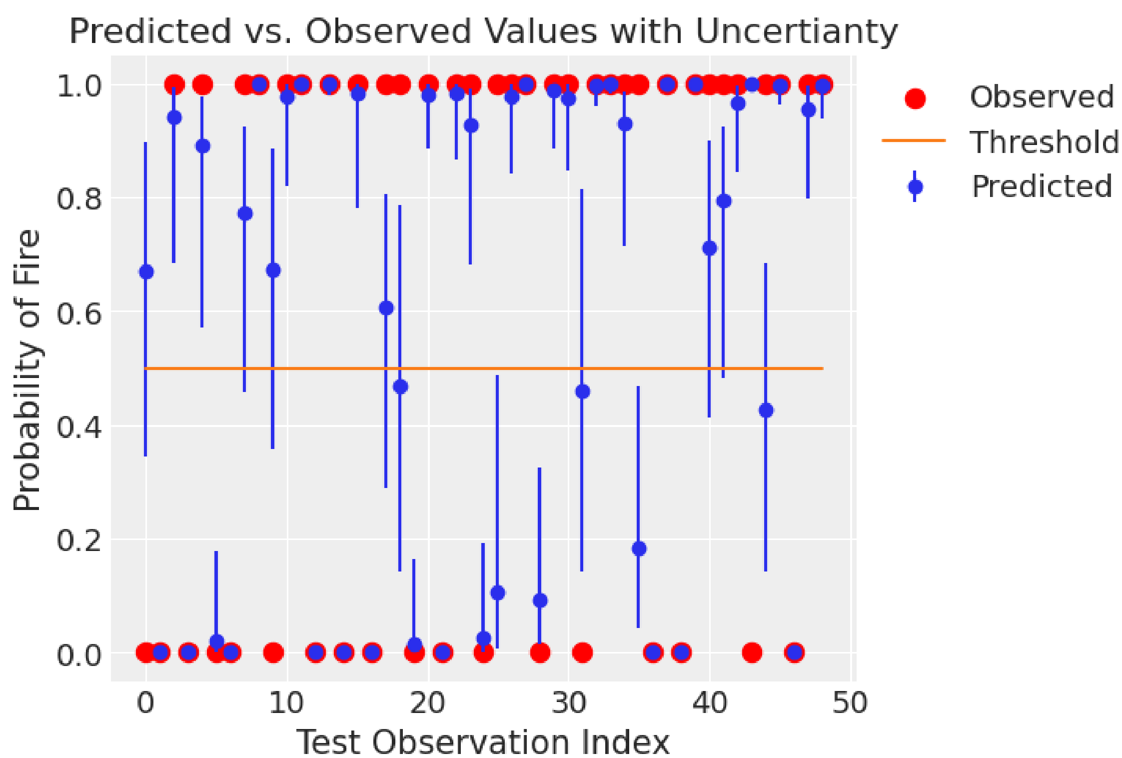


Figure 6.7 - Hierarchical Non-Linear Measurement Model - Predicted vs Observed Plot



Model Comparisons (Using WAIC)

MC1: Model 1 (pooled) vs Model 2 (unpooled)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
unpooled	0	-77.410803	16.526498	0.000000	0.818622	12.411316	0.000000	True	log
pooled	1	-87.477481	12.957641	10.066678	0.181378	16.316792	6.830921	True	log

MC2: Model 1 (pooled) vs Model 3 (hierarchical)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
hierarchical	0	-76.253370	15.234220	0.000000	0.7966	11.294549	0.000000	True	log
pooled	1	-87.477481	12.957641	11.224112	0.2034	16.316792	8.040822	True	log

MC3: Model 2 (unpooled) vs Model 3 (hierarchical)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
hierarchical	0	-76.253370	15.234220	0.000000	1.0	11.294549	0.000000	True	log
unpooled	1	-77.410803	16.526498	1.157433	0.0	12.411316	1.521519	True	log

MC4: Model 4 (pooled FWI) vs Model 5 (hierarchical FWI)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
Hierarchical FWI	0	-24.075804	2.688700	0.000000	0.545222	4.960111	0.000000	False	log
Pooled FWI	1	-24.199289	1.608203	0.123485	0.454778	5.336402	1.466881	False	log

MC5: Model 3 (hierarchical) vs Model 5 (hierarchical FWI)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
FWI (Hierarchical)	0	-24.075804	2.68870	0.000000	1.000000e+00	4.960111	0.000000	False	log
Measurements (Hierarchical)	1	-76.253370	15.23422	52.177566	9.215739e-12	11.294549	11.308366	True	log

MC6: Model 3 (hierarchical) vs Model 6 (non-linear hierarchical)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
Measurements (Nonlinear Hierarchical)	0	-59.35426	12.453843	0.00000	1.000000e+00	8.715021	0.000000	True	log
Measurements (Linear Hierarchical)	1	-76.25337	15.234220	16.89911	2.735590e-13	11.294549	8.119373	True	log

MC7: Model 5 (hierarchical FWI) vs Model 6 (non-linear hierarchical)

	rank	elpd_waic	p_waic	elpd_diff	weight	se	dse	warning	scale
FWI (Hierarchical)	0	-24.075804	2.688700	0.000000	1.0	4.960111	0.000000	False	log
Measurements (Nonlinear Hierarchical)	1	-59.354260	12.453843	35.278456	0.0	8.715021	9.086822	True	log