

Assignment 2-3 The range of values in 2's Complement Number System

1. **Show all numbers in 2's complement number systems from 0000 0000 0000 0000 to 1111 1111 1111 1111. Show the decimal number and binary number together.** You can skip part of the middle range.

Binary	Decimal
0000 0000 0000 0000	0
0000 0000 0000 0001	1
0000 0000 0000 0010	2
0000 0000 0000 0011	3
...	...
0111 1111 1111 1111	32767
1000 0000 0000 0000	-32768
...	...
1111 1111 1111 1101	-3
1111 1111 1111 1110	-2
1111 1111 1111 1111	-1

2. **Elaborate on the Detailed steps to convert -127 to 2's complement numbers.**

Step 1: Convert 127 to binary by dividing the quotients by 2 (2 for binary), and then the remainder is what the actual binary bit will be.

- a. $127/2 = 63$ with remainder 1
- b. $63/2 = 31$ with remainder 1
- c. $31/2 = 15$ with remainder 1
- d. $15/2 = 7$ with remainder 1
- e. $7/2 = 3$ with remainder 1
- f. $3/2 = 1$ with remainder 1
- g. $1/2 = 0$ with remainder 1
- h. Write all the remainders backwards (0 for padding)= 0111 1111 = 127

Step 2: Invert all bits

- a. 0111 1111 => 1000 0000
- b. Add 1 because it's complemented by 2's and adding 1 fixes the offset caused by the initial inversion => 1000 0001

Final: -127 in 2's complement is 1000 0001

3. Let's assume the the computer uses n-bit 2's complement number systems.

a. Show the smallest number and largest number in decimal and binary number

In 2's complement structure, the most significant bit (left-most bit) indicates the sign (0 for pos, 1 for neg), while the remaining bits represents the magnitude.

So we know that the smallest number is negative so $-(2^{n-1})$ for binary. And because MSB is already taken up to represent the sign, we have to subtract 1 from n where n is the number of bits. This part is an exponent because of the binary structure and its place values as seen in the conversion calculations. **So my final answer is -2^{n-1} .**

b. Elaborate on those numbers. Why is the range of negative numbers different from the range of positive numbers?

Same thing with a but opposite, where 0 is the MSB representing a positive sign, so it's only $2^{n-1}-1$, and $(n-1)$ is still the same. The last difference is to subtract the whole thing by 1, so we can ensure symmetry in the range of positive and negative numbers.

So my final answer is $(2^{n-1}) - 1$.