Game Theory

Matthew Williams and Kolton Burkhalter

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1 Introduction

A real-life example of game theory that comes as a scare to many Americans is the nuclear war. Many nations are capable of harnessing nuclear power to create weapons and some nations have proven it. Having Intercontinental Ballistic Missiles (ICBM) in your nation's arsenal shows a sense of power to other nations. These weapons have the ability to wipe out civilizations with a click of a button. Therefore, if a missile launch is provoked, it could incite a retaliation of equal or greater proportion and could destroy life as we know it.

A specific case of a nuclear war scare is the Cold War, a combat-less war between the United States and the former Soviet Union. These two nations were neck and neck in a race for nuclear weapons, with a virtually equal playing field and equal deception. These rivals possessed enough armed and operational weapons to potentially destroy the other one's entire country. They also can make more if they please, to show dominance and present a fear to the other player that they are willing to pull the trigger. However nuclear warheads cost a great amount of money to build and maintain, however you don't want to be perceived as weak and unable to retaliate. There is also the unclear natures interference of imperfect information of the capacity of the enemy's arsenal and the willingness to cooperate. With imperfect information, there is no signal to show how many weapons the opponent has. Therefore, if they discuss negotiations, it will result in cheap talk and unrealistic facts. We predict that perfect information will give each side the ability to know their aggressor's arsenal and become more willing to come to an agreement to disarm.

As conflict between nuclear powers like the United States, China, Russia, Iran, and North Korea grows more common, deterrence and disarmament theory are again practical. This game demonstrates how, regardless of the intentions of the actors, the game of nuclear deterrence makes it difficult to disarm or to trust eachother.

2 The Game

This game is played by two players who progress sequentially through an indefinite number of rounds. Each round consists of two decisions:

Step	Decision	Choices
1	Adjust Nuclear Stockpile	Integer between -15 and 15
2	Whether to Begin Nuclear War	Yes or No

These decisions will repeat indefinitely until either both players have 0 nuclear warheads in their stockpile or either player chooses to begin nuclear war.

Players will begin with an equal amount of ICBM warheads, or nukes (50). Players will simultaneously choose to what extent they want to add to or remove from their stockpile, and move on to the next round. These players will then be given the question of whether to Launch Nukes or Don't Launch Nukes. During these two rounds, the players are given the option to communicate with each other to negotiate or determine how many missiles they have. If one player decides to launch, nuclear war is started. It does not matter which one started the war - payoff will be calculated the same either way. Once one player or the other (or both) decide to launch missiles, the game is over. We will then analyze the damage done to each player, determined by the number of missiles launched (or kept in arsenal) at them. If nukes are not launched, the game continues and it returns to the first question, unless both players have 0 nukes, in which case it will end.

Payoff is determined by two things - the war outcome and the nuke costs. It takes at least 20 nukes to completely destroy the enemy, and both players can choose to build or disarm up to 15 nukes a round. Players begin with a nuclear arsenal of 50 and must pay a per-unit maintenance cost every round on the nukes in their arsenal, as well as a per-unit construction or deconstruction cost for any nukes they build/destroy. When nuclear war starts, both sides receive a final payoff which is a function of their relative numbers of nuclear warheads. The best payoff is having at least 20 missiles while your opponent has 0. After that is the payoff received when both players have 0 missiles remaining. Third is the payoff of mutually assured destruction (MAD), where both sides have equivalent numbers of missiles. The worst payoff is having 0 missiles and getting attacked by at least 20.

We will observe two versions of this game: one with complete information and one with incomplete information. The complete information game will let each side signal to the other the quantity of missiles that they possess as well as their motives. This will give full information of whether each player will launch or not and with how many warheads that they each carry. We will also play a game with incomplete information, with the quantity of weapons clouded. This will result in cheap talk and inaccurate information.

3 Payoffs

There is a cost to building and maintaining nuclear missiles. It costs more money to build one than it does to get rid of or maintain it, but the maintenance cost is applied to all missiles in the stockpile and can add up quickly. There is an incentive to keep the missiles rather than dismantling them, as it costs less.

Strategies	Per-Unit Cost
Increase	-0.5
Decrease	-0.2
Same	-0.1

Building nuclear missiles is an extremely expensive undertaking. It is only worth it if it lets you win a war, or deter action by your opponent. Deconstructing nuclear weapons is expensive due to the political forces at play and the cost of disposing of radioactive material. Even maintaining the stockpile has some costs associated with it, which disincentivizes an infinite Cold War.

The players get payoffs when the nuclear war starts as a function of the relative amounts of nukes.

P1/P2 Nukes	P1/P2 Payoffs	Result (P1)
Same Amount	-100, -100	Equilibrium Outcome
≥ 20, 0	150	Best Individual Payoff
0, ≤ 20	-200	Worst Individual Payoff
0, 0	50	Social Optimum

These only represent certain constant-value lines or points on the payoff graph. The actual payoff is linear between all of these defined parts. That is, the payoff for player 1, where x is player 1's nukes and y is player 2's nukes, is as follows:

$$f(x,y) = \begin{cases} x = 0, y < 20 & x * -10 \\ x = 0, y \geq 20 & -200 \\ x < 20, y = 0 & x * 5 \\ x \geq 20, y = 0 & 100 \\ x = y & -100 \\ x < y & (1 - \frac{x}{y}) * -100 - 100 \\ x > y & (1 - \frac{y}{x}) * 250 - 100 \end{cases}$$

The social optimum is for both to disarm before nuclear war begins. However, individually, their best payoffs are received by destroying the other person without being destroyed themselves.

4 Information

We are testing two different versions of this game to explore how information access changes the game. In one version, there is perfect information between players and both will know the other person's nuke count at all times. In the other, there is imperfect information, and players have a random chance to learn the other person's last choice (increase, decrease, maintain nuke counts) or the other player's nuke count. In the imperfect version, chances are as follows:

Information	Likelihood
Other Player's Last Decision	25%
Other Player's Nuke Count	10%

5 Analysis

It is unlikely that human players will do this, at least the first time they play, but an ideal rational actor should pick a strategy and stick to it. Therefore, we can make a simple model of each player's decisions by defining whether they decide to 'Attack' or 'Not Attack'. In this case, 'Attack' means spend some number of rounds building more nuclear missiles and then starting nuclear war, and 'Not Attack' means spending some number of rounds removing nuclear missiles until 0 is reached. With these definitions, we can make a payoff table:

	Attack	Not Attack
Attack	-100, -100	150, -200
Not Attack	-200, 150	50, 50

This resembles a Prisoner's Dilemma. The individual incentives of the players leads to a Nash Equilibrium of 'Attack', 'Attack', which is not the optimal social outcome. The players must cooperate to remove their nukes, but cooperation is hard to achieve because there is no trust. In the imperfect information game, if they say that they will deescalate, they should actually escalate. If the other player believes them, they will remove nukes until they, with a random chance, learn that the first player is not actually removing nukes. At this point they too will begin building more nukes. Thus, the player who was deceptive will have a small advantage, from which the second player can not recover. Therefore, whatever the other player says they will do, both players should choose the 'Attack' strategy.

The second player has a $> 50\%$ chance of learning of the first player's betrayal after 3 turns. The payoff the first player would receive if they betray the second player, and the second player doesn't realize for 3 turns, will be

$$f(50 + 45, 50 - 45) = (1 - \frac{5}{95}) * 250 - 100 = 136.84$$

Because they have a 50% chance of getting at least this payoff, we can estimate their payoff as $0.5 * 136.84 = 67.42$, which is greater than the 50 they would earn from achieving 0 nukes. This is actually below the value they will earn on average, because they get incremental payoffs for every round that their defection is not noticed, but proves the point. Therefore, they should always betray the second player. The second player should realize this, and thus should also choose the 'Attack' strategy. Both players should disregard any conversation as cheap talk.

In the perfect information version, each player is guaranteed to know after 1 turn if the other player betrayed them. Assuming that the first player betrays the second player, and the second player changes to the 'Attack' strategy once they learn they were betrayed, player one's payoff will cap after the first round at

$$f(50+15, 50-15) = (1 - \frac{35}{65}) * 250 - 100 = 15.38$$

This is less than the 50 they would receive by disarming. Therefore, both sides should disarm. When the players have perfect information, the Nash Equilibrium become 'Not Attack'. 'Attack' is not a dominated strategy, because it can outperform 'Not Attack' if the other player chooses 'Not Attack'. However, because each player will have perfect knowledge of what strategy the other player chose, they should always choose 'Not Attack'.

6 Implementation

We have implemented our game in the *otree* framework using Python. We are hosting our game on an online Heroku server, and will ask people to play the game during our report. We have configured two different 'treatments', one with perfect information and one with imperfect, to which participants will be randomly assigned. They will be automatically assigned to group of two and will go through this game together. Graphs will be generated on the fly to show the class the results. The game can be accessed at `http://tree-game.herokuapp.com`, where a user can configure sessions, play with a demo, or set up persistant rooms.