
HPC for numerical methods and data analysis

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Solutions – Introduction to Matlab® /Octave

Solution I (MATLAB)

We consider the following MATLAB code:

Solution II (MATLAB)

a) We consider the following MATLAB script `logplot.m`:

b) For the function $f(x) = [x - \log(x+1)]^4$, we have that:

$$\log(f(x)) = 4\log(x - \log(x+1)) \quad \begin{cases} \leq 4\log x \\ \geq 4\log(x-7)\,, \end{cases} \quad \text{for } x \in [0, 1000],$$

so that the curve $(x, f(X))$ is close to a straight line in the plot with the logarithmic scale on both the axes; its slope is approximately $p = 4$, being $f(x) = \mathcal{O}(x^p)$. Therefore, the plot obtained with the command `loglog` is the most useful to estimate the order of growth of $f(x)$.

c) We add the following MATLAB commands to the code suggested in a) to obtain the indicated figure:

Solution III (MATLAB)

a) Since we conjecture that the error reads $E_{c,h_i} = Ch_i^p$, then we have that $\log E_{c,h_i} = \log C + p\log h_i$, for $h = 1, \ldots, n$. Therefore, when using a logarithmic scale on both axes (*log-log scale*) for plotting the errors E_{c,h_i} vs. h_i, we should obtain a straight line $\log E_{c,h_i}$ of slope p, where p is the order of convergence. In order to graphically determine p, we plot the errors E_{c,h_i} vs. h_i in log-log scale and compare the line obtained with those corresponding to the lines (h_i, h_i^q) for some values of q, e.g. $q = 1, 2, 3$. The value of q for which (h_i, h_i^q) is *parallel* in log-log scale to (h_i, E_{c,h_i}) corresponds to the convergence order of the method p. We use the following MATLAB commands to *graphically* determine p:

We deduce that the convergence order of the method is $p = 2$ since (h_i, E_{c,h_i}) is *parallel* to (h_i, h_i^2) in the log-log scale plot.

We remark that in practical cases the conjecture $E_{c,h_i} = Ch_i^p$ can be typically used only for "small" values of $h_i(h_i \to 0)$, for which it suffices to verify that the lines (h_i, E_{c,h_i}) and (h_i, h_i^p) in log-log scale are parallel in the left part of the previous plot.

b) Still using the conjecture $E_{c,h_i} = Ch_i^p$, let us consider the errors $E_{c,h_{i-1}} = Ch_{i-1}^p$ and $E_{c,h_i} = Ch_i^p$ corresponding to $h = h_{i-1}$ and h_i for $i = 2, \ldots, n$, respectively. Then, we have:

$$\frac{E_{c,h_i}}{E_{c,h_{i-1}}} = \left(\frac{h_i}{h_{i-1}}\right)^p, \qquad p = \frac{\log\left(\frac{E_{c,h_i}}{E_{c,h_{i-1}}}\right)}{\log\left(\frac{h_i}{h_{i-1}}\right)} \qquad \text{for } i = 2, \ldots, n.$$

Since in practical cases the conjecture $E_{c,h_i} = Ch_i^p$ holds for "small" values of h_i, we typically select $i = n$ to determine the convergence order of the method p:

$$p = \frac{\log\left(\frac{E_{c,h_n}}{E_{c,h_{n-1}}}\right)}{\log\left(\frac{h_n}{h_{n-1}}\right)}.$$

We use the following MATLAB command to *algebraically* determine that $p = 2$:

Solution IV (MATLAB)

We use the MATLAB commands:

a) We extract the element in position (1,3) (first row, third column) as:

b) We extract the second row as:

c) We extract the first two columns from the matrix as:

d) We extract the vector containing all the elements of the second row of the matrix except for the third element as:

Exercise V (MATLAB)

We use the following MATLAB code:
We can clearly see that the expression $f(x) = (\sqrt{1+x} - 1)/x$ suffers from severe round-off errors as x approaches machine precision, because we compute the difference of two numbers with similar values, $\sqrt{1+x}$ and x, which causes a cancellation of significant digits (*hint*: type `eps` in the console to visualize the best relative precision that you can get using MATLAB).