

Pseudo-Polynomial Time Problems

As a parting comment on this subject, the book, *Computers and Intractability: A Theory of NP-Completeness*, by Garey and Johnson, also describes a rather interesting situation that can arise in algorithms both within P and NP. A pseudo-polynomial time algorithm is one whose time complexity is bound from above by a polynomial function in two variables: $Length[I]$ (where I is the input) and $Max[I]$ (i.e., the maximum value of the any value in the input). From Garey and Johnson:

A pseudo-polynomial-time algorithm . . . will display 'exponential behavior' only when confronted with instances containing 'exponentially large' numbers, [which] might be rare for the application we are interested in. If so, this type of algorithm might serve our purposes almost as well as a polynomial time algorithm.

When considering or developing such pseudo-polynomial time algorithms for NP-complete problems, one must be careful to recognize this is the case. Specifically, just because an algorithm has been developed for an NP-complete problem that exhibits pseudo-polynomial time behavior does not mean that the algorithm has a polynomial-time solution, which would thus prove that $P = NP$.