Minimum Spanning Trees

The next topic to consider has application in networking and vehicle routing—finding the minimum spanning tree of a graph.

Definition: Given a graph G, a spanning tree of G is a subgraph of G such that the subgraph is a tree, and all connected components from G are still connected in the subgraph.

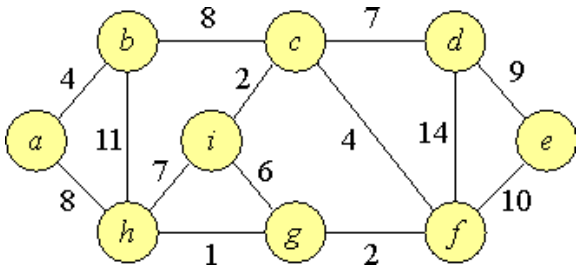
Now let's assume we associate a weight $w(u, v)$ on each edge (u, v) in the graph G. Let $w(T) = \sum_{(u,v)\in T} w(u, v)$ be the weight of spanning tree T.

Definition: Given a graph G, a minimum spanning tree of G is the spanning tree T that minimizes $w(T)$.

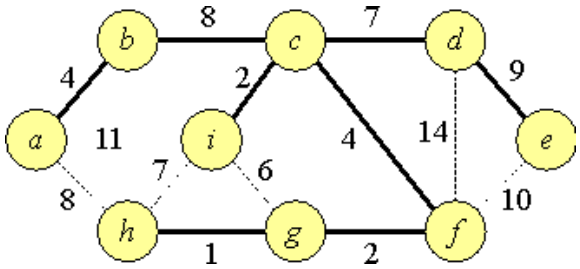
Thus there are two properties for a minimum spanning tree:

1. $T$is a tree.
2. $w(T)$is minimum.

For example, consider the following graph:



Our task will be to find a spanning tree of a graph such as this that minimizes the cost $w(T)$ given the set of weights on the edges of that tree. For this graph, the following is one example of a minimum spanning tree. As we will see a particular graph may have more than one minimum spanning tree.



In this figure, the heavy black lines indicate the edges that are part of the spanning tree, and the dashed lines are edges in the original graph that have been removed to form the spanning tree.