Some Famous NP-Complete Problems

The book, *Computers and Intractability: A Theory of NP-Completeness*, by Garey and Johnson, provides an extensive catalog of NP-complete problems. Some prominent NP-complete problems are as follows.

1. The 0/1 Knapsack Problem—As described earlier, this problem attempts to fill a fixed-capacity container (e.g., a knapsack) with a set of whole objects and maximize the value of the contents of the knapsack once capacity is reached.

2. The Maximum Clique Problem—This is a graph-theoretic problem. A clique is a subgraph that is a complete graph. Finding the maximum clique in a graph involves finding this maximum-sized complete subgraph.

3. The Vertex Cover Problem—This is another graph-theoretic problem. The object is to find the fewest vertices in a graph that, when taken together, contains at least one of the endpoints of all of the edges in the graph.

4. The Subset Sub Problem—As a combinatorial optimization problem, this one has consider application as a part of more complex problems. In the simplest case, this problem involves finding a subset of members of a set of numbers that sum exactly to some target value.

5. The Minimum Cost Binary Decision Tree Problem—Moving into more "real-world" problems, this one involves developing a decision procedure (such as a classification tree or a diagnostic fault tree) that minimizes the expected cost of using that tree. In this case, costs are associated with the interior nodes, corresponding to a test to be performed. The expected cost is determined as the average path cost over the entire tree.

6. The Job Shop Scheduling Problem—This problem involves determining a minimal cost schedule of using a set of manufacturing or assembly machines to produce a product. It involves sequencing the use of the machines so that products are assembled properly while taking advantage of parallel use of the machines.

7. The Integer Programming Problem—Many numeric optimization algorithms exist. One of the more common problems is referred to linear programming in which some objective function is being minimized or maximized subject to a set of linear constraints. The integer programming problem is a version of the linear programming problem except that the variables in the function are limited to taking on integers only.

8. Crossword Puzzle Construction—Consider something like the New York Times crossword puzzle. Finding a crossword puzzle configuration for a set of words that minimizes the number of black squares is extremely difficult. (In fact, it is NP-complete.) Some would claim that this is the reason for so many "strange" words in these more popular and well laid-out puzzles.