Recitation week 9

November 11, 2022

1. Given the following LP:

$$\begin{aligned} \min_x \ & c^\top x \\ \text{s.t. } & 3x + y \geq 3 \\ & x + 2y \geq 3 \\ & x, y \geq 0 \end{aligned}$$

build a vector c such that this problem:

- Has only one optimal solution
- Has a lot of optimal solutions
- Is unbounded

(*Hint: draw*)

2. Consider the following optimization problem:

$$\begin{aligned} \min \ & -xy \\ \text{s.t. } & x + y^2 \leq 2 \\ & x, y \geq 0. \end{aligned}$$

- Plot the feasible region.
- Plot some level sets for $-xy$.
- State the Lagrangian function for this problem.
- State the KKT conditions for this problem.
- Using those conditions solve the problem.
- Plot the optimal point you found. What can you tell with respect to the level sets of the function and the feasible region?

3. Consider the following optimization problem:

$$\begin{aligned} \min\ & x^2 + y^2 - 4x - 4y \\ & x^2 \leq y \\ & x + y \leq 2. \end{aligned}$$

- Find the minimum value of $f(x, y) = x^2 + y^2 - 4x - 4y$ (unconstrained optimization).
- Is this optimum in the feasible set? What do you expect from the Lagrange multipliers in this case?
- State the Lagrangian function for this problem.
- State the KKT conditions for this problem.
- Using those conditions, solve the problem.

4. Consider the following 3D optimization problem:

$$\begin{aligned} \min\ & 2e^{x-1} + (y-x)^2 + z^2 \\ \text{s.t. } & xyz \leq 1 \\ & x + z \geq c. \end{aligned}$$

- Write down the Lagrangian function.
- For this problem, state the KKT conditions.
- For which values of c does $X = [1, 1, 1]^\top$ filfill the KKT conditions?
- For these values of c and this X, which constraints are active?
- For this values of c, is X the optimum?