

Price Competition Online: Platforms vs. Branded Websites

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Abstract

The focus of this theoretical study is price competition when some firms operate their own branded website while others sell their products through an online platform, such as Amazon Marketplace. On one hand, selling through Amazon expands a firm's reach to more customers, but on the other hand, starting a website can help the firm to increase the perceived value of its product, that is, to build brand equity. In the short run the composition of firms is fixed, whereas in the long run each firm chooses between Amazon and its own website. I derive the equilibrium prices and profits, analyze the firms' behavior in the long run, and compare the equilibrium outcome with the social optimum. Comparative statics analysis reveals some interesting results. For example, I find that the number of firms that choose Amazon may go down in response to an increase in the total number of firms. A pure-strategy Nash equilibrium may not exist; I show that price dispersion among firms of the same type is more likely in less concentrated markets and/or when the increase in the perceived value of the product is relatively small.

Key words: pricing, competition, platforms, online marketplace, Amazon, brand equity

JEL codes: C72, D43, L11, L13, M31

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1 Introduction

Online marketplaces, where consumers can purchase from a variety of retailers on one website, have become an integral part of many eCommerce sellers' businesses. These marketplaces represent an opportunity for retailers to expand their reach to more customers, both nationally and internationally. The most prominent examples of online marketplaces are eBay, Amazon Marketplace, Google, Flipkart (India), and Alibaba and Taobao (China). Selling through an online marketplace generally comes at a cost to retailers. For example, Amazon charges its third-party retailers a referral fee for each sale. The retailers who use Amazon's fulfillment service pay extra. Yet, over two million sellers are willing to pay these fees to gain access to Amazon's more than 300 million customers.¹ According to Internet Retailer, in the first half of 2019 over half of the products sold on Amazon were from the marketplace sellers.²

Selling through an online marketplace (henceforth, I will collectively refer to such marketplaces as Amazon) seems like a good idea. All you have to do is source your product and Amazon handles the rest by providing you with an endless stream of consumer traffic. The catch is that people who shop on Amazon think they are buying from Amazon even though it is your product they are buying. This means that with Amazon, you have low brand equity. What is worse, you do not even have access to your customer list. On the other hand, if you start your own website, you can provide more information about your business or how to use your products. For example, food companies offer recipes and tips to consumers on their website. BILLY Footwear has dedicated its homepage to the brand's story. Lululemon's website includes in-depth product descriptions with "why we made this" pitches. Once you compile a database of your customers, you can implement a variety of marketing strategies. You can create a special promotion for all customers who purchased a specific product. You can alert your customers of an upcoming sale. You can reward repeated customers. You can take advantage of customer acquisition platforms, such as Google Shopping and Google AdWords, Bing Shopping and Bing Ads, Facebook, Instagram, Pinterest and YouTube, while simultaneously building your brand equity.

Lululemon, Timberland, Under Armour, and REI are examples of companies whose main online distribution channel is their website. In the same sports apparel industry there are firms that gravitated towards Amazon: OYANUS, Running Girl, and Camelsports. In bedding and home improvement industry, Crane & Canopy operates its own website, while Utopia Bedding is among the top 100 sellers on Amazon Marketplace.³

In this theoretical study, I consider a setting in which firms can sell their products through

¹<https://www.statista.com/statistics/476196/number-of-active-amazon-customer-accounts-quarter/>

²<https://www.statista.com/statistics/259782/third-party-seller-share-of-amazon-platform/>

³<https://www.marketplacepulse.com/amazon/top-amazon-usa-sellers>

an online marketplace, such as Amazon, or through their own website. On one hand, selling through Amazon expands a firm's reach to more customers, but on the other, starting a website can help the firm to increase the perceived value of its product, that is, to build brand equity. The aforementioned list of Amazon Marketplace top sellers skews towards unknown firms, backing up the low brand equity assumption. My research questions are: (i) How are firms' pricing choices affected by the presence of the e-commerce platform? (ii) Do firms choose the platform or their own branded website? (iii) Are there too many or too few firms on Amazon in equilibrium compared to the social optimum?

To address these questions, I use the spokes model of spatial competition proposed by Chen and Riordan (2007). The spokes model extends the linear city model (Hotelling, 1929) to arbitrary numbers of product varieties and of firms. Unlike the circular city model (Salop, 1979), competition is non-localized. The preference space consists of N lines, indexed by i , that start at the same central point (resembling the spokes on a bicycle wheel). Variety i is located at the extreme end of spoke i . There are $n \leq N$ firms, each producing a single variety. Consumers are uniformly distributed on the network of spokes and incur transportation costs—utility losses from imperfect matching—as they travel to a firm to purchase the firm's product. For a consumer located on spoke i variety i is her first preferred brand, and each of the other $N - 1$ varieties is equally likely to be her second preferred brand. The consumer has value v for one unit of either her first or second preferred brand, and zero value for other brands.

In my variation of the spokes model there are two types of firms, a-firms that sell through Amazon and w-firms that sell through their own website. Their numbers are n_a and $n - n_a$, respectively. Consider a consumer located on spoke i and suppose that an a-firm (a w-firm) is located at its extreme end. The consumer derives value v (value $v + \Delta$) from consuming one unit of variety i . As in the spokes model, each of the other $N - 1$ varieties is equally likely to be her second preferred brand. However, the consumer is only aware of the varieties that are sold on Amazon. Put differently, if you are an a-firm, every consumer can “see” your product. If you are a w-firm selling variety i through your own branded website, only consumers located on spoke i are aware of your existence. The good thing is that they are fans of your brand, which is captured by the brand equity parameter $\Delta > 0$.

The firms set their prices simultaneously. In the short run (Section 3) the composition of firms, n_a , is fixed. I first show that in any symmetric pure-strategy Nash equilibrium the difference between the price charged by w-firms and the price charged by a-firms must exceed Δ . I then calculate the equilibrium prices. I find that the prices do not depend on the composition of firms when the number of a-firms is small, and that both prices are decreasing in n_a when n_a is intermediate. The equilibrium involves mixed strategies when the number of a-firms is large. I employ the concept of an ϵ -Nash equilibrium and show that as n_a approaches n , the

mixed-strategy Nash equilibrium can be approximated by a pure-strategy profile.⁴

In the long run (Section 4) each firm can choose whether to build its own website or join Amazon Marketplace. The number of a-firms becomes endogenous. Comparative statics analysis reveals some interesting results. For example, the equilibrium number of a-firms may go down as the total number of firms increases. I also show that price dispersion among firms of the same type is more likely for smaller values of n and/or Δ .

From the social planner's perspective, a branded website increases the perceived value of the product; the drawback is that consumers who lack their first choices and whose second choice is a variety produced by a w-firm end up not purchasing. I show that the social optimum will be achieved if either Δ is so high that in equilibrium all firms choose to operate their own website, or if it is so low that all firms choose Amazon. In all other situations the equilibrium number of firms on Amazon tends to be excessive for higher values of n , and deficient in less concentrated markets.

1.1 Related Literature

The primary focus of my paper is price competition between firms, when some sell through an online marketplace while others run their own website. The existing theoretical papers on platform-based Internet retailing address complementary research questions. Jiang, Jerath, and Srinivasan (2011) investigate how a platform owner such as Amazon learns from the third-party retailers' early sales which of the products it should procure and sell directly and which it should leave for others to sell. Abhishek, Jerath, and Zhang (2016) analyze online retailers' incentives to adopt the agency selling format, under which they allow manufacturers direct access to their customers for a fee. Whether e-tailers end up giving control over prices to manufacturers depends on the degree of competition between e-tailers, as well as on the extent of the demand spillover between the electronic channel and the traditional brick-and-mortar channel. The strategic choice by an intermediary between agency selling and reselling is also studied in Hagiu and Wright (2015). Drivers of the optimal intermediation mode include the relative importance of the suppliers' versus the intermediary's private information, whether the products are long tail or short tail, and the presence of spillovers across products generated by marketing activities.

The closest to the present study is the paper by Ryan, Sun, and Zhao (2012). The authors consider a single online firm that can choose to contract with Amazon to sell its product through the marketplace system. Selling through Amazon expands the firm's customer base, but comes

⁴A strategy profile is an ϵ -Nash equilibrium if it is not possible for any player to gain more than ϵ in expected payoff by unilaterally deviating from his/her strategy. The concept is important in algorithmic game theory (Vazirani, Nisan, Roughgarden, and Tardos, 2007). In economics, it is often used when the mixed-strategy approach is seen as unrealistic (Tijs, 1981, Tan, Yu, and Yuan, 1995, Dixon, 1987, Ziad, 1997).

at a cost (a fixed participation fee plus revenue sharing). A key question for the firm is whether to sell through Amazon, and if so, at what price. The firms in my model face the same decision, but there is more than one firm—an arbitrary number of them. Ryan, Sun, and Zhao (2012) are also interested in the marketplace firm’s decision whether to sell a competing product and how to design the contract with the participating retailer.

In all of the above papers the platform is an active player. In my model it is exogenous, with the platform’s fee assumed away. I focus on firms instead: some sell through the platform, others through their own website, and the firms compete in prices. As I explained in the Introduction, I use the spokes model of spatial competition proposed by Chen and Riordan (2007). This model was also adopted in other theoretical studies of price competition, which I review below.

Amaldoss and He (2010) incorporate informative advertising into the spokes model. Specifically, the authors assume that consumers are completely uninformed about a firm’s product unless they are exposed to its advertising. The informed customers constitute fraction ϕ of the market. Thus, selling through Amazon in my model translates into $\phi = 1$. (Selling through own website is different from $\phi = 0$, because the website, while not visible to non-fans, is visible to consumers located on the same spoke.) An important difference is that I consider two types of firms, while in Amaldoss and He (2010) the firms are identical. To study the effect of advertising reach on equilibrium prices, Amaldoss and He (2010) initially treat ϕ as exogenous. In equilibrium the firms charge the same price and this price decreases in ϕ for high valuation products, but can increase when consumer valuations are low. In my model consumer valuations are high and I, too, find that a-firms—the firms with the highest possible level of customer reach—charge lower prices than w-firms. Amaldoss and He (2010) endogenize ϕ in the second part of their paper. Since the firms possess the same advertising technology, they choose the same ϕ in equilibrium.

Caminal and Claici (2007) use the spokes model to study loyalty-rewarding pricing schemes, like frequent flyer programs, in highly competitive environments ($n = N$ and the latter is large). The authors argue that these programs are business-stealing devices that tend to reduce average prices and increase consumer surplus. Caminal and Granero (2012) introduce multi-product firms into the spokes model to examine their role in the provision of product variety. In the presence of economies of scope, only a small number of firms are active. The authors find that under some parameter conditions the firms restrict their product range to relax price competition. Granero (2013) and Granero (2019) consider a multi-product duopoly along the lines of Caminal and Granero (2012). Granero (2013) examines the welfare impact of MFC clauses. Granero (2019) focuses on strategic choice of product quality.

Reggiani (2014) analyzes location choices of firms in the spokes model under the assumption that the firms can price discriminate. The author shows that when the number of firms is

large, an asymmetric pattern arises in equilibrium in which one firm supplies a generally appealing product whereas the others focus on niche customers. When the number of firms is intermediate, multiplicity of equilibrium location patterns arise, and they do not always minimize the sum of transport costs.

As the above literature review shows, the existing studies of the spokes model consider homogenous firms and, with the exception of Reggiani (2014), arrive at a symmetric outcome. My model is different in that it has two types of firms. In a symmetric pure-strategy Nash equilibrium the firms of the same type charge the same price, but this price differs across the types. When the equilibrium is in mixed strategies, even the firms of the same type charge different prices.

2 Model Setup

Consider a market along the lines of Chen and Riordan (2007). There are N possible varieties of a differentiated product, indexed by $i = 1, \dots, N$. The preference space consists of N lines (spokes) of length $1/2$, also indexed by i , that start at the same central point. Variety i is located at the extreme end of spoke i . Consumers of total mass $N/2$ are uniformly distributed over the spokes network. For a consumer located on spoke i variety i is her first preferred brand, and each of the other $N - 1$ varieties is equally likely to be her second preferred brand. The consumer has value v for one unit of her first or second preferred brand, and zero value for other brands.⁵

In Chen and Riordan's setup there are $n \leq N$ identical firms, each producing a single variety. A consumer located on spoke i at distance x_i from variety i obtains utility $v - tx_i$ from consuming it, where t is the unit transportation cost. The distance to any other variety is $1/2 - x_i + 1/2 = 1 - x_i$. This is because the consumer covers $1/2 - x_i$ units of distance to get to the center of the spokes network and then $1/2$ units of distance to get to variety $j \neq i$. Thus, the consumer obtains utility $v - t(1 - x_i)$ from consuming her second preferred brand.

I consider a variation of the spokes model. In my setup there are two types of firms, a-firms that sell through an online marketplace platform (henceforth, Amazon) and w-firms that run their own website. While Amazon provides firms with endless stream of consumer traffic, starting a website helps a firm to increase the perceived value of its product, that is, to build brand equity. Let n_a denote the number of a-firms, then the number of w-firms will be $n - n_a$. If you are an a-firm, everyone can "see" you. If you are a w-firm selling variety i through your own branded website, only consumers located on spoke i are aware of your existence. The good

⁵Chen and Riordan assume that each consumer only cares about two brands to assure the existence of a symmetric pure-strategy equilibrium in prices.

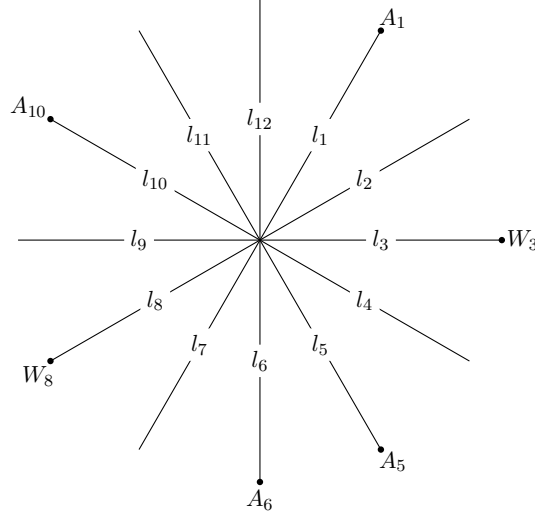


Figure 1: Spokes Model with $N = 12$, $n = 6$ and $n_a = 4$

thing is that they are fans of your product, which is captured by the brand equity parameter $\Delta > 0$ that transforms the aforementioned value v into $v + \Delta$.

Figure 1 depicts a radial network with twelve spokes (lines l_1, l_2, \dots, l_{12}), four a-firms and two w-firms. The firms that sell through Amazon are firms A_1, A_5, A_6 and A_{10} : A_1 produces variety 1, A_5 variety 5, A_6 variety 6 and A_{10} variety 10. The firms that run their own website are firms W_3 and W_8 : W_3 produces variety 3 and W_8 produces variety 8.

Consider a consumer located on l_1 at distance x_1 from variety 1. Variety 1 is her first preferred brand; she obtains utility $v - tx_1$ from consuming it. The consumer's second preferred brand is variety $j \neq 1$ chosen by nature with probability $1/(N - 1)$. Too bad if $j = 2, 4, 7, 9, 11$ or 12 , as these varieties are not produced by any of the firms. If $j = 3$ or 8 , that is not good either, because our consumer only sees varieties offered on Amazon. Only if $j = 5, 6$ or 10 the consumer's second preferred brand is available to her; she obtains utility $v - t(1 - x_1)$ from consuming it.

Next, consider a consumer located on l_2 at distance x_2 from variety 2. Unfortunately for this consumer, variety 2 is not produced by any of the firms. The consumer's second preferred brand is available to her if it is sold on Amazon by firm A_j , $j = 1, 5, 6$ or 10 ; she obtains utility $v - t(1 - x_2)$ from consuming it. Finally, consider a consumer located on l_3 at distance x_3 from variety 3. Variety 3 is her first preferred brand; she obtains utility $v + \Delta - tx_3$ from consuming it. The consumer's second preferred brand is available to her if it is sold by firm A_j , $j = 1, 5, 6$ or 10 ; she obtains utility $v - t(1 - x_3)$ from consuming it.

I assume that each firm's variable production cost equals c , regardless of whether it sells through its own website or through Amazon. The firms set their prices simultaneously. Consumers observe the prices and make their purchasing decisions.

3 Equilibrium Analysis

We will start the equilibrium analysis with consumers, then analyze the firms's pricing decisions.

3.1 Consumer Demand

Consider a consumer whose first preferred brand is variety i and whose second preferred brand is variety j . Suppose variety i is sold by a w-firm (firm W_i) and variety j is sold by an a-firm (firm A_j). The consumer compares her payoff from purchasing from firm W_i ,

$$v + \Delta - tx_i - p_i,$$

with that from purchasing from firm A_j ,

$$v - t(1 - x_i) - p_j,$$

where p_i and p_j are the prices charged by the firms. If $p_i \leq p_j + \Delta$, the consumer will purchase variety i irrespective of her location on l_i . If $p_i > p_j + \Delta$, she will purchase from firm W_i if and only if

$$x_i \leq \frac{1}{2t} (t + \Delta - p_i + p_j).$$

Next, suppose that both varieties i and j are sold on Amazon (by firms A_i and A_j). If $p_i \leq p_j$, the consumer will purchase from firm A_i irrespective of her location on l_i . If $p_i > p_j$, the consumer compares

$$v - tx_i - p_i$$

with

$$v - t(1 - x_i) - p_j,$$

and will purchase from firm A_i if and only if

$$x_i \leq \frac{1}{2t} (t - p_i + p_j).$$

The consumer's purchasing decision is trivial when variety i or variety j is available to her but not both. That is, when either variety j is not available on Amazon or variety i is not

produced by any of the firms.

We now derive the demand that the firm located at the end of spoke i faces, be it a firm that operates its own website (firm W_i), or a firm that sells its product through Amazon (firm A_i). Consider consumers on l_i and l_j for whom both variety i and variety j are the two desired brands. There are two types of marginal consumers. If variety i is produced by a w-firm and variety j is produced by an a-firm, the marginal consumer is located on l_i at distance

$$\hat{x}(p_i, p_j) = \max \left\{ \min \left\{ \frac{1}{2t} (t + \Delta - p_i + p_j), \frac{1}{2} \right\}, 0 \right\} \quad (1)$$

from variety i . If variety i and variety j are both produced by a-firms, the marginal consumer is located at distance

$$\hat{y}(p_i, p_j) = \max \left\{ \min \left\{ \frac{1}{2t} (t - p_i + p_j), 1 \right\}, 0 \right\} \quad (2)$$

from variety i . It is easy to see that the marginal consumer is located on l_i ($\hat{y} < 1/2$) if $p_i < p_j$, and on l_j ($\hat{y} > 1/2$) if $p_i > p_j$.

Firm W_i sells to two groups of consumers: consumers on l_i who have an alternative available on Amazon, and those who do not. Summing up over these groups yields firm W_i 's total demand:

$$q_i = \frac{1}{N-1} \sum_{j: \text{all a-firms}} \hat{x}(p_i, p_j) + \frac{1}{2} \frac{N - n_a - 1}{N-1}. \quad (3)$$

Firm A_i sells to four groups: (i) consumers whose desired brands are both sold on Amazon, one of which is variety i , (ii) consumers whose first choice is produced by a w-firm and whose second choice is variety i , (iii) consumers on l_i who do not have an alternative available on Amazon, and (iv) consumers who lack their first choices but for whom variety i is their second preferred brand. Summing up yields firm A_i 's total demand:

$$\begin{aligned} q_i = & \frac{1}{N-1} \sum_{j: \text{all a-firms but } A_i} \hat{y}(p_i, p_j) \\ & + \frac{1}{N-1} \sum_{j: \text{all w-firms}} \left(\frac{1}{2} - \hat{x}(p_j, p_i) \right) + \frac{1}{2} \frac{N - n_a}{N-1} + \frac{1}{2} \frac{N - n}{N-1}. \end{aligned} \quad (4)$$

When deriving the demand functions (3) and (4), I assumed that the prices are not prohibitively high so that all consumers whose desired varieties are available purchase. I did so for the ease of exposition. The situations where some consumers do not purchase because the prices are too high will not arise in equilibrium.

3.2 Pricing

I will focus on symmetric pure-strategy Nash equilibria in which a-firms charge the same price p_a^* and w-firms charge the same price p_w^* . To keep the model tractable and the results focused, I will assume

$$c + 2\frac{N-1}{n-1}t \leq v \leq c + \left(2\frac{N-1}{n-1} + \frac{1}{2}\frac{2N-n-1}{N-n}\right)t \quad (5)$$

throughout the paper. This assumption will be discussed in more detail after Lemma 2. Note that the upper limit placed on v is $+\infty$ when $n = N$.

Let us first solve the game for the two extreme values of n_a . When all firms operate their own website ($n_a = 0$), they are essentially monopolists. Because v is relatively high under the assumption (5), each w-firm finds it optimal to serve all of its fans—consumers located on the same spoke—by charging

$$p_w^* = v + \Delta - \frac{t}{2}.$$

We have our first result:

Lemma 1 (Equilibrium prices and profits when $n_a = 0$). *When all firms operate their own website, each sets its price equal to*

$$p_w^* = v + \Delta - \frac{t}{2},$$

serves consumers located on the same spoke, and earns the profit

$$\Pi_w^* = \frac{1}{2} \left(v + \Delta - \frac{t}{2} - c \right).$$

When all firms sell their products through Amazon ($n_a = n$), the equilibrium price is forced down by competition for the marginal consumers described in (2). The proof of Lemma 2 is relegated to the Appendix.

Lemma 2 (Equilibrium prices and profits when $n_a = n$). *When all firms sell their products through Amazon, they set their prices equal to*

$$p_a^* = c + \frac{2N-n-1}{n-1}t.$$

In equilibrium, each firm A_i serves consumers located on spoke i as well as those who lack their first choices but for whom variety i is their second choice. The resulting profit is

$$\Pi_a^* = \frac{(2N-n-1)^2}{2(N-1)(n-1)}t.$$

Remark 1. *The present model reduces to the spokes model when all firms sell their products through Amazon.*

In the spokes model, the nature of the equilibrium differs for different regions in the parameter space (Chen and Riordan, 2007, pp. 903-905). The assumed parameter space (5) matches the region that corresponds to the normal oligopoly competition: all consumers whose desired brands are available purchase and enjoy a strictly positive profit. (See the proof of Lemma 2.) Chen and Riordan call it normal because an increase in market concentration has the familiar effect of lowering the equilibrium prices and profits. As we see, both p_a^* and Π_a^* in Lemma 2 are decreasing in n .

What happens when n_a takes intermediate values? The next lemma establishes that in any symmetric pure-strategy Nash equilibrium the difference between the price that w-firms charge and the price that a-firms charge must exceed Δ .

Lemma 3 (Price difference $p_w^* - p_a^*$). *In a symmetric pure-strategy Nash equilibrium $p_w^* - p_a^* > \Delta$.*

That $p_w^* - p_a^* < \Delta$ cannot happen in equilibrium is easy to see. If a w-firm increases its price to $p_a^* + \Delta$, it will not lose any of its customers. Thus, the deviation is profitable. Now suppose $p_w^* - p_a^* = \Delta$. Under such prices each firm serves consumers located on the same spoke; a-firms also serve consumers who lack their first choices but whose second choices are available on Amazon. If an a-firm decreases its price, it will steal consumers from all the other firms (a-firms and w-firms). If the firm increases its price, it will lose some of its customers only to the other a-firms. Thus, there is a kink in demand. In the Appendix, I first derive the condition under which any deviation to a lower price is unprofitable:

$$p_a^* \leq c + \frac{2N - n - 1}{n - 1}t.$$

I then derive the condition that takes care of the firm's incentives not to increase its price:

$$p_a^* \geq c + \frac{2N - n - 1}{n_a - 1}t.$$

Since

$$c + \frac{2N - n - 1}{n - 1}t < c + \frac{2N - n - 1}{n_a - 1}t$$

for any $n_a < n$, a price that satisfies both conditions does not exist. Therefore, $p_w^* - p_a^* = \Delta$ cannot happen in equilibrium.

In Proposition 1, I show that a unique symmetric pure-strategy Nash equilibrium exists when n_a is below some threshold n_a^{III} . The nature of competition changes as n_a crosses two other

thresholds, n_a^I and n_a^{II} , with $n_a^I < n_a^{II} < n_a^{III}$. While the expression for n_a^I is rather simple,

$$n_a^I = \frac{N-1}{\frac{v+\Delta-c}{t} + \frac{1}{2}},$$

and n_a^{II} is the solution to a quadratic equation,

$$n_a^{II} = N + n - \frac{3}{2} + \frac{(n-1)\Delta}{2t} - \sqrt{\left(N + n - \frac{3}{2} + \frac{(n-1)\Delta}{2t}\right)^2 - (N-1)(n-1)},$$

n_a^{III} can only be defined implicitly. (Please see the proof of Proposition 1 in the Appendix.)

Proposition 1 (Equilibrium prices when $n_a \in (0, n_a^{III}]$). *For any $n_a \in (0, n_a^{III}]$, the pricing game has a unique symmetric pure-strategy Nash equilibrium in which a-firms charge the same price p_a^* and w-firms charge the same price p_w^* .*

(i) When $n_a \in (0, n_a^I]$ (Region I), the prices are

$$p_w^* = v + \Delta - \frac{t}{2} \quad \text{and} \quad p_a^* = v - \frac{3t}{2}.$$

(ii) When $n_a \in (n_a^I, n_a^{II}]$ (Region II), the prices are

$$p_w^* = c + \frac{N - n_a - 1}{n_a}t \quad \text{and} \quad p_a^* = c + \frac{N - 2n_a - 1}{n_a}t - \Delta.$$

(iii) When $n_a \in (n_a^{II}, n_a^{III}]$ (Region III), the prices are

$$p_w^* = c + \frac{\left(N - n + \frac{(N-1)(2n-1)}{n_a}\right)t + \Delta}{3n - n_a - 1}$$

and

$$p_a^* = c + \frac{\left(3N - 2n - 1 + \frac{(N-1)n}{n_a}\right)t - (n - n_a)\Delta}{3n - n_a - 1}.$$

Remark 2. When $n_a \in (n_a^{III}, n)$ (Region IV), the equilibrium is in mixed strategies.

Figure 2 plots the equilibrium prices against the number of a-firms; in this figure $c = 2$, $v = 7$, $t = 1$, $\Delta = 1$, $N = 100$, and $n = 90$. When the number of a-firms is small, w-firms adopt the same pricing strategy as in Lemma 1. A-firms focus on capturing consumers whose first choice is produced by a w-firm, so in Region I they set their prices equal to

$$p_a^* = p_w^* - t - \Delta = v - \frac{3t}{2}.$$

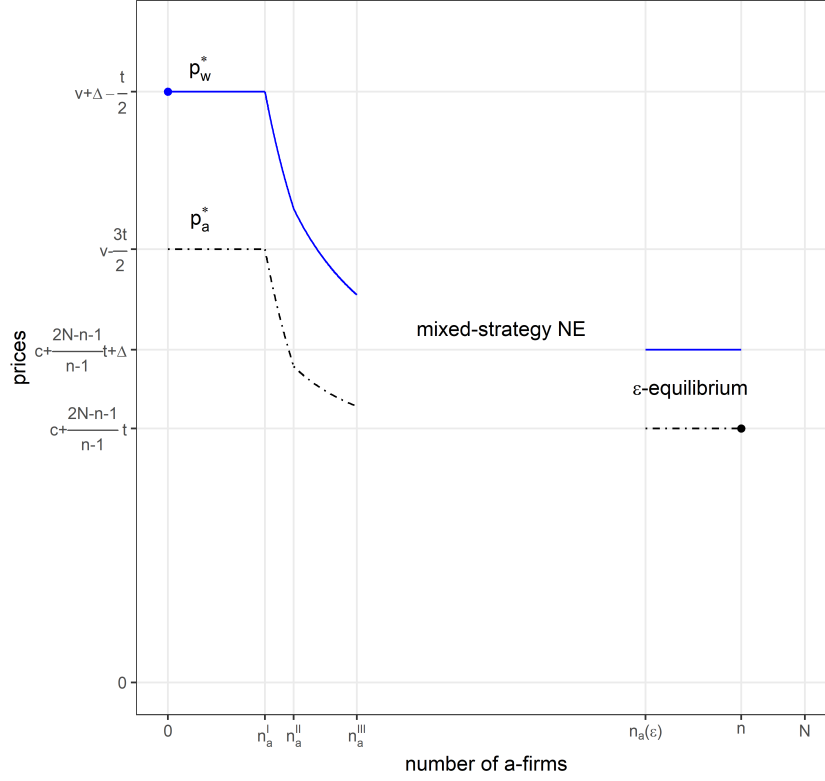


Figure 2: Equilibrium Prices

In equilibrium each w-firm earns the profit

$$\Pi_w^* = \frac{1}{2} \frac{N - n_a - 1}{N - 1} (p_w^* - c). \quad (6)$$

Here $1/2$ is the number of consumers located on the same spoke. Only those whose second preferred brand is not available on Amazon will purchase from the firm, which explains why $1/2$ is multiplied by $(N - n_a - 1)/(N - 1)$. As to a-firms, each earns

$$\Pi_a^* = \left(\frac{1}{2} + \frac{1}{2} \frac{N - n_a}{N - 1} \right) (p_a^* - c). \quad (7)$$

In addition to consumers located on spoke i , firm A_i sells to consumers whose first choice is produced by a w-firm and whose second choice is variety i , hence the terms $1/2$ and $1/2 \times (N - n_a)/(N - 1)$ in (7).

As n_a increases, the fraction of consumers that a w-firm loses to the firms on Amazon increases (the term $(N - n_a - 1)/(N - 1)$ in (6) decreases). When n_a crosses the threshold

n_a^I (Region II), w-firms start lowering their prices. A-firms still find it worthwhile to capture consumers whose first choice is produced by a w-firm, so $p_a^* = p_w^* - t - \Delta$.

When $n_a \in (n_a^{II}, n_a^{III}]$ (Region III), the equilibrium prices are forced down by competition for the marginal consumers in (1) as well as in (2). While the price difference $p_w^* - p_a^*$ is constant (equals $t + \Delta$) in Regions I and II, in Region III it decreases from $t + \Delta$ at $n_a = n_a^{II}$ to some number between $t + \Delta$ and Δ at $n_a = n_a^{III}$.

Finally, when $n_a \in (n_a^{III}, n)$ (Region IV), a symmetric pure-strategy Nash equilibrium does not exist. This is because the price difference $p_w^* - p_a^*$ gets too close to Δ , which creates incentives for an a-firm to deviate globally to a price above $p_w^* - \Delta$. (See the proof of Proposition 1 in the Appendix.) The equilibrium is in mixed strategies, which means firms of the same type charge different prices. This result is important because it rationalizes price dispersion observed in online markets.

Recall that $n_a = n$ yields a symmetric equilibrium in pure strategies (Lemma 2). A natural question arises: is there a pure-strategy profile that is “close” to the (potential) symmetric mixed-strategy equilibrium when all but a few firms are a-firms? I answer this question in Section 3.3 where I employ the concept of an ϵ -Nash equilibrium.

Equilibrium profits in Regions I-III are calculated easily from Proposition 1. Figure 3 plots the equilibrium profits against the number of a-firms for the same parameter values Figure 2 does: $c = 2$, $v = 7$, $t = 1$, $\Delta = 1$, $N = 100$, and $n = 90$. Both $\Pi_w^*(n_a)$ and $\Pi_a^*(n_a)$ are decreasing functions of n_a , as expected.

Corollary 1 (Equilibrium profits when $n_a \in (0, n_a^{III}]$). *The profits of each w-firm and each a-firm in the unique symmetric pure-strategy Nash equilibrium are:*

$$\Pi_w^*(n_a) = \frac{N - n_a - 1}{2(N - 1)} \left(v + \Delta - \frac{t}{2} - c \right) \quad \text{and} \quad \Pi_a^*(n_a) = \frac{2N - n_a - 1}{2(N - 1)} \left(v - \frac{3t}{2} - c \right)$$

in Region I,

$$\Pi_w^*(n_a) = \frac{(N - n_a - 1)^2}{2(N - 1)n_a} t \quad \text{and} \quad \Pi_a^* = \frac{2N - n_a - 1}{2(N - 1)} \left(\frac{N - 2n_a - 1}{n_a} t - \Delta \right).$$

in Region II,

$$\Pi_w^*(n_a) = \frac{n_a}{2(N - 1)t} \left(\frac{\left(N - n + \frac{(N-1)(2n-1)}{n_a} \right) t + \Delta}{3n - n_a - 1} \right)^2$$

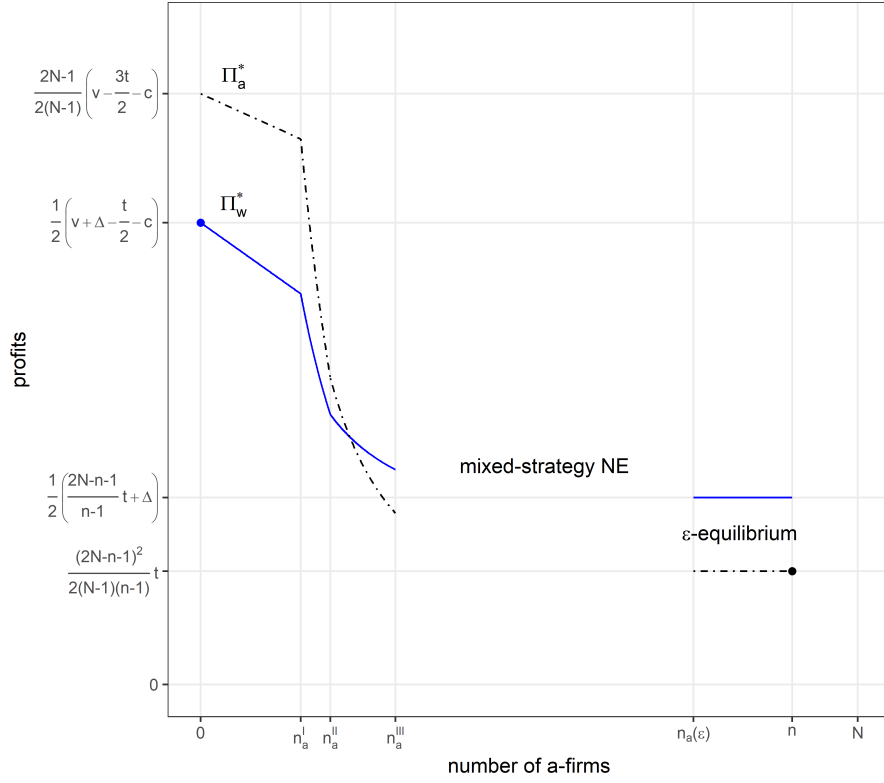


Figure 3: Equilibrium Profits

and

$$\Pi_a^*(n_a) = \frac{n-1}{2(N-1)t} \left(\frac{\left(3N - 2n - 1 + \frac{(N-1)n}{n_a} \right) t - (n - n_a)\Delta}{3n - n_a - 1} \right)^2$$

in Region III.

How does an increase in n_a affect consumers? First, an increase in n_a enables consumers whose first choice is not produced and whose second choice was produced by a w-firm and now is produced by an a-firm to obtain a positive surplus. Second, when a w-firm becomes an a-firm, the consumers located on the same spoke no longer receive Δ when they purchase from the firm. Because the new price is lower by more than Δ (Lemma 3), they still benefit. There is also a group of consumers who switch from w-firms to this converted firm. They obviously benefit (would not switch otherwise). As to the rest of the consumers, they continue paying the same prices for the same products if an increase in n_a happens in Region I. If an increase in n_a happens in Region II or Region III, they enjoy lower prices.

3.3 ϵ -Nash Equilibrium

In Region IV a pure-strategy Nash equilibrium does not exist. In this section I take an approach that is different from mixed strategies. I consider an ϵ -Nash equilibrium in which each player is satisfied to get close to (but not necessarily achieve) his/her best response. Formally, an ϵ -Nash equilibrium is a strategy profile such that it is not possible for any player to gain more than ϵ by unilaterally deviating from his/her strategy.

Proposition 2 (ϵ -Nash equilibrium when $n \in [n(\epsilon), n)$). *For any fixed positive ϵ there is a number $n_a(\epsilon)$ such that for all $n_a \geq n_a(\epsilon)$ the prices*

$$p_w^\epsilon = c + \frac{2N - n - 1}{n - 1}t + \Delta \quad \text{and} \quad p_a^\epsilon = c + \frac{2N - n - 1}{n - 1}t$$

constitute a symmetric pure-strategy ϵ -Nash equilibrium. The resulting profits are

$$\Pi_w^\epsilon = \frac{1}{2} \left(\frac{2N - n - 1}{n - 1}t + \Delta \right) \quad \text{and} \quad \Pi_a^\epsilon = \frac{(2N - n - 1)^2}{2(N - 1)(n - 1)}t.$$

It is easy to show that a w-firm does not have an incentive to deviate. The competition in Region IV is so intense that the best a w-firm can do is to make sure that all of its fans—consumers located on the same spoke—purchase the firm's product. The price $p_w^\epsilon = p_a^\epsilon + \Delta$ achieves exactly that. An a-firm does have an incentive to deviate, but this incentive tends to zero as n_a approaches n . In the Appendix I show that an a-firm's most profitable deviation is to a higher price

$$\tilde{p}_a(n_a) = \frac{p_a^\epsilon + c}{2} + \frac{2N - n - 1}{2(n_a - 1)}t.$$

The profit of the deviating firm is

$$\tilde{\Pi}_w(n_a) = \frac{n_a - 1}{2(N - 1)t} (\tilde{p}_a(n_a) - c)^2 > \Pi_w^\epsilon.$$

I then demonstrate $\tilde{\Pi}_w(n_a) - \Pi_w^\epsilon \rightarrow 0$ as $n_a \rightarrow n$.⁶ In other words, for any $\epsilon > 0$ there exists a number $n_a(\epsilon)$ such that for all $n_a \geq n_a(\epsilon)$

$$\tilde{\Pi}_a(n_a) - \Pi_a^\epsilon \leq \epsilon.$$

For large values of ϵ any price configuration is an equilibrium. The smaller ϵ becomes, the more price configurations are ruled out, but the number of ϵ -Nash equilibria will still be a continuum. Even if we restrict our attention to symmetric pure-strategy ϵ -Nash equilibria, we

⁶Here, I treat n_a as a continuous variable.

will not have a unique ϵ -Nash equilibrium. In my symmetric pure-strategy ϵ -Nash equilibrium

$$p_a^\epsilon = c + \frac{2N - n - 1}{n - 1}t,$$

which is the price the a-firms charge in Lemma 2. (It seems like the most natural choice.) Because $p_w^\epsilon = p_a^\epsilon + \Delta$, the a-firms in Proposition 2 sell to the same number of consumers, hence earn the same profit as the firms in Lemma 2. The fact that Π_w^ϵ does not depend on n_a makes it easier to prove the convergence of $\tilde{\Pi}_w(n_a)$ to Π_w^ϵ .

Region IV comes as a rather large gap in Figures 2 and 3. The ϵ -Nash equilibrium approach bridges this gap and confirms the “continuity” of the model’s insights in this region. Proposition 2 turns out to be useful later in Section 4, where the firms choose between selling through Amazon and operating their own website. Without it, I would not be able to check whether an a-firm has incentives to become a w-firm when $n_a = n$ and obtain condition (10).

3.4 Social Optimum

In all four regions the Nash equilibrium prices fail to induce all consumers on the n spokes to receive their first preferred brands. Take $n_a \in (0, n_a^{III}]$ and consider consumers whose first choice is produced by a w-firm (firm W_i) and whose second choice is available on Amazon. Not everybody on l_i will purchase from firm W_i . Since $p_w^* - p_a^* > \Delta$, consumers described by

$$x_i \in \left(\frac{1}{2t}(t + \Delta - p_w^* + p_a^*), \frac{1}{2} \right]$$

will purchase from Amazon. This is inefficient, as it results in higher transport costs and lost Δ .

Next, take $n_a \in (n_a^{III}, n)$ and consider consumers whose desired brands (varieties i and j) are both sold on Amazon. Because the equilibrium is in mixed strategies, firms A_i and A_j will end up charging different prices. Suppose firm A_i ’s price is higher, then some consumers on l_i will purchase variety j , which is inefficient.

For the n brands to be allocated to consumers efficiently, the following two conditions must be satisfied: (i) each type of firms charges the same price, and (ii) the price difference does not exceed Δ . Any set of efficient prices allows the social planner to achieve the welfare

$$\begin{aligned} W(n_a) = & n_a \frac{1}{2} \left(v - \frac{t}{4} - c \right) + (n - n_a) \frac{1}{2} \left(v + \Delta - \frac{t}{4} - c \right) \\ & + (N - n) \frac{1}{2} \frac{n_a}{N - 1} \left(v - \frac{3t}{4} - c \right). \end{aligned} \quad (8)$$

The first term are consumers on the n_a spokes whose first preferred brand is offered on Amazon;

they travel $t/4$ on average. The second term are consumers on the $n - n_a$ spokes whose first preferred brand is a product produced by a w-firm; they travel $t/4$ on average. The third term are consumers on the $N - n$ spokes whose first choice is not produced and whose second choice is available on Amazon; they travel $3t/4$ on average.

Since $W(n_a)$ is linear in n_a , it is maximized at one of the two extremes. When n_a equals 0 or n , condition (ii) is mute, and the equilibrium prices obtained in Lemmas 1 and 2 are efficient. This means that once the social planner sets n_a to 0 or n , he/she does not need to adjust the prices.

Proposition 3 (Social optimum). *The socially optimal number of a-firms is*

$$n_a^{SO} = \begin{cases} 0, & \text{if } \Delta > \frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right), \\ n, & \text{if } \Delta < \frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right). \end{cases}$$

At these two possible values of n_a^{SO} the equilibrium prices induce efficiency.

This is intuitive. A branded website increases the perceived value of the product; the drawback is that consumers who lack their first choices and whose second choice is a variety produced by a w-firm end up not purchasing. When $n = N$, it is socially optimal that all firms sell through their own website, no matter what Δ is. When $n < N$, then all firms should sell through Amazon if Δ is low, and operate their own website if Δ is high.

4 Long Run: Amazon or Website?

In the previous section the composition of firms was fixed. In this section I allow firms to choose between selling through Amazon and operating their own website. The number of a-firms becomes endogenous.

Suppose all firms operate their own website. For $n_a = 0$ to be an equilibrium, no firm should benefit from becoming an a-firm. We evaluate $\Pi_a^*(n_a)$ at $n_a = 1$ to obtain the profit of the deviating firm:

$$\frac{2N - n_a - 1}{2(N - 1)} \left(v - \frac{3t}{2} - c \right) \Big|_{n_a=1} = v - \frac{3t}{2} - c.$$

We do not want this number to exceed Π_w^* calculated in Lemma 1. That is,

$$\Pi_w^* = \frac{1}{2} \left(v + \Delta - \frac{t}{2} - c \right) \geq v - \frac{3t}{2} - c,$$

or

$$\Delta \geq v - c - \frac{5t}{2}. \quad (9)$$

Now suppose Δ is below $v - c - 5t/2$. How many firms will be selling through Amazon? Let n_a^* be the equilibrium number of a-firms. It must satisfy the following two conditions:

$$\Pi_a^*(n_a^*) \geq \Pi_w^*(n_a^* - 1) \quad \text{and} \quad \Pi_w^*(n_a^*) \geq \Pi_a^*(n_a^* + 1).$$

The first condition makes sure an a-firm does not have incentives to become a w-firm, while the second condition says a switch in the other direction is not profitable. For the parameter values used in Figures 2 and 3, we find that $n_a^* = 21$. If we treat n_a as a continuous variable, the equilibrium number of a-firms will be determined by

$$\Pi_w^*(n_a^*) = \Pi_a^*(n_a^*).$$

In Figure 3 the intersection occurs in Region III at $n_a^* = 22.69$. The intersection could have happened in Region II, but not in Region I. This is because in Region I $\Pi_w^*(n_a)$ and $\Pi_a^*(n_a)$ are linear in n_a , and the former is always steeper than the latter:

$$\Pi_w^{*'}(n_a) = -\frac{v + \Delta - \frac{t}{2} - c}{2(N-1)} < \Pi_a^{*'}(n_a) = -\frac{v - \frac{3t}{2} - c}{2(N-1)}.$$

When Δ is close to zero, all firms will be selling through Amazon. For $n_a = n$ to be an equilibrium, $\Pi_w^*(n-1)$ should not exceed Π_a^* from Lemma 2. Since the firms' Nash equilibrium profits in Region IV are beyond our reach, we substitute $\Pi_w^*(n-1)$ by its approximation Π_w^ϵ . The condition is, therefore,

$$\Pi_a^* = \frac{(2N - n - 1)^2}{2(N-1)(n-1)}t \geq \Pi_w^\epsilon = \frac{1}{2} \left(\frac{2N - n - 1}{n-1}t + \Delta \right),$$

or

$$\Delta \leq \frac{(N-n)(2N-n-1)}{(N-1)(n-1)}t. \quad (10)$$

Comparing

$$\frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right)$$

from Proposition 3 with the thresholds in (9) and (10) reveals that in the assumed parameter space (5)

$$\frac{(N-n)(2N-n-1)}{(N-1)(n-1)}t < \frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right) < v - c - \frac{5t}{2}.$$

Table 1: Equilibrium number of a-firms: n_a^*

	n						
	70	75	80	85	90	95	100
2.5	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>
2.4	13.55	13.55	13.55	13.55	13.55	13.55	13.55
2.3	13.93	13.93	13.93	13.93	13.93	13.93	13.93
⋮							
1.6	-	17.63	17.45	17.32	17.29	17.29	17.29
1.5	-	18.41	18.19	18.03	17.91	17.91	17.91
1.4	-	-	19.01	18.81	18.67	18.57	18.57
1.3	-	-	19.93	19.68	19.50	19.37	19.28
1.2	-	-	20.97	20.66	20.44	20.27	20.13
1.1	-	-	22.15	21.78	21.49	21.27	21.11
1.0	-	-	23.53	23.05	22.69	22.42	22.20
0.9	-	-	25.15	24.54	24.08	23.73	23.45
0.8	-	-	27.10	26.31	25.71	25.26	24.90
0.7	-	-	-	28.45	27.66	27.06	26.59
0.6	-	-	-	31.13	30.06	29.25	28.63
0.5	<u>70</u>	-	-	34.63	33.10	31.98	31.12
0.4	<u>70</u>	<u>75</u>	-	39.53	37.17	35.52	34.29
0.3	<u>70</u>	<u>75</u>	<u>80</u>	-	43.10	40.41	38.52

Notes. Underlined are the instances where $n_a^* = 0$ or n . Region II intersections are shaded gray. An entry of “-” indicates a Region IV intersection. The rest are Region III intersections. Above the diagonal $n_a^{SO} = 0$, below it $n_a^{SO} = n$.

It follows that $n_a^* = n_a^{SO}$ when $n_a^* = 0$ or n .

Proposition 4 (Equilibrium number of a-firms). *All firms choose to operate their websites if $\Delta \geq v - c - 5t/2$, and they all choose to sell through the platform if*

$$\Delta \leq \frac{(N - n)(2N - n - 1)}{(N - 1)(n - 1)}t.$$

Intermediate values of Δ yield $n_a^ \in (n_a^I, n)$. Whenever $n_a^* = 0$ or n , the equilibrium number of a-firms is socially optimal.*

Table 1 calculates the equilibrium number of a-firms for various values of Δ and n , while the other parameters stay at $c = 2, v = 7, t = 1$, and $N = 100$. The aforementioned $n_a^* = 22.69$ (framed) is in the $\Delta = 1, n = 90$ cell. The first row is all zeros because by Proposition 4 $n_a^* = 0$ when $\Delta \geq v - c - 5t/2 = 2.5$.

The underlined numbers in the bottom left corner of Table 1 correspond to $n_a^* = n$. This is

because the threshold

$$\frac{(N-n)(2N-n-1)}{(N-1)(n-1)}t$$

equals 0.57 when $n = 70$, 0.42 when $n = 75$, and 0.30 when $n = 80$.

An entry of “-” indicates that $\Pi_w^*(n_a)$ and $\Pi_a^*(n_a)$ intersect somewhere in Region IV (the equilibrium of the pricing game is in mixed strategies). All Region II intersections are shaded gray. Because in Region II neither $\Pi_w^*(n_a)$ nor $\Pi_a^*(n_a)$ depend on n , the shaded numbers in each row stay the same. The rest are Region III intersections.

The broken diagonal line divides the table into two parameter regions:

$$\Delta > \frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right) \quad \text{and} \quad \Delta < \frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right).$$

The threshold

$$\frac{N-n}{N-1} \left(v - \frac{3t}{4} - c \right)$$

equals 1.29 when $n = 70$, 1.07 when $n = 75$, 0.86 when $n = 80$, 0.64 when $n = 85$, 0.43 when $n = 90$, 0.21 when $n = 95$, and 0 when $n = 100$. The socially optimal number of a-firms is zero above the diagonal; $n_a^{SO} = n$ below the diagonal (Proposition 3).

Table 1 reveals:

- As n increases, the equilibrium number of a-firms decreases or, if the equilibrium switches from Region III to Region II, remains the same. Put differently, there may be fewer firms on Amazon in more concentrated markets. This is counterintuitive; it would be reasonable to expect that as new firms enter the market, some become a-firms and the others w-firms, so both n_a^* and $n - n_a^*$ increase. Here is what actually happens in Region III. Suppose a new firm enters the market as a w-firm. This firm is a direct competitor to the a-firms, but not to the w-firms. As a result, Π_a^* goes down more than Π_w^* does. This has a snowball effect: Each subsequent entrant will prefer to join the w-firms. As the difference between Π_w^* and Π_a^* grows, some a-firms would become w-firms to restore the equality of profits in equilibrium. The equilibrium number of a-firms decreases.
- The higher is Δ , the lower is the equilibrium number of a-firms, which is in line with the economics intuition.
- Price dispersion among firms of the same type is more likely for smaller values of n and/or Δ . This interesting result is a consequence of the aforementioned negative effects of n and Δ on the equilibrium number of a-firms. A decrease in n or Δ pushes n_a^* into Region IV, where the equilibrium is in mixed strategies.

Table 2: Equilibrium prices: p_w^*, p_a^*

		n						
		70	75	80	85	90	95	100
Δ	1.6	-	6.77, 4.33	6.75, 4.23	6.73, 4.15	6.72, 4.12	6.72, 4.12	6.72, 4.12
	1.4	-	-	6.38, 4.15	6.36, 4.06	6.35, 3.99	6.33, 3.92	6.33, 3.92
	\vdots							
	0.8	-	-	5.15, 3.84	5.17, 3.77	5.18, 3.71	5.19, 3.65	5.20, 3.60
	0.6	-	-	-	4.72, 3.65	4.74, 3.59	4.76, 3.54	4.77, 3.49

- Unless $\Delta \geq 2.5$, the equilibrium number of a-firms is bounded away from zero. This is because the intersection cannot happen in Region I.

What is the effect of market concentration on the equilibrium prices? In the normal oligopoly case of Chen and Riordan (2007) the higher is n , the lower is the equilibrium price. Will the equilibrium prices be decreasing functions of n in the present model, with n_a determined endogenously?

I calculated the equilibrium prices for the rows $\Delta = 0.6$, $\Delta = 0.8$, $\Delta = 1.4$, and $\Delta = 1.6$ in Table 1, and recorded them in Table 2. The prices in the $\Delta = 1.6$, $n = 90$ cell coincide with those in the $\Delta = 1.6$, $n = 95$ and $\Delta = 1.6$, $n = 100$ cells. This is because the three cells share the same $n_a^* = 17.29$, and the prices in Region II are independent of the total number of firms (Proposition 1). The same can be said about the two shaded cells in the $\Delta = 1.4$ row.

In the $\Delta = 1.4$ and $\Delta = 1.6$ rows both p_w^* and p_a^* go down and then plateau as n increases. In the $\Delta = 0.6$ and $\Delta = 0.8$ rows, p_w^* goes up while p_a^* goes down as n increases. The unusual result that a price can be increasing in the total number of firms is a consequence of the negative relationship between n_a^* and n . As the gap between p_w^* and p_a^* widens, this exacerbates the inefficiency described in the first paragraph of Section 3.

The interesting thing is that while the exogenous effect of an increase in product variety is a welfare improvement, the ultimate effect on welfare is not straightforward because of strategic effects. Thus the analysis shows that whether more competition is welfare-enhancing must be carefully considered as an empirical matter.

5 Concluding Remarks

In this theoretical paper I considered a setting in which firms can sell their products through an online marketplace (Amazon Marketplace) or through their own website. While selling through Amazon expands a firm's customer reach, running a website can help the firm to build brand

equity. I derived the equilibrium prices and profits, analyzed the firms' behavior in the long run, and compared the equilibrium outcome with the social optimum.

I used the spokes model of spatial competition proposed by Chen and Riordan (2007) because it can easily accommodate more than one type of firms. The reason being that competition in the spokes model is non-localized. The Salop's circular city model would not work, because with more than one type of firms there are numerous distinct ways of locating the firms equidistantly around the circle. Chen and Riordan's assumption that each consumer cares about a limited number of brands is important to the present study, too. When some w-firms become a-firms, consumers whose first choice is not produced and whose second choice was produced by a w-firm and now is produced by an a-firm now obtain a positive surplus. This expansion effect allows for a detailed study of how the composition of firms affects prices, consumers, firms and welfare.

My approach to modeling w-firms as firms that non-fans do not see, as opposed to a-firms, is novel and offers new insights. Despite the fact that I restricted my attention to the parameter space that coincides with the normal oligopoly case of Chen and Riordan (2007) when $n_a = n$ with "foreseeable" comparative statics, I obtained a few surprising results. For example, I found that the price that w-firms charge can be increasing in the total number of firms and that a symmetric pure-strategy Nash equilibrium might not exist. The latter means price dispersion among the firms of the same type.

I abstracted from costs associated with creating and running a website, as well as the fees that Amazon charges its third-party retailers.⁷ Incorporating these costs will make the model more complex, taking the focus away from the strategic effects of Amazon Marketplace. I also assumed that a firm cannot be an a-firm and a w-firm at the same time. In reality, some companies sell through both rented and owned platforms. These are firms that are capable of synchronizing the inventory over multiple channels; many of them started out on a rented platform like Amazon to make some sales until their own branded website kicks into high gear. Once you created a website and invested in building your brand, you might not want it to be commoditized with the rest of the listings on Amazon. Finally, I assumed that all changes in the composition of firms are seamless. I do not account for the fact that when a firm leaves Amazon Marketplace and builds its own website, it may at first have a substantially smaller market.

⁷Amazon offers individual seller and pro merchant accounts. Individual seller accounts are geared to occasional, low-volume sellers. Pro merchant accounts are designed to meet the needs of businesses and provide many volume-selling features for \$39.99 per month fee, including unlimited product listings and inventory management. All Amazon sellers, both individual and pro, pay a referral fee for every item that sells on Amazon. Referral fees range from 6% up to 20% depending on the product's category.

References

- [1] Abhishek, Vibhanshu, Kinshuk Jerath, and Z. John Zhang, 2016, “Agency Selling or Re-selling? Channel Structures in Electronic Retailing,” *Management Science*, 62(8), pp. 2259-2280.
- [2] Amaldoss, Wilfred, and Chuan He, 2010, “Product Variety, Informative Advertising, and Price Competition,” *Journal of Marketing Research*, 47(1), pp. 146-156.
- [3] Caminal, Ramon, and Adina Claici, 2007, “Are Loyalty-Rewarding Pricing Schemes Anti-Competitive?” *International Journal of Industrial Organization*, 25(4), pp. 657-674.
- [4] Caminal, Ramon, and Lluís Granero, 2012, “Multi-Product Firms and Product Variety,” *Economica*, 79(314), pp. 303-328.
- [5] Chen, Yongmin, and Michael H. Riordan, 2007, “Price and Variety in the Spokes Model,” *Economic Journal*, 117(522), pp. 897-921.
- [6] Dixon, Huw, 1987, “Approximate Bertrand Equilibria in a Replicated Industry,” *Review of Economic Studies*, 54(1), pp. 47-62.
- [7] Granero, Lluís, 2013, “Most-Favored-Customer Pricing, Product Variety, and Welfare,” *Economics Letters*, 120(3), pp. 579-582.
- [8] Granero, Lluís, 2019, “Strategic Product variety and Quality Choice,” *Economics Letters*, 182(1), pp. 10-14.
- [9] Hagiu, Andrei, and Julian Wright, 2015, “Marketplace or Reseller?” *Management Science*, 61(1), pp. 184-203.
- [10] Hotelling, Harold, 1929, “Stability in Competition,” *Economic Journal*, 39(153), pp. 41-57.
- [11] Jiang, Baojun, Kinshuk Jerath, and Kannan Srinivasan, 2011, “Firm Strategies in the ‘Mid Tail’ of Platform-Based Retailing,” *Marketing Science*, 30(5), pp. 757-775.
- [12] Reggiani, Carlo, 2014, “Spatial Price Discrimination in the Spokes Model,” *Journal of Economics and Management Strategy*, 23(3), pp. 628-649.
- [13] Ryan, Jennifer K., Daewon Sun, and Xuying Zhao, 2012, “Competition and Coordination in Online Marketplaces,” *Production and Operation Management*, 21(6), pp. 997-1014.

- [14] Salop, Steven C., 1979, “Monopolistic Competition with Outside Goods,” *Bell Journal of Economics*, 10(1), pp. 141-156.
- [15] Tan, Kok-Keong, Jian Yu, and Xian-Zhi Yuan, 1995, “Existence Theorems of Nash Equilibria for Non-Cooperative n -Person Games,” *International Journal of Game Theory*, 24(3), pp. 217-222.
- [16] Tijs, Steve J., 1981, “Nash Equilibria for Noncooperative n -Person Games in Normal Form,” *SIAM Review*, 23(2), pp. 225-237.
- [17] Vazirani, Vijay V., Noam Nisan, Tim Roughgarden, and Eva Tardos, 2007, *Algorithmic Game Theory*, Cambridge University Press.
- [18] Ziad, Abderrahmane, 1997, “Pure-Strategy ϵ -Nash Equilibrium in n -Person Nonzero-Sum Discontinuous Game,” *Games and Economic Behavior*, 20(2), pp. 238-249.

Appendix

Proof of Lemma 2

Let p_a^* be the price that a-firms charge in equilibrium. Firm A_i faces the demand

$$\frac{n-1}{N-1} \frac{1}{2t} (t - p_i + p_a^*) + \frac{N-n}{N-1}$$

in the neighborhood of p_a^* . The profit of firm A_i is, therefore,

$$\left(\frac{n-1}{N-1} \frac{1}{2t} (t - p_i + p_a^*) + \frac{N-n}{N-1} \right) (p_i - c),$$

or

$$\frac{1}{2(N-1)} \left(\frac{n-1}{t} (t - p_i + p_a^*) + 2(N-n) \right) (p_i - c). \quad (11)$$

The first-order condition is

$$\frac{n-1}{t} (t - p_i + p_a^*) + 2(N-n) - \frac{n-1}{t} (p_i - c) = 0. \quad (12)$$

Substituting $p_i = p_a^*$ yields

$$2N - n - 1 - \frac{n-1}{t} (p_a^* - c) = 0,$$

$$p_a^* = c + \frac{2N - n - 1}{n-1} t.$$

We can use (11) and (12) to calculate the equilibrium profit:

$$\Pi_a^* = \frac{n-1}{2(N-1)t} (p_a^* - c)^2 = \frac{(2N-n-1)^2}{2(N-1)(n-1)} t.$$

The requirement that $v - t - p_a^* > 0$ is satisfied when v exceeds

$$c + 2 \frac{N-1}{n-1} t,$$

the lower limit placed on v by (5). It is also easy to show that firm A_i does not have incentives to deviate globally to $p_i = v - t$ when v is less than

$$c + \left(2 \frac{N-1}{n-1} + \frac{1}{2} \frac{2N-n-1}{N-n} \right) t,$$

the upper limit in (5).

Proof of Lemma 3

That $p_w^* - p_a^* < \Delta$ cannot happen in equilibrium is obvious. If a w-firm increases its price to $p_a^* + \Delta$, the number of consumers it attracts will not change. Thus, the deviation is profitable. Now suppose $p_w^* - p_a^* = \Delta$. Under such prices each firm serves consumers located on the same spoke; a-firms also serve consumers who lack their first choices but whose second choices are available on Amazon. Consider a deviation by an a-firm to $p_a^* - \epsilon$. The firm will lose

$$\epsilon \left(\frac{1}{2} + \frac{1}{2} \frac{N-n}{N-1} \right)$$

on its existing customers and gain

$$\frac{\epsilon}{2t} \frac{n-1}{N-1} (p_a^* - \epsilon - c)$$

on the customers it steals from the other firms. Equilibrium requires

$$\epsilon \left(\frac{1}{2} + \frac{1}{2} \frac{N-n}{N-1} \right) \geq \frac{\epsilon}{2t} \frac{n-1}{N-1} (p_a^* - \epsilon - c)$$

for all $\epsilon > 0$, or

$$p_a^* \leq c + \frac{2N-n-1}{n-1} t.$$

Next, consider a deviation by an a-firm to $p_a^* + \epsilon$. The firm will gain

$$\epsilon \left(\frac{1}{2} + \frac{1}{2} \frac{N-n}{N-1} \right)$$

on its existing customers and lose

$$\frac{\epsilon}{2t} \frac{n_a - 1}{N - 1} (p_a^* + \epsilon - c)$$

as some of its customers switch to the other a-firms. Equilibrium requires

$$\epsilon \left(\frac{1}{2} + \frac{1}{2} \frac{N-n}{N-1} \right) \leq \frac{\epsilon}{2t} \frac{n_a - 1}{N - 1} (p_a^* + \epsilon - c)$$

for all $\epsilon > 0$, or

$$p_a^* \geq c + \frac{2N - n - 1}{n_a - 1} t.$$

It is easy to see that the two conditions cannot be satisfied simultaneously.

Proof of Proposition 1

We prove each of the three parts in turn.

(i) When $n_a \in [0, n_a^I)$, w-firm adopt the same strategy as in the case of $n_a = 0$,

$$p_w^* = v + \Delta - \frac{t}{2}.$$

A-firms focus on capturing the consumers whose first choice is produced by a w-firm, so they set their prices equal to

$$p_a^* = p_w^* - t - \Delta = v - \frac{3t}{2}.$$

The resulting profits are

$$\Pi_w^* = \frac{1}{2} \frac{N - n_a - 1}{N - 1} (p_w^* - c) = \frac{N - n_a - 1}{2(N - 1)} \left(v + \Delta - \frac{t}{2} - c \right)$$

and

$$\Pi_a^* = \left(\frac{1}{2} + \frac{1}{2} \frac{N - n_a}{N - 1} \right) (p_a^* - c) = \frac{2N - n_a - 1}{2(N - 1)} \left(v - \frac{3t}{2} - c \right).$$

(ii) Like in Region I, in Region II a-firms focus on capturing the consumers whose first choice is produced by a w-firm, so they set their prices equal to $p_a^* = p_w^* - t - \Delta$. Firm W_i faces

the demand

$$\frac{n_a}{(N-1)} \frac{1}{2t} (t + \Delta - p_i + (p_w^* - t - \Delta)) + \frac{1}{2} \frac{N - n_a - 1}{N - 1}$$

in the neighborhood of p_w^* . The profit of firm W_i is, therefore,

$$\left(\frac{n_a}{(N-1)} \frac{1}{2t} (t + \Delta - p_i + (p_w^* - t - \Delta)) + \frac{1}{2} \frac{N - n_a - 1}{N - 1} \right) (p_i - c),$$

or

$$\frac{1}{2(N-1)} \left(\frac{n_a}{t} (-p_i + p_w^*) + N - n_a - 1 \right) (p_i - c). \quad (13)$$

The first-order condition is

$$\frac{n_a}{t} (-p_i + p_w^*) + N - n_a - 1 - \frac{n_a}{t} (p_i - c) = 0. \quad (14)$$

Substituting $p_i = p_w^*$ yields

$$N - n_a - 1 - \frac{n_a}{t} (p_w^* - c) = 0,$$

$$p_w^* = c + \frac{N - n_a - 1}{n_a} t.$$

Thus,

$$p_a^* = p_w^* - t - \Delta = c + \frac{N - 2n_a - 1}{n_a} t - \Delta.$$

We can use (13) and (14) to calculate Π_w^* :

$$\Pi_w^* = \frac{n_a}{2(N-1)t} (p_w^* - c)^2 = \frac{(N - n_a - 1)^2}{2(N-1)n_a} t.$$

As to a-firms, they earn

$$\Pi_a^* = \left(\frac{1}{2} + \frac{1}{2} \frac{N - n_a}{N - 1} \right) (p_a^* - c) = \frac{2N - n_a - 1}{2(N-1)} \left(\frac{N - 2n_a - 1}{n_a} t - \Delta \right).$$

The threshold n_a^I can be obtained by setting p_w^* to $v + \Delta - t/2$:

$$c + \frac{N - n_a - 1}{n_a} t = v + \Delta - \frac{t}{2},$$

$$n_a^I = \frac{N - 1}{\frac{v + \Delta - c}{t} + \frac{1}{2}}.$$

(iii) In Region III $\Delta < p_w^* - p_a^* < t + \Delta$, so $\hat{x} \in (0, 1/2)$. Firm A_i faces the demand

$$\frac{n_a - 1}{N - 1} \frac{1}{2t} (t - p_i + p_a^*) + \frac{n - n_a}{N - 1} \frac{1}{2t} (p_w^* - p_i - \Delta) + \frac{1}{2} \frac{N - n_a}{N - 1} + \frac{1}{2} \frac{N - n}{N - 1}$$

in the neighborhood of p_a^* . The profit of firm A_i is, therefore,

$$\left(\frac{n_a - 1}{N - 1} \frac{1}{2t} (t - p_i + p_a^*) + \frac{n - n_a}{N - 1} \frac{1}{2t} (p_w^* - p_i - \Delta) + \frac{1}{2} \frac{N - n_a}{N - 1} + \frac{1}{2} \frac{N - n}{N - 1} \right) (p_i - c),$$

or

$$\frac{1}{2(N - 1)} \left(\frac{n_a - 1}{t} (-p_i + p_a^*) + \frac{n - n_a}{t} (p_w^* - p_i - \Delta) + 2N - n - 1 \right) (p_i - c). \quad (15)$$

The first-order condition is

$$\frac{n_a - 1}{t} (-p_i + p_a^*) + \frac{n - n_a}{t} (p_w^* - p_i - \Delta) + 2N - n - 1 - \frac{n - 1}{t} (p_i - c) = 0. \quad (16)$$

Substituting $p_i = p_a^*$ yields

$$\frac{n - n_a}{t} (p_w^* - p_a^* - \Delta) + 2N - n - 1 - \frac{n - 1}{t} (p_a^* - c) = 0,$$

or

$$\frac{2n - n_a - 1}{n - 1} p_a^* - \frac{n - n_a}{n - 1} p_w^* = c + \frac{2N - n - 1}{n - 1} t - \frac{n - n_a}{n - 1} \Delta.$$

Firm W_i faces the demand

$$\frac{n_a}{N - 1} \frac{1}{2t} (t + \Delta - p_i + p_a^*) + \frac{1}{2} \frac{N - n_a - 1}{N - 1}$$

in the neighborhood of p_w^* . The profit of firm W_i is, therefore,

$$\left(\frac{n_a}{N - 1} \frac{1}{2t} (t + \Delta - p_i + p_a^*) + \frac{1}{2} \frac{N - n_a - 1}{N - 1} \right) (p_i - c),$$

or

$$\frac{1}{2(N - 1)} \left(\frac{n_a}{t} (\Delta - p_i + p_a^*) + N - 1 \right) (p_i - c). \quad (17)$$

The first-order condition is

$$\frac{n_a}{t} (\Delta - p_i + p_a^*) + N - 1 - \frac{n_a}{t} (p_i - c) = 0. \quad (18)$$

Substituting $p_i = p_w^*$ yields

$$\frac{n_a}{t} (\Delta - p_w^* + p_a^*) + N - 1 - \frac{n_a}{t} (p_w^* - c) = 0,$$

or

$$2p_w^* - p_a^* = c + \frac{N-1}{n_a} t + \Delta.$$

To find p_w^* and p_a^* , we solve the system of equations

$$\begin{cases} \frac{2n - n_a - 1}{n - 1} p_a^* - \frac{n - n_a}{n - 1} p_w^* = c + \frac{2N - n - 1}{n - 1} t - \frac{n - n_a}{n - 1} \Delta \\ 2p_w^* - p_a^* = c + \frac{N-1}{n_a} t + \Delta \end{cases}$$

Let

$$\begin{aligned} \gamma &= \frac{n - n_a}{n - 1}, \\ \Phi &= \frac{2N - n - 1}{n - 1} t - \frac{n - n_a}{n - 1} \Delta, \\ \Psi &= \frac{N-1}{n_a} t + \Delta. \end{aligned}$$

Then the system can be rewritten as

$$\begin{cases} (1 + \gamma)p_a^* - \gamma p_w^* = c + \Phi \\ 2p_w^* - p_a^* = c + \Psi \end{cases}$$

Solving it yields

$$\begin{aligned} p_w^* &= c + \frac{\Phi + (1 + \gamma)\Psi}{2 + \gamma} = c + \frac{\frac{2N-n-1}{n-1}t - \frac{n-n_a}{n-1}\Delta + \left(1 + \frac{n-n_a}{n-1}\right) \left(\frac{N-1}{n_a}t + \Delta\right)}{2 + \frac{n-n_a}{n-1}} \\ &= c + \frac{\left(N - n + \frac{(N-1)(2n-1)}{n_a}\right) t + \Delta}{3n - n_a - 1} \end{aligned}$$

and

$$\begin{aligned} p_a^* &= c + \frac{2\Phi + \gamma\Psi}{2 + \gamma} = c + \frac{2\left(\frac{2N-n-1}{n-1}t - \frac{n-n_a}{n-1}\Delta\right) + \frac{n-n_a}{n-1} \left(\frac{N-1}{n_a}t + \Delta\right)}{2 + \frac{n-n_a}{n-1}} \\ &= c + \frac{\left(3N - 2n - 1 + \frac{(N-1)n}{n_a}\right) t - (n - n_a)\Delta}{3n - n_a - 1}. \end{aligned}$$

We want to make sure that $p_w^* - p_a^* < t + \Delta$:

$$c + \frac{\Phi + (1 + \gamma)\Psi}{2 + \gamma} - c - \frac{2\Phi + \gamma\Psi}{2 + \gamma} < t + \Delta,$$

$$\Psi - \Phi < (t + \Delta)(2 + \gamma).$$

Substituting for γ , Φ and Ψ yields

$$\frac{N-1}{n_a}t + \Delta - \frac{2N-n-1}{n-1}t + \frac{n-n_a}{n-1}\Delta < (t + \Delta)\left(2 + \frac{n-n_a}{n-1}\right),$$

$$\frac{N-1}{n_a}t - \frac{2N-n-1}{n-1}t < \Delta + t\left(2 + \frac{n-n_a}{n-1}\right),$$

$$n_a > \frac{N-1}{\frac{\Delta}{t} + \frac{2N+2n-n_a-3}{n-1}}.$$

The threshold n_a^{II} can be found from

$$n_a^{II} = \frac{N-1}{\frac{\Delta}{t} + \frac{2N+2n-n_a^{II}-3}{n-1}},$$

$$n_a^{II} = N + n - \frac{3}{2} + \frac{(n-1)\Delta}{2t} - \sqrt{\left(N + n - \frac{3}{2} + \frac{(n-1)\Delta}{2t}\right)^2 - (N-1)(n-1)}.$$

We can use (17) and (18) to calculate the profit of a w-firm in equilibrium:

$$\Pi_w^* = \frac{n_a}{2(N-1)t}(p_w^* - c)^2 = \frac{n_a}{2(N-1)t} \left(\frac{\left(N - n + \frac{(N-1)(2n-1)}{n_a}\right)t + \Delta}{3n - n_a - 1} \right)^2.$$

Similarly, we can use (15) and (16) to calculate the profit of an a-firm in equilibrium:

$$\Pi_a^* = \frac{n-1}{2(N-1)t}(p_a^* - c)^2 = \frac{n-1}{2(N-1)t} \left(\frac{\left(3N - 2n - 1 + \frac{(N-1)n}{n_a}\right)t - (n - n_a)\Delta}{3n - n_a - 1} \right)^2.$$

Finally, we want to make sure that firm A_i does not have incentives to deviate to a significantly higher price $p_i \in (p_w^* - \Delta, p_a^* + t)$, in which case it would be maximizing

$$\left(\frac{n_a - 1}{N-1} \frac{1}{2t} (t - p_i + p_a^*) + \frac{1}{2} \frac{N - n_a}{N-1} + \frac{1}{2} \frac{N - n}{N-1} \right) (p_i - c),$$

or

$$\frac{1}{2(N-1)} \left(\frac{n_a-1}{t} (-p_i + p_a^*) + 2N - n - 1 \right) (p_i - c). \quad (19)$$

The first-order condition is

$$\frac{n_a-1}{t} (-p_i + p_a^*) + 2N - n - 1 - \frac{n_a-1}{t} (p_i - c) = 0, \quad (20)$$

implying

$$p_i = \frac{p_a^* + c}{2} + \frac{2N - n - 1}{2(n_a - 1)} t.$$

We use (19) and (22) to calculate the profit of the deviating firm:

$$\Pi_a = \frac{n_a-1}{2(N-1)t} (p_i - c)^2 = \frac{n_a-1}{2(N-1)t} \left(\frac{p_a^* - c}{2} + \frac{2N - n - 1}{2(n_a - 1)} t \right)^2.$$

The threshold n_a^{III} is defined by equating Π_a and Π_a^* ,

$$\frac{n_a-1}{2(N-1)t} \left(\frac{p_a^* - c}{2} + \frac{2N - n - 1}{2(n_a - 1)} t \right)^2 = \frac{n-1}{2(N-1)t} (p_a^* - c)^2,$$

or

$$(n_a - 1) \left(\frac{p_a^* - c}{2} + \frac{2N - n - 1}{2(n_a - 1)} t \right)^2 = (n - 1) (p_a^* - c)^2,$$

where

$$p_a^* = c + \frac{\left(3N - 2n - 1 + \frac{(N-1)n}{n_a} \right) t - (n - n_a) \Delta}{3n - n_a - 1}.$$

Proof of Proposition 2

Suppose firm A_i deviates to a higher price $p_i > p_a^\epsilon$. Since $p_w^\epsilon = p_a^\epsilon + \Delta$, this price will also exceed $p_w^\epsilon - \Delta$. The firm faces the demand

$$\frac{n_a-1}{N-1} \frac{1}{2t} (t - p_i + p_a^\epsilon) + \frac{1}{2} \frac{N - n_a}{N-1} + \frac{1}{2} \frac{N - n}{N-1},$$

or

$$\frac{1}{2(N-1)} \left(\frac{n_a-1}{t} (-p_i + p_a^\epsilon) + 2N - n - 1 \right).$$

Firm A_i 's profit is, therefore,

$$\frac{1}{2(N-1)} \left(\frac{n_a-1}{t} (-p_i + p_a^\epsilon) + 2N - n - 1 \right) (p_i - c). \quad (21)$$

The first-order condition is

$$\frac{n_a - 1}{t} (-p_i + p_a^\epsilon) + 2N - n - 1 - \frac{n_a - 1}{t} (p_i - c) = 0, \quad (22)$$

implying

$$\tilde{p}_a(n_a) = \frac{p_a^\epsilon + c}{2} + \frac{2N - n - 1}{2(n_a - 1)} t.$$

We use (21) and (22) to calculate the deviating firm's profit:

$$\tilde{\Pi}_a(n_a) = \frac{n_a - 1}{2(N - 1)t} (\tilde{p}_a(n_a) - c)^2.$$

It is left to show that $\tilde{\Pi}_a(n_a) \rightarrow \Pi_a^\epsilon$ as $n_a \rightarrow n$. Since $\tilde{p}_a(n) = p_a^\epsilon$,

$$\begin{aligned} \tilde{\Pi}_a(n_a) &= \frac{n_a - 1}{2(N - 1)t} (\tilde{p}_a(n_a) - c)^2 \rightarrow \frac{n - 1}{2(N - 1)t} (p_a^\epsilon - c)^2 \\ &= \frac{n - 1}{2(N - 1)t} \left(\frac{2N - n - 1}{n - 1} t \right)^2 \\ &= \frac{(2N - n - 1)^2}{2(N - 1)(n - 1)} t = \Pi_a^\epsilon \quad \text{as } n_a \rightarrow n, \end{aligned}$$

by continuity. This means that for any $\epsilon > 0$ there exists a number $n_a(\epsilon)$ such that for all $n_a \geq n_a(\epsilon)$

$$\tilde{\Pi}_a(n_a) - \Pi_a^\epsilon \leq \epsilon.$$

The proposed prices, therefore, constitute an ϵ -Nash equilibrium.