IOTA5104 Project I

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1 Introduction

This project aim to reproduce the video shown in the class that using several circle or rotating vectors to draw any patterns. The idea is treat the trajectory of the pattern as a function of t, so that the function is a periodic function which can be represented by Fourier Series(FS).

2 Methodology

In this project, we only consider the 2D trajectory. Assuming there is a continue periodic trajectory $f(t) = (x, y)$, we can consider this function has a complex dependent variable which real part is the x coordinate and imaginary part is the y coordinate. According to the FS, any periodic function can be represented by a sum of sine and cosine functions shown below:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right) \right)$$

With Eular equation $e^{ix} = \cos(x) + i\sin(x)$, the function can be represented into another form.

$$f(t) = \cdots c_{-2}e^{2jt} + c_{-1}e^{jt} + c_0 + c_1e^{-jt} + c_2e^{-2jt} + \cdots$$

By representing $f(x)$ into such a form, we only need to determine the coefficients c_i in order to calculate the coordinates of the pattern in the complex plane at any given moment. We can easily calculate c_i because the integral of e^{ijt} from 0 to 2π is zero.

$$f(t)e^{ijt} = \cdots + c_{i-1}e^{jt} + c_i + c_{i+1}e^{-jt} + \cdots$$

$$\int_0^{2\pi} f(t)e^{ijt}dt = \int_0^{2\pi} c_i dt = 2\pi c_i$$

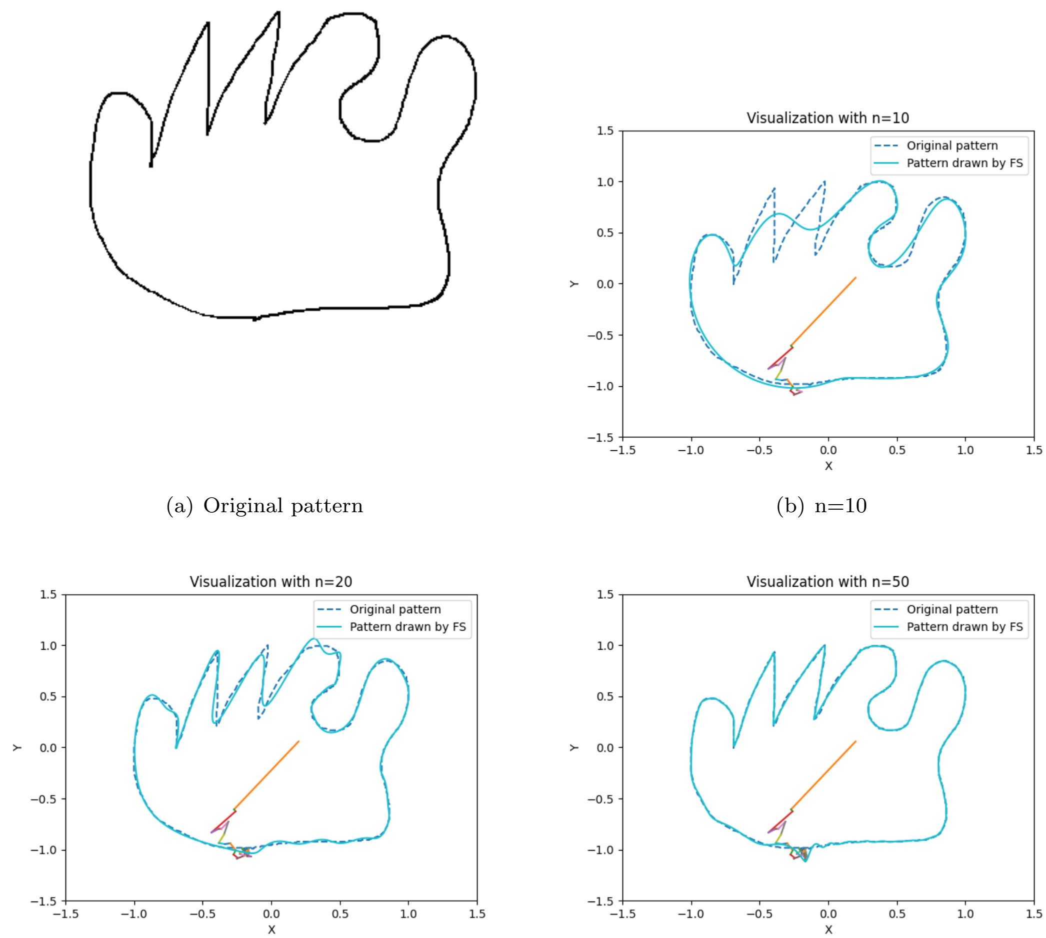
Once we determined all c_i, each term c_ie^{-ijt} equals to a vector in complex plane. By drawing all vectors end to end, we can get the coordinate at any time.

3 Implementation and Results

The first file **draw_trajectory.py** is used for drawing any pattern in a window and save the trajectory into **trajectory.txt**.

In the **generate_fs.py**, firstly normalize the trajectory to make the trajectory of the pattern is located in the center. Than according to the x and y coordinate to construct a complex function respect to t. Because the signal is discrete, t should equals to $\frac{2\pi n}{N}$ where N is the length of the trajectory and the integral become accumulation.

After using accumulation to calculate all the coefficients c_i, for each term c_ie^{-ijt} we calculate the corresponding vector according to the t. By changing the number of coefficients, the recovered pattern will have different shape. For one pattern, the results of using different number of coefficients (n) are shown below:



(a) Original pattern

(b) n=10

(c) n=20

(d) n=50

Figure 1: Results

From the results, we can see that using an increasing number of coefficients, or frequency components, to reconstruct the pattern leads to progressively better outcomes. If the pattern or trajectory contains high-frequency components, using only low-frequency components for reconstruction will result in information loss. As shown in Fig. 1(b), the sharp features cannot be recovered, while the smoother parts fit well.

It's important to note that at the beginning and end of the trajectory, a Gibbs phenomenon may occur. As we increase the number of frequency components used, the Gibbs phenomenon becomes more pronounced. This is not expected to happen, and I am uncertain about the underlying reasons for this issue. It may be caused by the discontinuity of the manually drawn trajectory at the starting and ending points or may because of the sampling rate.

4 Reference

`https://www.bilibili.com/video/BV1Nb411T7Fu`