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Higgs Boson self-coupling measurements using ratios of cross sections

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Abstract: We consider the ratio of cross sections of double-to-single Higgs boson production at the Large Hadron Collider at 14 TeV. Since both processes possess similar higher-order corrections, leading to a cancellation of uncertainties in the ratio, this observable is well-suited to constrain the trilinear Higgs boson self-coupling. We consider the scale variation, parton density function uncertainties and conservative estimates of experimental uncertainties, applied to the viable decay channels, to construct expected exclusion regions. We show that the trilinear self-coupling can be constrained to be positive with a 600 fb^{-1} LHC dataset at 95% confidence. Moreover, we demonstrate that we expect to obtain a +30% and −20% uncertainty on the self-coupling at 3000 fb^{-1} without statistical fitting of differential distributions. The present article outlines the most precise method of determination of the Higgs trilinear coupling to date.

Keywords: Standard Model, Higgs Physics, Hadronic Colliders, Beyond Standard Model.

Contents

1. Introduction

One of the aims of the Large Hadron Collider (LHC) is to search for the agent of electroweak symmetry breaking (EWSB), which in its minimal form is the Standard Model (SM) Higgs boson (H). Recently, both the ATLAS and the CMS collaborations have observed a new state with a mass of about 125 GeV, whose properties are in substantial agreement with the SM Higgs boson [1–5]. The quest for understanding the mechanism behind EWSB does not end with the discovery of this particle. It is crucial to test the Higgs sector to its full extent, measuring the couplings of the Higgs boson to gauge bosons and matter fields [6–30], and also to probe its self-interactions [31–35]. After EWSB, the Higgs potential can be written as

$$V(H) = \frac{1}{2}M_H^2 H^2 + \lambda_{HHH} v H^3 + \frac{1}{4}\lambda_{HHHH} H^4 \,. \tag{1.1}$$

In the SM, $\lambda_{HHH}^{SM} = \lambda_{HHHH}^{SM} = (M_H^2/2v^2) \approx 0.13$ for a Higgs mass of $M_H \simeq 125$ GeV and a vacuum expectation value of $v \simeq 246$ GeV. We can also define normalized couplings $\lambda \equiv \lambda_{HHH}/\lambda_{HHH}^{SM}$ and $\tilde{\lambda} \equiv \lambda_{HHHH}/\lambda_{HHHH}^{SM}$. With an extended Higgs sector, as is common in many new physics models beyond the SM, these couplings will deviate from the SM values. Therefore, measuring these two couplings is crucial to reveal the true nature of the Higgs boson. At the LHC, the quartic coupling $\tilde{\lambda}$ may be probed via triple Higgs boson production. However, its tiny cross section [36] makes it very difficult, if not impossible, to do so. On the other hand, the trilinear coupling λ can be measured in Higgs boson pair production, $pp \to HH$, which may be discovered at a large luminosity phase of the LHC.

The discovery potential for Higgs boson pair production at the LHC has been studied in [32–35, 37]. In Refs. [32, 37], constraints were placed on λ using statistical fits to the shape of the visible mass distributions of the final decay products of the Higgs pairs, whereas Refs. [33,34] focused on the establishment of the Higgs pair production process using improved techniques originating mainly from developments in the understanding of boosted jet substructure [38,39]. In Ref. [35] the final state $b\bar{b}\gamma\gamma$ was revisited as well as $b\bar{b}\tau^+\tau^-$ and $b\bar{b}W^+W^-$ (fully leptonic), without making use of jet substructure techniques (although boosted Higgs bosons were required). The present article concentrates on using the results from the available phenomenological studies along with the best available theoretical cross section calculations and conservative estimates of the experimental uncertainties, to demonstrate the possibility of constraining the trilinear Higgs self-coupling at the LHC.

The article is organised in the following way: in Section 2 we dissect the Higgs boson production cross sections and in Section 3 we examine the theoretical uncertainties on the ratio of cross sections of double-to-single Higgs production. Then, in Section 4, we present the expected constraints obtained at integrated luminosities of 600 fb^{-1} and 3000 fb^{-1}. We conclude in Section 5.

2. Dissection of the cross sections

The Higgs boson pair production cross section is dominated by gluon fusion, as is the single production cross section [40, 41]. For the pair production, other modes, like $qq \to qqHH$,VHH, $t\bar{t}HH$ are a factor of 10-30 smaller [42, 43], and thus we do not consider them in the rest of our analysis. At leading order (LO), there are two main contributions: a diagram containing a 'triangle' loop, and one containing a 'box' loop of top (or bottom) quarks, as shown in Fig. 1. The production of a single, on-shell Higgs boson only contains a diagram of the 'triangle' type. The triangle diagram can only contain initial-state gluons in a spin-0 state, whereas the box contribution can contain both spin-0 and spin-2 configurations. Therefore, there are two Lorentz structures involved in the box diagram matrix element. At LO, we may write, schematically:

$$\sigma_{HH}^{LO} = |\alpha_1 C_{\rm tri}^{(1)} + \beta_1 C_{\rm box}^{(1)}|^2 + \gamma_1^2 |C_{\rm box}^{(2)}|^2 \,, \tag{2.1}$$

where $C_{\rm tri}^{(1)}$ represents the matrix element for the triangle contribution and $C_{\rm box}^{(i)}$ represents the matrix element for the two Lorentz structures ($i = 1, 2$) originating from the box contribution [40, 44]. The parameters α_1, β_1 and γ_1 are given in terms of the Standard Model Lagrangian parameters, by:

$$\alpha_1 = \lambda y_t \ ,$$
$$\beta_1 = \gamma_1 = y_t^2 \ , \tag{2.2}$$

where λ is the (normalized) Higgs triple coupling defined in the previous section and y_t is the Standard Model $Ht\bar{t}$ coupling (as defined after electroweak symmetry breaking and assumed to be real) normalized to the SM value.[1] In contrast, the single Higgs cross section, again, schematically, will only contain the matrix element squared $|C_{\rm tri}^{(1)}|^2$.

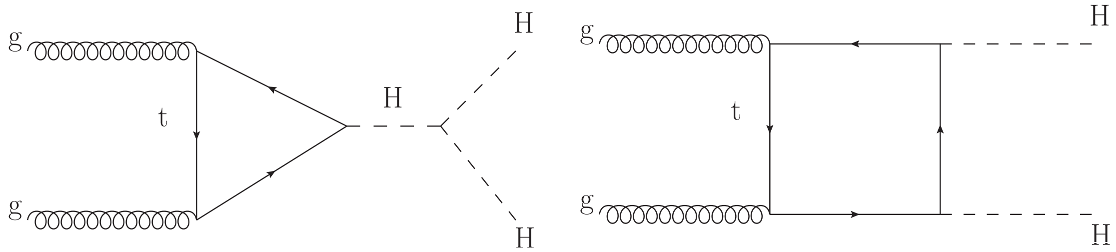

Figure 1: The Higgs pair production diagrams contributing to the gluon fusion process at LO are shown.

We have performed numerical fits using the results of the `hpair` program [45], used to calculate the total cross section for Higgs boson pair production at leading and approximate next-to-leading (NLO) orders. The fits were done employing MSTW2008lo68cl and MSTW2008nlo68cl parton density functions [46] and using top and bottom quark masses of 174.0 GeV and 4.5 GeV respectively. We have obtained:

$$\sigma_{HH}^{\rm LO} = 5.15\lambda^2 y_t^2 - 25.1\lambda y_t^3 + 38.1 y_t^4 \ ,$$
$$\sigma_{HH}^{\rm NLO} = 9.54\lambda^2 y_t^2 - 46.9\lambda y_t^3 + 71.6 y_t^4 \ . \tag{2.3}$$

It is evident from Eqs. (2.1) and (2.3) that the Higgs pair production cross section contains an interference term proportional to (λy_t^3). Hence, for positive values of (λy_t^3) the cross section is reduced, whereas for negative values, it is enhanced. The box squared term is dominant, and scales as y_t^4, whereas the triangle squared term is subdominant due to the off-shell Higgs boson which then decays to the Higgs boson

[1] In the previous discussion, Eqs. (2.1) and (2.2), assume implicitly that the contribution from bottom loops are neglible. In the SM case they contribute to the total cross section less than 0.2%. We have checked numerically that the variation of the bottom Yukawa coupling effect on each form factor is less than 1%. In the rest of this article we include the bottom loops in our numerical calculations assuming that the bottom Yukawa coupling has the SM value. For the sake of simplicitly we assume the validity of Eqs. (2.1) and (2.2) in what follows.

pairs, and scales as $\lambda^2 y_t^2$. Also note that there exists a minimum value of $\sigma_{HH}^{\rm NLO}$ at $\lambda \simeq 2.46 y_t$.

We note that the structure and hence the different contributions to the cross section can of course be modified if new physics that allows new resonances to run in the triangle and box loops is present [47–51]. Here, we are assuming that λ and y_t can be modified, but no other particles enter the diagrams.

3. Ratios of cross sections

It has been pointed out in Ref. [26] that the ratio of cross sections between Higgs pair production and single Higgs production:

$$C_{HH} = \frac{\sigma(gg \to HH)}{\sigma(gg \to H)} \equiv \frac{\sigma_{HH}}{\sigma_H}, \tag{3.1}$$

could be more accurately determined theoretically than the Higgs-pair production cross section itself. This is based on the fact that the processes are both gluon-initiated and the respective higher-order QCD corrections could be very similar. Hence, it is assumed that a large component of the QCD uncertainties drop out in the ratio C_{HH}. Moreover, experimental systematic uncertainties that affect both cross sections may cancel out by taking the ratio. An example is the luminosity uncertainty, which should cancel out provided the same amount of data is used in both measurements.

Here we investigate the extent to which the above assumptions are correct, using the available calculations for the cross sections. We start by considering the LO and NLO calculations for $\sigma(gg \to HH)$ and $\sigma(gg \to H)$ at the LHC at 14 TeV. Using the MSTW2008lo68cl and MSTW2008nlo68cl parton density functions [46], we show in Figs. 2 and 3 the cross sections as well as their ratios, C_{HH}, as a function of the Higgs mass at both LO and NLO.[2] We present the scale uncertainty obtained by varying the factorisation and renormalization scales (set to be equal) between $[0.5\mu_0, 2.0\mu_0]$, where $\mu_0 = M_H$ for the `higlu` program, used to obtain the single Higgs cross sections [53], and $\mu_0 = M_{HH}$ for the `hpair` program, used for the Higgs pair production cross sections [45]. The scale choices are the natural ones for each of the processes but we verified that the conclusions are not altered substantially by changing the `hpair` scale, i.e. the numerator, to equal the scale that appears in the denominator, $\mu_0 = M_H$. Implicit in the calculation of the scale uncertainty of the ratio C_{HH}, is the fact that the scale variation of the single and double Higgs cross sections between $0.5\mu_0$ and $2.0\mu_0$ is fully correlated: i.e., we obtain the upper and lower variations of the ratio by dividing the cross sections with the same magnitude

[2]It is important to note that the NLO calculation has been performed in the heavy top mass limit, and hence in the case of Higgs boson pair production, it is expected to be approximate. At LO, the accuracy of the large top mass approximation is $\mathcal{O}(10\%)$ [31, 50, 52].

of variation of the scale. This is an approximation that is justified since the two processes possess similar topologies, and is in fact one of the main insights in favour of using C_{HH}. We also show, in the ratio, the resulting PDF uncertainty, calculated using the MSTW2008nlo68cl error sets according to the prescription found in [54].

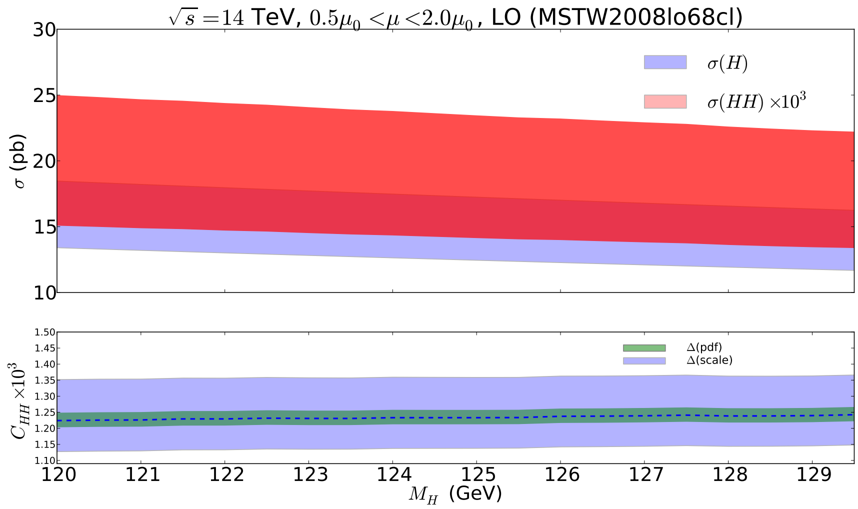

Figure 2: The cross sections for single and double Higgs boson production at leading order using the MSTW2008lo68cl PDF set. In the lower plot, the fractional uncertainty due to scale variation is shown in the blue band, as well as the PDF uncertainty in the green band.

Several observations on the behaviour of the C_{HH} ratio can be made. First of all, it is evident that the fractional uncertainty due to scale variation is reduced with respect to the individual calculations in both leading and next-to-leading orders: for the LO case, the individual cross sections have a $\sim \pm 16\%$ (single Higgs boson production) and $\sim \pm 25\%$ (double Higgs boson production) scale uncertainty, whereas the ratio has a $\sim \pm 9\%$ scale uncertainty. For the NLO case, it is reduced from $\sim \pm 17\%$ (single and double Higgs boson production) to $\sim \pm 1.5\%$ for the ratio.[3]

Furthermore, we can explicitly see that the uncertainty due to the QCD corrections partially cancels out: even though the individual K-factors in the cross sections σ_H and σ_{HH} are large, they are also very similar, both being ~ 2. As a consequence, the central value of the ratio only decreases by a small amount from $\sim$1.25 to $\sim$1.0. This is an indication that higher order corrections are quite likely to change the ratio by an even smaller fraction than the change from LO to NLO, when it is considered at NNLO, whereas the single cross section has a K-factor of about $\sim$1.5 [59]

[3]Note that in Ref. [55], threshold resummation effects in SM Higgs pair-production in soft-collinear effective theory were considered. The authors claim a reduction of the scale uncertainty to 3%. For other resummation studies in single Higgs production see, for example [56–58].

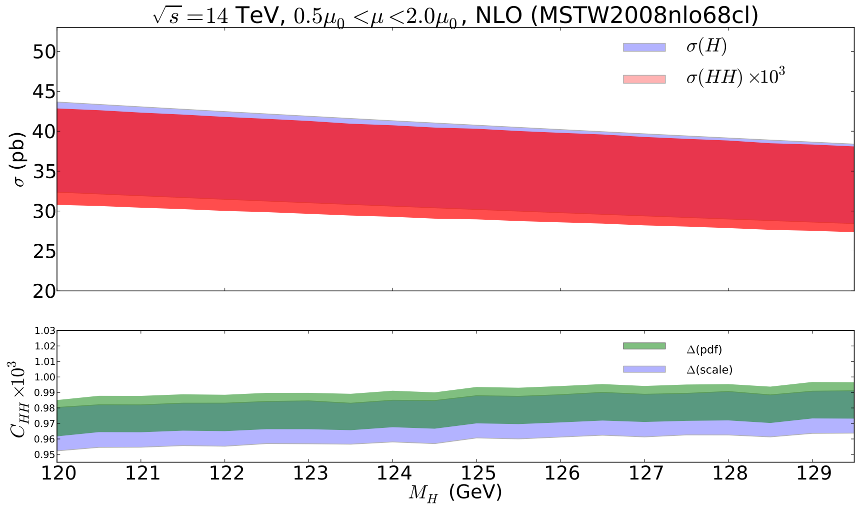

Figure 3: The cross sections for single and double Higgs boson production at next-to-leading order using the MSTW2008nlo68cl PDF set. In the lower plot, the fractional uncertainty due to scale variation is shown in the blue band, as well as the PDF uncertainty in the green band.

with respect to the NLO calculation.[4] These findings support the idea of employing the fully correlated scale variation described before as a realistic estimate for the theoretical error.[5]

The PDF uncertainties for the cross sections themselves are not shown since they are of the order of a few % and hence subdominant. The PDF uncertainty is also sub-dominant in the case of the LO ratio, as shown in Fig. 2. In the case of the NLO ratio, the PDF uncertainty becomes comparable to the scale uncertainty as can be seen in Fig. 3. Combining the two errors in quadrature would induce an error of $\pm\mathcal{O}(3\%)$, still smaller than the $\sim \pm 17\%$ error on the NLO Higgs pair production cross section. To remain conservative, we will assume that the theoretical errors on C_{HH} and σ_{HH} are $\pm 5\%$ and $\pm 20\%$, respectively, in what follows.

4. Constraining the self-coupling

In the studies conducted in Refs. [32, 37], the Higgs self-coupling was constrained using the final states $b\bar{b}\gamma\gamma$, $b\bar{b}\mu^+\mu^-$ and $W^+W^-W^+W^-$ (in the high Higgs mass region). The constraints were obtained by fitting the visible mass distributions in each process for the signal and backgrounds.

Here we chose to follow a different strategy: taking into account the facts that the different signal channels possess a relatively low number of events and that the

[4] An equivalent calculation at NNLO does not presently exist for Higgs pair production.

[5] Note that a detailed study of the scale and parton density function uncertainties in Higgs pair production can be found in Ref. [35].

shapes of distributions for the backgrounds (and even the signal) are not always very well known, we employ only information originating from the rates. Furthermore, we use the theoretically more stable ratio C_{HH} between the double and single Higgs production cross sections, examined in the previous section. We focus on luminosities of 600 fb^{-1} and 3000 fb^{-1} that can be respectively obtained by ATLAS and CMS together in the first long-term 14 TeV run, or by the individual experiments in an even longer-term run at the same energy. We do not attempt to combine between the individual channels, as this will require a more detailed study from the experimental collaborations.

4.1 Variation with self-coupling and top quark Yukawa

To quantify the possible region that can be constrained using the ratio C_{HH}, we first examine the behaviour of the cross section for Higgs pair production and the ratio C_{HH} at 14 TeV, when varying the self-coupling λ, as well as the top Yukawa, y_t. It is important to consider variation of the top quark Yukawa determination, since the production rates of both double and single Higgs production can be substantially affected. Moreover, the expected accuracy on the top quark Yukawa is expected to be $\pm\mathcal{O}(15\%)$ at 300 fb^{-1} of LHC data at 14 TeV [60].

We show the cross section σ_{HH} and ratio C_{HH} at $y_t = 1$ as a function of λ, as well as both quantities at $\lambda = 1$ as a function of y_t in Figs. 4 and 5, respectively. Evidently, the effects of both λ and y_t are significant: the cross section varies from ~ 30 fb at $(\lambda, y_t) = (1, 1)$ (i.e. the SM values) to ~ 125 fb at $(\lambda, y_t) = (-1, 1)$ and ~ 300 fb at $(\lambda, y_t) = (1, 1.6)$. The ratio itself varies from ~ 1 at $(\lambda, y_t) = (1, 1)$ to ~ 3.5 at $(\lambda, y_t) = (-1, 1)$ and $(\lambda, y_t) = (1, 1.6)$. It is obvious that negative values of λ can be excluded sooner than the positive values, since the cross section and ratio of cross sections both increase fast with decreasing λ.

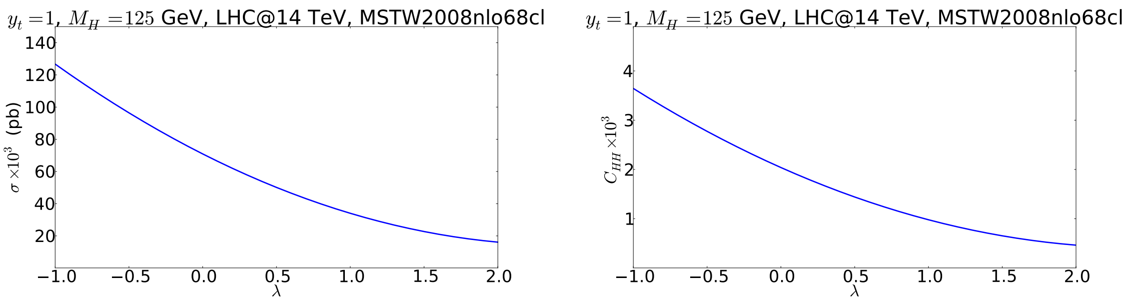

Figure 4: The cross section for double Higgs production and the ratio C_{HH} at next-to-leading order using the MSTW2008nlo68cl PDF set, as a function of λ at $y_t = 1$.

We note that negative values of y_t are currently viable [21] and physical, and could arise in beyond-the-SM physics models. Since Higgs pair production only depends on the sign of the product (λy_t), the corresponding values for $y_t < 0$, $\lambda > 0$

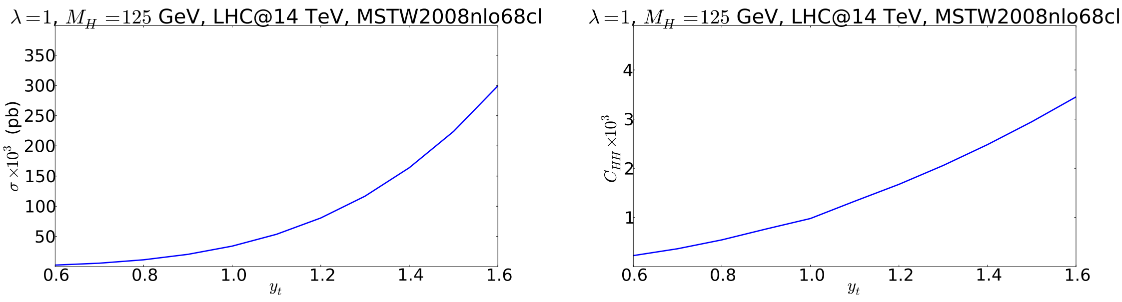

Figure 5: The cross section for double Higgs production and the ratio C_{HH} at next-to-leading order using the MSTW2008nlo68cl PDF set, as a function of y_t at $\lambda = 1$.

are equivalent to those for the points with the same absolute values of the parameters but $y_t > 0$, $\lambda < 0$.[6]

4.2 Deriving constraints

The ratio C_{HH} can be used to derive the expected constraints that can be obtained at a 14 TeV LHC. Certain assumptions on the systematic uncertainties need to be made for the branching ratios related to each mode. We first define the following quantities:

$$\begin{aligned} \sigma_{HH}^{b\bar{b}xx} &\equiv \sigma_{HH} \times \mathrm{BR}(b\bar{b}) \times \mathrm{BR}(xx) \ , \\ \sigma_{H}^{b\bar{b}} &\equiv \sigma_{H} \times \mathrm{BR}(b\bar{b}) \ , \end{aligned} \tag{4.1}$$

where xx denotes the $H \to xx$ decay mode in question. Hence, we can invert the above relations to obtain:

$$C_{HH}^{\text{exp.}} = \left. \frac{\sigma_{HH}^{b\bar{b}xx}}{\sigma_{H}^{b\bar{b}} \times BR(xx)} \right|_{\text{exp.}} \ , \tag{4.2}$$

which is the experimental measurement of the theoretical quantity C_{HH}.

Since the scope of this article is not a detailed experimental study, we now make several assumptions on the measurement uncertainties for each of the quantities in the ratio of Eq. (4.2). We focus on the region $\lambda \in (-1.0, 2.0)$, since the cross section becomes too small for $\lambda > 2$.[7] According to Ref. [63], the branching ratio of $H \to b\bar{b}$ times the cross section for single Higgs is expected to be known to $\pm 20\%$

[6]Note that the degeneracy that appears in Higgs pair production may be resolved through the study of different processes long before the Higgs self-coupling is probed. See, for example, Refs. [61, 62].

[7]Note that there exists a minimum of the Higgs pair production cross section at $\lambda \sim 2.46$ (for $y_t = 1$). Beyond this, the cross section values are symmetric with respect to values below $\lambda \sim 2.46$. However, values of the coupling above this region can be considered to be 'large' from the perturbative point of view, and we do not consider them here.

after 300 fb^{-1} of data at 14 TeV, and hence we assume that the uncertainty on $\sigma_H^{b\bar{b}}$ is $\pm 20\%$. Similarly, according to [63], the uncertainties on BR($\tau^+\tau^-$), BR(W^+W^-) and BR($\gamma\gamma$) are expected to be $\pm 12\%$, $\pm 12\%$ and $\pm 16\%$, respectively, at 300 fb^{-1}. To remain conservative, we assume that going beyond 300 fb^{-1} of luminosity, there will be *no* improvement on these uncertainties. This can be true, for example, if the measurements are dominated by systematic uncertainties that cannot be improved further. Moreover, the uncertainty on the cross section of the measured final state, $\Delta\sigma_{HH}^{b\bar{b}xx}$, is estimated by assuming that the Poisson distribution of the obtained number of events can be approximated by a Gaussian, for simplicity. Hence, if we expect a number of B background events and we experimentally measure N events, the error on the signal estimate, $S = N - B$, is given by $\Delta S = \sqrt{N+B}$. The expected number of events for the studies we consider below were taken from [33, 34, 37]. We combine all the estimates of the uncertainties in quadrature for each mode to obtain an estimate of the total uncertainty:

$$\left(\frac{\Delta C_{HH}}{C_{HH}}\right)^2 = \left(\frac{\Delta\sigma_{HH}^{b\bar{b}xx}}{\sigma_{HH}^{b\bar{b}xx}}\right)^2 + \left(\frac{\Delta \mathrm{BR}(xx)}{\mathrm{BR}(xx)}\right)^2 + \left(\frac{\Delta\sigma_H^{b\bar{b}}}{\sigma_H^{b\bar{b}}}\right)^2 . \tag{4.3}$$

In what follows we also add the theoretical error estimates in quadrature to the above.

As a first approximation we assume that the top quark Yukawa coupling has the SM value and is determined without error, i.e., that $y_t = 1$. Thus, we consider a situation where the Standard Model is valid everywhere but in the Higgs potential. Inevitably, this introduces a degree of model-dependence on the derived λ exclusion. We will consider relaxing the $y_t = 1$ assumption below. Note that the possibility of $y_t = -1$ can be studied simply by reflection: $\lambda \to -\lambda$.

One can form the following question:

> Given an assumption for the 'true' value of the Higgs self-coupling, λ_{true}, what is the constraint we *expect* to impose on λ?

The question is more elaborate than the one that was being asked in searches of the Higgs boson itself, as in that situation the absence of the Higgs Boson at a certain mass resulted in a specific prediction for the background rate. In the case of λ, we have a continuous variable that has to be assessed, and the same experimental results could be due to different true values of λ. The answer to the above question results in an 'exclusion' plot, calculated by drawing the curves that result in expected measurements that are one or two standard deviations away from the central value which is assumed to be equal to that given by λ_{true}. By virtue of this definition, it is obvious that the central value itself is, of course, not expected to be excluded.

Using C_{HH} we draw such curves for 600 fb^{-1} of data in Figs. 6, 7 and 8 for the final states $b\bar{b}\tau^+\tau^-$, $b\bar{b}W^+W^-$ and $b\bar{b}\gamma\gamma$, respectively. To bring the three channels to an equal footing, we have rescaled the $b\bar{b}\tau^+\tau^-$ cross section in [33] by employing a factor of 32.4/28.4 accounting for the central value of the NLO production cross section used in [34], and moreover, rescaled by $0.7^2/0.8^2$ for a reduced τ-jet tagging efficiency. For the $b\bar{b}W^+W^-$ mode in [34] we use the tauonic decays of the W bosons, and for the $b\bar{b}\gamma\gamma$ result in [37] we average between the 'hi' and 'lo' LHC results to get 6 versus 12.5 events at 600 fb^{-1}. We have not rescaled the $b\bar{b}\gamma\gamma$ analysis, since this was done for a Higgs of mass 120 GeV in [37]. In the lower panel of Fig. 6 we also show the exclusion regions extracted by using the Higgs pair production cross section measurement itself, with an associated uncertainty of $\pm 20\%$. We assume that the uncertainty on BR($b\bar{b}$) is the same as that on $\sigma_H^{b\bar{b}}$, namely $\pm 20\%$. It is obvious that the exclusion obtained from the cross section is expected to be weaker than that obtained by the ratio, due to the larger theoretical systematic uncertainty on the cross section itself. Moreover, the expected exclusion from σ_{HH} will be more affected by experimental systematic uncertainties which would add to the errors. For completeness, we show the estimated fractional uncertainty on the ratio, $\Delta C_{HH}/C_{HH}$, used to extract the exclusion regions, for the different processes and investigated luminosities in Table 1. At high luminosity the uncertainties all tend to similar numbers since we have assumed that the other contributing uncertainties (ΔBR(xx) and $\Delta\sigma_H^{b\bar{b}}$) do not improve and they become systematic-dominated. These values are provided for completeness, as an indication, and merit further investigation by the experimental collaborations.

Process	$\Delta C_{HH}/C_{HH}$ (600 fb^{-1})	$\Delta C_{HH}/C_{HH}$ (3000 fb^{-1})
$b\bar{b}\tau^+\tau^-$	0.335	0.257
$b\bar{b}W^+W^-$	0.558	0.325
$b\bar{b}\gamma\gamma$	0.738	0.401

Table 1: The fractional uncertainties on the ratio of double-to-single Higgs boson production cross sections, $\Delta C_{HH}/C_{HH}$, for the different channels and the two investigated LHC luminosities, 600 fb^{-1} and 3000 fb^{-1}, using $M_H = 125$ GeV. These include the theoretical error due to the scale/parton density functions uncertainties, assumed to be 5%.

The interpretation of the 'exclusion' curves is simple: as an example, if we assume or believe that the 'true' value of the triple Higgs coupling is $\lambda_{\rm true} = 1$, then by examining Fig. 6 for the $b\bar{b}\tau^+\tau^-$ mode at 600 fb^{-1}, we can conclude that the expected experimental result should lie within $\lambda \in (0.62, 1.52)$ with $\sim$68% confidence (1σ), and $\lambda \in (0.31, 3.08)$ at $\sim$95% (2σ) confidence. We expect to exclude any values outside this range after 600 fb^{-1}, given the value $\lambda_{\rm true} = 1$. We show the collected exclusion limits for $\lambda_{\rm true} = 1$ and $y_{t,\rm true} = 1$ (i.e. the SM values) at 1σ and 2σ at 600 fb^{-1} as well as the end-of-run LHC integrated luminosity of 3000 fb^{-1} in Table 2. The 3000 fb^{-1} values have also been calculated by assuming no improvement in the uncertainty estimates that we have assumed at 600 fb^{-1}. The table demonstrates an important conclusion: it is possible, using the discovery of the three viable channels, to constrain the trilinear coupling λ to be positive at 95% confidence at 600 fb^{-1} (provided $y_{t,\mathrm{true}} = 1$). Moreover, a naive combination of the 'uncertainties', at 1σ about λ_{true}, over the three channels indicates that a measurement of accuracy +30% and −20% is possible simply by using the rates at 3000 fb^{-1}.

Process	600 fb^{-1} (2σ)	600 fb^{-1} (1σ)	3000 fb^{-1} 2σ	3000 fb^{-1} 1σ
$b\bar{b}\tau^+\tau^-$	(0.31, 3.08)	(0.62, 1.52)	(0.45,1.99)	(0.70, 1.37)
$b\bar{b}W^+W^-$	(-0.04, 19.37)	(0.41, 2.20)	(0.33, 2.88)	(0.63,1.50)
$b\bar{b}\gamma\gamma$	(-0.29, 65.56)	(0.26, 3.99)	(0.20, 5.22)	(0.56, 1.66)

Table 2: The expected limits on λ at 1σ and 2σ confidence levels, provided that λ_{true} and $y_{t,\mathrm{true}}$ have their SM values: $\lambda_{\mathrm{true}} = 1$, $y_{t,\mathrm{true}} = 1$. The results have been derived using C_{HH} and are shown for 600 fb^{-1} and 3000 fb^{-1}.

It is interesting to compare the regions obtained by the above method with those obtained in Ref. [37], where the authors used the only viable mode for a low mass Higgs boson at the time ($M_H = 120$ GeV), $b\bar{b}\gamma\gamma$, to extract λ from the visible mass distribution. After background subtraction, their best limit at 600 fb^{-1} was $\lambda \in (0.26, 1.94)$ at 1σ. Here, for the $b\bar{b}\tau^+\tau^-$ we obtain $\lambda \in (0.62, 1.52)$, for the $b\bar{b}W^+W^-$ mode we obtain $\lambda \in (0.41, 2.20)$ and for the $b\bar{b}\gamma\gamma$ mode, $\lambda \in (0.26, 3.99)$. It is evident that the ratio provides a comparable exclusion region, especially considering the fact that Ref. [37] considers relatively optimistic background subtraction. However, the ratio possesses advantages over the distribution analysis that may contain systematic uncertainties induced by the modelling of the shapes of both the signal and background. Note that an interesting study of the theoretical sensitivity of different initial states ($gg \to HH$, $qq' \to HHqq'$, $q\bar{q}' \to WHH$ and $q\bar{q} \to ZHH$) on the trilinear coupling can be found in [35].

It is important to consider the more general case when the top quark coupling to the Higgs boson is $y_t \neq 1$ [64–72]. This, for example, can be the case in a model where the top quark does not obtain its mass from a SM Higgs boson alone. However, one must keep in mind that additional heavy states potentially present in such models can further modify the predictions. A direct measurement of the magnitude of the $Ht\bar{t}$ coupling can be made by observation of associated production of a single Higgs with top quark pairs [73] using boosted jet techniques that exploit the substructure of so-called 'fat' jets. Since the cross section for Higgs pair production, as well as the single Higgs cross section, both depend on the top coupling, y_t, determination of y_t and the triple coupling, λ, cannot be done independently through measurement of the ratio C_{HH}. Since the error on a determination of y_t is expected to be $\mathcal{O}(15\%)$ [60], an investigation of the possible constraints in the $y_t - \lambda$ plane is essential. This

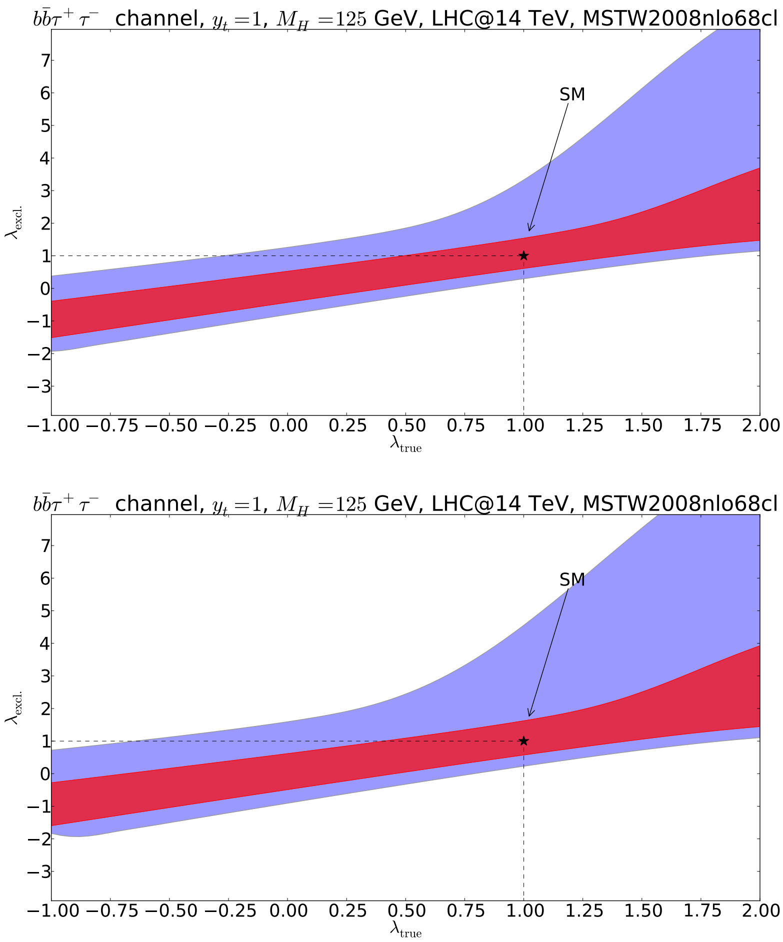

Figure 6: The expected exclusion for λ at one and two standard deviations for a given value of $\lambda_{\rm true}$ at 600 fb^{-1} for the $b\bar{b}\tau^+\tau^-$ decay mode. The exclusion constructed from the ratio, C_{HH}, is shown on the top panel, whereas the exclusion obtained from the cross section, σ_{HH}, is shown on the bottom panel.

can be done with a certain assumption on the underlying true value of λ and y_t. For simplicity, here we assume that $\lambda_{\rm true} = 1$ and $y_{t,{\rm true}} = 1$: i.e., they both have their respective SM values. We then calculate the induced error as we did before and calculate the 1σ and 2σ confidence levels on where the actual measurement will likely end up in the $y_t - \lambda$ plane. The results are shown in Figs. 9, 10 and 11 for $b\bar{b}\tau^+\tau^-$, $b\bar{b}W^+W^-$ and $b\bar{b}\gamma\gamma$ respectively, given an integrated luminosity of 600 fb^{-1}. The figures illustrate an important point: for a model-independent determination of the Higgs triple self-coupling, a good measurement of y_t is crucial. If, for example, we consider y_t at the edges of the expected $\mathcal{O}(15\%)$ error, then $y_t = 0.85$ yields $\lambda \in (0.2, 1.03)$ whereas $y_t = 1.15$ yields $\lambda \in (1.18, \sim 2.5)$, both at 1σ. This is a result of the sensitivity of the single and double cross sections on y_t (given by Eq. (2.3)).

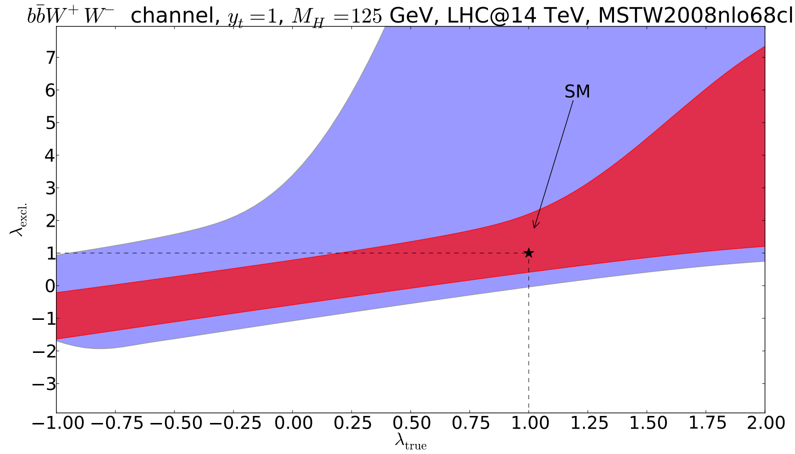

Figure 7: The expected exclusion for λ at one and two standard deviations for a given value of λ_{true} at 600 fb^{-1} for the $b\bar{b}W^+W^-$ decay mode, constructed by using the ratio of cross sections C_{HH}.

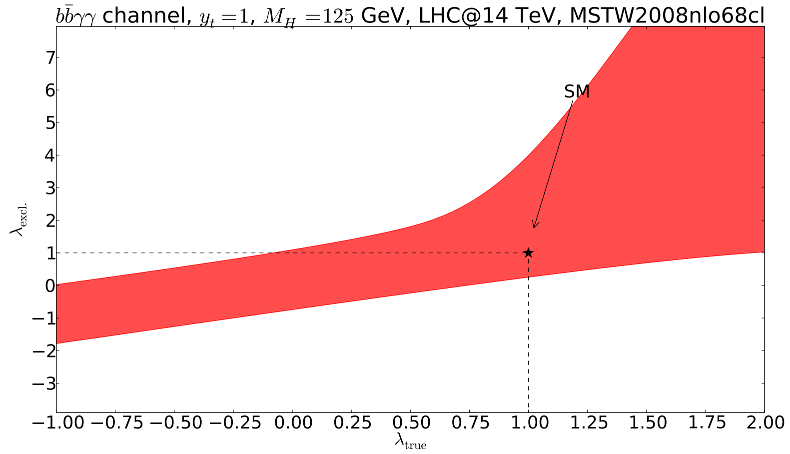

Figure 8: The expected exclusion for λ at one standard deviations for a given value of λ_{true} at 600 fb^{-1} for the $b\bar{b}\gamma\gamma$ decay mode, constructed by using the ratio of cross sections C_{HH}. The two standard deviations exclusion is not shown since it is weak.

5. Conclusions

We have considered the theoretical error on the ratio of cross sections of double-to-single Higgs production, C_{HH}, at a 14 TeV LHC, including scale variation and parton density function uncertainties. Under the assumption that the double and single Higgs boson production cross sections possess a similar form of higher-order corrections, which we motivated in Section 3, we showed in the same section that the ratio is a more theoretically stable quantity than the cross section itself. Subsequently, assuming a 5% total theoretical error on C_{HH}, and using conservative assumptions

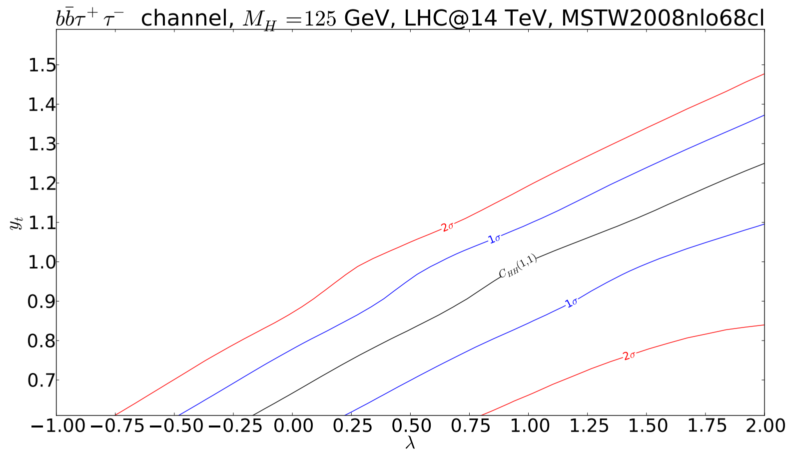

Figure 9: The 1σ and 2σ confidence regions in the $y_t - \lambda$ plane at 600 fb^{-1} for the $b\bar{b}\tau^+\tau^-$ decay mode, derived using C_{HH}, under the assumption that $\lambda_{\text{true}} = 1$ and $y_{t,\text{true}} = 1$.

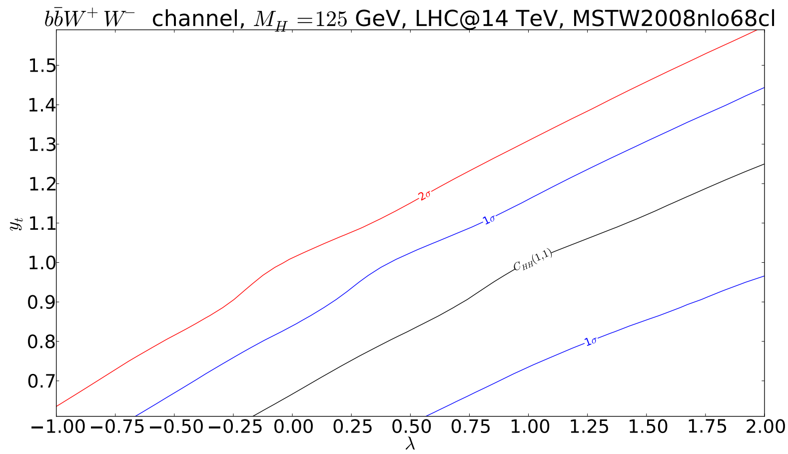

Figure 10: The 1σ and 2σ confidence regions in the $y_t - \lambda$ plane at 600 fb^{-1} for the $b\bar{b}W^+W^-$ decay mode, derived using C_{HH}, under the assumption that $\lambda_{\text{true}} = 1$ and $y_{t,\text{true}} = 1$.

on the experimental uncertainties of the quantities involved in measuring the ratio, we used this ratio to construct possible exclusions given a true value of the Higgs self-coupling at a 14 TeV LHC and integrated luminosities of 600 fb^{-1} and 3000 fb^{-1}. In the SM ($\lambda_{\text{true}} = 1$ and $y_{t,\text{true}} = 1$), we concluded that it is possible to constrain the trilinear coupling to be positive, at 95% confidence at 600 fb^{-1}, only using the discovery of the three viable channels. We also showed that a naive combination of the 'uncertainties' at 1σ over the three channels indicates that a measurement of accuracy $+30\%$ and -20% is possible simply by using the ratio C_{HH} at 3000 fb^{-1}. The present work outlines the most precise method of determination of the Higgs triple self-coupling to date. We have also considered the more model-independent

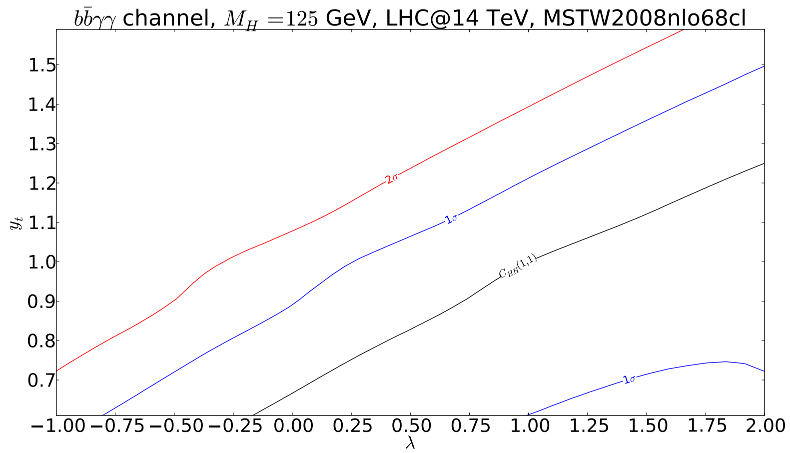

Figure 11: The 1σ and 2σ confidence regions in the $y_t - \lambda$ plane at 600 fb^{-1} for the $b\bar{b}\gamma\gamma$ decay mode, under the assumption that $\lambda_{\mathrm{true}} = 1$ and $y_{t,\mathrm{true}} = 1$.

scenario where the top-Higgs coupling is not assumed to have the SM value, and we have considered the possible exclusion region in the $y_t - \lambda$ plane in that case. Thus we concluded that an accurate, model-independent, determination of the $Ht\bar{t}$ coupling, y_t, is crucial to the determination of the Higgs boson triple self-coupling.

We stress that the derived expected exclusion regions are based on the assumption that the SM is valid everywhere else except in the Higgs sector itself, and we assume no higher dimensional operators. Thus, deviations from expected exclusions would be an indication of some inconsistency in these assumptions that would require further assessment in the form of new physics models. Furthermore, it is obvious from the present study, as well as previous ones, that the measurement of the Higgs boson trilinear self-coupling is a challenging task, and further effort, both on behalf of theorists and experimentalists, should be made in order to obtain the best possible measurement during the lifetime of the LHC.

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