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Portfolio Optimization : Mean Absolute Deviation & Maximum Drawdown

1 Mean Absolute Deviation (MAD) Model

The Mean Absolute Deviation (MAD) model is a portfolio optimization framework used as an alternative to the traditional Mean-Variance Optimization (Markowitz) model. In this approach, investors aim to maximize the portfolio's return for a given level of risk. The utility function for the Mean-Variance Optimization model is defined as :

$$\boxed{U(x) = x^T \mu - \frac{1}{2} x^T \Sigma x}$$

In contrast, the MAD model operates with the same return proxies and portfolio sets as the mean-variance framework but uses mean absolute deviation as the risk proxy instead of portfolio volatility.

Additionally, the MAD model does not impose any assumptions about the distribution of returns. Unlike the Markowitz model, which assumes that returns are normally distributed (a necessary condition for using variance as a risk measure), the MAD model remains valid regardless of the return distribution.

The primary objective of the MAD model is to construct a portfolio whose returns, over all periods, deviate as little as possible from a predetermined expected return. This approach makes the MAD model a flexible and computationally efficient alternative to traditional variance-based optimization methods.

The model operates as follows :

$$\boxed{\min \quad \frac{1}{m} \sum_{j=1}^{m} \left| R_j - \mu_p \right|}$$

Subject to :

$$\begin{cases} \displaystyle\sum_{i=1}^{n} x_i = 1, \\ \displaystyle\sum_{i=1}^{n} x_i \cdot \mu_i = \mu_p, \end{cases}$$

Definitions

— Portfolio return in period j :

$$R_j = \sum_{i=1}^{n} x_i \cdot r_{ij}$$

where :
— R_j : Portfolio return in period j.
— r_{ij} : Return of asset i in period j.
— x_i : Portfolio weight of asset i.
— μ_p : Portfolio's expected return.
— μ_i : Expected return of asset i.
— m : Number of time periods.
— n : Number of assets.
— x_i : Portfolio weight of asset i.

1.1 Equivalent MAD model

Solving the MAD model above is equivalent to solving the following linear programming model :

$$\boxed{\min \quad \frac{1}{m}\sum_{j=1}^{m} z_j}$$

Subject to :

$$\begin{cases} z_j \geq R_j - \mu_p \quad \text{or} \quad z_j \geq -R_j + \mu_p, \\ z_j \geq 0, \\ \sum_{i=1}^{n} x_i \cdot \mu_i = \mu_p, \\ \sum_{i=1}^{n} x_i = 1. \end{cases}$$

2 Maximum drawdown (MDD) model

The Maximum Drawdown (MDD) of an asset refers to the largest observed decline in the asset's value from a peak to a trough over a specified time period. In fact, it measures the worst loss an asset has experienced during a particular period. This is an important risk measure because it focuses on potential losses an investor may face in adverse market conditions.

The Maximum Drawdown (MDD) model is a portfolio optimization model to minimize the worst possible loss in the portfolio over a given time period. In fact, the model aims to combine stocks such that the worst overall performance of the portfolio in any given trading day is as good as possible.

In addition, it is reasonable to use drawdown as a measure of risk instead of standard deviation, because the latter also penalizes desirable outcomes, such as returns above the mean return.

At day t, the return of the portfolio is

$$\bar{r}_t = \sum_{i=1}^{n} r_{i,t} \cdot x_i$$

The optimization problem is formulated as :

$$\boxed{\max_{\mathbf{x}\in\mathbb{R}^n} \min_{t\in[[1,n]]} \bar{r}_t}$$

Subject to :

$$\begin{cases} \sum_{i=1}^{n} r_{i,t} \cdot x_i \geq \rho, & \forall t \in [[1,n]] \\ \sum_{i=1}^{n} x_i = 1 & \\ 0 \leq x_i \leq 50\%, & \forall i \in [[1,n]] \end{cases}$$

We use the constraint :

$$0 \leq x_i \leq 50\%, \forall i \in [[1,n]]$$

so there will be at least 2 stocks in the optimal portfolio, therefore avoid a trivial solution allocating 100% of the funds to the one stock with the minimum maximum drawdown

2.1 Equivalent MDD model

Solving the MDD model above, is equivalent to solving the following model :

$$\max_{\mathbf{x}\in\mathbb{R}^n, y\in\mathbb{R}} y$$

Subject to :

$$\begin{cases} y \leq \bar{r}_t, & \forall t \in [[1,n]] \\ \sum_{i=1}^{n} r_{i,t} \cdot x_i \geq \rho, & \forall t \in [[1,n]] \\ \sum_{i=1}^{n} x_i = 1 \\ 0 \leq x_i \leq 50\%, & \forall i \in [[1,n]] \end{cases}$$