Mustafa MAJJI

Portfolio Optimization : Risk Parity

1 Risk Parity model (RPP)

Risk parity is an approach to portfolio management that focuses on the allocation of risk rather than the allocation of capital. Unlike the minimum variance portfolio, which aims to minimize overall portfolio variance, the risk parity portfolio seeks to ensure that each asset contributes equally to the portfolio's overall volatility.

Furthermore, the risk parity model addresses the issue of risk concentration inherent in the Markowitz model. While the Markowitz model considers the risk of the portfolio as a whole, it often overlooks the importance of risk diversification among individual assets.

The model operates as follows :

$$\boxed{\min \sum_{i=1}^{N} \left(\frac{w_i(\Sigma w)_i}{w^T \Sigma w} - b_i \right)^2}$$

Where :

Definitions

The volatility of a portfolio is :

$$\sigma(w) = \sqrt{w^T \Sigma w} \quad (\Sigma \text{ is the covariance matrix of returns})$$

The risk contribution of the ith asset to the total risk $\sigma(w)$ is defined as :

$$RC_i = \frac{w_i(\Sigma w)_i}{\sqrt{w^T \Sigma w}} \quad \text{so that : } \sum_{i=1}^{N} RC_i = \sigma(w)$$

The relative risk contribution of the ith asset is defined as :

$$RRC_i = \frac{w_i(\Sigma w)_i}{w^T \Sigma w} \quad \text{so that : } \sum_{i=1}^{N} RRC_i = 1$$

The risk parity portfolio (RPP) equalizes the risk contributions :

$$RC_i = \frac{\sigma(w)}{N}$$

More generally, the risk budgeting portfolio (RBP) allocates the risk according to the risk profile determined by the weights b ($b^T 1 = 1$ and $b \geq 0$) :

$$RC_i = b_i \sigma(w)$$

The risk budgeting equation is :

$$RRC_i = b_i, \quad \text{so} : \frac{w_i(\Sigma w)_i}{w^T \Sigma w} = b_i$$

or :

$$w_i(\Sigma w)_i = b_i \times w^T \Sigma w$$