Computer Vision

Interest Point detection
Corner Detection

Slides are created from the material By;
Oge Marques Practical Image and Video Processing Using Matlab
Mubarak Shah and Alper Yilmaz UCF, Fundamentals of Computer Vision
Richard Szeliski Computer Vision Algorithms and Applications

Interest Point Detection

Interest point

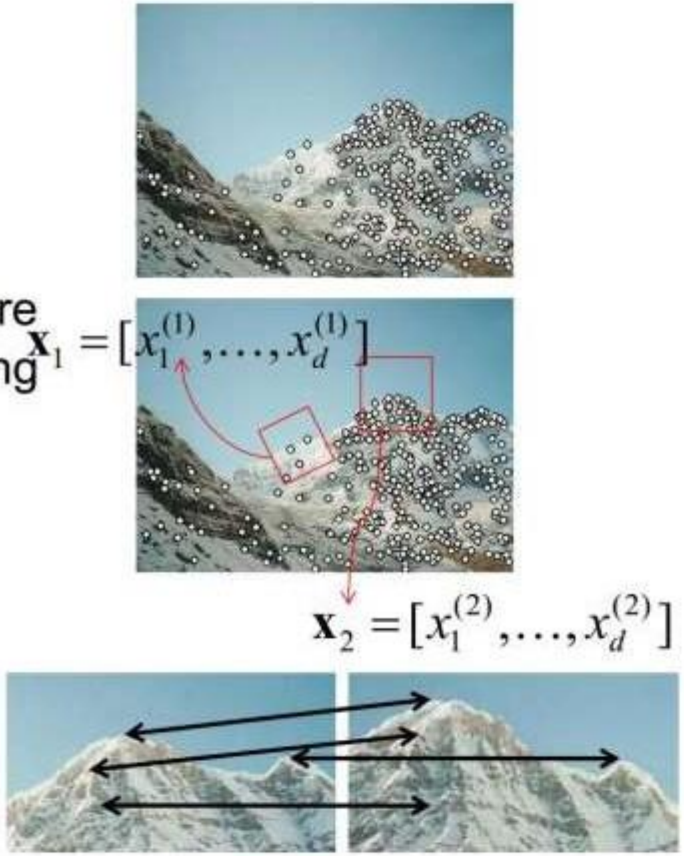
- Interest point or Feature Point is the point which is expressive in texture.
- Interest point is the point at which the direction of the boundary of the object changes abruptly or
- intersection point between two or more edge segments.

Local features: main components

1) **Detection:** Identify the interest points

2) **Description** :Extract feature vector descriptor surrounding each interest point.

3) **Matching:** Determine correspondence between descriptors in two views



Kristen Grauman

Where can we use it?

- Automate object tracking
- Point matching for computing disparity
- Stereo calibration
  - Estimation of fundamental matrix
- Motion based segmentation
- Recognition
- 3D object reconstruction
- Robot navigation
- Image retrieval and indexing

Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.

No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

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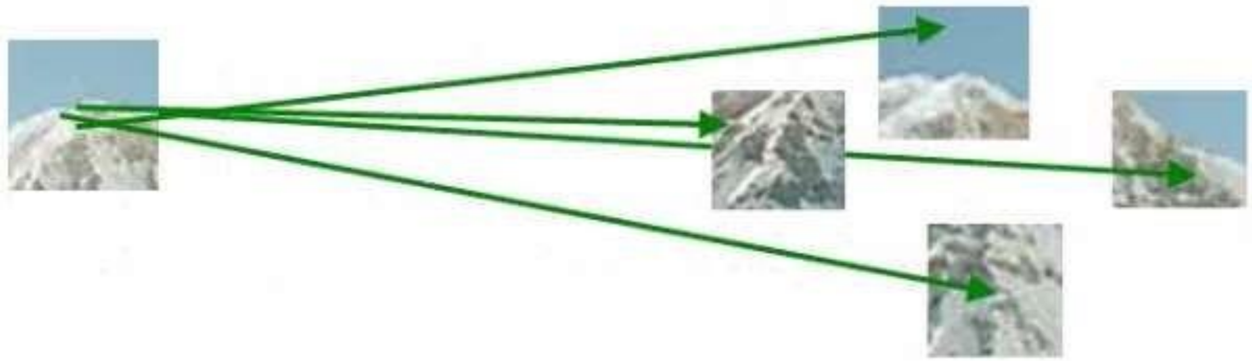
Goal: descriptor distinctiveness

- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

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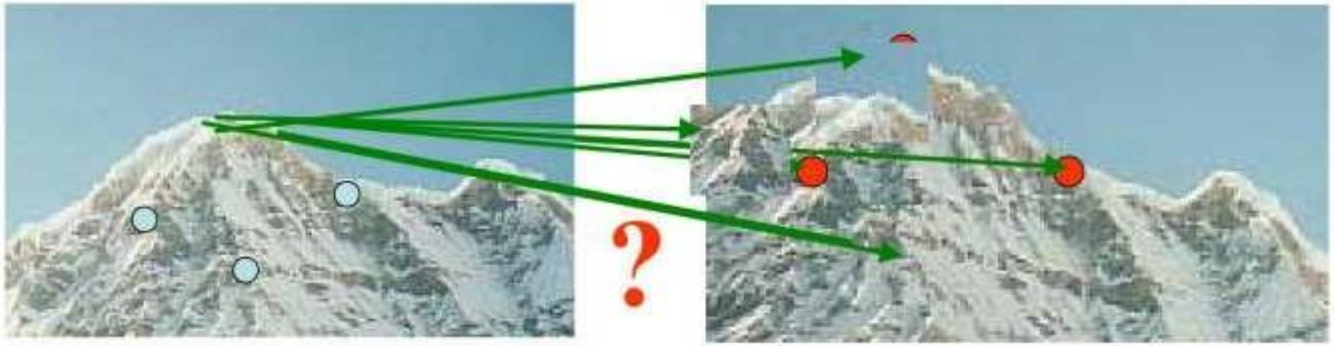

- Must provide some invariance to geometric and photometric differences between the two views.

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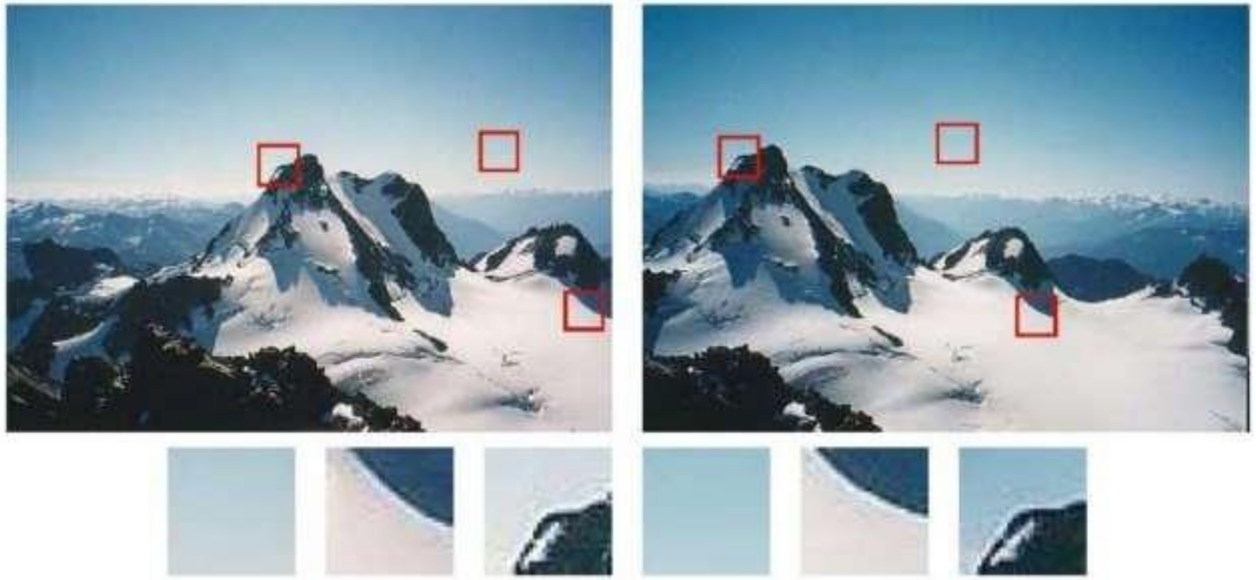
Some patches can be localized or matched with higher accuracy than others.



Local features: main components

1) Detection: Identify the interest points

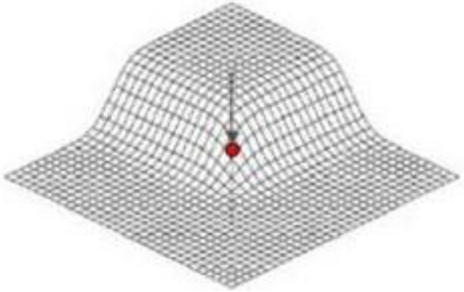
2) Description: Extract vector feature descriptor surrounding each interest point.

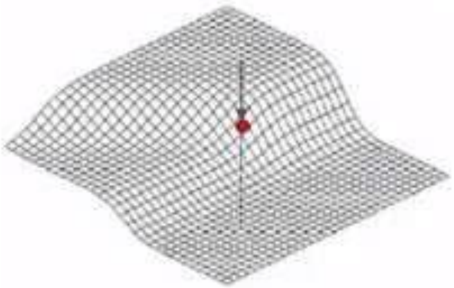
3) Matching: Determine correspondence between descriptors in two views

Kristen Grauman

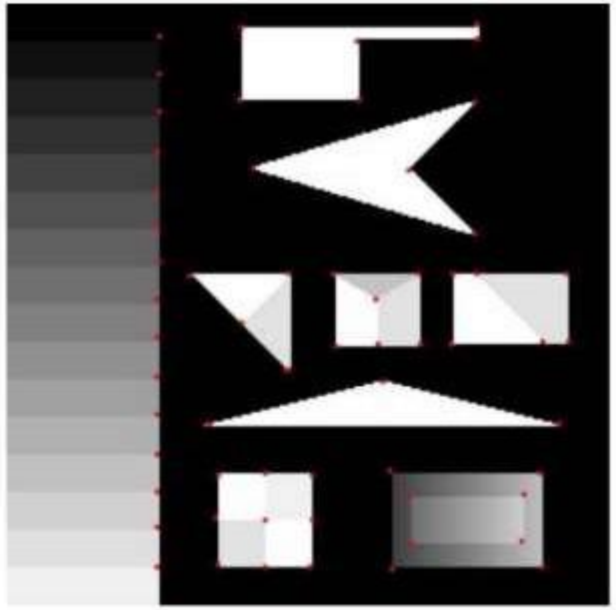
What is an interest point

- Expressive texture
  - The point at which the direction of the boundary of object changes abruptly
  - Intersection point between two or more edge segments





Synthetic & Real Interest Points



Corners are indicated in red

Properties of Interest Point Detectors

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- Detect all (or most) true interest points
- No false interest points
- Well localized.
- Robust with respect to noise.
- Efficient detection

Possible Approaches to Corner Detection

- Based on brightness of images
  - Usually image derivatives

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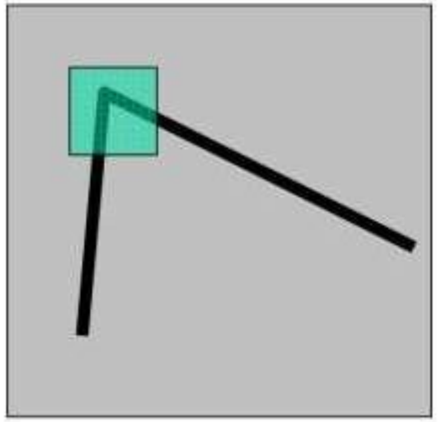
- Based on brightness of images
  - Usually image derivatives
- Based on boundary extraction
  - First step edge detection
  - Curvature analysis of edges

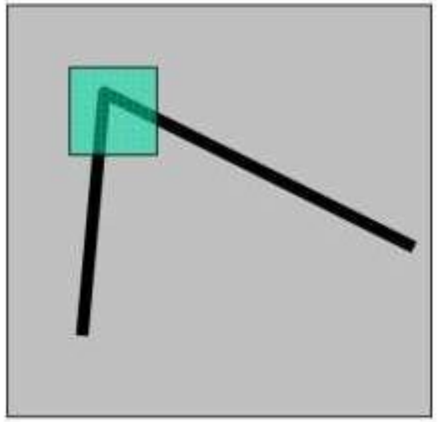
Harris Corner Detector



C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988

Harris Corner Detector

- Corner point can be recognized in a window



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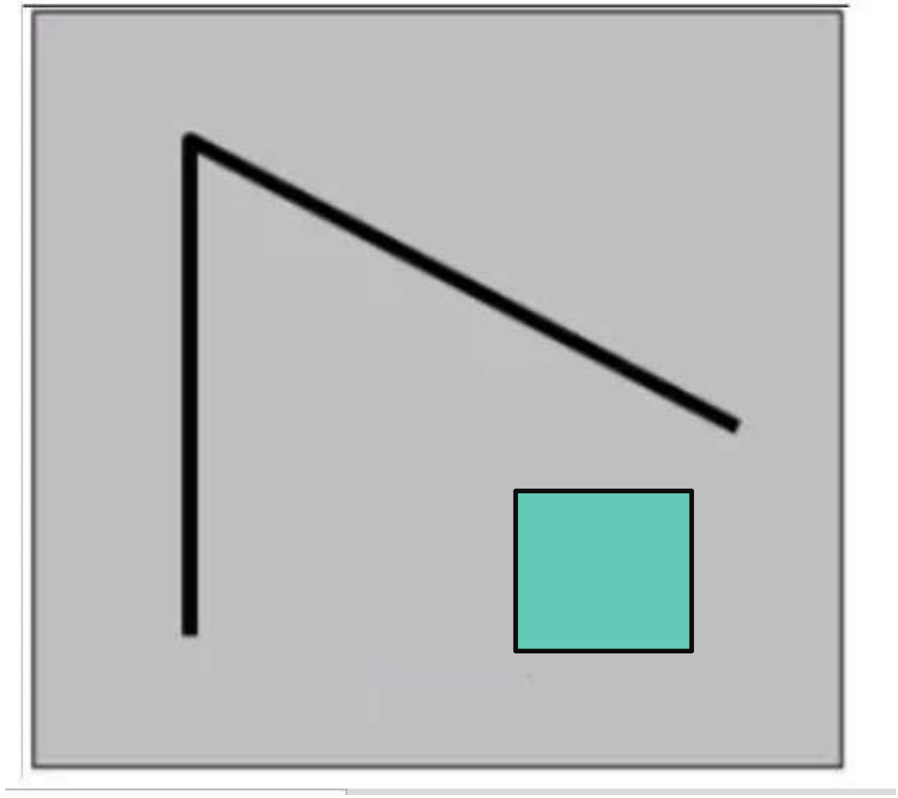
Basic Idea



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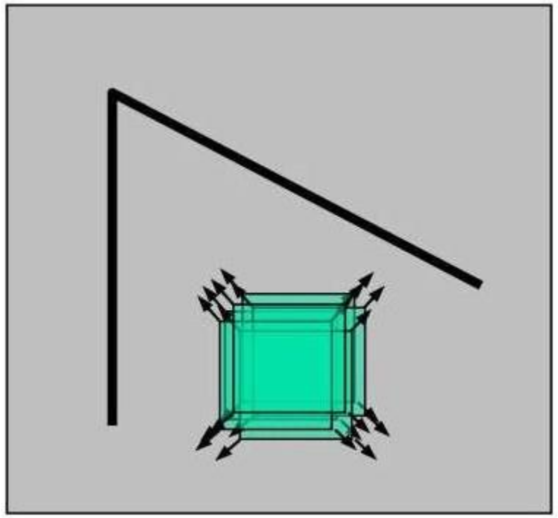

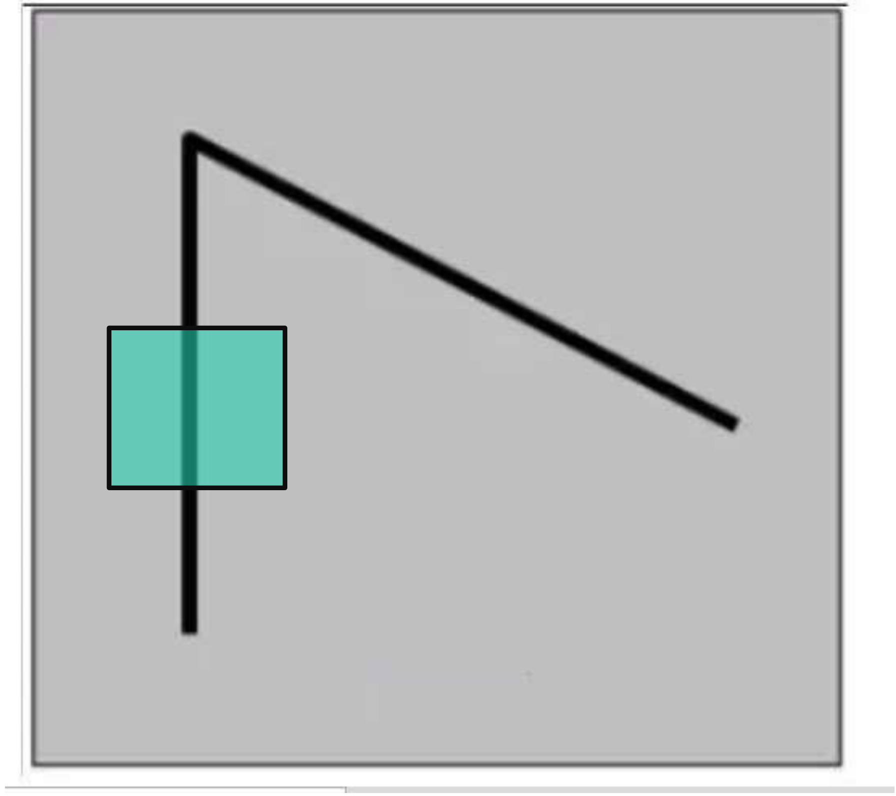

"flat" region:
no change in
all directions

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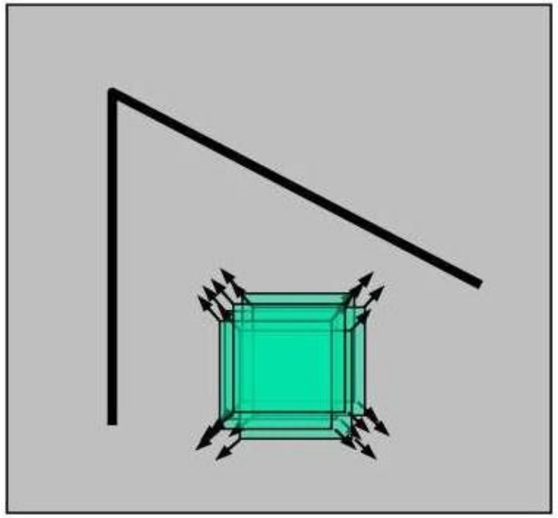

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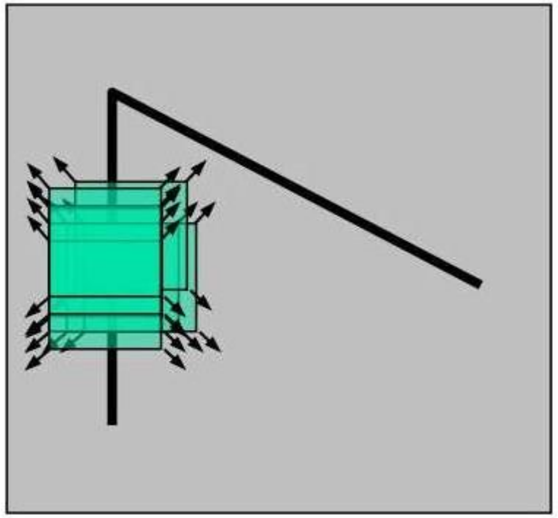
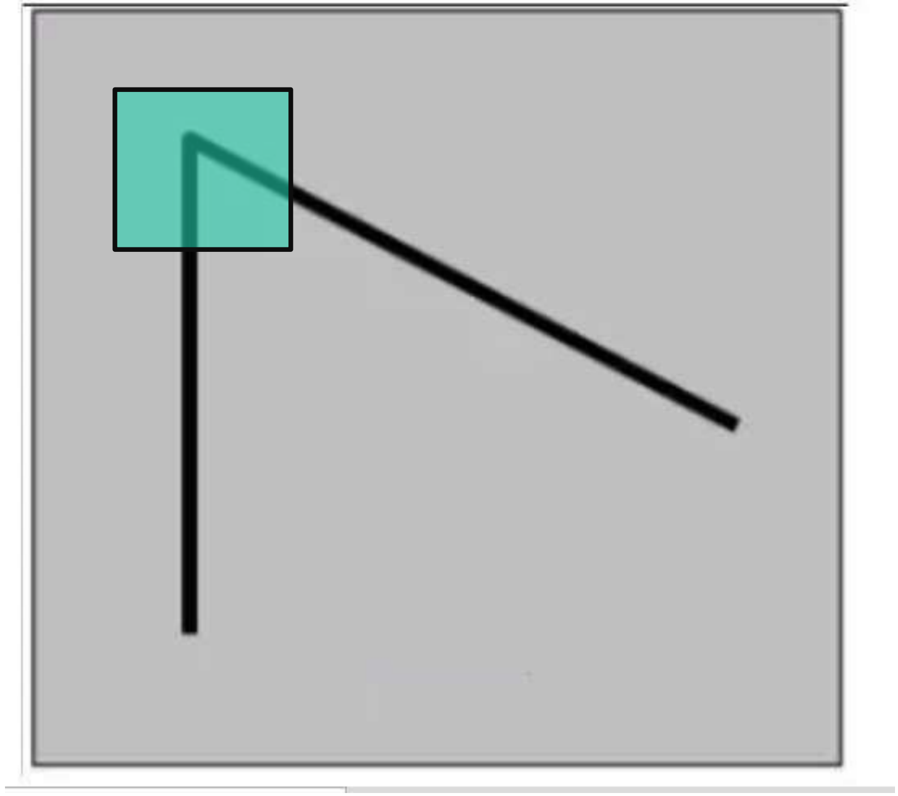


"edge":
no change along
the edge direction



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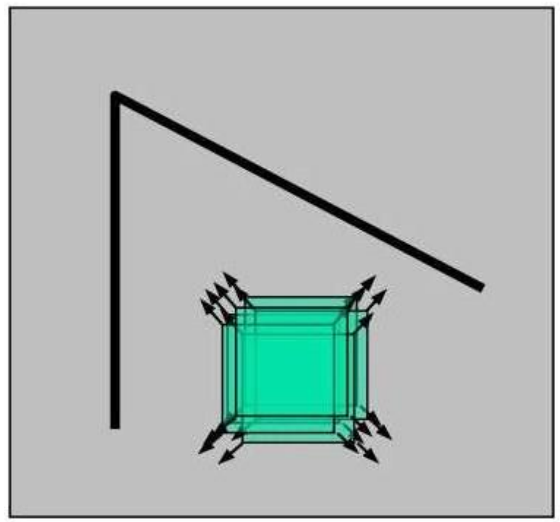

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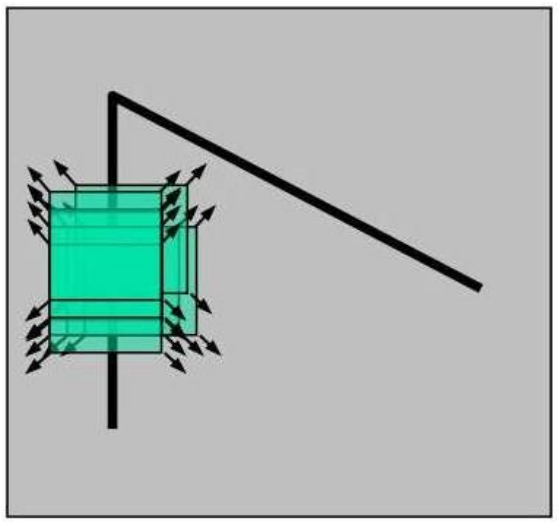

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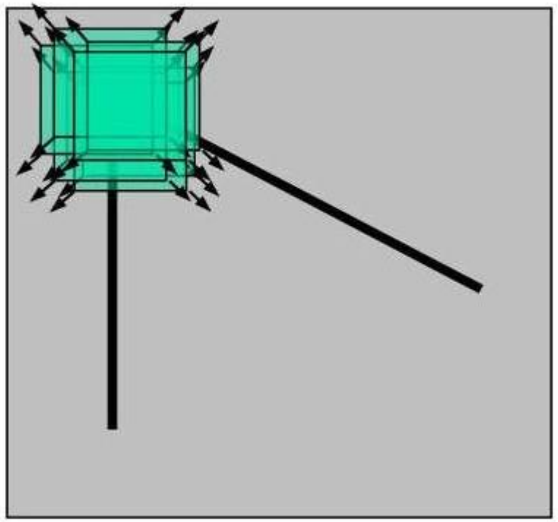

"corner":
significant change
in all directions

Mathematics of Harris Detector

$$E(u,v) = \begin{pmatrix} u & v \end{pmatrix} M \begin{pmatrix} u \\ v \end{pmatrix} \qquad M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

Mathematics of Harris Detector

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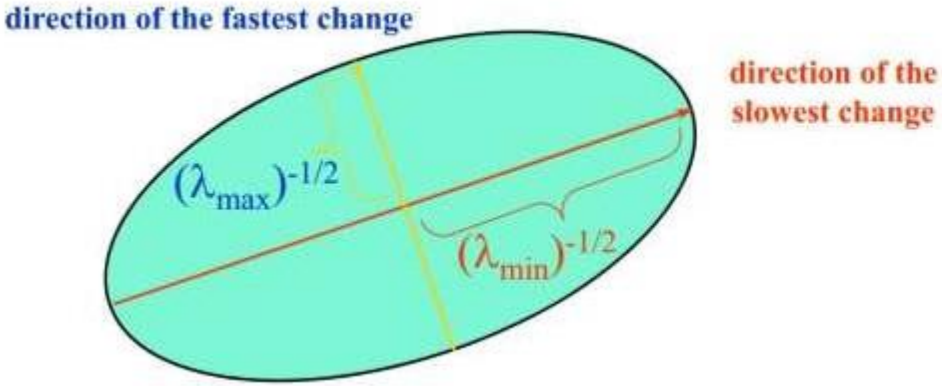

Eigen Vectors and Eigen Values

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To find eigen values of a matrix A first find the roots of:

$$\det(A - \lambda I) = 0$$

Then solve the following linear system for each eigen value to find corresponding eigen vector

$$(A - \lambda I)x = 0$$

Example

$$A=\begin{bmatrix}-1 & 2 & 0\\ 0 & 3 & 4\\ 0 & 0 & 7\end{bmatrix}$$

Eigen Values

$$\det(A-\lambda I)=0$$

$$\lambda_1=7,\ \lambda_2=3, \lambda_3=-1$$

Example

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Eigen Values

$$\det(A - \lambda I) = 0$$

$$\lambda_1 = 7,\ \lambda_2 = 3, \lambda_3 = -1$$

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}, \mathbf{x}_2 = \qquad \mathbf{x}_3 =$$

Eigen Vectors

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$$\det\left(\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

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$$\det\left(\begin{bmatrix} -1-\lambda & 2 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & 7-\lambda \end{bmatrix}\right) = 0$$

$$(-1-\lambda)((3-\lambda)(7-\lambda)-0) = 0$$
$$(-1-\lambda)(3-\lambda)(7-\lambda) = 0$$
$$\lambda = -1, \quad \lambda = 3, \quad \lambda = 7$$

Eigen Vectors

$\lambda=-1$ $(A-\lambda I)x=0$

Eigen Vectors

$\lambda = -1$

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$0 + 0 + 8x_3 = 0$$

$$x_1 = 1, \quad x_2 = 0, \quad x_3 = 0$$

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Eigen Vectors

Mathematics of Harris Detector

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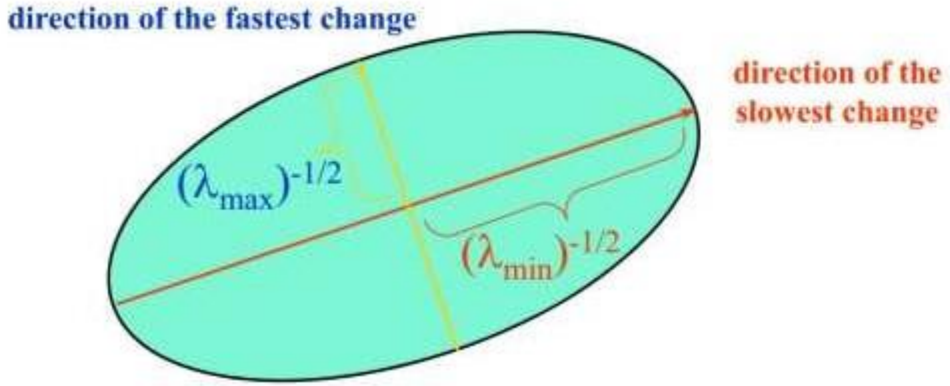


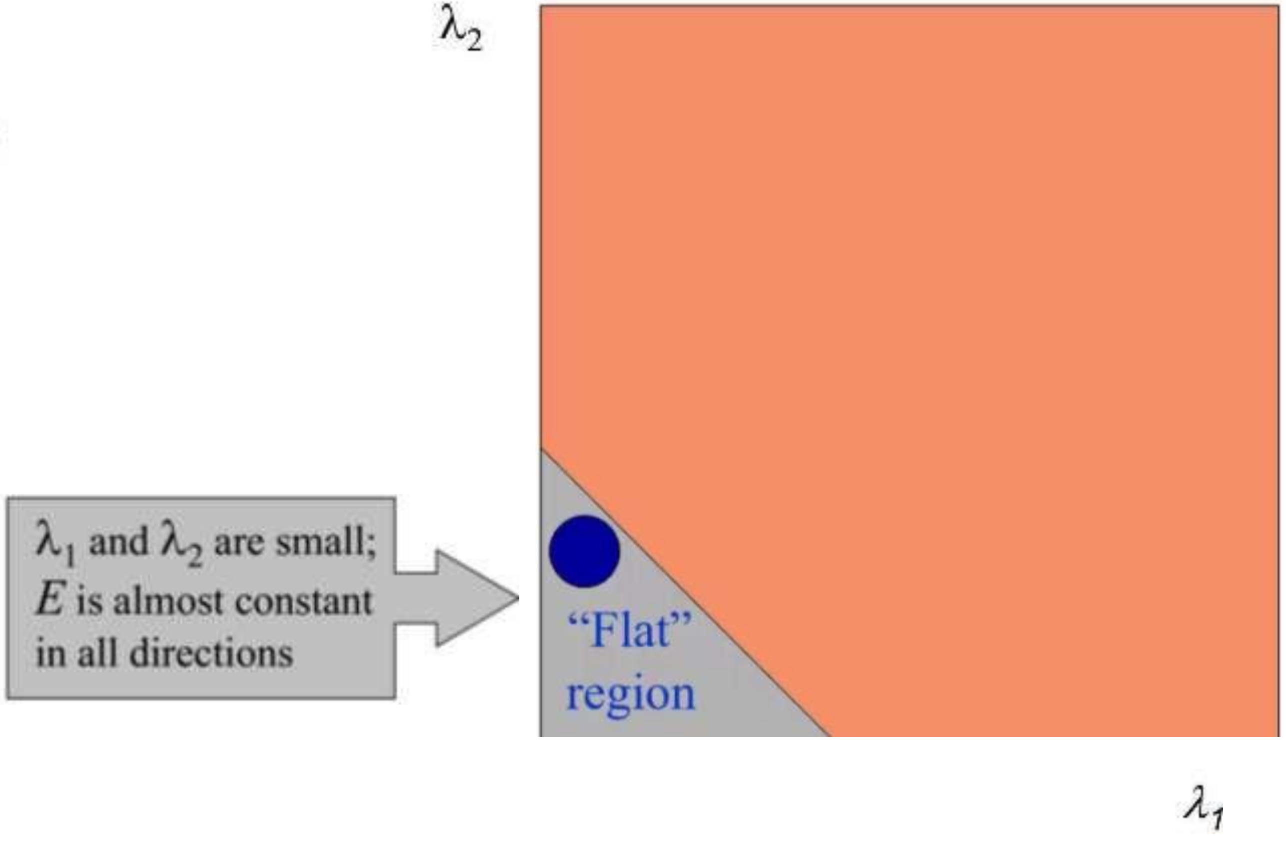
Mathematics of Harris Detector

Classification of image points using eigenvalues of M:

Now we will look into these eigen values and decide weather that point is interest point/corner or not which are actually the Derivative of Ix and Iy and their squares….

Mathematics of Harris Detector

Classification of image points using eigenvalues of M:



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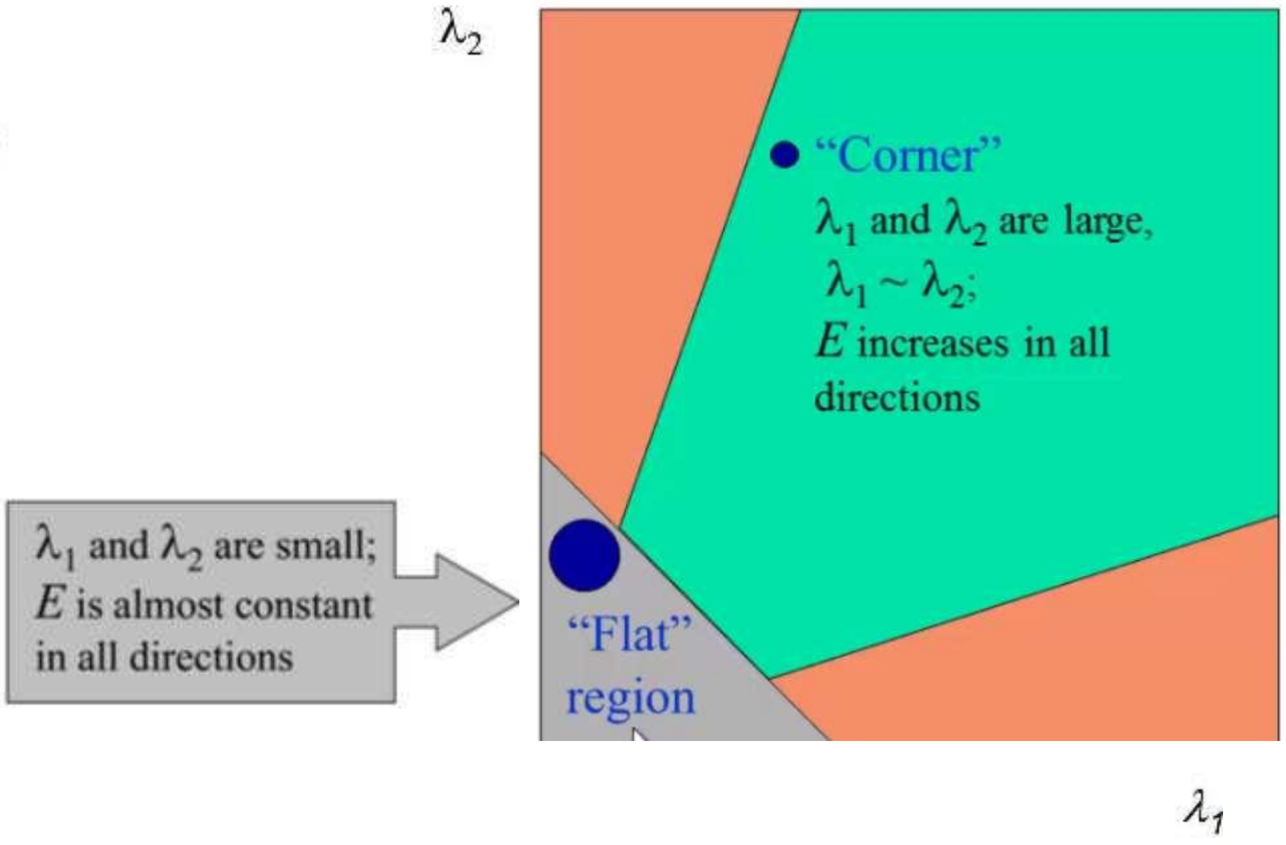

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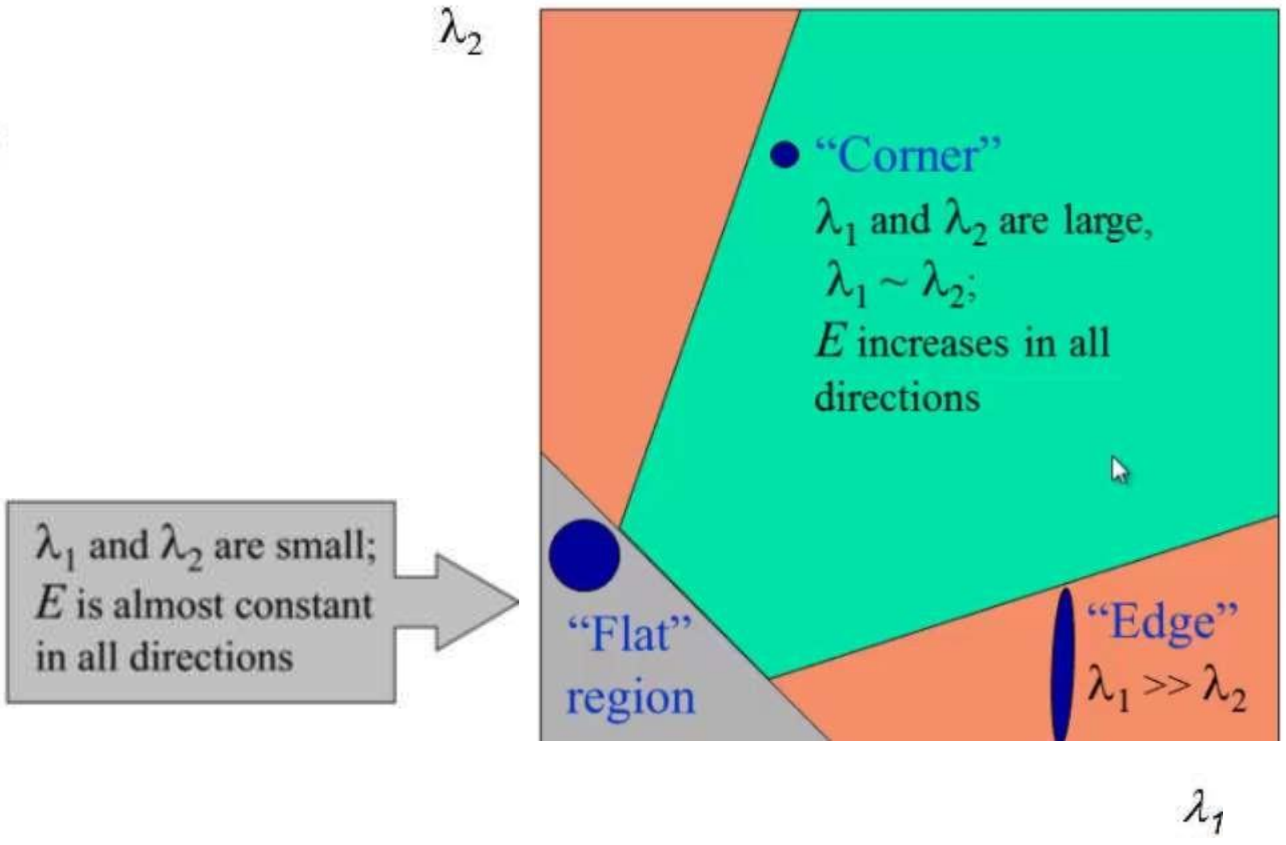

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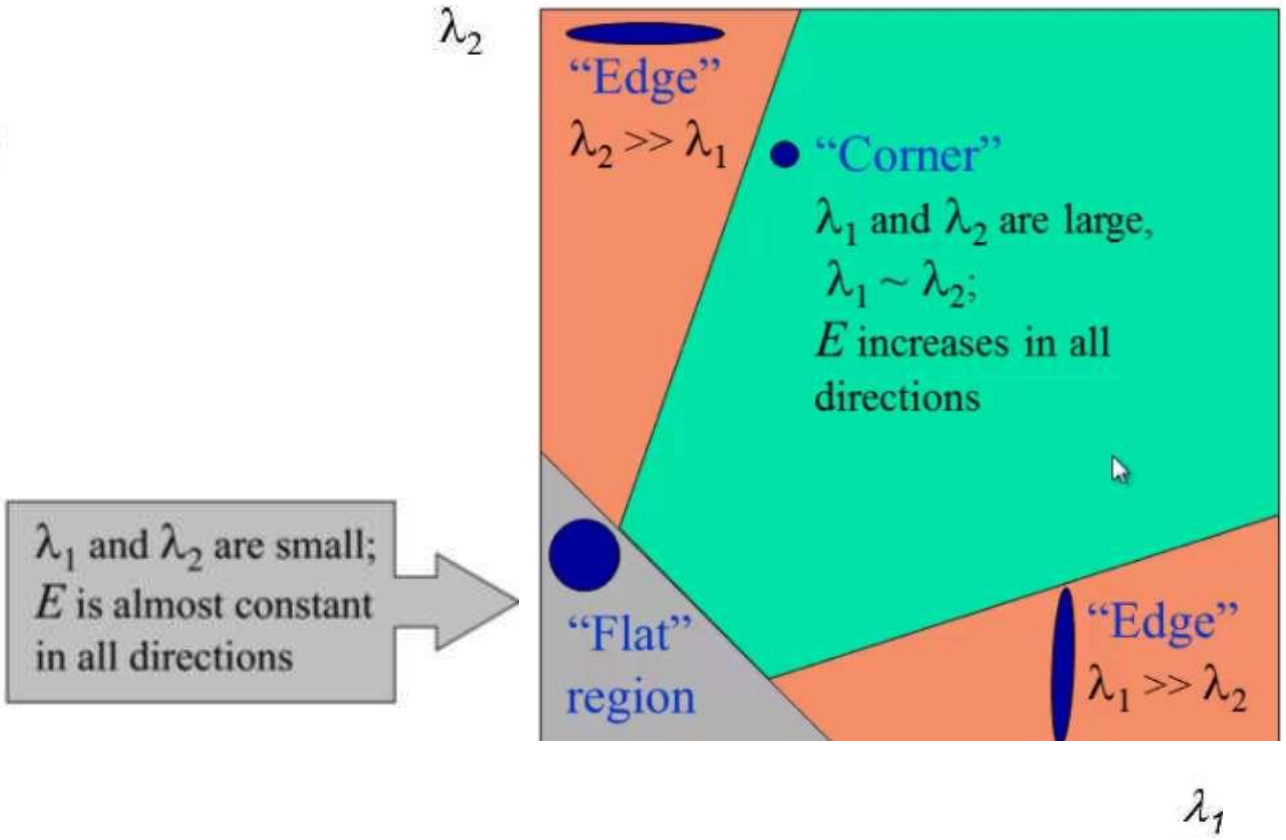

Mathematics of Harris Detector

- Measure of cornerness in terms of λ_1, λ_2

$$M = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$R = \det M - k(\mathit{trace}M)^2 \qquad R = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$$

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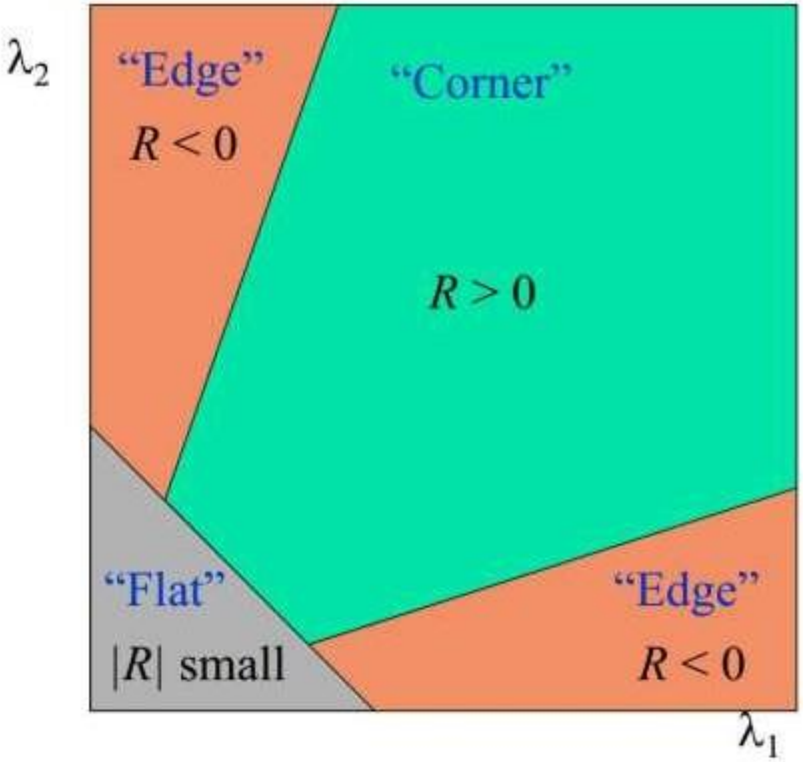
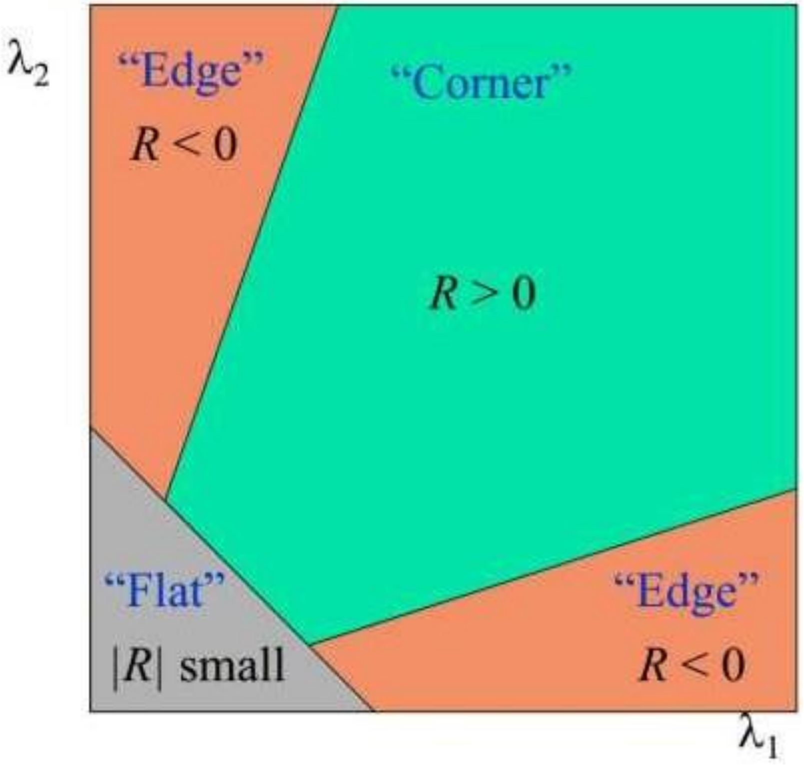


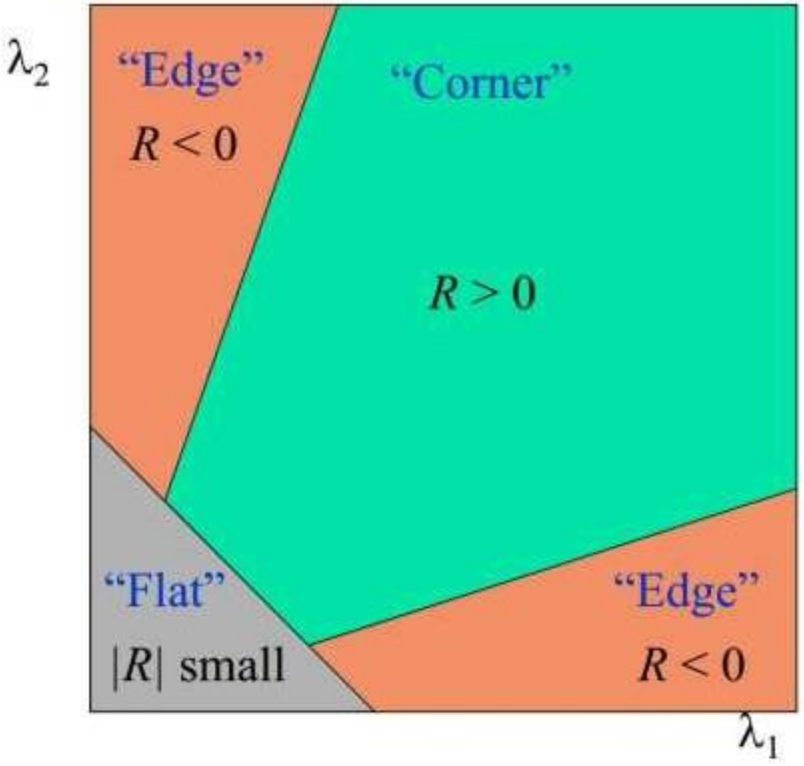
Mathematics of Harris Detector

- R depends only on eigenvalues of M



Mathematics of Harris Detector

- R depends only on eigenvalues of M
- R is large for a corner



Mathematics of Harris Detector

- R depends only on eigenvalues of M
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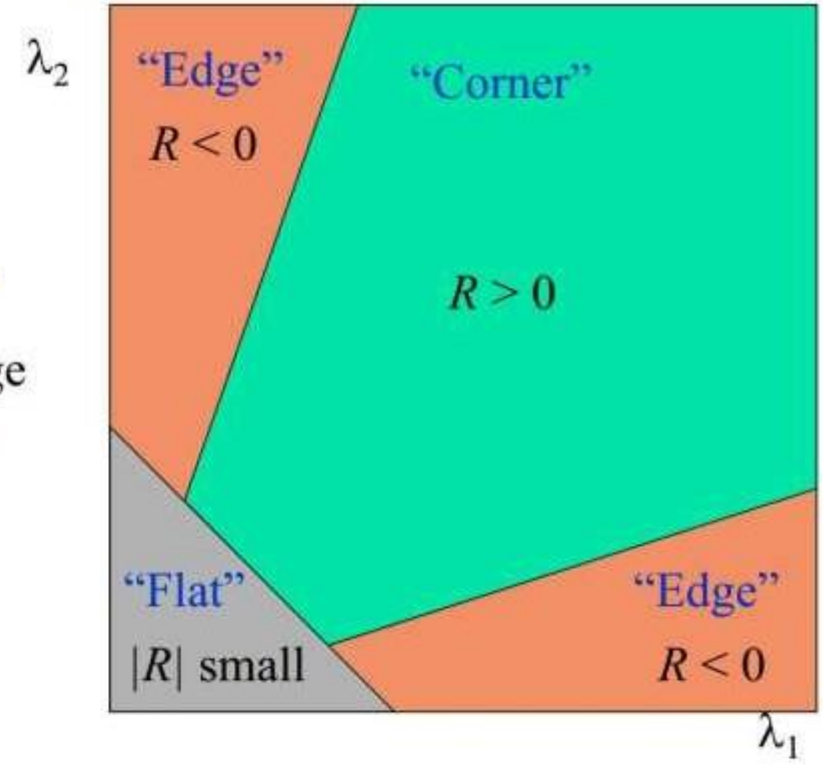
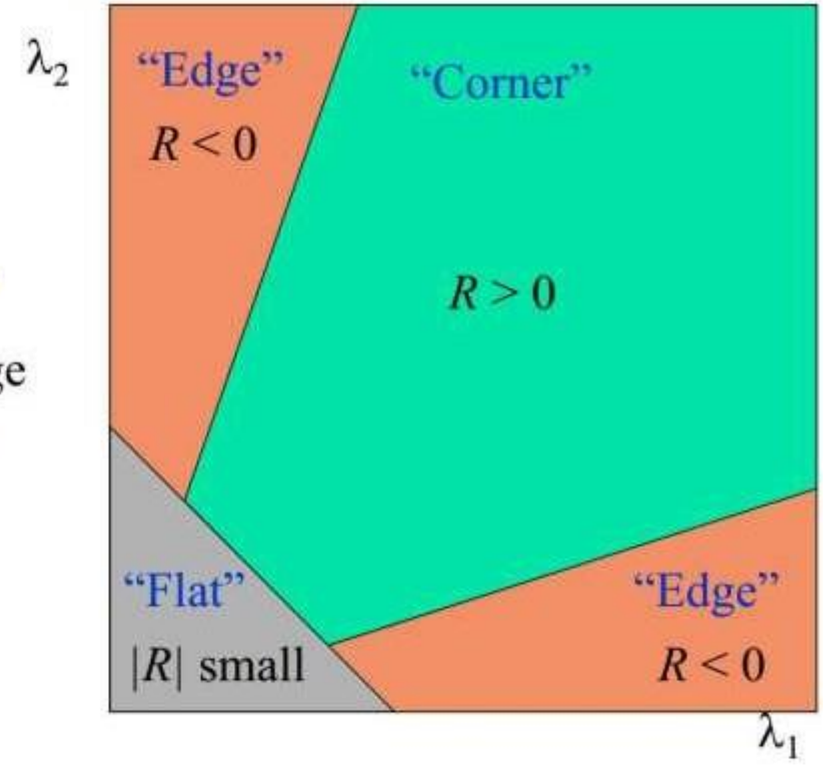
- R is negative with large magnitude for an edge

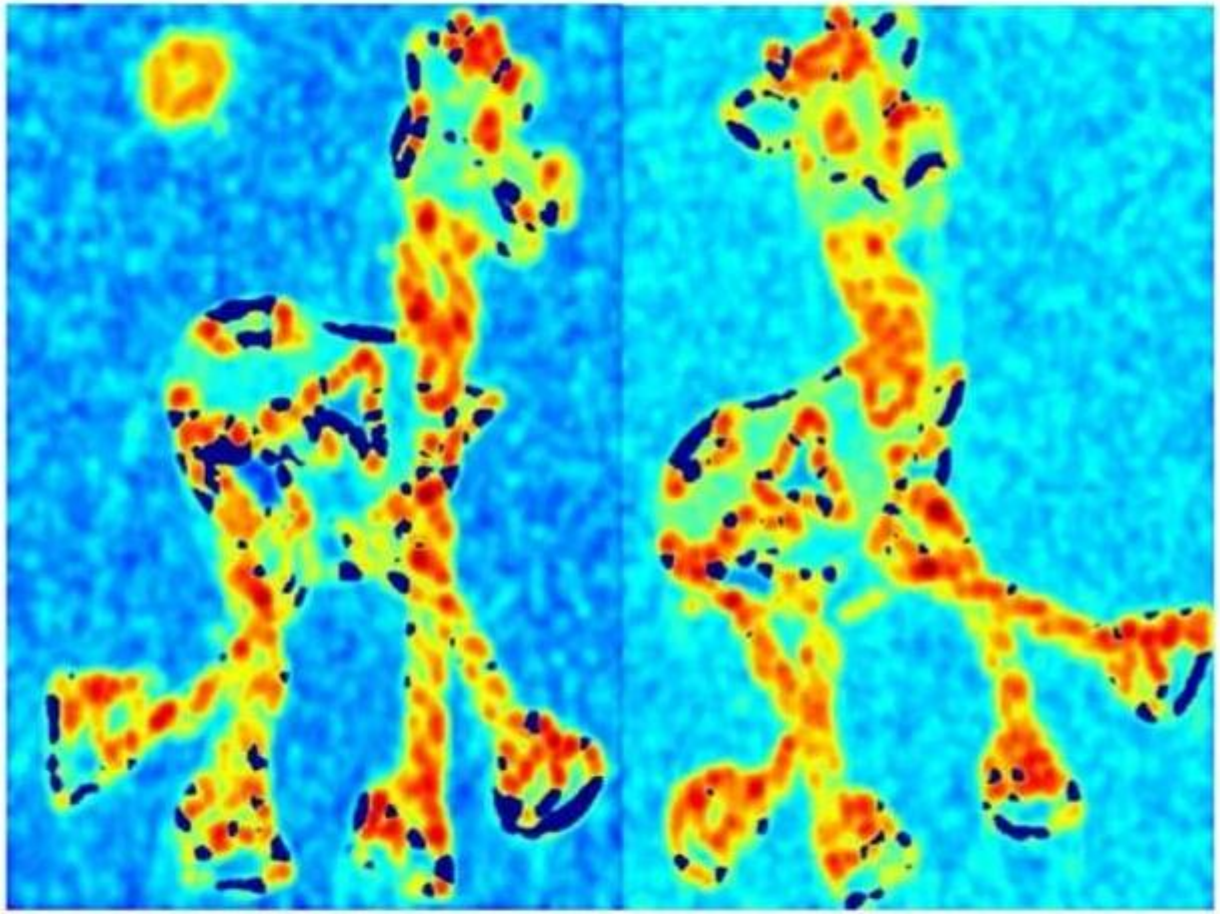


Mathematics of Harris Detector

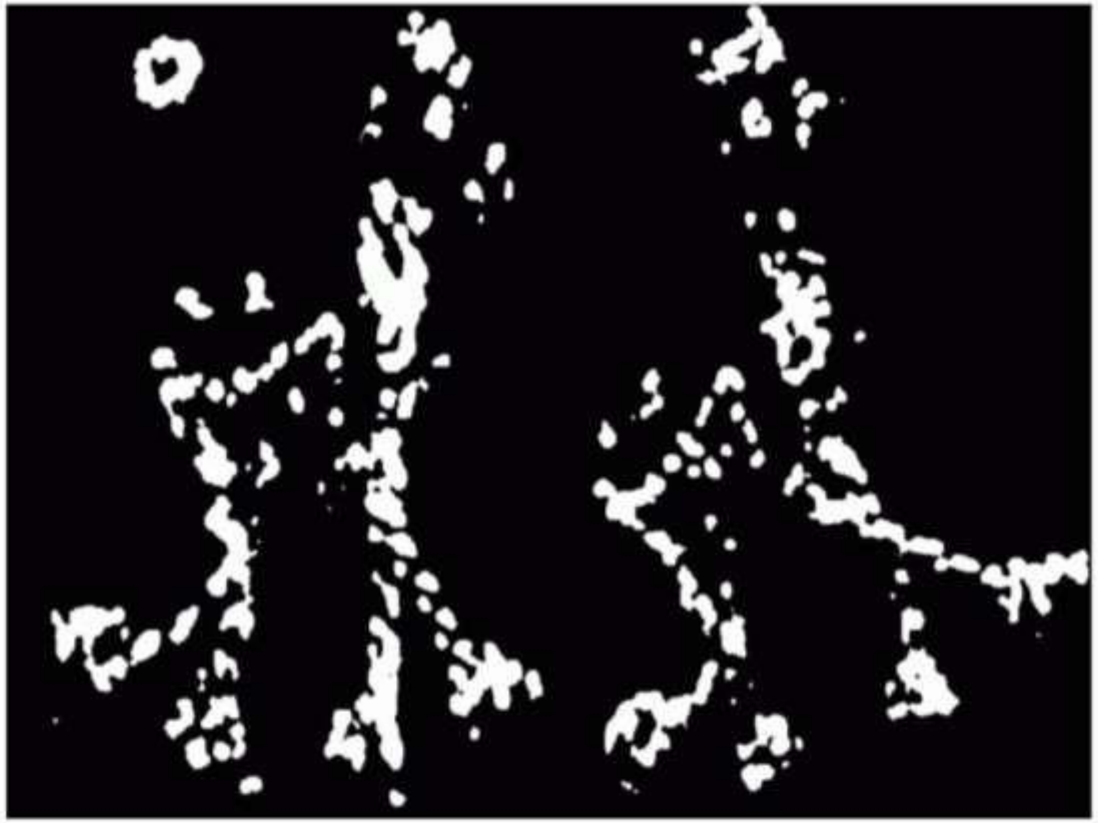
- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



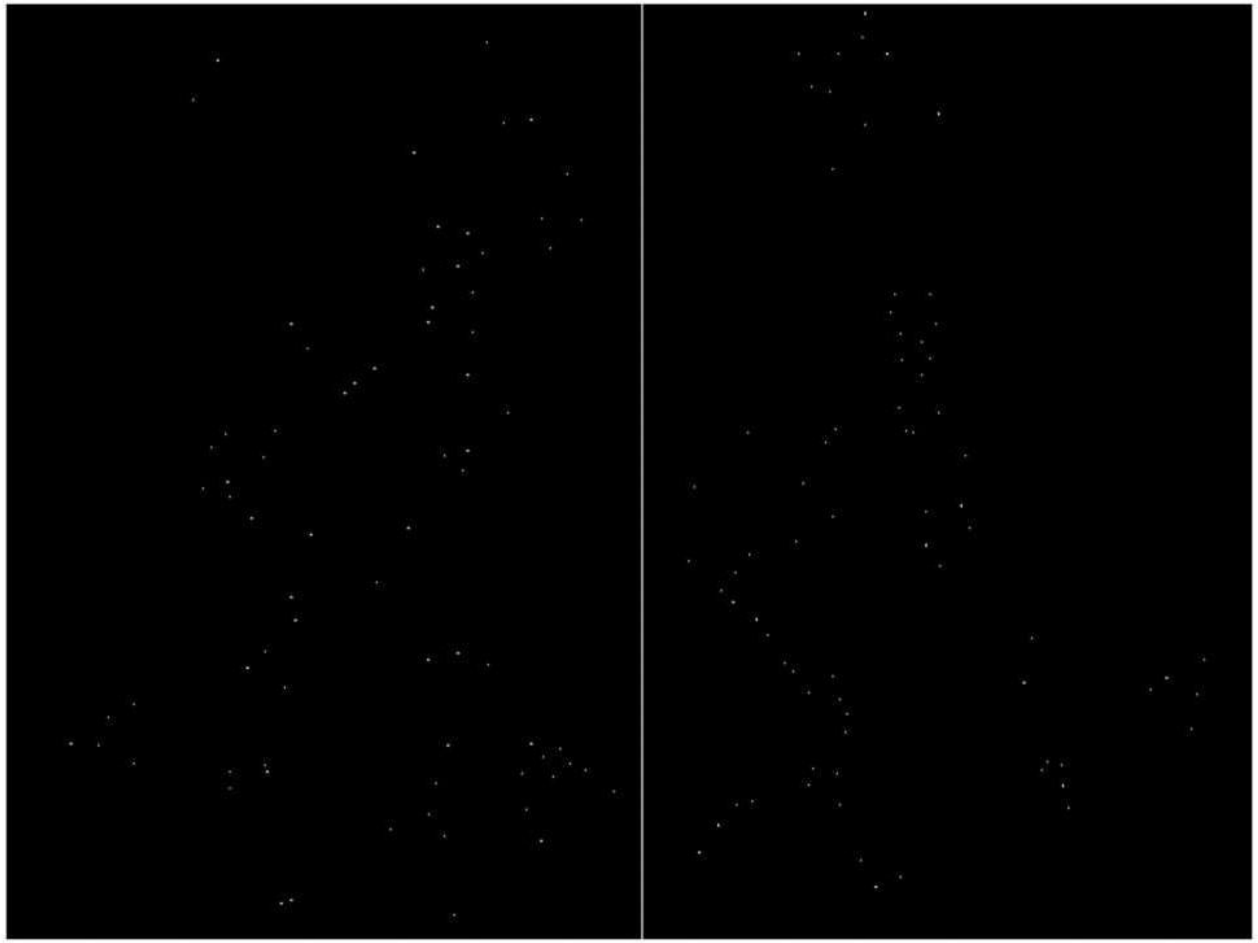
Compute corner response



Find points with large corner response: $R >$ threshold



Take only the points of local maxima of R



If pixel value is greater than its neighbors then it is a local maxima.

Other Version of Harris Detectors

$$R = \lambda_1 - \alpha\lambda_2$$

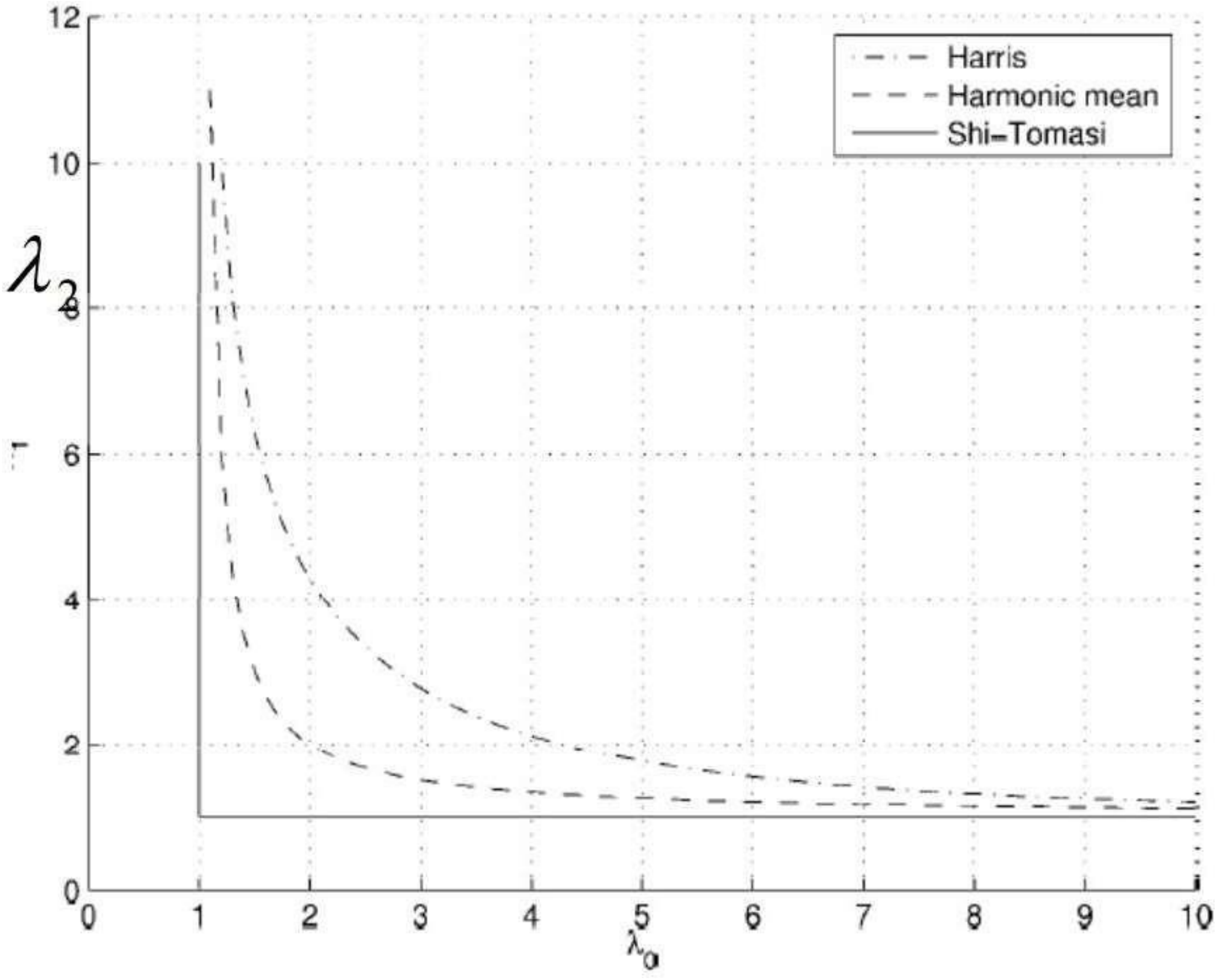
Triggs

$$R = \frac{\det(M)}{trace(M)_1} = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

Szeliski (Harmonic mean)

$$R = \lambda_1$$

Shi-Tomasi



Harris
Harmonic mean
Shi–Tomasi
λ_2
λ_0
0
2
4
6
8
10
12
0
1
2
3
4
5
6
7
8
9
10