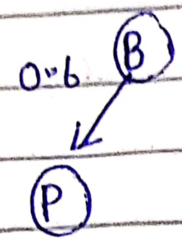
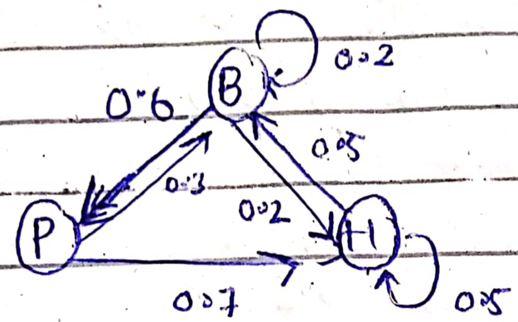
HMM

Markov Chain :

There is a way to predict tomorrow based on if you know today



// Today they are serving burger and there is 0.6 probability that ~~yester~~ tommorrow will be a pizza day



Future state depends on current state

$$P(X_{n+1}=x \mid X_1=x_1, X_2=x_2 \ldots X_n=x_n)$$

$$P(X_{n+1}=x \mid X_n=x_n)$$

(2) Sum of Arrows going out of a state Should be 1

How can we represent the markov chain in better way.

It is a directed graph we can built an adjancey matrix

$$\begin{array}{c|ccc} & B & P & H \\ \hline B & 0.2 & 0.6 & 0.2 \\ P & 0.3 & 0 & 0.7 \\ H & 0.5 & 0 & 0 \end{array} = A$$

Note: Our goal is to find pro of each state

Take row $\pi_0 = [0 \quad 1 \quad 0]$

// means we are own Pizza day and other are 0

Multiply π with A

$$= [0 \ 1 \ 0] \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0 \end{bmatrix}$$

$$= \pi_1 = [0.3 \quad 0 \quad 0.7]$$

Repeat

$$\pi_1 A = [0.3 \quad 0 \quad 0.7] \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0 \end{bmatrix}$$

$$\pi_2 = [0.41 \quad 0.18 \quad 0.41]$$

$$\pi_2 A = [0.34 \quad 0.25 \quad 0.41]$$

After some time output π will become equal to input π stationary state

$$\boxed{\pi A = \pi}$$

$$AV = \lambda V$$

Suppose π is an eigen vector with value 1

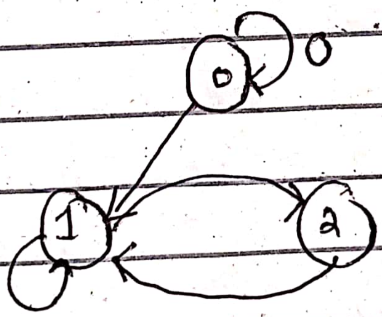
$$\pi[1] + \pi[2] + \pi[3] = 1$$

if $\pi = [\,0.35211 \quad 0.21127 \quad 0.43662\,]$

$$\pi A = \pi$$

we get

$$\pi A = [\,0.35211 \quad 0.211266 \quad 0.436621\,]$$

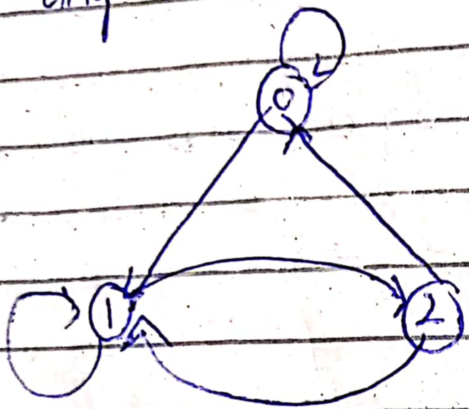


0 is Transient : 0 py dobara wapis anay ki Prob Less than 1 ha

1 is Recurrent : We are bound to revisit 1.
2 is Recurrent.

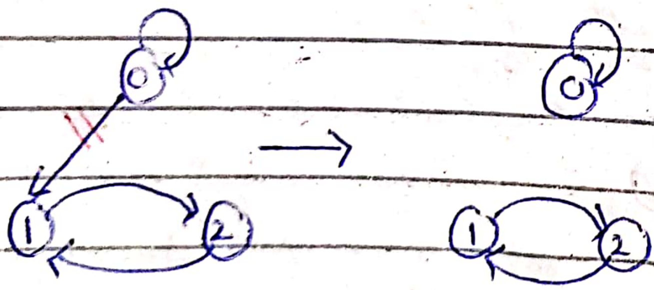
Reducable: When some states are unreachable from other.

Irreducable: Can go to any state from any other



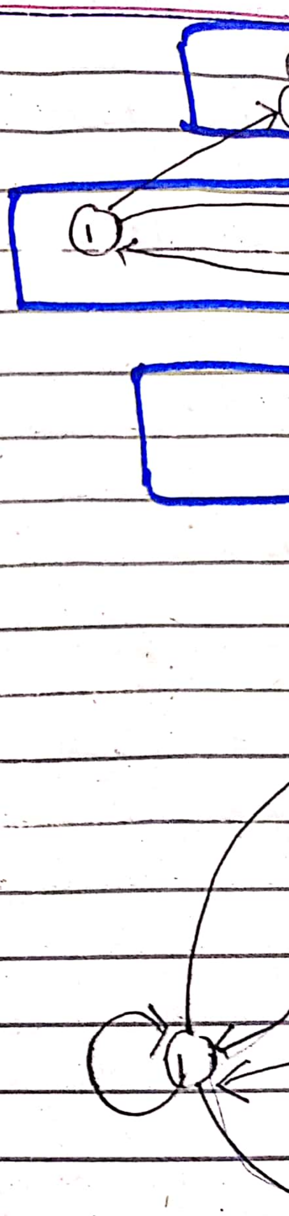
Why do we call it reducible

Cz we can reduce / divide this chain into smaller irreducable



both are irreducable

On the bases of two way communi dive it into classe.



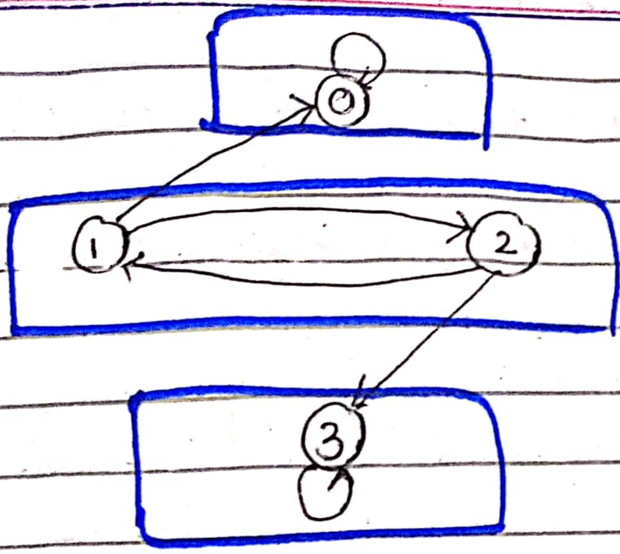
Prob to
in n

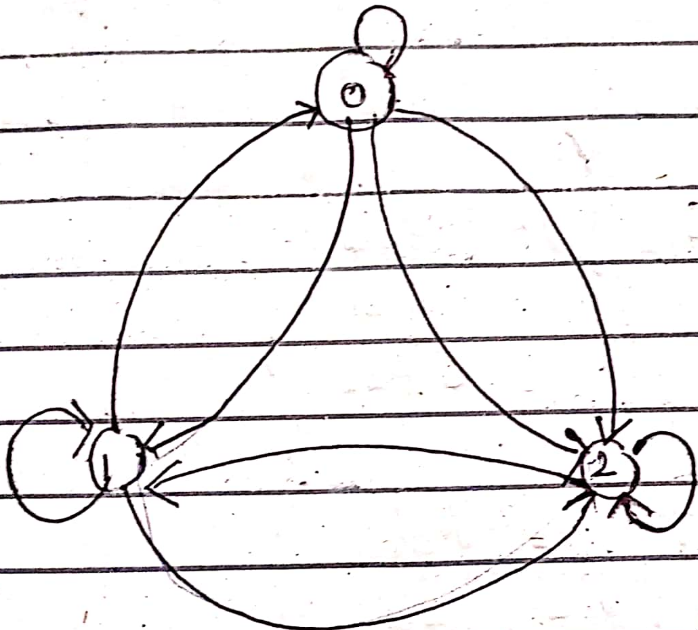
P_{ij}

$P_{02}($

$P02(2)$

0.
0
0.





$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & .5 & .2 & .3 \\
1 & .6 & 0.2 & .2 \\
2 & .1 & .8 & .1
\end{array}
$$

Prob to go to state j from i in n steps

$P_{ij}(n)$

$P_{02}(1) = 0.3$

$P_{02}(2) = ?$

$0 \to 0 \to 2$

$0 \to 1 \to 2$

$0 \to 2 \to 2$

$$= (A_{01} \times A_{12}) + (A_{00} \times A_{02}) + (A_{02} \times A_{22})$$

$$= (0.2 \times 0.2) + (0.5 \times 0.3) + (0.3 \times 0.1)$$

$$= 0.22$$

// we are not interested in answere but in the eq.

Rewrite the eq

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} \end{bmatrix} \times \begin{bmatrix} A_{02} \\ A_{12} \\ A_{22} \end{bmatrix}$$

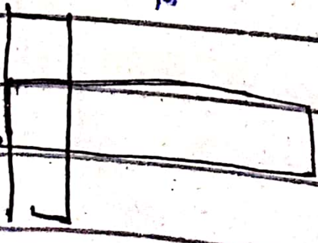
which is

$$\begin{bmatrix} \boxed{0.5 \quad 0.2} & 0.3 \\ 0.6 \quad 0.2 & 0.2 \\ 0.1 \quad 0.8 & 0.1 \end{bmatrix}$$

Try $P_{10}(2)$

$$\cancel{P_{10}} \; (A_{11} \times A_{10}) + (A_{12} \times A_{20}) + (A_{10} \times A_{00})$$

$$\begin{bmatrix} A_{10} & A_{11} & A_{12} \end{bmatrix} \times \begin{bmatrix} A_{00} \\ A_{10} \\ A_{20} \end{bmatrix}$$



$$A^2 = \begin{bmatrix} 0.40 & 0.38 \\ 0.44 & 0.32 \\ 0.54 & 0.26 \end{bmatrix}$$

$$P_{ij}(2) = (A)^2_{ij}$$

$$P_{ij}(n) = (A)^n_{ij}$$

HMM

- States are hidden
- but we have some

HMM = Hiden MC –

Obs var depend

$$A^2 = \begin{bmatrix} 0.40 & 0.38 & 0.22 \\ 0.44 & 0.32 & 0.24 \\ 0.54 & 0.26 & 0.20 \end{bmatrix}$$

yh brabar ayye

$$\cancel{P_{ij}(2) = (A_{ij})^2}$$

$$P_{ij}(2) = (A)^2_{ij}$$

$$P_{ij}(n) = (A)^n_{ij}$$

HMM

- States are hidden
- but we have some observed vari

HMM = Hiden MC + Obs var

Obs var depend on todays weather

$$
\begin{array}{c|ccc|}
 & R & C & S \\
\hline
R & .5 & .3 & .2 \\
C & .4 & .2 & .4 \\
S & 0 & .3 & .7 \\
\end{array}
\qquad
\begin{array}{|cc|}
S & H \\
\hline
0.9 & 0.1 \\
0.6 & 0.4 \\
0.2 & 0.8 \\
\end{array}
$$

$$
\begin{tikzcd}
S \arrow[r] \arrow[d] & C \arrow[r] \arrow[d] & S \arrow[d] \\
H & H & S
\end{tikzcd}
$$

$P(S) \times P(H|S)$

? $\quad$ 0.8

$P(C|S) \times P(H|C)$

0.3 $\quad$ 0.4

$P(S|C) \times P(S|S)$

0.4 $\quad$ 0.2

$$
\begin{array}{cccc}
 & R & C & S \\
\pi & [\,0.218, & 0.273 & 0.50
\end{array}
$$

$0.509 \times 0.8 \times 0.3 \times 0.4 \times 0.4 \times 0.2$