LAGRANGE INTERPOLATION - UNEQUAL INTERVAL

NEWTON DIVIDED DIFFERENCE

Q:- Find value of y when $x=10$ by Lagrange interpolation formula

x	5	6	9	11
y	12	13	14	16

$$f(x) = \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \times (12) + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)}(13) + \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14)$$

$$+ \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) +$$

$$\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13) +$$

$$\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}(14) +$$

$$\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}(16)$$

$$f(10) = \cancel{\frac{(4)(1)(-1)(12)}{(-1)(-4)(-6)}}$$

$$f(10) = 4 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = 14.66$$

Q:- Find Value of y when $x=10$ by Newton Divided Difference formula.

Data:

x	5	6	9	11
y	12	13	14	16

Sol:

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	5	12	$\frac{13-12}{6-5}=1$	$\frac{(1/3)-1}{9-5}=-1/6$	$\frac{(2/15)-(-1/6)}{11-5}$ $=1/20$
x_1	6	13	$\frac{14-13}{9-6}=1/3$	$\frac{1-(1/3)}{11-6}=2/15$	
x_2	9	14	$\frac{16-14}{11-9}=1$		
x_3	11	16			

$$f(x)=f(x_0)+(x-x_0)\Delta f(x_0)+$$

$$(x-x_0)(x-x_1)\Delta^2 f(x_0)+$$

$$(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)+$$

$$(x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x_0)+\cdots$$

$$f(x)=12+(x-5)(1)+(x-5)(x-6)(-1/6)+$$

$$(x-5)(x-6)(x-9)(1/20)$$

$$f(10)=12+(10-5)(1)+(10-5)(10-6)(-1/6)+$$
$$(10-5)(10-6)(10-9)(1/20)$$

$$f(10)=14.66$$

— x — x —

Q:- Use Lagrange's formula to fit polynomial to the following data. Hence find $y(-2)$, $y(1)$, $y(4)$

x	-1	0	2	3
y	-8	3	1	2

Sol:

$$\cancel{f(x) = \frac{(x+0)(x-2)(x-3)}{(3+0)(x+2)(2-3)}}$$

$$f(x) = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8) +$$

$$\frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}(3) +$$

$$\frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)}(1) +$$

$$\frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}(2)$$

$$f(x) = (x-2)(x-3)\left[\frac{-8x}{(-1)(-3)(-4)} + \frac{3(x+1)}{(1)(-2)(-3)}\right] +$$

$$x(x+1)\left[\frac{x-3}{(3)(2)(-1)} + \frac{2(x-2)}{(4)(3)(1)}\right]$$

$$f(x) = (x^2-5x+6)\left[\frac{2x}{3} + \frac{x+1}{2}\right] +$$

$$+(x^2+x)\left[\frac{-(x-3)}{6}+\frac{x-2}{6}\right]$$

$$f(x)=(x^2-5x+6)\left[\frac{4x+3x+3}{6}\right]+$$

$$(x^2+x)\left[\frac{-\cancel{x}+3+\cancel{x}-2}{6}\right]$$

$$f(x)=(x^2-5x+6)\frac{(7x+3)}{6}+\frac{x^2+x}{6}$$

$$f(x)=\frac{7x^3-31x^2+28x+18}{6}$$

$$f(-2)=\frac{7(-2)^3-31(-2)^2+28(-2)+18}{6}=-36.33$$

$$f(1)=\frac{7(1)^3-31(1)^2+28(1)+18}{6}=3.66$$

$$f(4)=\frac{7(4)^3-31(4)^3+28(4)+18}{6}=13.666$$

—x—x—

Inverse Lagrange's formula to find value of x

Q:- Apply Lagrange's formula inversely to find value of x when $y = 19$ given following

x	0	1	2
y	0	1	20

$\Rightarrow$

y	0	1	20
x	0	1	2

Sol:-

$$x = \frac{(y-1)(y-20)}{(0-1)(0-20)}(0) + \frac{(y-0)(y-20)}{(1-0)(1-20)}(1)$$

$$+ \frac{(y-0)(y-1)}{(20-0)(20-1)}(2)$$

x at $y = 19 = ?$

$$x_{at\ y=19} = 0 + \frac{(19-0)(19-20)}{(1)(-19)} + \frac{(19)(19-1)}{(20)(19)}(2)$$

$$x_{at\ y=19} = 2.8$$

—— x —— x ——