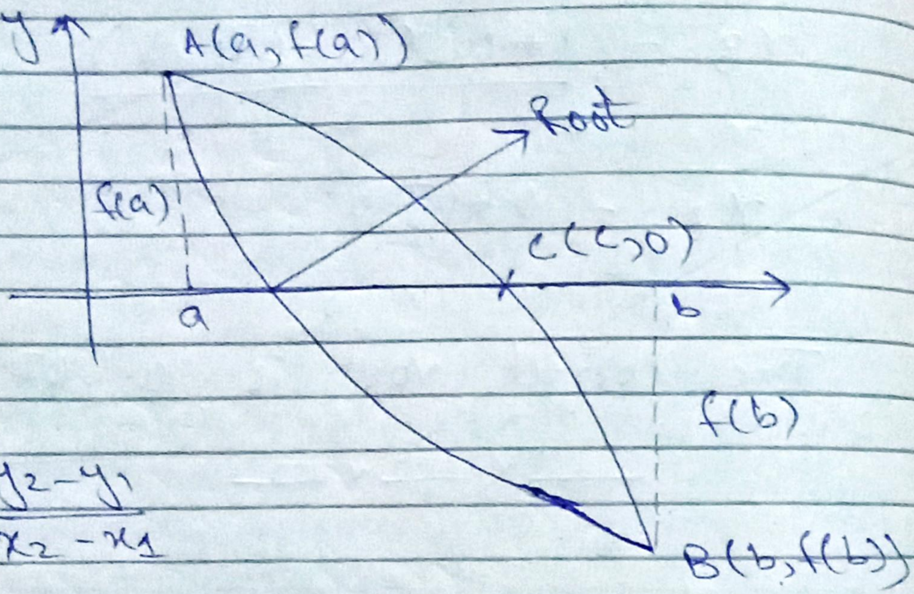
REGULA FALSI METHOD

FALSE POSITION METHOD

Slope of AB = Slope of AC



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{f(b) - f(a)}{b - a} = \frac{0 - f(a)}{c - a}$$

$$c - a = \frac{-f(a)(b - a)}{f(b) - f(a)}$$

$$c = a - \frac{f(a)(b - a)}{f(b) - f(a)}$$

$$c = \frac{af(b) - \cancel{af(a)} - bf(a) + \cancel{af(a)}}{f(b) - f(a)}$$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Working Procedure

1. find the interval $[a, b]$

$$f(a)f(b) < 0$$

2. find $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

3. $f(a)f(c) < 0$ roots lies in $[a, c]$

$f(b)f(c) < 0$ roots lies in (c, b)

4. Repeat step 1 & 2

Q:- find a real root of $x^3 - 2x - 5 = 0$ using the method false position upto four iterations.

Sol:-

$$f(x) = x^3 - 2x - 5$$
$$f(2) = (2)^3 - 2(2) - 5$$
$$f(2) = 8 - 4 - 5 = -1$$

(3)

$f(3) = (3)^3 - 2(3) - 5$

$f(3) = 27 - 6 - 5$

$f(3) = 16$

<u>I iteration</u>

$a = 2 \qquad b = 3$

$f(a) = -1 \qquad f(b) = 16$

$f(a)\,f(b) < 0$

$(-1)(16) < 0$

$-16 < 0$ True

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$c = \frac{(2)(16) - (3)(-1)}{16 - (-1)}$$

$$c = \frac{32 + 3}{16 + 1} = \frac{35}{17}$$

$\cancel{\frac{35}{17}}$ $\boxed{c = 2.0588}$

$f(c) = f(2.0588)$

$f(c) = (2.0588)^3 - 2(2.0588) - 5$

$f(c) = -0.3908 < 0$

Second Iteration

$a = c$ $\quad\quad$ $b = 3$

$f(a) = f(c)$ $\quad\quad$ $f(b) = 16$

$a = 2.0588$

$f(a) = -0.3908$

$f(a)f(b) < 0$

$(-0.3908)(16) < 0$

$-6.252 < 0$ True

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$c = \frac{(2.0588)\times(16) - (3)(-0.3908)}{16 - (-0.3908)}$$

$c = 2.0812$

$f(c) = f(2.0812)$

$f(c) = (2.0812)^3 - 2(2.0812) - 5$

$f(c) = -0.1479 < 0$

Thrid Iteration

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$a = 2.0812$ $\quad$ $b = 3$
$f(a) = -0.1479$ $\quad$ $f(b) = 16$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$c = \frac{16(2.0812) + 3(0.1479)}{16 - (-0.1479)}$$

$c = 2.0896$

$f(c) = f(2.0896)$

$= (2.0896)^3 - 2(2.0896) + 5$

$f(c) = -0.0551$

Fourth Iteration

$a = 2.0896$ $\quad$ $b = 3$
$f(a) = -0.0551$ $\quad$ $f(b) = 16$

$$c = \frac{16(2.0896) - (3)(-0.0551)}{16 - (-0.0551)}$$

$c = 2.0927$

Hence the required root is

≈ 2.0927

— x — x —

Q: Using False position method find a real root of the equation

$$x\log_{10} x - 1.2 = 0$$

in 3 steps.

Sol:-

$f(x) = x\log_{10} x - 1.2$

$f(1) = (1)\log_{10}(1) - 1.2 = -1.2 < 0$

$f(2) = (2)\log_{10}(2) - 1.2 = -0.59 < 0$

$f(3) = (3)\log_{10}(3) - 1.2 = 0.231 > 0$

$f(2.5) = (2.5)\log_{10}(2.5) - 1.2 = -0.2051 < 0$

$f(2.6) = 2.6\log_{10}(2.6) - 1.2 = -0.121 < 0$

$f(2.7) = 2.7\log_{10}(2.7) - 1.2 = -0.035 < 0$

$f(2.8) = 2.8\log_{10}(2.8) - 1.2 = 0.05207 > 0$

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First Iteration

$a = 2.7$ $\quad$ $b = 2.8$

$f(a) = -0.0353$ $\quad$ $f(b) = 0.0520$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$c = \frac{(2.7)(0.052) - (2.8)(-0.0353)}{0.0520 - (-0.0353)}$$

$c = 2.7404$

$f(c) = f(2.7404) = (2.7404)\log_{10}(2.7404) - 1.2$

$f(2.7404) = -0.00021 < 0$ True

Second Iteration

$a = c = 2.7404$ $\quad$ $b = 2.8$

$f(a) = f(c) = -0.00021$ $\quad$ $f(b) = 0.0520$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$c = \frac{(2.7404)(0.0520) - (2.8)(-0.00021)}{0.0520 - (-0.00021)}$$

$c = 2.7406$

$f(c) = f(2.7406) = (2.7406)\log_{10}(2.7406)$
$- 1.2$

$f(c) = f(2.7406) = -0.00004 < 0$

Thrid Iteration

$a = c = 2.7406$ $\quad$ $b = 2.8$

$f(a) = f(c) = -0.00004$ $\quad$ $f(b) = 0.0520$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$c = \frac{(2.7406)(0.052) - (2.8)(-0.00004)}{(0.0520) - (-0.00004)}$$

$\boxed{c = 2.7406}$

—— x —— x ——