CENTRAL DIFFERENCE INTERPOLATION

Q:- Find u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$

$u_{12} = 35$, $u_{16} = 40$

Sol:-

$u_9 = ?$

x	f(x)	Δf(x)	Δ²f(x)	Δ³f(x)	Δ⁴f(x)
0	$14\ y_{-2}$	$10\ \Delta y_{-2}$	$2\ \Delta^2 y_{-2}$		
4	$24\ y_{-1}$	$8\ \Delta y_{-1}$	$-5\ \Delta^2 y_{-1}$	$-3\ \Delta^3 y_{-2}$	$10\ \Delta^4 y_{-2}$
8	$32\ y_0$	$3\ \Delta y_0$	$2\ \Delta^2 y_0$	$7\ \Delta^3 y_{-1}$	
12	$35\ y_1$	$5\ \Delta y_{+1}$			
16	$40\ y_2$				

Gauss Forward:-

$$f(a+hu) = y_0 + \frac{u(\Delta y_0)}{1!} + \frac{u(u-1)(\Delta^2 y_{-1})}{2!} +$$

$$\frac{(u+1)(u)(u-1)(\Delta^3 y_{-1})}{3!} +$$

$$\frac{(u+1)(u)(u-1)(u-2)\ \Delta^4 y_{-2}}{4!}$$

Gauss Backward:-

(2)

$$f(a+hu) = Y_0 + \frac{u(\Delta Y_{-1})}{1!} + \frac{u(u+1)(\Delta^2 Y_{-1})}{2!}$$

$$+ \frac{(u-1)(u)(u+1)(\Delta^3 Y_{-2})}{3!}$$

$$+ \frac{(u-1)(u)(u+1)(u+2)(\Delta^4 Y_{-2})}{4!}$$

Stirling Formula :- $\frac{\text{gauss Forward} + \text{guass backward}}{2}$

stirling formula :-

$$f(a+hu) = Y_0 + \frac{u}{1!}\left(\frac{\Delta Y_0 + \Delta Y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 Y_{-1}$$

$$+ \frac{u(u^2-1)}{3!}\left(\frac{\Delta^3 Y_{-1} + \Delta^3 Y_{-2}}{2}\right)$$

$$+ \frac{u^2(u^2-1)}{4!}\left(\Delta^4 Y_{-2}\right)$$

Gauss Forward

$f(9) = ?$

$a + hu = 9, \quad a = 8, \quad h = 4$

$8 + 4u = 9$

$4u = 9 - 8$

$\boxed{u = 0.25}$

$$f(9) = 32 + \frac{(0.25)(3)}{1!} + \frac{(0.25-1)(-5)}{2}$$

$$+ \frac{(0.25+1)(0.25)(0.25-1)(7)}{6}$$

$$+ \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)(10)}{24}$$

$$f(9) = 33.1162$$

Gauss Backward :-

$f(9) = ?$ $\quad a=8, h=4, u=0.25$

$$f(9) = 32 + \frac{(0.25)(8)}{1} + \frac{(0.25+1)(0.25)(-5)}{2}$$

$$+ \frac{(0.25-1)(0.25)(0.25+1)(-3)}{6}$$

$$+ \frac{(0.25+1)(0.25)(0.25+1)(0.25+2)(10)}{24}$$

$$f(9) = 33.1162$$

Sterling Formula :-

$$f(9) = 32 + \frac{0.25}{1}\left(\frac{8+3}{2}\right) + \frac{(0.25)^2(-5)}{2}$$

(4)

$$+\frac{(0.25)\left((0.25)^2-1\right)}{6}\left(\frac{-3+7}{2}\right)$$

$$+\frac{(0.25)^2\left((0.25)^2-1\right)(10)}{24}$$

$f(9) = 33.1162.$

—— x —— x ——

BESSEL'S FORMULA

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	14 Y_{-2}	10 ΔY_{-2}			
4	24 Y_{-1}	8 ΔY_{-1}	$-2\ \Delta^2 Y_{-2}$	$-3\ \Delta^3 Y_{-2}$	
8	32 Y_0	3 ΔY_0	$-5\ \Delta^2 Y_{-1}$	$+7\ \Delta^3 Y_{-1}$	$10\ \Delta^4 Y_{-2}$
12	35 Y_1	5 ΔY_1	$2\ \Delta^2 Y_0$	-1	$\Delta^4 Y_{-1}$
16	40 Y_2				

BESSEL'S FORMULA

$$f(a+hu) = \frac{1}{2}(Y_0+Y_1) + \frac{(u-1/2)\Delta Y_0}{1!}$$

$$+\frac{u(u-1)}{2!}\left(\frac{\Delta^2 Y_0+\Delta^2 Y_{-1}}{2}\right)$$

$$+\frac{(u-\frac{1}{2})(u)(u-1)\Delta^3 Y_{-1}}{3!}$$

$$+\frac{(u+1)u(u-1)(u-2)}{4!}\left(\frac{\Delta^4 Y_{-1}+\Delta^4 Y_{-2}}{2}\right)$$

$f(9) = ?$

$a + hu = 9$

$8 + 4u = 9$

$4u = 9 - 8 \qquad u = 0.25$

$$f(9) = \frac{1}{2}\left(\frac{32+35}{3}\right) + \frac{(0.25)(-1/2)(3)}{1!}$$

$$+ \frac{(0.25)(0.25-1)}{2!}\left(\frac{-5+2}{2}\right)$$

$$+ \frac{(0.25)(-\frac{1}{2})(0.25)(0.25-1)(7)}{6}$$

$$+ \left\{\frac{(0.25)(+1)(0.25)(0.25-1)(0.25-2)}{24}\right\}\left[\frac{10+0}{2}\right]$$

$$\boxed{f(9) = 33.1162}$$

———X———X———