FIXED POINT ITERATION METHOD

→ Solution of Algebraic & Transcedental Equation by INTEGRATION METHOD

1) Fixed Point Iteration Method or Integration Method :-

Suppose we have equation $f(x) = 0$
The equation can be expressed as
$x = \phi(x)$

$$|\phi'(x)| < 1$$
at $x = x_0$

Then iterative method applied. The successive approximation is given by

$$x_n = \phi(x_{n-1})$$

$$x_1 = \phi(x_0)$$

$$x_2 = \phi(x_1)$$

Q:- Find a real root of equation

$f(x) = x^3 + x^2 - 1 = 0$

②

Solution:

$f(0) = (0)^3 + (0)^2 - 1 = -1$
$f(1) = (1)^3 + (1)^2 - 1 = 1$

$$x_0 = \frac{0+1}{2} = 0.5$$

(I) $x^3 + x^2 - 1 = 0$
$x^3 = 1 - x^2$
$x = (1-x^2)^{1/3} = \phi(x)$

$$\phi'(x) = \frac{1}{3} \frac{(-2x)}{(1-x^2)^{2/3}}$$

$$\phi'(x)_{\text{at } x_0 = 0.5} = \left| \frac{1}{3} \frac{(-2(0.5))}{(1-(0.5)^2)^{2/3}} \right|$$

$$\phi'(x)_{\text{at } x_0 = 0.5} = 0.4038 < 1$$

(II) $x^3 + x^2 - 1 = 0$
$x^2 = 1 - x^3$
$x = (1-x^3)^{1/2} = \phi(x)$

$$\phi'(x) = \frac{1}{2} \frac{-3x^2}{(1-x^3)^{1/2}}$$

$$\phi'(x)_{\text{at } x_0 = 0.5} = \left| \frac{1 \times (-3(0.5)^2)}{2 \times (1-(0.5)^3)^{1/2}} \right|$$

(3)

$\phi'(x)$ at $x_0 = 0.5$ $= 0.4008 < 1$

(III) $x^3 + x^2 - 1 = 0$

$x^2(x+1) - 1 = 0$

$x^2(x+1) = 1$

$x^2 = \frac{1}{x+1}$

$\cancel{x = \frac{1}{\sqrt{x+1}} = \phi(x)}$

$\cancel{\phi'(x) = -\frac{1}{2}(x+1)^{1/2}}$

$x = \frac{\sqrt{1}}{\sqrt{1+x}} = 1(1+x)^{-1/2} = \phi(x)$

$\phi'(x) = \left| -\frac{1}{2}(x+1)^{-\frac{3}{2}}(1) \right|$

$\phi'(x)$ at $x_0 = 0.5$ $= \left| -\frac{1}{2}(0.5+1)^{-\frac{3}{2}} \right|$

$\phi'(x)$ at $x_0 = 0.5 = 0.2721 < 1$

④

We will go with case III because

$\left| \phi'(x)_{\text{at } x_0 = 0.5} \right|$ giving the

smallest values amoung these

three cases.

So,

$$x_n = \frac{1}{\sqrt{1 + x_{n-1}}}$$

Put $n = 1$

$$x_1 = \frac{1}{\sqrt{1 + x_0}}$$

$$x_1 = \frac{1}{\sqrt{1 + 0.5}} = 0.81649$$

$$x_2 = \frac{1}{\sqrt{1 + x_1}} = \frac{1}{\sqrt{1 + 0.81649}}$$

$x_2 = 0.74196$

$$x_3 = \frac{1}{\sqrt{1 + x_2}} = \frac{1}{\sqrt{1 + 0.74196}} = 0.7576$$

$$x_4 = \frac{1}{\sqrt{1+x_3}} = \frac{1}{\sqrt{1+0.75767}} = 0.75427$$

$$x_5 = \frac{1}{\sqrt{1+x_4}} = \frac{1}{\sqrt{1+0.75427}} = 0.75500$$

$$x_6 = \frac{1}{\sqrt{1+x_5}} = \frac{1}{\sqrt{1+0.75500}} = 0.75485$$

$$x_7 = \frac{1}{\sqrt{1+x_6}} = \frac{1}{\sqrt{1+0.75485}} = 0.75488$$

The root of equation is 0.7548.

—— x —— x ——

Q2: Find the roots of $\cos x = 3x - 1$, correct to four decimal places by using iterative method

Solution:-

$[\ ,\ ]$

$$f(x) = \cos(x) - 3x + 1$$
$$f(0) = \cos(0) - 3(0) + 1 = 2$$
$$f(\tfrac{\pi}{2}) = \cos(\tfrac{\pi}{2}) - 3(\tfrac{\pi}{2}) + 1 = -2.7127$$

Hum $x_0 = ?$ wo value lien gay jo zero k zyada qareeb ho.

(6)

$x_0 = 0$

$x = \phi(x)$

$\cos x = 3x - 1$

$3x = \cos x + 1$

$x = \dfrac{\cos x + 1}{3}$

$\phi(x) = \dfrac{1 + \cos x}{3}$

$\phi'(x) = 0 + \left(-\dfrac{\sin x}{3}\right)$

$|\phi'(x)| = \left|+\dfrac{\sin x}{3}\right|$

$|\phi'(x)|_{\text{at } x_0 = 0} = \left|\dfrac{\sin(0)}{3}\right| < 1$

$= 0 < 1$

$|\phi'(x)|_{\text{at } x_0 = 0} < 1$

$$x_n = \frac{1+\cos x_{n-1}}{3}$$

put $n=1$

$$x_1 = \frac{1+\cos x_0}{3}$$

$$x_1 = \frac{1+\cos(0)}{3} = \frac{2}{3} = 0.66667$$

put $n=2$

$$x_2 = \frac{1+\cos(x_1)}{3} = \frac{1+\cos(2/3)}{3}$$

$$x_2 = 0.595296$$

put $n=3$

$$x_3 = \frac{1+\cos(x_2)}{3}$$

$$= \frac{1+\cos(0.595296)}{3}$$

$$= 0.609328$$

put $n=4$

$$x_4 = \frac{1+\cos(x_3)}{3}$$

$$x_4 = \frac{1+\cos(0.609328)}{3}$$

$$x_4 = 0.606678$$

put $n=5$

$$x_5 = \frac{1+\cos(x_4)}{3}$$

$$x_5 = \frac{1+\cos(0.606678)}{3}$$

$$x_5 = 0.607182$$

put $n=6$

$$x_6 = \frac{1+\cos(x_5)}{3}$$

$$x_6 = \frac{1+\cos(0.607182)}{3}$$

$$x_6 = 0.607086$$

put $n=7$

$$x_7 = \frac{1+\cos(x_6)}{3}$$

$$x_7 = \frac{1+\cos(0.607086)}{}$$

$x_7 = 0.607105$

put $n=8$

$$x_8 = \frac{1+\cos(x_7)}{3}$$

$$x_8 = \frac{1+\cos(0.607105)}{3}$$

$x_8 = 0.607101$

The correct root is 0.6071

—x—x—