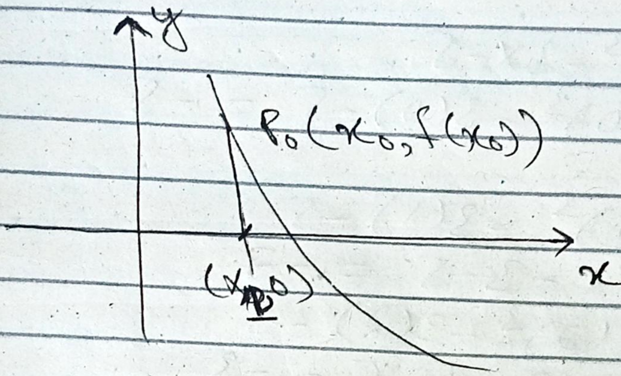
NEWTON RAPHSON METHOD

Solution of Algebraic & Transcedental Equation by

NEWTON RAPHSON METHOD



$$y - y_1 = \frac{dy}{dx}(x - x_1)$$ formula of tangent

Equation of Tangent

$$y - f(x_0) = f'(x_0)(x - x_0)$$
$$0 - f(x_0) = f'(x_0)(x_1 - x_0)$$

$$x_1 - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Formula of Newton Raphson Method

②

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x_n) \neq 0$$

Q:- Find by Newton Raphson Method a root of equation $x^3 - 3x - 5 = 0$

Sol

$f(x) = x^3 - 3x - 5$

$f(0) = (0)^3 - 3(0) - 5 = -5$

$f(1) = (1)^3 - 3(1) - 5$

$f(1) = 1 - 3 - 5 = -7$

$f(2) = (2)^3 - 3(2) - 5$

✓ $f(2) = 8 - 6 - 5 = -3$

$f(3) = (3)^3 - 3(3) - 5$

$f(3) = (3)^3 - 3(3) - 5$

✓ $f(3) = 27 - 9 - 5 = 16$

→ x_0 wo hoga jo zero k qareeb hoga $\boxed{x_0 = 2}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = x^3 - 3x - 5$

$f'(x) = 3x^2 - 3$

$$x_{n+1} = x_n - \frac{x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

put $n=0$

$$x_1 = x_0 - \frac{x_0^3 - 3x_0 - 5}{3x_0^2 - 3}$$

$$x_1 = 2 - \frac{(2)^3 - 3(2) - 5}{3(2)^2 - 3}$$

$$\boxed{x_1 = 2.3333}$$

put $n=1$

$$x_2 = x_1 - \frac{x_1^3 - 3x_n - 5}{3x_1^2 - 3}$$

$$= 2.3333 - \frac{(2.3333)^3 - 3(2.3333) - 5}{3(2.3333)^2 - 3}$$

$$\boxed{x_2 = 2.2805}$$

put $n=2$

$$x_3 = x_2 - \frac{(x_2)^3 - 3(x_2) - 5}{3(x_2)^2 - 3}$$

$$x_3 = (2.2805) - \frac{(2.2805)^3 - 3(2.2805)^2 - 5}{3(2.2805)^2 - 3}$$

$$\boxed{x_3 = 2.2790}$$

put $n = 3$

$$x_4 = x_3 - \frac{(x_3)^3 - 3(x_3)^2 - 5}{3(x_3)^2 - 3}$$

$$x_4 = 2.2790 - \frac{(2.2790)^3 - 3(2.2790)^2 - 5}{3(2.2790)^2 - 3}$$

$$\boxed{x_4 = 2.2790}$$

— x — x —

Q: Use Newton Raphson Method to find a real root of

$\cos x - xe^x = 0$

corrected to four decimal points.

Sol:-

$$f(x) = \cos x - x e^{x}$$

$$f(0) = \cos(0) - (0) e^{0}$$
$$f(0) = 1 - 0 = 1$$
$$f(1) = \cos(1) - (1) e^{1}$$
$$f(1) = -2.1779$$

Transdental equation mein +ev and -ev value ka mid lay lien.

$$x_0 = 0.5$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = \cos x - x e^{x}$$
$$f'(x) = -\sin x - e^{x} - x e^{x}$$
$$f'(x) = -\sin x - e^{x}(x+1)$$

$$x_{n+1} = x_n + \frac{\cos x_n - x_n e^{x_n}}{\sin x_n + e^{x_n}(x_n+1)}$$

put $n=0$

$$x_1 = x_0 + \frac{\cos x_0 - x_0 e^{x_0}}{\sin x_0 + e^{x_0}(x_0+1)}$$

6

$$x_1 = 0.5 + \frac{\cos(0.5) - (0.5)e^{(0.5)}}{\sin(0.5) + e^{(0.5)}(0.5+1)}$$

$$\boxed{x_1 = 0.5182}$$

put $n=1$

$$x_2 = x_1 + \frac{\cos(x_1) - x_1 e^{x_1}}{\sin(x_1) + e^{x_1}(x_1+1)}$$

$$x_2 = 0.5182 + \frac{\cos(0.5182) - (0.5182)e^{(0.5182)}}{\sin(0.5182) + e^{(0.5182)}(0.5182+1)}$$

$$\boxed{x_2 = 0.518}$$

—— x —— x ——

Q: Apply Newton Raphson Method to solve the equation

$$2(x-3) = \log_{10} x$$

Sol:-

$$\boxed{\log_{10} x = 0.4343 \log_e x}$$

e base h

$$2x - 6 - \log_{10} x = 0$$

$$f(3) = 2(3) - 6 - \log_{10}(3) = -0.47712$$

$$f(4) = 2(4) - 6 - \log_{10}(4) = 1.39794$$

→ x_0 hm 3 and 4 k average lay lay lay hm

$x_0 = \frac{3+4}{2}$

$\boxed{x_0 = 3.5}$

Put $n=1$

$f(x) = 2x - 6 - \log_{10} x$

$f(x) = 2x - 6 - 0.43431 \log_e x$

$f'(x) = 2 - \frac{0.43431}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{2x_n - 6 - 0.43431 \log_e x_n}{2 - \frac{0.43431}{x_n}}$

put $n=1$

$x_1 = x_0 - \frac{2x_0 - 6 - 0.43431 \log_e x_0}{2 - \frac{0.43431}{x_0}}$

$x_1 = 3.5 + \frac{2(3.5) - 6 - 0.43431 \log_e 3.5}{2 - \frac{0.43431}{3.5}}$

$$\boxed{x_1 = 3.25696}$$

put $n = 1$

$x_2 = ?$

put $n = 2$

$x_3 = ?$

—— x —— x ——