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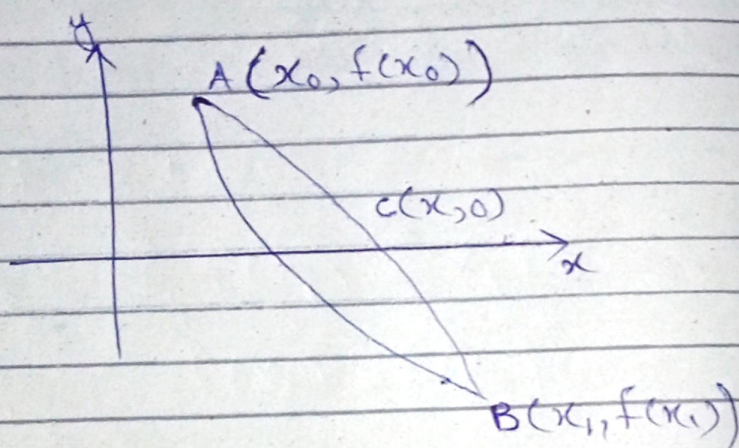
SECANT METHOD

CHORD METHOD

Solution of Algebraic & Transcendental Equation by SECANT METHOD.

→ Secant Method (Chord Method)

This method is quite similar to regula falsi method except for the condition $f(x_1)f(x_2) < 0$.



slope of AB = slope of AC

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} \neq \frac{0 - f(x_0)}{x - x_0}$$

$$x - x_0 = \frac{-f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

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$$x = x_0 + \frac{f(x_0)(x_0 - x_1)}{f(x_1) - f(x_0)}$$

$$x = \frac{x_0 f(x_1) - x_0 \cancel{f(x_0)} + \cancel{f(x_0)}(x_0) - f(x_0)(x_1)}{f(x_1) - f(x_0)}$$

$$x = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_{n+2} = \frac{x_n f(x_{n+1}) - x_{n+1} f(x_n)}{f(x_{n+1}) - f(x_n)}$$

Note:- This method fails when $f(x_n) = f(x_{n-1})$

Q:- A real root of the equation $x^3 - 5x + 1 = 0$ lies in the interval $(0, 1)$. Perform four iterations of the secant method.

Sol:-

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$f(x) = x^3 - 5x + 1$$

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$$f(0) = (0)^3 - 5(0) + 1$$

$$f(0) = 0 - 0 + 1 = 1$$

$$f(1) = (1)^3 - 5(1) + 1$$

$$f(1) = 1 - 5 + 1 = -3$$

$$x_0 = 0$$

$$f(x_0) = 1$$

$$x_1 = 1$$

$$f(x_1) = -3$$

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

→ put $n=1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0)(-3) - (1)(1)}{-3 - 1} = \frac{-1}{-4} = \frac{1}{4}$$

$$\boxed{x_2 = 0.25}$$

$$f(x_2) = f(0.25) = (0.25)^3 - 5(0.25) + 1$$

$$f(x_2) = -0.234375$$

→ put $n=2$

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$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = \frac{(1)(-0.234375) - (0.25)(-3)}{(-0.234375) - (-3)}$$

$$x_3 = 0.186441$$

$$f(x_3) = f(0.18644) = (0.18644)^3 - 5(0.18644) + 1$$

$$f(x_3) = 0.074281$$

→ put $n=3$

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

~~$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$~~

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$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$x_4 = \frac{(0.25)(0.074281) - (0.18644)(-0.234375)}{(0.07428) - (-0.234375)}$$

$$x_4 = 0.20174$$

$$\begin{aligned} f(x_4) &= f(0.20174) \\ &= (0.20174)^3 - 5(0.20174) + 1 \\ f(x_4) &= -0.00048 \end{aligned}$$

→ Put $n=4$

$$x_5 = 0.20081$$

— x — x —