INTERPOLATION

The Technique or Method of estimating unknown values from given set of observation is known as ~~integ~~ interpolation.

Equal Interval

x	f(x)
10	100
20	200
30	300

1) Newton Forward & Backward
2) Gauss Forward & Backward / Stirling / Bessel's

Unequal Interval

x	f(x)
5	670
10	685
17	750
20	800
24	950

1) Lagrange Interpolation
2) Newton Divided Difference

INTERPOLATION FOR EQUAL INTERVAL

→ Newton Forward and Backward Interpolation

Q:- Estimate the population in ~~1985~~ 1895 & 1925 from the following

Year	1981	1901	1911	1921	1931
Population	46	66	81	93	101

Sol:-

$\Delta f(x) = f(x+h) - f(x)$

x	f(x)	Δf(x)	Δ²f(x)	Δ³f(x)	Δ⁴f(x)
1891	46				
1901	66	20			
1911	81	15	-5		
1921	93	12	-3	2	
1931	101	8	-4	-1	-3

Population in ~~1985~~ 1895 = ?

Newton Forward Formula :-

$$f(a+hu) = f(a) + \frac{u}{1!}\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 f(a) + \dots$$

$a+hu = 1895$, $a = 1891$, $h = 10$

a = base h = interval ka difference

$a+hu = 1895$

$1891 + 10(u) = 1895$

$10(u) = 1895 - 1891$

$$u = \frac{4}{10} \qquad \boxed{u = 0.4}$$

$$f(1895) = 46 + \frac{0.4 \times 20}{1} + \frac{0.4(0.4-1)(-5)}{2\times 1}$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)(2)}{3\times 2\times 1}$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)(-3)}{4\times 3\times 2\times 1}$$

$$\boxed{f(1895) = 54.8528}$$

Population in 1925 = ?

Newton Backward Formula :-

$$\nabla f(x) = f(x) - f(x-h)$$

$$f(a+hu) = f(a) + \frac{u\nabla f(a)}{1!} + \frac{u(u+1)\nabla^2 f(a)}{2!}$$

$$+ \frac{u(u+1)(u+2)\nabla^3 f(a)}{3!}$$

$$+ \frac{u(u+1)(u+2)(u+3)\nabla^4 f(a)}{4!} + \cdots +$$

$f(1925) = ?$

(4)

$a = 1931 \quad h = 10$

~~$f(a+hu) = ?$~~
~~$f(1931 + 10u) = 1925$~~

$a + hu = 1925$
$1931 + 10(u) = 1925$
$10u = 1925 - 1931$

$$\boxed{u = -0.6}$$

$$f(1925) = 101 + \frac{(-0.6)(8)}{1!} + \frac{(-0.6)(-0.6+1)(-4)}{2\times1}$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-1)}{3\times2\times1}$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-3)}{4\times3\times2\times1}$$

$$\approx \boxed{f(1925) = 96.8368}$$

——x——x——

Q:- Find number of men getting wages between Rs 10 & Rs 15 from the following data.

Wages	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

x wages	f(x) CF	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
below 10	9			
below 20	30+9=39	30	5	
below 30	35+39=~~55~~ 74	35	7	2
below 40	42+74=116	42		

$f(15) = ?$

Newton Forward Formula :-

$$f(a+hu) = f(a) + \frac{u\,\Delta f(a)}{1!} + \frac{u(u-1)\Delta^2 f(a)}{2!} + \frac{u(u-1)(u-2)\Delta^3 f(a)}{3!} + \frac{u(u-1)(u-2)(u-3)\Delta^4 f(a)}{4!} + \cdots$$

$a+hu=15 \Rightarrow a=10 \qquad h=10$

$10 + 10(u) = 15$

$10(u) = 15-10$

$u = \frac{5}{10} = 0.5$

$$f(15) = 9 + \frac{(0.5)(30)}{1!} + \frac{(0.5)(0.5-1)(5)}{2\times1} + \frac{(0.5)(0.5-1)(0.5-2)(2)}{3\times2\times1}$$

$f(15) = 23.5$

(6).

$f(15) = 23.5 = 24$

Hence number of men getting wages between 10 & 15 rupees is

$24 - 9 = 15$

—— x —— x ——

Q:- Find the lowest degree polynomial $y(x)$ that fit the data, find $y(5)$

x	0	2	4	6	8
y	5	9	61	209	501

$y = f(x)$

x	f(x)	Δf(x)	Δ²f(x)	Δ³f(x)	Δ⁴f(x)
0	5				
		4			
2	9		48		
		52		48	
4	61		96		0
		148		48	
6	209		144		
		292			
8	501				

$f(x) = ?$

Newton Forward Formula:-

$$f(a+hu) = f(a) + \frac{u}{1!} \Delta f(a) + \frac{u(u-1)}{2!} \Delta^2 f(a)$$

$$+ \frac{u(u-1)(u-2)}{3!} \Delta^3 f(a) + \cdots +$$

$a+hy = x$

$0 + 2y = x$

$y = \frac{x}{2}$

$$f(x) = 5 + \frac{(x/2)(4)}{1!} + \frac{(x/2-1)(x/2)(48)}{2!}$$

$$= \frac{(x/2)(x/2-1)(x/2-2)(48)}{3!}$$

$$= 5 + 2x + \left(\frac{x}{2}-1\right)\left(\frac{x}{2}\right)(24)$$

$$+ \left(\frac{x}{2}\right)\left(\frac{x}{2}-1\right)\left(\frac{x}{2}-2\right)16$$

$$= 5 + 2x + \frac{(x-2)(x)(24)}{2\times 3}$$

$$+ x\frac{(x-2)}{2}\frac{(x-4)}{2}16$$

$$= 5 + 2x + (x^2 - 2x)8$$

$$+ x(x^2 - 4x - 2x + 8)(8)$$

$$= 5 + 2x + 8x^3 - 16x + 8x^3 - 48x + 64$$

$f(5) = ?$

$$f(5) = 5 + \frac{(5/2)(4)}{1} + \frac{(0.5-1)(0.5)(48)}{2!}$$

(8)

$$+\frac{(0.5)(0.5-1)(0.5-2)(48)}{3!}$$

$f(x) = x^3 - 2x + 5$

$y(5) = (5)^3 - 2(5) + 5$

$= 125 - 10 + 5$

$\boxed{y(5) = 120}$

— x — x —