JACOBI METHOD - ITERATIVE METHOD

Q:- $27x + 6y - z = 85$
$6x + 15y + 2z = 72$
$x + y + 54z = 110$

Sol

$|27| > |6| + |-1| \Rightarrow |x| > |y| + |z|$
$|15| > |6| + |2| \Rightarrow |y| > |x| + |z|$
$|54| > |1| + |1| \Rightarrow |z| > |x| + |y|$

So we can use Jacobi's method for given system of equation.

$27x + 6y - z = 85$

$$x = \frac{-6y + z + 85}{27}$$

$6x + 15y + 2z = 72$

$$y = \frac{-6x - 2z + 72}{15}$$

$x + y + 54z = 110$

$$z = \frac{-x - y + 110}{54}$$

$x_0 = 0, \; y_0 = 0, \; z_0 = 0$

(2)

First Iteration

$$x_1 = \frac{-6y+z+85}{27} = \frac{-6(0)+0+85}{27}$$

$$x_1^1 = \frac{85}{27} = 3.148$$

$$y = \frac{-6x-2z+72}{15} = \frac{-6(0)-2(0)+72}{15}$$

$$y_1^1 = \frac{72}{15} = 4.8$$

$$z = \frac{-x-y+110}{54} = \frac{-0-0+110}{54}$$

$$z_1^1 = \frac{110}{54} = 2.037$$

Second Iteration

$$x_2^2 = \frac{1}{27}(-6(4.8)+2.037+85) = 2.157$$

$$y^2 = \frac{1}{15}\left[-6(3.148)-2(2.037)+72\right] = 3.269$$

$$z^2 = \frac{1}{54}\left[-3.148-4.8+110\right] = 1.89$$

Thrid Iteration

$$x^3 = \left[-6(3.269) + 1.89 + 85\right]\frac{1}{27} = 2.492$$

$$y^3 = \frac{1}{15}\left[-6(2.157) - 2(1.89) + 72\right] = 3.685$$

$$z^3 = \frac{1}{54}\left[-2.157 - 3.269 + 110\right] = 1.937$$

Fourth Iteration

$$x^4 = \frac{1}{27}\left[85 - 6(3.685) + 1.937\right] = 2.401$$

$$y^4 = \frac{1}{15}\left[72 - 6(2.492) - 2(1.937)\right] = 3.545$$

$$z^4 = \frac{1}{54}\left[110 - 2.492 - 3.685\right] = 1.925$$

Fifth Iteration

$$x^5 = \frac{1}{27}\left[85 - 6(3.545) + 1.925\right] = 2.432$$

$$y^5 = \frac{1}{15}\left[72 - 6(2.401) - 2(1.925)\right] = 3.583$$

$$z^5 = \frac{1}{54}\left[110 - 2.401 - 3.685\right] = 1.927$$

Sixth Iteration

$$x^6 = 2.423 \quad y^6 = 3.57 \quad z^6 = 1.926$$

Seventh Iteration

$x^7 = 2.426$

$y^7 = 3.574$

$z^7 = 1.926$

Sixth and seventh Iteration give same values therefore we can stop the iteration. Hence the solution of equation is

$x = 2.426$
$y = 3.574$
$z = 1.926$

—— x —— x ——