~~Diagonal Matrix~~

(1)

BISECTION METHOD

→ Solution of Algebraic & Transcedental Equation by Bisection Method.

1) Bisection Method : (Bolzano Method)

This method is based on the repeated application of intermediate value property.

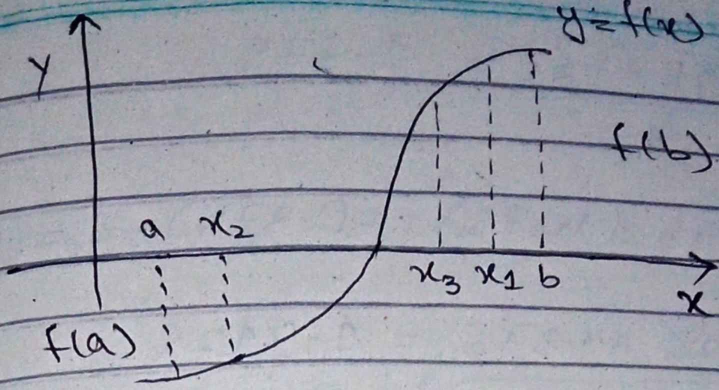
Let $f(x)$ be continous between a & b
let $f(a)$ be -ive.
& $f(b)$ be +ive.

Then the first approximation of the root is

$x_1 = \frac{a+b}{2}$, if $f(x_1) = 0$, then

x_1 is the root of $f(x_1) = 0$

Otherwise, the root lies between a & x_1 or x_1 & b, according as $f(x_1)$ is positive or negative. Then we bisect the interval as before & continous the process until the root is found accuracy



Q1: Find the real root of the equation

$$f(x) = x^3 - x - 1 = 0$$

Solution

$$f(-1) = -1 + 1 + (-1) = -1$$
$$f(1) = 1 - 1 - 1 = -1$$
$$f(0) = 0 - 0 - 1 = -1$$
$$f(2) = 8 - 2 - 1 = +5$$

$$x_1 = \frac{1+2}{2} = 1.5$$

$$f(1.5) = (1.5)^3 - (1.5) - 1 = \frac{7}{8}$$

$$x_2 = \frac{1 + 1.5}{2} = 1.25$$

$$f(1.25) = (1.25)^3 - (1.25) - 1 = \frac{-19}{64}$$

$$x_3 = \frac{1.25 + 1.5}{2} = 1.375$$

$$f(1.375) = (1.375)^3 - (1.375) - 1 = \frac{115}{512}$$

$$x_4 = \frac{1.25 + 1.375}{2} = 1.3125$$

$$f(1.3125) = (1.3125)^3 - (1.3125) - 1 =$$

$$= \frac{-211}{4096}$$

$$x_5 = \frac{1.3125 + 1.375}{2} = 1.34375$$

$f(x_5) = ?$

—— x —— x ——

Q2:- Find the real root of the equation $x^3 - 2x - 5 = 0$

Solution:-

$$f(x) = x^3 - 2x - 5$$

$$f(0) = (0)^3 - 2(0) - 5 = -5$$
$$f(1) = (1)^3 - 2(1) - 5 = -6$$
$$f(2) = (2)^3 - 2(2) - 5 = -1$$

4

$f(3) = (3)^3 - 2(3) - 5 = 16$

$x_1 = \frac{2+3}{2} = 2.5$

$f(2.5) = (2.5)^3 - 2(2.5) - 5 = 5.6250$

$x_2 = \frac{2+2.5}{2} = 2.25$

$f(2.25) = (2.25)^3 - 2(2.25) - 5 = \frac{121}{64}$

$x_3 = \frac{2+2.25}{2} = 2.125$

$f(2.125) = (2.125)^3 - 2(2.125) - 5 = \frac{177}{512}$

$x_4 = \frac{2+2.125}{2} = 2.0625$

$f(2.0625) = (2.0625)^3 - 2(2.0625) - 5 = -0.35$

$x_5 = \frac{2.0625 + 2.125}{2} = 2.09375$

$f(2.09375) = (2.09375)^3 - 2(2.09375)$
$- 5$

$= \frac{-293}{32768}$

(5)

$$x_6 = \frac{2.09375 + 2.125}{2} = 2.109375$$

$$f(2.109375) = (2.109375)^3 - 2(2.109375) - 5 = 0.16$$

$$x_7 = \frac{2.09375 + 2.109375}{2}$$

$$= 2.1015625$$

$$f(2.1015625) = (2.1015625)^3 - 2(2.1015625) - 5 = 0.07856225967$$

$$x_8 = \frac{2.09375 + 2.1015625}{2}$$

$$= 2.09765625$$

$$f(2.09765625) = (2.09765625)^3 - 2(2.1015625) - 5$$

$$= 0.0269017815б$$

$$x_9 = \frac{2.09375 + 2.09765625}{2}$$

$$= 2.095703125$$

$$f(2.095703125) = (2.095703125)^3 - 2(2.095703125) - 5$$
$$= 0.01286233217$$

$$x_{10} = \frac{2.09375 + 2.095703125}{2}$$

$$= 2.094726563$$

$$f(2.094726563) = (2.094726563)^3 - 2(2.094726563) - 5 =$$

$$= 1.954353408 \times 10^{-3}$$

$$x_{11} = \frac{2.09375 + 2.094726563}{2}$$

$$= 2.094238282$$

$$f(2.094238282) = (2.094238282)^3 - 2(2.094238282) - 5$$
$$= -3.49514083 \times 10^{-3}$$

$$x_{12} = \frac{2.094238282 + 2.094726563}{2}$$

$$= 2.094482423$$

$$f(2.094482423) = (2.094482423)^3 - 2(2.094482423) - 5$$
$$= -7.707626527 \times 10^{-4}$$

⑦

$$x_{13} = \frac{2.094482423 + 2.094726563}{2}$$

$$= 2.094604493$$

The root is 2.094

——x——x——