GAUSS SEIDAL METHOD

Q: Solve the system by using of equation
   Gauss - Seidal iterative method.

$$2x_1 - x_2 + 0x_3 = 7 \qquad |2| > |-1| + |0|$$
$$-x_1 + 2x_2 - x_3 = 1 \qquad |2| > |-1| + |-1|$$
$$0x_1 - x_2 + 2x_3 = 1 \qquad |2| > |0| + |-1|$$

$$2x_1 - x_2 + 0x_3 = 7$$

$$x_1 = \frac{x_2 + 0x_3 + 7}{2} = \frac{x_2 + 7}{2}$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$x_2 = \frac{+x_1 + x_3 + 1}{2}$$

$$-0x_1 - x_2 + 2x_3 = 1$$

$$x_3 = \frac{+0x_1 + x_2 + 1}{2} = \frac{x_2 + 1}{2}$$

Initial Approximation

$$x_1^{(0)} = 0 \qquad x_2^{(0)} = 0 \qquad x_3^{(0)} = 0$$

First Iteration

$$\rightarrow x_1^1 = \frac{0+7}{2} = \frac{7}{2} = 3.5$$

$$\rightarrow x_2^1 = \frac{1}{2}(1 + x_1^{(1)} + x_3^0)$$

$$\rightarrow x_2^1 = \frac{(\frac{7}{2}) + 0 + 1}{2} = 2.25$$

$$\rightarrow x_3^1 = \frac{1}{2}(1 + x_2^{(1)}) / 2$$

$$\rightarrow x_3^1 = \frac{1}{2}(1 + \overline{2.25}) = \overline{2.25} \ 1.625$$

Second Iteration

$$x_1^2 = \frac{1}{2}(7 + x_2^1)$$

$$x_1^2 = \frac{1}{2}(7 + 2.25) = \frac{9.25}{2} = 4.625$$

$$\overline{x_2^3 = \frac{1}{2}(1 + 3.8 + 2.25)} =$$

$$x_2^2 = \frac{1}{2}(1 + x_1^2 + x_3^1)$$

$$= \frac{1}{2}(1 + 4.625 + 1.625)$$

$$= 3.625$$

$$x_3^2 = \frac{1}{2}(1 + x_2^2)$$

$$= \frac{1}{2}(1 + 3.625) = 2.3125$$

Thrid Iteration

$$x_1^{(3)} = \frac{1}{2}(x_2^2 + 7)$$

$$= \frac{1}{2}(3.625 + 7) = 5.3125$$

$$x_2^{(3)} = \frac{1}{2}(x_1^{(3)} + x_3^{(2)} + 1)$$

$$= \frac{1}{2}(5.3125 + 2.3125 + 1)$$

$$= 4.3125$$

$$x_3^{(3)} = \frac{1}{2}(1 + x_2^{(3)})$$

$$= \frac{1}{2}(1 + 4.3125) = 2.65625$$

Fourth Iteration

$$x_1^{(4)} = \frac{1}{2}(x_2^3 + 7)$$

$$= \frac{1}{2}(4.3125 + 7) = 5.65625$$

$$x_2^{(4)} = \frac{1}{2}(x_1^{(4)} + x_3^{(3)} + 1)$$

$$= \frac{1}{2}(1 + x_1^{(4)} + x_3^{(3)})$$

$$x_3^{(3)} + 7 = \frac{1}{2}(1 + x_2^{(4)})$$

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$$x_2^{(4)} = \frac{1}{2}(1 + 5.65625 + 2.65625)$$

$$= 4.65625$$

$$x_3^{(4)} = \frac{1}{2}(1 + x_2^{(4)})$$

$$= \frac{1}{2}(1 + 4.65625)$$

$$= \frac{1}{2}(5.65625)$$

$$= 2.828125$$

Fifth Iteration

$$x_1^{(5)} = 5.8281$$

$$x_2^{(5)} = 4.8281$$

$$x_3^{(5)} = 2.9106$$

Sixth Iteration

$$x_1^{(6)} = 5.9140$$

$$x_2^{(6)} = 4.9140$$

$$x_3^{(6)} = 2.9570$$

Seventh Iteration

$$x_1^{(7)} = 5.9570$$

$$x_2^{(7)} = 4.9570$$

$$x_3^{(7)} = 2.9785$$

$$x_1 = 5.9570 \simeq 6$$
$$x_2 = 4.9570 \simeq 5$$
$$x_3 = 2.9785 \simeq 3$$

$$\boxed{\begin{array}{l} x_1 = 6 \\ x_2 = 5 \\ x_3 = 3 \end{array}} \longrightarrow \text{roots}$$

— x — x —