NUMERICAL INTEGRATION

The Area bounded by the curve $f(x)$ & x-axis between limit a & b is denoted by

$$I = \int_a^b f(x)\,dx \quad \text{— (I)}$$

divide the interval (a,b) into n equal interval with length h (step size)

i.e $(a,b) = (a = x_0, x_1, x_2, \dots, x_{n-1}, x_n = b)$

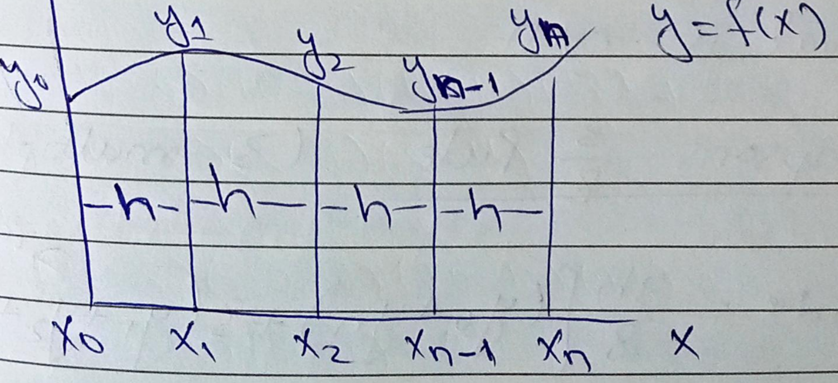
$a = x_0$

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$$\boxed{n = \frac{b-a}{h}} \qquad \boxed{h = \frac{b-a}{n}}$$

$x_n = x_{n-1} + h$



② Equation ① can be solved by using

① **Trapezoidal Rule**

$$\int_a^b f(x)\,dx = h\left[\frac{y_0+y_n}{2} + y_1 + y_2 + \cdots + y_{n-1}\right]$$

It is applicable on any number of interval

② **Simpsons $\frac{1}{3}$ Rule** (EVEN INTERVAL)

sum of odd terms

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots)\right]$$

sum of even terms

It is applicable if the total No of interval is even.

③ **Simpsons $\frac{3}{8}$ Rule** (3 multiple)

remaining terms

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\cdots) + 2(y_3+y_6+\cdots)\right]$$

sum of multiple of 3

It is applicable if total number of interval is multiple of 3.

Q: Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using

1) Trapezodial Rule
2) Simpson 1/3 Rule
3) Simpson 3/8 Rule

$y = \frac{1}{1+x^2}$

Also find values of π in each case.

$h = \frac{b-a}{n}$ $\quad h = \frac{1-0}{6}$ $\quad \boxed{h = \frac{1}{6}}$

$x_0 = 0$ $\quad y_0 = \frac{1}{1+x_0^2} = \frac{1}{1+(0)^2} = 1$

$x_1 = 1/6$ $\quad y_1 = \frac{1}{1+x_1^2} = \frac{1}{1+(1/6)^2} = \frac{36}{37}$

$x_2 = 2/6$ $\quad y_2 = \frac{1}{1+x_2^2} = \frac{1}{1+(2/6)^2} = 0.9$

$x_3 = 3/6$ $\quad y_3 = \frac{1}{1+x_3^2} = \frac{1}{1+(3/6)^2} = 0.8$

$x_4 = 4/6$ $\quad y_4 = \frac{1}{1+x_4^2} = \frac{1}{1+(4/6)^2} = 9/13$

$x_5 = 5/6$ $\quad y_5 = \frac{1}{1+x_5^2} = \frac{1}{1+(5/6)^2} = \frac{36}{61}$

4

$x_6 = 1 \quad y_6 = \frac{1}{1+x_6^2} = \frac{1}{1+(1)^2} = 0.5$

By <u>Trapezodial Formula</u>

$$\int_a^b f(x)\,dx = h\left[\frac{y_0+y_n}{2} + y_1 + y_2 + \dots + y_{n-1}\right]$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = h\left[\frac{y_0+y_6}{2} + y_1 + y_2 + y_3 + y_4 + y_5\right]$$

$$= \frac{1}{6}\left(\frac{1+0.5}{2} + \frac{36}{37} + 0.9 + 0.8 + \frac{9}{13} + \frac{36}{61}\right]$$

$$= \cancel{0.7783921} \; 0.78423$$

By <u>Simpson $\frac{1}{3}$ Rule</u>

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\dots) + 2(y_2+y_4+\dots)\right]$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$=\frac{(1/6)}{3}\left[(1+0.5)+4\left(\frac{36}{37}+0.8+\frac{36}{61}\right)+2\left(0.9+\frac{9}{13}\right)\right]$$

$$= 0.785396$$

By Using Simpson $\frac{3}{8}$ rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+\cdots)+2(y_3+y_6+\cdots)\right]$$

$$\cancel{= \frac{3(1/6)}{8}}$$

$$\int_a^b \frac{1}{1+x^2}\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

⑥

$$= \frac{3(1/6)}{8}\left[(1+0.5)+3\left(\frac{36}{37}+0.9+\frac{9}{13}+\frac{36}{61}\right)+2(0.8)\right]$$

$$= 0.785394$$

By Direct Integration

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left(\tan^{-1}x\right)_0^1$$

$$= \left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4} - 0$$

$$\boxed{\int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4}} \quad - (4)$$

I)

$$0.78423 = \frac{\pi}{4}$$

$$\pi = 0.78423 \times 4 = 3.1369$$

II)

$0.785396 = \frac{\pi}{4}$

$\pi = 0.785396 \times 4 = 3.14158$

III)

$0.785394 = \frac{\pi}{4}$

$\pi = 0.785394 \times 4 = 3.14157$

—— x —— x ——

Q: Given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
log x	1.3863	1.9351	1.9816	1.5261	1.5686	1.6094	1.6487

Evaluate $\int_4^{5.2} \log x \, dx$ By simpson $\frac{3}{8}$ Rule

Sol:

$$h = \frac{5.2-4}{6} = \frac{1.2}{6} = 0.2$$

use $\boxed{\ln}$ in calculator because it base is e

$$\int_4^{5.2} \log_e x \, dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3}{6}(0.2)\left[(1.3863+1.6487) + 3(1.4351+1.4816+1.5686 + 1.6094) + 2(1.5261)\right]$$

$$= 1.82784$$

—— x —— x ——

Q:- Use Trapezodial Rule to compute $\int_1^2 \frac{1}{x} dx$ using three intervals. compare it with exact value

$$h = \frac{b-a}{n}$$

$$h = \frac{2-1}{3} = \frac{1}{3}$$

$x_0 = 1$ $\qquad y_0 = \frac{1}{x_0} = 1/1 = 1$

$x_1 = 1 + 1/3 = 4/3$ $\qquad y_1 = \frac{1}{x_1} = \frac{1}{4/3} = 3/4$

$x_2 = 4/3 + 1/3 = 5/3$ $\qquad y_2 = \frac{1}{x_2} = \frac{1}{5/3} = 3/5$

$x_3 = 5/3 + 1/3 = 2$ $\qquad y_3 = \frac{1}{x_3} = 1/2 = \frac{1}{2}$

$$\int_1^2 \frac{1}{x} dx = \left[\frac{1+0.5}{2} + \frac{3}{4} + \frac{3}{5} \right]$$

$$= \cancel{0.69} \quad 0.70$$

By Direct Integration $\qquad \log_e x$

$$\int_1^2 \frac{1}{x} dx = (\log x)_1^2 = \log(2) - \log(1)$$

$$= \log_e 2 = 0.693$$

Approximate value = 0.70

Exact value = 0.693

—— x —— x ——

Q:- Calculate using simpson $\frac{1}{3}$ Rule the value $\int_{0}^{\pi/2} \sqrt{\sin x}\, dx$

Sol:-