Unsteady-State Pressure Distribution Created By a Directionally Drilled Well

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Introduction

Many methods[1-4] have been developed to analyze transient wellbore pressure data to determine the formation permeability, porosity, average pressure, and well condition. These methods usually are based on solutions[5,6] of unsteady-state flow problems that consider a fluid flowing toward a fully penetrating well that is perpendicular to the upper and lower formation boundary planes. Actually, most wells do not penetrate the producing formation perpendicularly. Instead, there is a certain angle between the normal to the formation plane and the well axis, such as when a vertical well penetrates a dipping formation or when a directionally drilled well penetrates a horizontal formation. These kinds of wells are called "slanted wells." Although such wells are common, there appears to have been only one study of the performance of such completions.

Roemershauser and Hawkins[7] studied steady-state flow in a reservoir producing through a fully penetrating, slanted well using an electrical model. They considered a circular reservoir of finite extent and concluded that the slant of a fully penetrating well causes an increase in the well productivity. The increase in well productivity results from the decrease in the resistance to flow around the wellbore caused by an increase in the producing-interval area exposed to flow. This increase in well productivity indicates that a fully penetrating, slanted well creates a negative skin effect. Roemershauser and Hawkins graphed the increase in well productivity vs the angle of slant of the well. There appears to have been no study of the unsteady-state performance of slanted wells.

Mathematical Derivation

The unsteady-state laminar flow of a slightly compressible fluid through an anisotropic, homogeneous, porous medium can be described after assuming small pressure gradients everywhere in the reservoir and neglecting gravity effects:[9]

$$\frac{\partial^2 p(r,\Theta,z,t)}{\partial r^2}+\left(\frac{1}{r}\right)\left[\frac{\partial p(r,\Theta,z,t)}{\partial r}\right]+\left(\frac{1}{r^2}\right)\left[\frac{\partial^2 p(r,\Theta,z,t)}{\partial \Theta^2}\right]$$

$$+\left(\frac{k_z}{k_r}\right)\left[\frac{\partial^2 p(r,\Theta,z,t)}{\partial z^2}\right]=\left(\frac{1}{\eta}\right)\left[\frac{\partial p(r,\Theta,z,t)}{\partial t}\right], \quad \ldots (1)$$

where

$$\eta = \frac{k_r}{\phi\,\mu\,c_t} = \text{constant.}$$

In Eq. 1, it is also assumed that the horizontal permeabilities k_x and k_y are equal and constant, thus equaling k_r. This assumption is not necessary, and the following results can be generalized to the case of simple anisotropy, where k_x, k_y, and k_z are all constant but are not equal, by redefinition of the horizontal variables x and y. For example, we define z' as

$$z' = z\sqrt{k_r/k_z}\ . \quad \ldots\ldots (2)$$

The permeability ratio in the fourth term on the left of

Analysis of a solution derived to study the unsteady-state pressure distribution created by a directionally drilled well indicates that the slant of a fully penetrating well creates a negative skin effect that is a function of the angle of slant and the formation thickness. Calculation of this pseudoskin factor permits evaluation of the actual well condition.

Eq. 1 is combined with a new vertical scale in z' and Eq. 1 becomes the diffusivity equation in cylindrical coordinates for a homogeneous medium.

To determine the initial and boundary conditions of the problem, consider the following system.

1. A horizontal, anisotropic, homogeneous, infinite porous medium of thickness h, porosity ϕ, and permeability k_r and k_z, all of which are independent of pressure and time.

2. A slightly compressible fluid of constant viscosity μ and compressibility c_t.

3. A slanted well of finite producing interval h_w and radius r_w. The angle of inclination of the well, measured from a normal to the formation bedding plane, is Θ_w (angle of slant). The well production rate, q_w, is constant and the middle point of the producing interval is located at an elevation $z = z_w$. The reservoir thickness and the interval length must satisfy the following conditions.

$$h_w \leqslant \frac{h}{\cos\Theta_w};\ \frac{h_w}{2} \leqslant \frac{z_w}{\cos\Theta_w};\ \frac{h_w}{2} \leqslant \frac{(h - z_w)}{\cos\Theta_w} \quad \text{..(3)}$$

Well production will be simulated by a line source located at the wellbore axis.

4. There is no flow across the upper and lower boundaries. Hence,

$$\left.\frac{\partial p(r,\Theta,z,t)}{\partial z}\right|_{z=h} = 0,$$

and

$$\left.\frac{\partial p(r,\Theta,z,t)}{\partial z}\right|_{z=0} = 0 \quad \text{..............(4)}$$

5. The initial reservoir pressure is p_i and does not change as $r \rightarrow \infty$. This condition is expressed as

$$p(r,\Theta,z,0) = p_i, \quad \text{......................(5)}$$

and

$$\lim_{r \rightarrow \infty} p(r,\Theta,z,t) = p_i \quad \text{......................(6)}$$

The solution for this problem was derived[8] by using the point-source solution as a starting point (see the Appendix).

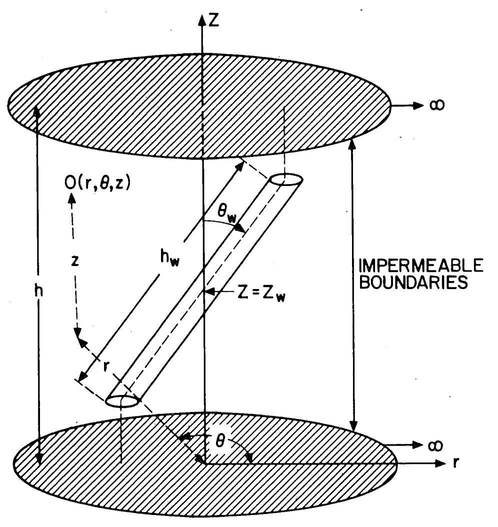

Fig. 1 — Slanted well in an infinite slab reservoir.

The dimensionless pressure at any point (r_D,Θ,z_D) at dimensionless time t_D, created by a continuous, slanted line source in an infinite slab reservoir, may be expressed as follows (see Fig. 1 for coordinate system).

$$\begin{aligned} P_D(r_D, \Theta, z_D, t_D, \Theta_w', z_{wD}, h_{wD}, h_D) = \\ \frac{h_D}{4\, h_{wD}\, \pi^{1/2} \cos\Theta_w'} \int_0^{t_D} \frac{e^{-\frac{r_D^2}{4\tau}}}{\tau^{3/2}} \\ \sum_{n=-\infty}^{\infty} \int_{-\frac{h_{wD}}{2}\cos\Theta_w'}^{\frac{h_{wD}}{2}\cos\Theta_w'} \Big\{ \exp\Big[- \\ \frac{z^2\tan^2\Theta_w' - 2r_D z\tan\Theta_w'\cos\Theta + (z_D + 2nh_D - z_{wD} - z)^2}{4\tau}\Big] \\ + \exp\Big[- \\ \frac{z^2\tan^2\Theta_w' + 2r_D z\tan\Theta_w'\cos\Theta + (z_D + 2nh_D + z_{wD} - z)^2}{4\tau}\Big]\Big\} \\ dz\, d\tau, \quad \text{.................................(7)} \end{aligned}$$

where

$$p_D(r_D,\Theta,z_D,t_D,\Theta_w',z_{wD},h_{wD},h_D) = \frac{k_r \sqrt{k_r/k_z}\, h\Delta p(r,\Theta,z,t,\Theta_w',z_w,h_w,h)}{141.2\, q_w B\mu}, \quad \text{....(8)}$$

$$t_D = \frac{0.000264\, k_r t}{\phi\, \mu c_t r_w^2}, \quad \text{......................(9)}$$

$$h_D = \frac{h}{r_w}\sqrt{\frac{k_r}{k_z}}, \quad \text{......................(10)}$$

$$r_D = \frac{r}{r_w}, \quad \text{............................(11)}$$

$$z_D = \frac{z}{r_w}\sqrt{\frac{k_r}{k_z}}, \quad \text{......................(12)}$$

$$z_{wD} = \frac{z_w}{r_w}\sqrt{\frac{k_r}{k_z}}, \quad \text{......................(13)}$$

$$h_{wD} = \frac{h_w}{r_w}\sqrt{\cos^2\Theta_w \frac{k_r}{k_z} + \sin^2\Theta_w}, \quad \text{........(14)}$$

$$\Theta_w' = \tan^{-1}\left(\sqrt{\frac{k_z}{k_r}}\tan\Theta_w\right) \quad \text{.............(15)}$$

Eq. 7 applies to partially and fully penetrating wells. For fully penetrating wells, $z_{wD} = ½h_D$, and

$$h_{wD} = \frac{h_D}{\cos\Theta_w'}.$$

Eq. 7 simplifies to

$$p_D(r_D,\Theta,z_D,t_D,\Theta_w',h_D) = \left(\frac{1}{4\,\pi^{1/2}} \int_0^{t_D} \frac{e^{-\frac{r_D^2}{4\tau}}}{\tau^{3/2}}\right)$$

$$\sum_{n=-\infty}^{\infty} \int_{-\frac{h_D}{2}}^{\frac{h_D}{2}} \left\{ \exp\left[- \frac{z^2\tan^2\Theta_w' - 2r_D z\tan\Theta_w'\cos\Theta + (z_D + 2nh_D - \frac{h_D}{2} - z)^2}{4\tau} \right] + \exp\left[- \frac{z^2\tan^2\Theta_w' + 2r_D z\tan\Theta_w'\cos\Theta + (z_D + 2nh_D + \frac{h_D}{2} - z)^2}{4\tau} \right] \right\} dz\, d\tau \quad \text{.......(16)}$$

Eq. 16 indicates that the dimensionless pressure created by a fully penetrating, slanted well is a function of position, dimensionless time, angle of slant, and dimensionless formation thickness.

Long Time Approximation

At large values of the dimensionless time, the dimensionless pressure at any point in the reservoir may be expressed as

$$p_D(r_D,\Theta,z_D,t_D,\Theta_w',h_D) = \frac{1}{2}\left[\ln\left(\frac{t_D}{r_D^2}\right) + 0.80907\right] + s_\Theta(r_D,\Theta,z_D,\Theta_w',h_D) \quad \text{.......(17)}$$

for $t_D > t_{D1}$,

where

$$s_\Theta(r_D,\Theta,z_D,\Theta_w',h_D) \approx p_D(r_D,\Theta,z_D,t_{D1},\Theta_w',h_D) - \frac{1}{2}\left[\ln\left(\frac{t_{D1}}{r_D^2}\right) + 0.80907\right], \quad \text{.......(18)}$$

and

$$t_{D1} = \max \left| \begin{array}{l} 70\ r_D^2 \\ \frac{25}{3}(r_D\cos\Theta + \frac{h_D}{2}\tan\Theta_w')^2 \\ \frac{25}{3}(r_D\cos\Theta - \frac{h_D}{2}\tan\Theta_w')^2 \end{array} \right| \quad \text{...(19)}$$

Eq. 17 indicates that the dimensionless pressure for a slanted well may be expressed as a sum of two terms for times greater than t_{D1}. One term is the familiar semilog approximation for a fully penetrating, vertical well, and the other term is a pseudoskin factor by analogy with the skin-factor concept of van Everdingen[10] and Hurst.[11] The pseudoskin factor, s_Θ, is a function of geometry only. Eq. 17 is discussed further in Ref. 8; however, the validity of this equation may be checked by means of Tables 1 through 5, and Figs. 2 through 6, which present evaluations of Eq. 16.

Wellbore Pressure

Analysis of Eq. 16 shows that the pressure is not uniform at the wellbore. This situation was caused by two facts: the well was simulated by a line source, and fluid withdrawal along this line source was considered uniform and independent of time (uniform-flux solution). Pressures along the surface of the wellbore cylinder of radius r_w depend on both Θ and z. In Ref. 8, it is shown that there are four symmetrical points on the wellbore surface that exist at the average wellbore pressure (average of the cylindrical surface). These points are located at Θ values of 90° and 270° and at distances 20 and 80 percent from the bottom of the formation along the well cylinder. Wellbore pressures were computed from Eq. 16 at such an average point. Eq. 16 becomes

$$p_{wD}(t_D,\Theta_w',h_D) \approx \left(\frac{1}{2h_D}\int_0^{t_D} \frac{e^{-\frac{1}{4\tau}}}{\tau}\right) \int_{-\frac{h_D}{2}}^{\frac{h_D}{2}} \exp\left[-\frac{\tan^2\Theta_w'(z+0.3h_D)^2}{4\tau}\right] \left[1 + 2\sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2\tau}{h_D^2}} \cos(0.8n\pi) \cos n\pi\left(\frac{1}{2} - \frac{z}{h_D}\right)\right] dz\, d\tau \quad \text{.......(20)}$$

Although it is easy to verify that Eq. 20 presents a good approximation of the average pressure for the constant-flux, slanted, line-source problem, it is not obvious that it also is correct for the more important infinite-conductivity source problem (p_{wf} constant over r_w, z at any time). Appendix B of Ref. 8 presents an analysis of the infinite-conductivity problem in the manner introduced by Gringarten *et al.*[12] It is shown that the locations for the average pressure on the wellbore cylinder also correspond to appropriate locations for constant-flux solutions that match the wellbore pressures for infinite-conductivity source solutions. This was done by dividing an infinite-conductivity source into M elements and determining the flux distribution required to cause a constant wellbore pressure.

Discussion of Results

The transient wellbore pressure of a fully penetrating, slanted well was computed for several values of angle of slant and dimensionless formation thickness. The results are given in Tables 1 through 5. Figs. 2 through 6 show dimensionless wellbore pressure vs the logarithm of dimensionless time. It can be seen from these figures that, for each case, there are three well defined flow periods: (1) a radial flow period whose pressure-log time curve is a straight line of slope equal to 1.151 $\cos\Theta_w'$, (2) a transition flow period, and (3) a pseudoradial flow period whose pressure-log time curve has the common slope of 1.151.

These flow periods are limited by light and heavy dashed lines in Figs. 2 through 6. We only consider the pseudoradial flow period ($t_D > t_{D1}$) since the radial and transition flow periods usually are affected by wellbore storage. Standard well-test analysis methods are valid to analyze transient pressure data of a slanted well during the pseudoradial flow period because the pressure-time relationship for this case is similar to that of a vertical well. However, the calculation of skin factor must be modified because of the difference between the pressure

TABLE 1 — DIMENSIONLESS WELLBORE PRESSURE VS DIMENSIONLESS TIME FOR A FULLY PENETRATING, SLANTED WELL IN AN INFINITE SLAB RESERVOIR

$h_D = 100$

	Θ_w				
t_D	15°	30°	45°	60°	75°
1×10^{-1}	0.0120	0.0108	0.0088	0.0062	0.0032
2×10^{-1}	0.0707	0.0634	0.0518	0.0366	0.0189
5×10^{-1}	0.2703	0.2424	0.1979	0.1399	0.0724
7×10^{-1}	0.3767	0.3378	0.2758	0.1950	0.1009
1	0.5043	0.4522	0.3602	0.2611	0.1351
2	0.7841	0.7030	0.5740	0.4059	0.2101
5	1.1919	1.0686	0.8725	0.6170	0.3194
7	1.3477	1.2083	0.9866	0.6976	0.3611
1×10	1.5148	1.3581	1.1089	0.7841	0.4059
2×10	1.8436	1.6529	1.3496	0.9543	0.4940
5×10	2.2829	2.0472	1.6714	1.1816	0.6116
7×10	2.4452	2.1937	1.7914	1.2659	0.6551
1×10^2	2.6179	2.3507	1.9211	1.3571	0.7017
2×10^2	2.9551	2.6612	2.1841	1.5473	0.7981
5×10^2	3.4032	3.0796	2.5508	1.8319	0.9528
7×10^2	3.5681	3.2344	2.6879	1.9421	1.0183
1×10^3	3.7431	3.3991	2.8338	2.0597	1.0911
2×10^3	$^*p_D(1,t_D)-0.123$	3.7227	3.1228	2.2912	1.2375
5×10^3		4.1617	3.5291	2.6291	1.4506
7×10^3		4.3259	3.6859	2.7678	1.5432
1×10^4		4.5012	3.8555	2.9225	1.6528
2×10^4		$^*p_D(1,t_D)-0.516$	4.1916	3.2397	1.9005
5×10^4			4.6434	3.6793	2.2855
7×10^4			$^*p_D(1,t_D)-1.175$	3.8439	2.4382
1×10^5				4.0195	2.6046
2×10^5				$^*p_D(1,t_D)-2.148$	2.9368
5×10^5					3.3861
7×10^5					3.5526
1×10^6					$^*p_D(1,t_D)-3.586$

$^*p_D(1,t_D)$ = line-source solution at the wellbore.

TABLE 2 — DIMENSIONLESS WELLBORE PRESSURE VS DIMENSIONLESS TIME FOR A FULLY PENETRATING, SLANTED WELL IN AN INFINITE SLAB RESERVOIR

$h_D = 200$

	Θ_w				
t_D	15°	30°	45°	60°	75°
1×10^{-1}	0.0120	0.0108	0.0088	0.0062	0.0032
2×10^{-1}	0.0707	0.0634	0.0518	0.0366	0.0189
5×10^{-1}	0.2703	0.2424	0.1979	0.1399	0.0724
7×10^{-1}	0.3767	0.3378	0.2758	0.1950	0.1009
1	0.5043	0.4522	0.3692	0.2611	0.1351
2	0.7841	0.7030	0.5740	0.4059	0.2101
5	1.1919	1.0686	0.8725	0.6170	0.3194
7	1.3477	1.2083	0.9866	0.6976	0.3611
1×10	1.5148	1.3581	1.1089	0.7841	0.4059
2×10	1.8436	1.6529	1.3496	0.9543	0.4940
5×10	2.2825	2.0465	1.6709	1.1815	0.6116
7×10	2.4443	2.1915	1.7894	1.2653	0.6550
1×10^2	2.6161	2.3455	1.9151	1.3542	0.7010
2×10^2	2.9506	2.6459	2.1602	1.5273	0.7905
5×10^2	3.4950	3.0495	2.4937	1.7623	0.9107
7×10^2	3.5590	3.2007	2.6221	1.8553	0.9578
1×10^3	3.7332	3.3625	2.6618	1.9599	1.0117
2×10^3	4.0726	3.6798	3.0409	2.1785	1.1322
5×10^3	4.5222	4.1029	3.4159	2.4801	1.3171
7×10^3	4.6879	4.2601	3.5567	2.5925	1.3883
1×10^4	4.8640	4.4286	3.7090	2.7153	1.4655
2×10^4	$^*p_D(1,t_D)-0.152$	4.7620	4.0193	2.9757	1.6301
5×10^4		5.2117	4.4531	3.3692	1.9071
7×10^4		5.3783	4.6166	3.5239	2.0289
1×10^5		$^*p_D(1,t_D)-0.608$	4.7913	3.6919	2.1685
2×10^5			5.1336	4.0260	2.4649
5×10^5			$^*p_D(1,t_D)-1.378$	4.4765	2.8900
7×10^5				$^*p_D(1,t_D)-2.494$	3.0516
1×10^6					3.2249
2×10^6					3.5656
5×10^6					$^*p_D(1,t_D)-4.099$

$^*p_D(1,t_D)$ = line-source solution at the wellbore.

of a slanted well and the pressure of a vertical well.

The pseudoskin factor, s_Θ, was defined as the difference between the dimensionless pressure created by a slanted well and that created by a vertical well. Fig. 7 shows a graph of pseudoskin factor, s_Θ, during the pseudoradial flow period vs the logarithm of dimensionless formation thickness for several values of angle of slant. The pseudoskin factor may be approximated as

$$s_\Theta(\Theta_w', h_D) = -\left(\frac{\Theta_w'}{41}\right)^{2.06} - \left(\frac{\Theta_w'}{56}\right)^{1.865} \times \log\left(\frac{h_D}{100}\right), \quad \ldots\ldots\ldots\ldots (21)$$

for $0° \leqslant \Theta_w' \leqslant 75°$ and $t_D \geqslant t_{D1}$.

From conventional pressure analysis of field data, we obtain a total value for the skin factor, s_t, that includes the true skin factor, s_d, and the pseudoskin factor caused by the inclination of the well:

$$s_t = s_d + s_\Theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

The true skin factor, s_d, actually is a sum of many factors that appear to cause a steady-state pressure drop in the vicinity of the well (partial penetration, perforations, non-Darcy flow, etc.). Values for the pseudoskin factor caused by the slant of the well may be obtained from either Fig. 7 or Eq. 21 and may be used to correct the apparent total skin factor to the true skin factor using Eq. 22.

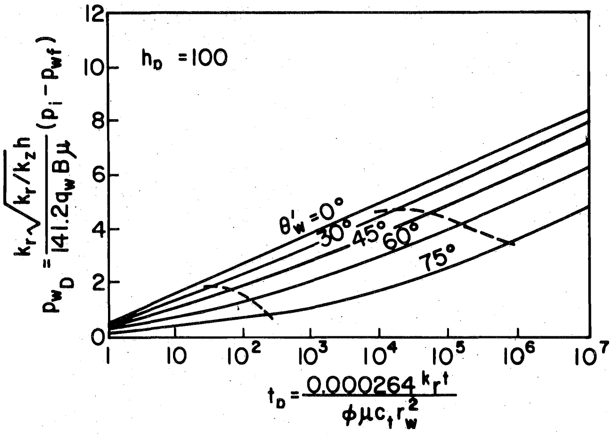

Fig. 2 — Dimensionless wellbore pressure vs dimensionless time for a fully penetrating, slanted well.

Applications

The most obvious application of the solutions lies in computation of the effect of directional drilling on the performance of a well. Tables 1 through 5 present sufficient pressure-time data to cover many cases of practical interest. Another application lies in performance matching to determine the existing condition of a direc-

TABLE 3 — DIMENSIONLESS WELLBORE PRESSURE VS DIMENSIONLESS TIME FOR A FULLY PENETRATING, SLANTED WELL IN AN INFINITE SLAB RESERVOIR

h_D = 500

	Θ_w				
t_D	15°	30°	45°	60°	75°
1×10^{-1}	0.0120	0.0108	0.0088	0.0062	0.0032
2×10^{-1}	0.0707	0.0634	0.0518	0.0366	0.0189
5×10^{-1}	0.2703	0.2424	0.0979	0.1399	0.0724
7×10^{-1}	0.3767	0.3378	0.2758	0.1950	0.1009
1	0.5043	0.4522	0.3692	0.2611	0.1351
2	0.7841	0.7030	0.5740	0.4059	0.2101
5	1.1919	1.0686	0.8725	0.6170	0.3194
7	1.3477	1.2083	0.9866	0.6976	0.3611
1×10	1.5148	1.3581	1.1089	0.7841	0.4059
2×10	1.8436	1.6529	1.3496	0.9543	0.4940
5×10	2.2829	2.0465	1.6709	1.1815	0.6116
7×10	2.4443	2.1915	1.7894	1.2653	0.6550
1×10^2	2.6161	2.3455	1.9151	1.3542	0.7010
2×10^2	2.9503	2.6541	2.1597	1.5272	0.7905
5×10^2	3.3924	3.0416	2.4834	1.7560	0.9090
7×10^2	3.5549	3.1873	2.6024	1.8401	0.9525
1×10^3	3.7272	3.3419	2.7286	1.9293	0.9987
2×10^3	4.0629	3.6448	2.9766	2.1037	1.0886
5×10^3	4.5091	4.0545	3.3218	2.3518	1.2145
7×10^3	4.6736	4.2076	3.4588	2.4525	1.2671
1×10^4	4.8483	4.3708	3.5982	2.5646	1.3282
2×10^4	5.1880	4.6897	3.8805	2.7909	1.4618
5×10^4	5.6388	5.1165	4.2609	3.0960	1.6541
7×10^4	5.8051	5.2761	4.4063	3.2138	1.7279
1×10^5	*$p_D(1,t_D)-0.183$	5.4474	4.5653	3.3463	1.8111
2×10^5		5.7853	4.8872	3.6297	1.9994
5×10^5		*$p_D(1,t_D)-0.731$	5.3298	4.0446	2.3171
7×10^5			5.4950	4.2040	2.4521
1×10^6			5.6710	4.3757	2.6036
2×10^6			*$p_D(1,t_D)-1.647$	4.7144	2.9167
5×10^6				*$p_D(1,t_D)-2.953$	3.3534
7×10^6					3.5175
1×10^7					3.6927
2×10^7					*$p_D(1,t_D)-4.778$

*$p_D(1,t_D)$ = line-source solution at the wellbore.

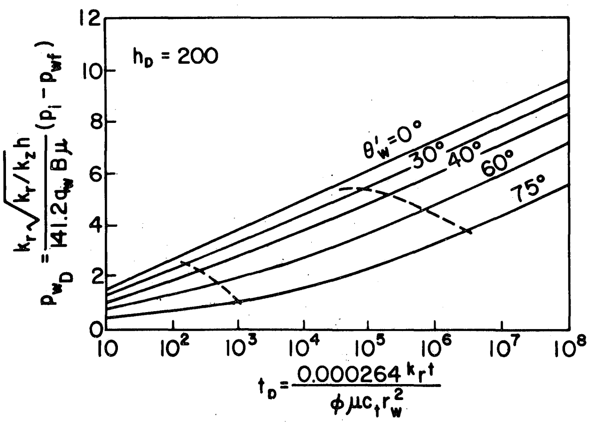

Fig. 3 — Dimensionless wellbore pressure vs dimensionless time for a fully penetrating, slanted well.

tionally drilled well. Consider the following example. A pressure buildup test indicates that the skin factor for a directionally drilled well is 0.8, indicating mild damage. The semilog straight-line slope is 125 psi/log cycle. Thus, the pressure drop across the apparent skin is 0.87(125) (0.8), or 87 psi. The average angle of inclination is 45°, the formation thickness is 100 ft, the wellbore radius is 0.3 ft, and core analysis indicates that the horizontal-to-vertical permeability ratio is about 5. Determine the portion of the apparent skin factor caused by directional drilling. The well fully penetrates the formation.

First determine the pseudoskin factor caused by directional drilling:

From Eq. 10,

$$h_D = \frac{100 \text{ ft}}{0.3 \text{ ft}} 5^{1/2} = 745.4.$$

From Eq. 15,

$$\Theta_w' = \tan^{-1}\left(\frac{\tan 45°}{5^{1/2}}\right) = 24.1°.$$

From Eq. 21,

$$s_\Theta = -(24.1/41)^{2.06} - (24.1/56)^{1.865} \times \log_{10}(745.4/100) = \underline{\underline{-0.66}}.$$

Thus, from Eq. 23, the portion of the total skin effect caused by factors other than directional drilling is

$$s_d = 0.8 - (-0.66) = \underline{\underline{+1.46}}.$$

This indicates that the well is more damaged than origi-

TABLE 4 — DIMENSIONLESS WELLBORE PRESSURE VS DIMENSIONLESS TIME FOR A FULLY PENETRATING, SLANTED WELL IN AN INFINITE SLAB RESERVOIR

h_D = 1,000

	Θ_w				
t_D	15°	30°	45°	60°	75°
1×10^{-1}	0.0120	0.0108	0.0088	0.0062	0.0032
2×10^{-1}	0.0707	0.0634	0.0518	0.0366	0.0189
5×10^{-1}	0.2703	0.2424	0.1979	0.1399	0.0724
7×10^{-1}	0.3767	0.3378	0.2758	0.1950	0.1009
1	0.5043	0.4522	0.3692	0.2611	0.1351
2	0.7841	0.7030	0.5740	0.4059	0.2101
5	1.1919	1.0686	0.8725	0.6170	0.3194
7	1.3477	1.2083	0.9866	0.6976	0.3611
1×10	1.5148	1.3581	1.1089	0.7841	0.4059
2×10	1.8436	1.6529	1.3496	0.9543	0.4940
5×10	2.2825	2.0465	1.6709	1.1815	0.6116
7×10	2.4443	2.1915	1.7894	1.2653	0.6550
1×10^2	2.6161	2.3455	1.9151	1.3542	0.7010
2×10^2	2.9503	2.6451	2.1597	1.5272	0.7905
5×10^2	3.3924	3.0416	2.4834	1.7560	0.9090
7×10^2	3.5549	3.1872	2.6023	1.8401	0.9525
1×10^3	3.7271	3.3416	2.7284	1.9293	0.9987
2×10^3	4.0618	3.6417	2.9734	2.1025	1.0884
5×10^3	4.5046	4.0392	3.2979	2.3317	1.2069
7×10^3	4.6676	4.1863	3.4183	2.4163	1.2506
1×10^4	4.8408	4.3437	3.5485	2.5078	1.2973
2×10^4	5.1787	4.6548	3.8119	2.6984	1.3939
5×10^4	5.6271	5.0736	4.1789	2.9832	1.5487
7×10^4	5.7921	5.2284	4.3160	3.0934	1.6143
1×10^5	5.9671	5.3931	4.4620	3.2111	1.6871
2×10^5	6.3084	5.7168	4.7511	3.4426	1.8335
5×10^5	$^*p_D(1,t_D)-0.207$	6.1559	5.1574	3.7805	2.0467
7×10^5		6.3201	5.3142	3.9192	2.1392
1×10^6		6.4954	5.4838	4.0739	2.2488
2×10^6		$^*p_D(1,t_D)-0.824$	5.8199	4.3911	2.4965
5×10^6			6.2717	4.8307	2.8815
7×10^6			$^*p_D(1,t_D)-1.850$	4.9953	3.0343
1×10^7				5.1709	3.2006
2×10^7				$^*p_D(1,t_D)-3.299$	3.5329
5×10^7					3.9821
7×10^7					4.1487
1×10^8					$^*p_D(1,t_D)-5.292$

$^*p_D(1,t_D)$ = line-source solution at the wellbore.

nally thought and that it is a candidate for stimulation. The true pressure drop across the skin is 159 psi, rather than the previous value of 87 psi.

Conclusions

The unsteady flow of a slightly compressible fluid toward a slanted line source in an infinite slab reservoir has three well defined flow periods: an initial radial flow period, a transition flow period, and a pseudoradial flow period.

The analytical solution for the dimensionless pressure distribution created by a slanted well shows the existence of a pseudoradial flow period at large values of time; therefore, all methods commonly used to analyze transient pressure data that assume radial flow are applicable to this case.

The inclination of a fully penetrating, slanted well creates a negative skin effect that is a function of the angle and is proportional to the logarithm of the formation thickness. The apparent skin factor should be corrected for this pseudoskin factor.

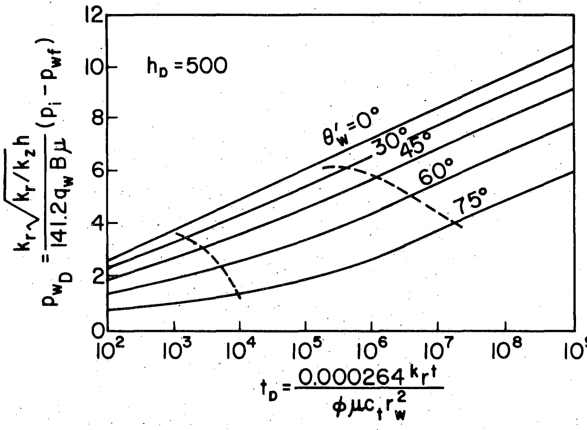

Fig. 4 — Dimensionless wellbore pressure vs dimensionless time for a fully penetrating, slanted well.

Nomenclature

B = formation volume factor, res vol/std vol

c_t = total system effective compressibility, psi^{-1}

TABLE 5 — DIMENSIONLESS WELLBORE PRESSURE VS DIMENSIONLESS TIME FOR A FULLY PENETRATING, SLANTED WELL IN AN INFINITE SLAB RESERVOIR

h_D = 5,000

	Θ_w				
t_D	15°	30°	45°	60°	75°
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7	1.3477	1.7083	0.9866	0.6976	0.3611
1×10	1.5148	1.3581	1.1089	0.7841	0.4059
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7×10	2.4443	2.1915	1.7894	1.2653	0.6550
1×10^2	2.6161	2.3455	1.9151	1.3542	0.7010
2×10^2	2.9503	2.6451	2.1597	1.5272	0.7905
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7×10^2	3.5549	3.1872	2.6023	1.8401	0.9525
1×10^3	3.7271	3.3416	2.7284	1.9293	0.9987
2×10^3	4.0618	3.6417	2.9734	2.1025	1.0889
5×10^3	4.5043	4.0384	3.2974	2.3316	1.2069
7×10^3	4.6668	4.1841	3.4163	2.4157	1.2505
1×10^4	4.8390	4.3386	3.5424	2.5049	1.2966
2×10^4	5.1738	4.6387	3.7875	2.6781	1.3863
5×10^4	5.6163	5.0355	4.1114	2.9072	1.5049
7×10^4	5.7789	5.1812	4.2304	2.9913	1.5484
1×10^5	5.9512	5.3359	4.3567	3.0805	1.5946
2×10^5	6.2870	5.6389	4.6047	3.2550	1.6845
5×10^5	6.7332	6.0486	4.9500	3.5031	1.8105
7×10^5	6.8978	6.2017	5.0829	3.6038	1.8630
1×10^6	7.0724	6.3649	5.2264	3.7159	1.9242
2×10^6	7.4122	6.6838	5.5086	3.9422	2.0578
5×10^6	7.8630	7.1106	5.8891	4.2473	2.2500
7×10^6	8.0293	7.2702	6.0345	4.3651	2.3238
1×10^7	*$p_D(1,t_D)-0.261$	7.4415	6.1935	4.4976	2.4070
2×10^7		7.7794	6.5154	4.7810	2.5953
5×10^7		*$p_D(1,t_D)-1.040$	2.9580	5.1959	2.9131
7×10^7			7.1232	5.3553	3.0480
1×10^8			7.2992	5.5270	3.1996
2×10^8			*$p_D(1,t_D)-2.321$	5.8657	3.5127
5×10^8				*$p_D(1,t_D)-4.104$	3.9494
7×10^8					4.1134
1×10^9					4.2886
2×10^9					*$p_D(1,t_D)-6.485$

*$p_D(1,t_D)$ = line-source solution at the wellbore.

h = formation thickness, ft
h_w = length of the producing interval, ft
k = formation permeability, md
p_D = dimensionless pressure drop
p_{wD} = dimensionless wellbore pressure
Δp = pressure drop, psi
q_w = total flow rate of a well, STB/D
r = radial distance, ft
r_w = wellbore radius, ft
s_d = true skin factor excluding slant effect, dimensionless
s_t = apparent skin factor, dimensionless
s_Θ = pseudoskin factor caused by inclination, dimensionless
t = producing time, hours
t_D = dimensionless producing time
t_{D1} = dimensionless time of start of semi-logarithmic period
z = elevation of a point, ft
z_{wD} = dimensionless elevation of the middle point of the producing interval of a slanted well
η = hydraulic diffusivity based on the radial permeability $(k_r/\phi\mu c_t)$
Θ = horizontal angle in polar coordinates (see Fig. 1)
Θ_w = angle of slant (see Fig. 1)
Θ_w' = see Eq. 15
μ = viscosity, cp
ϕ = porosity, fractional

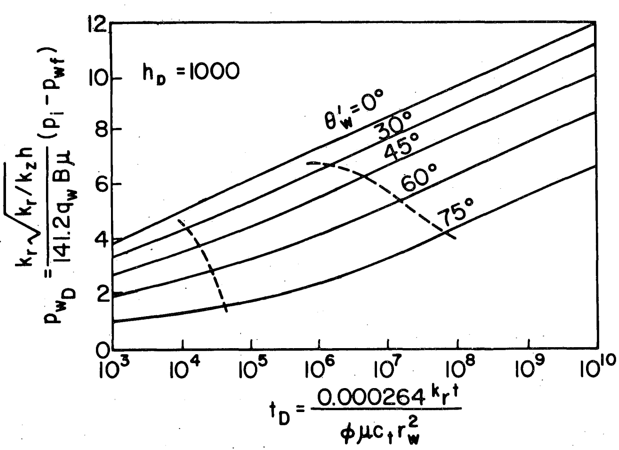

Fig. 5 — Dimensionless wellbore pressure vs dimensionless time for a fully penetrating, slanted well.

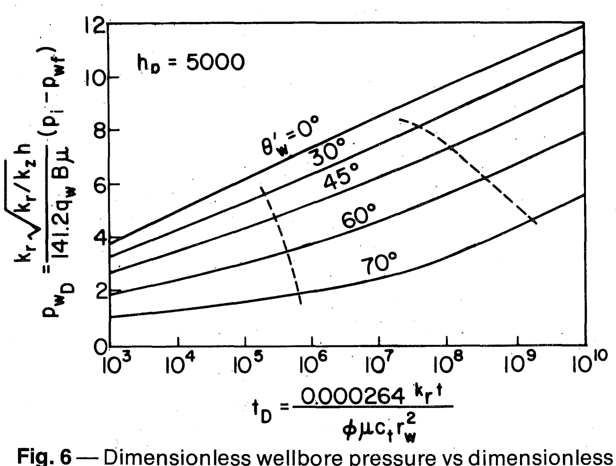

Fig. 6 — Dimensionless wellbore pressure vs dimensionless time for a fully penetrating, slanted well.

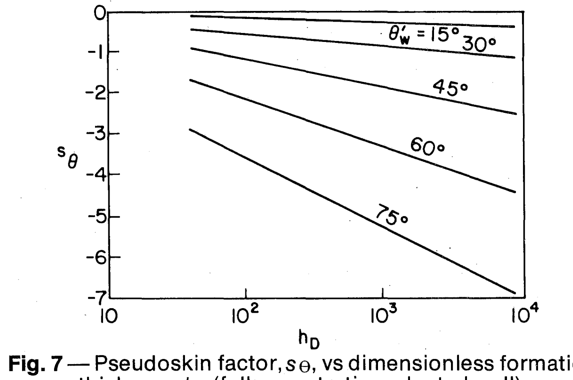

Fig. 7 — Pseudoskin factor, s_Θ, vs dimensionless formation thickness, h_D (fully penetrating, slanted well).

Subscripts

D = dimensionless
r = radial
w = well
z = vertical

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APPENDIX

Unsteady-State Pressure Distribution Created by a Slanted Line Source in An Infinite Slab Reservoir

The instantaneous point-source solution is expressed by

$$\Delta p_0\,(d,\,t) = \frac{q}{8\phi c\,(\pi\eta t)^{3/2}}\,\exp\left(-\frac{d^2}{4\eta t}\right), \quad \ldots \text{(A-1)}$$

and gives the pressure drop, Δp_0, created at Point M at time t by an instantaneous point source, P, of strength

q; d is the distance between Point P and Point M.

The pressure drop at Point M created by an instantaneous line source, L, can be obtained by integrating the point-source solution along Line L; thus,

$$\Delta p_1 \, (\mathrm{L,M},t) = \frac{q}{8\phi c(\pi\eta t)^{3/2}} \int_{\mathrm{L}} \exp\left(-\frac{\overline{\mathrm{PM}}^2}{4\eta t}\right) d\mathrm{L} \; , \quad \text{(A-2)}$$

where q is the instantaneous fluid withdrawal per unit length of the line source; in this case, the fluid withdrawal, q, is considered uniform along the line source.

The pressure drop at a point (r,Θ,z) and time t, created by an instantaneous line source of finite length h_w in an infinite porous medium in cylindrical coordinates, is given by

$$\Delta p_1 \, (r,\Theta,z,t) = \frac{q}{8\phi c(\pi\eta t)^{3/2}\cos\Theta_w} \int_{z_w - \frac{h_w}{2}\cos\Theta_w}^{z_w + \frac{h_w}{2}\cos\Theta_w} \exp\left[-\frac{r^2+(z_p-z_w)^2\tan^2\Theta_w - 2r(z_p-z_w)\tan\Theta_w\cos\Theta+(z-z_p)^2}{4\,t}\right] dz_p \quad \text{(A-3)}$$

The pressure drop created by a slanted line source in a semi-infinite reservoir can be found by superposing the effect of two line sources symmetrical to each other with respect to the impermeable boundary. Thus,

$$\Delta p_2 \, (r,\Theta,z,t,\Theta_w,z_w,h_w) = \left(\frac{q\, e^{-\frac{r^2}{4\eta t}}}{8\phi c\;(\pi\eta t)^{3/2}\cos\Theta_w}\right) \int_{-\frac{h_w}{2}\cos\Theta_w}^{\frac{h_w}{2}\cos\Theta_w} \left\{\exp\left[-\frac{z'^2\tan^2\Theta_w - 2rz'\tan\Theta_w\cos\Theta+(z-z_w-z')^2}{4\eta t}\right] + \exp\left[-\frac{z'^2\tan^2\Theta_w + 2rz'\tan\Theta_w\cos\Theta+(z+z_w-z')^2}{4\eta t}\right]\right\} dz' \quad \text{(A-4)}$$

The upper impermeable boundary in the system shown in Fig. 1 can be generated by superimposing an infinite number of pairs of slanted line-source systems. So, the pressure drop at a point (r,Θ,z,t), created by an *instantaneous* slanted line source of finite length h_w in an infinite slab porous medium of thickness h (Fig. 1), is given by

$$\Delta p_3 \, (r,\Theta,z,t,\Theta_w,z_w,h_w,h) = \left(\frac{q\, e^{-\frac{r^2}{4\eta t}}}{8\phi c\;(\pi\eta t)^{3/2}\cos\Theta_w}\right) \sum_{n=-\infty}^{\infty} \left\{ \int_{-\frac{h_w}{2}\cos\Theta_w}^{\frac{h_w}{2}\cos\Theta_w} \left[\exp\left(-\frac{z'^2\tan^2\Theta_w - 2rz'\tan\Theta_w\cos\Theta+(z+2nh-z_w-z')^2}{4\eta t}\right) + \exp\left(-\frac{z'\tan\Theta_w + 2rz'\tan\Theta_w\cos\Theta+(z+2nh+z_w-z')^2}{4\eta t}\right) dz'\right\} \right. \quad \text{(A-5)}$$

Integration of Eq. A-5 with respect to time gives the pressure drop created by a *continuous* slanted line source:

$$\Delta p(r,\Theta,z,t,\Theta_w,z_w,h_w,h) = \left(\frac{q}{8\phi c(\pi\eta)^{3/2}\cos\Theta_w}\right) \int_0^t \frac{e^{-\frac{r^2}{4\eta(t-\tau')}}}{(t-\tau')^{3/2}} \sum_{n=-\infty}^{\infty} \int_{-\frac{h_w}{2}\cos\Theta_w}^{\frac{h_w}{2}\cos\Theta_w} \left\{\exp\left[-\frac{z'^2\tan^2\Theta^w - 2rz'\tan\Theta_w\cos\Theta+(z+2nh-z_w-z')^2}{4\eta(t-\tau')}\right] + \exp\left[-\frac{z'^2\tan^2\Theta_w + 2rz'\tan\Theta_w\cos\Theta+(z+2nh-z_w-z')^2}{4\eta(t-\tau')}\right]\right\} dz'd\tau' \quad \text{(A-6)}$$

Define $\tau = t-\tau'$ and $q_w = q\; h_w$, and Eq. A-6 becomes

$$\Delta p(r,\Theta,z,t,\Theta_w,z_w,h_w,h) = \left[\frac{q_w}{8\phi c h_w \cos\Theta_w(\pi\eta)^{3/2}}\right] \int_0^t \frac{e^{-\frac{r^2}{4\eta\tau}}}{\tau^{3/2}} \sum_{n=-\infty}^{\infty} \int_{-\frac{h_w}{2}\cos\Theta_w}^{\frac{h_w}{2}\cos\Theta_w} \left\{\exp\left[-\frac{z'^2\tan^2\Theta_w - 2rz'\tan\Theta_w\cos\Theta+(z+2nh-z_w-z')^2}{4\eta\tau}\right] + \exp\left[-\frac{z'^2\tan^2\Theta_w + 2rz'\tan\Theta_w\cos\Theta+(z+2nh+z_w-z')^2}{4\eta\tau}\right]\right\} dz'd\tau \quad \text{(A-7)}$$

This derivation has been presented for a homogeneous medium. Substitution of Eq. 2 for all vertical parameters and of Eq. 15 for Θ_w generalizes Eq. A-7 to the case of simple $k_r - k_z$ anisotropy. Introduction of dimensionless parameters leads directly to Eq. 7.

The case where both areal and vertical anisotropy exists is of some importance in interference testing. Ref. 8 presents the source function for a slanted plane (zig-zag plane), which may be used to write a solution analogous to Eq. A-7 that considers that $k_x \neq k_y \neq k_z$. **JPT**

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