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Performance of Slanted and Horizontal Wells on an Anisotropic Medium

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ABSTRACT

Although the technology of highly inclined and horizontal wells has recently progressed significantly, a reliable and practical formula to estimate the productivity of the well with respect to any angle of slant and anisotropy of permeability is still missing. This paper provides a method of estimation which fills the gap.

The well pressure decline curves are generated by a semi-analytical in-house simulator. They lead to a geometrical pseudo skin factor which is matched for slanted wells with an analytically-derived equation. For horizontal wells, comparison is made with several existing formulae.

An unrestrictive approach of anisotropy is possible through a spatial transformation from real medium into equivalent isotropic medium : this is achieved by modifying the length, azimuth, inclination and radius of the well. Equations of the pseudo skin factor are modified according to this transformation.

Calculations reveal that a slanted well is less affected by anisotropy than a horizontal well : with a low vertical permeability, it can be more advantageous than a horizontal well of the same length. Graphic charts are displayed to easy the choice between horizontal and highly inclined wells.

INTRODUCTION

The recent development of horizontal wells technology can be associated to a long list of advantages and fields of application.

For low permeability reservoirs, the major interest in drilling horizontally is to increase productivity. But in predicting horizontal well performance, one should not forget a possible unfortunate effect : a low vertical-to-horizontal permeability ratio. The uncertainties linked to the difficulties to quantify this ratio (local heterogeneities, core sampling, etc..) may lead to abandon a horizontal well project. In order to minimize the risks, an appropriate answer is to drill almost horizontally, but not parallel to the stratigraphy or system of deposition. A wide range of trajectories is available, from the more complex "snake well" to the simple linear, highly inclined well.

Such a well has a double purpose : increase the producing length, and dispatch the production upon the whole thickness of the reservoir.

Several authors [2,3,6] have proposed formulae to predict the performance of a horizontal well, taking into account permeability anisotropy and various shapes of the drainage volume. CINCO et al.[1] have introduced the definition of a pseudo skin factor to estimate the long-time performance of a conventional slanted well (less than 75°). CHANG[8] has built a gridded model suitable to non vertical wells, and compared performance of horizontal and slanted wells under isotropic permeability conditions. But a reliable and practical formula to estimate the productivity of the well with respect to any angle of slant and degree of anisotropy is still missing.

The proceeding of this work is divided into four steps :

- a semi-analytical simulator generates pressure decline curves for a set of slanted and horizontal wells producing in a homogeneous, isotropic, isopach, infinite reservoir. Comparison with a fully penetrating vertical well leads to a geometrical pseudo skin factor ;

- for horizontal wells, pseudo skin factors are validated by comparison with published results. The accuracy of several analytically-derived equations commonly used in the literature is then discussed ;

- for slanted wells, direct comparison with horizontal wells of the same length affords a new ready-to-use equation ;
- both selected formulae are then transformed for anisotropic reservoirs. This is made possible through a spatial transformation from real medium into equivalent isotropic medium, modifying length, azimuth, inclination and radius of the well.

An obvious application of the new equation is to optimize the geometry of the well for different assumptions on the anisotropy ratio : graphic charts are displayed to easy the choice between horizontal and highly inclined wells.

A method is then proposed to adapt the existing productivity-index equations to slanted wells, with the division of the pseudo skin factor into two components.

MATHEMATICAL MODEL :

A semi-analytical simulator has been developed to predict the performance of a well producing in an homogeneous, isotropic, isopach reservoir, under monophasic flow conditions. The well can be declared as a succession of linear segments, jointed as well as disjointed.

The reservoir can be limited laterally by constant - pressure or no-flow boundaries. The reason for its "semi-analytical" appellation is that, unlike most well-test pressure models, every line-source is divided into a sufficiently large number of point-sources. The pressure drop is measured at one point of the wellbore face, by addition of the continuous point-source solutions (instead of a line-source solution) :

$$P(r,t) = \frac{1}{r}\,\frac{QB\mu}{4\pi k}\,\mathrm{ERFC}\left(\frac{r}{\sqrt{\dfrac{4kt}{\phi\mu c_t}}}\right), \quad \ldots\ldots\ldots [1]$$

(in S.I. units)

Where ERFC accounts for the complementary error function.

In order to reduce the computing times, the number of point-sources is reduced by a gathering algorithm based upon their distances to the point of measurement. Additional virtual sources simulate the external boundary conditions. Two options are available, according to uniform-flux of infinite-conductivity solutions :

- The uniform-flux option assumes a constant distribution of flow along the producing intervals. Hence, the pressure varies on the wellbore surface and an equivalent point[1,4] has to be determined for the pressure measurement. The location of this point depends on the geometry of the well and the instant t of measurement.
- The infinite conductivity option is a refinement of the first option : the well is divided into n segments, and the pressure is measured in n points. Each of these segments produces under uniform flux conditions, but the n flows per unit length are modified until convergence of the n pressures is obtained. This option is supposed to realize a good approximation of the real, ideal infinite-conductivity solution (flows per unit length vary in space and time, for a given total constant flow rate).

ROSA et al.[4] have showed that, for a horizontal well, pressure responses converge for n > 125 if the division of the well is isometric. But along the well axis, $\frac{dQ}{dx}$ varies more towards the extremities than in its center (see figs. 4a/b/c). Hence, for a given n, the solution will be more accurate if the well is divided more finely at its extremities.

The exact solution is known in 2-D (infinite-conductivity fracture). The distribution of flow for a line-source producing Q between x = 0 and x = +L is, under steady-state conditions :

$$\frac{dQ}{dx} = \frac{Q}{L}\,\frac{1/\pi}{\sqrt{\frac{x}{L} - \left(\frac{x}{L}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

The division of the well for the present 3-D model is directly inspired by the 2-D distribution. For a linear well producing Q between x = 0 and x = L, each segment $[x_{i-1}, x_i]$ is defined by $x_i = \frac{L}{2}\left(1 - \cos\frac{\pi i}{n}\right),\ 0 \le i \le n$.. [3]

PSEUDO SKIN FACTOR

Set the following dimensionless variables, in S.I. units :

$$P_D = \frac{2\,\pi\,k\,H}{Q\,B\,\mu}\,\Delta P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

$$t_D = \frac{k\,t}{\phi\,\mu\,c_t\,r_w^{\,2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

$$r_D = \frac{r}{r_w}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

where r is the distance from the well axis to the point of measurement.

For a vertical well producing in an infinite slab reservoir, the logarithmic approximation of the dimensionless pressure is :

for $t_D \geq 25{r_D}^2$,

$$P_D = \tfrac{1}{2}(\ln t_D + 0.809) + (S - \ln r_D) \quad \text{[7]}$$

where S is the apparent skin effect : S = 0 if the well is fully penetrating and without skin damage. At the wellbore face, $r_D = 1$ and eq.[7] becomes :

$$P_D = \tfrac{1}{2}(\ln t_D + 0.809) + S \quad \text{[8]}$$

It has been shown that, for slanted wells[1] and horizontal wells[4], the long-term approximation for the pressure solution has the same form :

$$P_D = \tfrac{1}{2}(\ln t_D + 0.809) + Sg \quad \text{[9]}$$

where Sg is the geometrical pseudo skin factor.

The time needed to reach the logarithmic pressure decline, and use eq.[9], is many orders of magnitude greater than for a vertical well : $t_D \geq 10^4$ to 10^{10}, depending on the geometry of the well[1,4].

Results

Results of the simulations are summarized in tables 1a/b/c/d/e. Dimensionless times have been chosen in logarithmic progression, to highlight the beginning of the logarithmic pressure decline.

Other parameters are :

$L_D = \frac{L}{H}$ = well - length to reservoir-thickness ratio.

P_D = dimensionless pressure for uniform - flux solution, for an equivalent - point ordinate $x_{eq.} = 0.85$ L (corresponding to a dimensionless distance of 0.7 to the center of the well, by analogy with well-test analysis[4]).

P_D^* = dimensionless pressure for infinite-conductivity solution, the well being divided into 10 segments. Dimensionless lengths of the segments are (eq.[3]) :

$L_{iD} = \frac{L_i}{L}$ = 0.024, 0.071, 0.111, 0.139, 0.155, 0.155, 0.139, 0.111, 0.071 and 0.024 for $1 \leq i \leq 10$.

S_{gH}^*, S_{gs}^* = Geometrical pseudo skin factors for horizontal and slanted wells, obtained from the infinite-conductivity solution P_D^*. The dimensionless pressure response for a vertical well appears at the top of tables 1b/d, for L/H = 1.

$H_D = \frac{H}{r_w}$ = reservoir-thickness to wellbore-radius ratio.

For horizontal wells, H_D ranges usually from 10^2 to 10^3. Hence, two values have been selected to check the sensitivity of the results to the dimensionless thickness H_D :

H_D = 100 for tables 1a/b and H_D = 1000 for tables 1c/d.

Tables 1a and 1c refer to a horizontal centered well ; additional runs have been performed for an off-centered well (see table 1e) :

$e_D = \frac{e}{H}$ = dimensionless eccentricity, e being the vertical distance between the well and the center of the reservoir ($-\frac{H}{2} \leq e \leq \frac{H}{2}$).

S_{gHc}^* = Geometrical pseudo skin factor for a horizontal centered well, already reported in tables 1a and 1c.

Distribution of flow

Figures 4a/b/c illustrate the influence of several parameters on the distribution of flow along the well axis.

For a horizontal well, the flow rate per unit length $\frac{dQ}{dx}$ is the lowest at the center of the well : $0.8 \frac{Q}{L} \leq \frac{dQ}{dx} \leq 0.9 \frac{Q}{L}$, and the ratio $\frac{dQ/dx}{Q/L}$ decreases when L/H increases (fig.4a).

Meanwhile, the point where $\frac{dQ}{dx} = \frac{Q}{L}$ remains close to the equivalent point ($0.825\ L \leq x_{eq} \leq 0.85\ L$ after ROSA et al.[4]). Although not mathematically exact, there is a strong similitude between the equivalent point, for which uniform-flux solution matches infinite - conductivity solution, and the average flowrate point for which $\frac{dQ}{dx} = \frac{Q}{L}$.

The same remarks command attention with slanted wells (fig.4c). CINCO et al.[1] have established that the equivalent points $x_{eq} = 0.2$ L and 0.8 L coincide with the average wellbore pressure points (average of the cylindrical surface in the uniform-flux solution).

Here they also coincide with the average flowrate points.

The distribution of flow is more equalized for a slanted well than for a horizontal well, even for a large L/H. It is explained by the compensating effect of the no-flow boundaries towards the extremities of the slanted well.

Eccentricity has little influence on the distribution of flow along a horizontal well (fig.4b). It will be showed thereafter that the additional pressure drop caused by eccentricity becomes neglectable when L/H is high.

HORIZONTAL WELL in isotropic medium

Three formulae will be presented here, all derived from analytical solutions based on the uniform-flux assumptions. They also use the same geometrical approach, consisting in dividing the 3D flow problem into two 2D problems, one in the horizontal plane (pseudo-radial flow) and one in the vertical plane perpendicular to the well axis (pseudo-linear flow).

GIGER[8] has expressed a formula for steady-state flow conditions :

$$P_{DH} = \ln\left(\frac{4r_e}{L}\right) + \frac{H}{L}\ln\left(\frac{h}{2\pi\, r_w \cos\frac{\pi e}{H}}\right) \quad [10]$$

where r_e accounts for the distance between the center of the well and the constant-pressure boundary, and $r_c > 2L$.

A vertical well under the same conditions would give :

$$P_{DV} = \ln\left(\frac{r_e}{r_w}\right) \quad [11]$$

Subtracting [11] to [10] yields the geometrical pseudo skin factor :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{h}{2\pi\, r_w}\,\frac{1}{\cos\frac{\pi e}{H}}\right) \quad [12]$$

S_{gH} may be divided into three components :

- $S_F = \ln\left(\frac{4r_w}{L}\right)$ represents the pseudo skin factor of a vertical fracture of the same length L as the horizontal well.

  It accounts for the reduction in pressure drop associated with the pseudo-radial horizontal flow : $S_F < 0$.

- $S_{VC} = \frac{H}{L}\ln\frac{h}{2\pi r_w}$ represents the additional pressure drop due to the vertical convergence of flow in the vicinity of a centered horizontal well : $S_{VC} > 0$.

- $S_e = \frac{H}{L}\ln\left(\frac{1}{\cos\frac{\pi e}{H}}\right)$ represents the additional pressure drop due to the eccentricity of the well : $S_e > 0$.

JOSHI[2] has proposed for the vertical convergence term $S_{VC} = \frac{H}{L}\ln\left(\frac{H}{2r_w}\right)$.

It only differs from GIGER'S formula by the constant π inside the logarithm. Pseudo skin factor for a centered well is then :

$$S_{gHc} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{h}{2r_w}\right) \quad [13]$$

JOSHI also expressed a corrective term for well eccentricity, written[2] as

$S_{VC} + S_e = \frac{H}{L}\ln\frac{(H/2)^2 - e^2}{Hr_w/2}$, which would lead to $S_e = \frac{H}{L}\ln(1-\left(\frac{2e}{H}\right)^2)$.

This is surprising indeed, since $S_e < 0$ comes against the belief that the effect of eccentricity is unfavourable to the productivity of the well. Therefore, it will be replaced by its opposite $S_e = -\frac{H}{L}\ln\left(1-\left(\frac{2e}{H}\right)^2\right)$ and the geometrical pseudo skin factor becomes, after JOSHI's :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{H}{2r_w}\,\frac{1}{1-\left(\frac{2e}{H}\right)^2}\right) \quad [14]$$

Finally, KUCHUCK et al.[7] have published an expression for a pseudo skin factor due to vertical convergence, rewritten as :

$$S_{VC} + S_e = \frac{H}{L}\left[-\ln\left(\frac{2\pi r_w}{H}\cos\left(\frac{\pi e}{H}\right)\right) - \frac{2}{L}\left(\frac{H}{12} + \frac{e^2}{H}\right)\right],\ \text{valid}$$

for $\frac{L}{H} > 0.4$. It leads to :

$$S_{gHc} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{h}{2\pi r_w}\right) - \frac{1}{6}\left(\frac{H}{L}\right)^2 \quad [15]$$

for a centered well, and :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{H}{2\pi r_w \cos\left(\frac{\pi e}{H}\right)}\right) - \left(\frac{H}{L}\right)^2\left(\frac{1}{6} + 2\left(\frac{e}{H}\right)^2\right) \quad [16]$$

for an off-centered well.

The difference with GIGER'S formula (eq.[12]) is a negative term in $\left(\frac{H}{L}\right)^2$, which will not be significant when the well length L is many times greater than thickness H.

Centered well

A necessary step of this work was to validate the results of the simulations, i.e. the approximate infinite conductivity solution obtained with the division of the well into 10 non-isometric segments. ROSA[4] has calculated another pseudo skin factor for a greater number of isometric segments (60 to 125), and the results Sp can be expressed with the present set of dimensionless variables as :

$$P_D = \frac{1}{2}(\ln t_D + 0.809) + \ln\frac{r_w}{L/2} + Sp^* \quad [17]$$

Compare with [9] to get :

$$S^*_{gH} = \ln\frac{2r_w}{L} + Sp^* \quad [18]$$

Transformation of Sp^* into S^*_{gH} is presented in table 2a and S_{gH} is compared with GIGER'S formula [12].

Fig. 1 displays a comparison between this work, ROSA'S data and the analytical formulae [12], [13] and [15]. The difference

$$\Delta S = S_{gHc} - \left[\ln\left(\frac{r_w}{L/4}\right) + \frac{H}{L}\ln\frac{H}{2\pi r_w}\right]$$

takes GIGER'S formula as an implicit reference. ΔS is plotted against dimensionless length $\frac{L}{H}$ and the graph calls for the following comments :

- The difference between ROSA'S data and this work is insignificant, hence the present infinite-conductivity solution offers excellent accuracy.
- GIGER'S formula is reasonably accurate ($\Delta S<0.1$) when $\frac{L}{H} \geq 1$, which is obviously the usual field of investigation since a horizontal well aims at increasing the producing length.
- JOSHI'S formula can be applied when the horizontal well is long, but could lead to wrong estimations for small values of L/H. In anisotropic reservoirs, this ratio changes into $\sqrt{\frac{kv}{kH}}\frac{L}{H}$ and is reduced drastically, as it will be discussed later.
- KUCHUK'S formula fits the correct trend for low values of L/H, and is recommended when $0.4 \leq \frac{L}{H} \leq 2$ (it has been checked also for $\frac{L}{H} = 0.5$ and 0.75) althrough this range presents little interest for horizontal wells application.

Off-centered well

Results for off-centered wells are presented in tables 1e and 2b.

The difference $S_e = S_{gH} - S_{gHc}$ between off-centered and centered wells is plotted against dimensionless eccentricity e/H, and compared with analytical formulae on fig.2.

The following remarks can be drawn :

- the influence of eccentricity is inversely proportional to ($\frac{L}{H}$) and it is in agreement with all quoted formulae : eccentricity acts as a nearby skin damage, the well length tends to reduce its effect ;
- GIGER'S formula ($S_e = -\frac{H}{L}\ln(\cos\frac{\pi e}{H})$) gives pessimistic results for small values of $\frac{L}{H}$, especially with a strong eccentricity when the well is very close to the upper or lower boundary of the reservoir ;
- JOSHI'S formula ($S_e = -\frac{H}{L}\ln(1-(\frac{2e}{H})^2)$) is accurate when $\frac{L}{H} \geq 2$ (see table 2b), but slightly pessimistic under 2 ;
- KUCHUK'S formula ($S_e = -\frac{H}{L}\ln(\cos\frac{\pi e}{H}) - 2(\frac{H}{L})^2(\frac{e}{H})^2$) is correct on the whole investigated range.

As a conclusion for horizontal wells, the following formulae are recommended :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left[\frac{H}{2\pi r_w}\,\frac{1}{1-(\frac{2e}{H})^2}\right] \quad \text{for } \frac{L}{H} \geq 2 \quad [19]$$

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left[\frac{H}{2\pi r_w}\,\frac{1}{\cos(\frac{\pi e}{H})}\right] - \left(\frac{H}{L}\right)^2\left[\frac{1}{6}+2\left(\frac{e}{H}\right)^2\right]$$

$$\text{for } \frac{L}{H} \geq 0.4 \quad [20]$$

SLANTED WELL in isotropic medium

KARCHER et al.[3] have proposed a formula to estimate the productivity of slanted wells under steady-state conditions, but its complexity prevents from a convenient use.

CINCO et al.[1] have set up a correlation for the geometrical pseudo skin factor of a slanted well :

$$S_\theta = -\left(\frac{\theta}{41}\right)^{2.06} - \left(\frac{\theta}{56}\right)^{1.865}\operatorname{Log}\left(\frac{H_D}{100}\right) \quad \text{for } \theta \leq 75° \quad [21]$$

This correlation is based on values listed in table 4, calculated with the uniform-flux solution and the equivalent point located at $x_{eq} = 0.8$ L.

The value S^*_{gs} obtained with the present model have been compared with eq.[12] for horizontal wells, then fitted by :

$$S_{gs} - S_{gH} = \ln\frac{\pi}{2} + \frac{1}{2}\ln\frac{L}{H} \quad [22]$$

Finally the pseudo skin factor S_{gs} can be written simply as :

$$S_{gs} = \ln\left(\frac{4r_w}{L}\right) + \frac{H}{L}\ln\left(\frac{\sqrt{LH}}{4r_w}\right) \quad [23]$$

Results for S^*_{gs} and S_{gs} are listed in table 3, and the difference $|S_{gs} - S^*_{gs}|$ never exceeds 0.03. Figure 3 shows that CINCO'S correlation (eq.[21]), does not apply for angles of slant $\theta \geq 75°$. GIGER'S formula (eq.[12]) has been drawn also to show that in an isotropic medium, a horizontal centered well is slightly better than a slanted well of the same length.

Finally, CINCO'S data S_θ are compared with eq.[23] in table 4 ; again, the agreement is excellent : $|S_{gs} - S_\theta| < 0.03$.

Hence, equation [23] can be applied to fully penetrating slanted wells from 0° to 90°.

ANISOTROPIC RESERVOIRS

Most authors have studied the influence of anisotropy on the performance of horizontal and slanted wells, and published formulae frequently use the horizontal-to-vertical anisotropy ratio $\sqrt{k_h/k_v}$.

By transforming both distances and permeabilities, one can adapt the flow equations from isotropic into anisotropic medium.

But such a spatial transformation implies a deformation of the circular well section into an ellipse, and the choice of an equivalent well radius in order to carry on the analytical approach.

Appendix A presents a general theory for spatial transformation, whose main innovations are :

- application to any 3-D anisotropy $k_x \neq k_y \neq k_z$
- transformation of the well radius for any azimuth and inclination of the well axis.

This section will be limited to the case of a basic anisotropy $k_V \neq k_H$. According to Appendix A, write the transformed parameters in the equivalent isotropic medium :

$$k' = k_h^{2/3}\, k_v^{1/3} \quad [24]$$

$$H' = \alpha^{2/3} H \quad [25]$$

$$tg\theta' = \frac{1}{\alpha} tg\theta \quad [26]$$

$$L' = \alpha^{2/3} \gamma L \quad [27]$$

$$r'_w = \frac{r_w}{\alpha^{1/3}} \frac{1 + 1/\gamma}{2}, \quad [28]$$

where :

$$\alpha = \sqrt{K_H/K_V} \text{ (anisotropy ratio)} \quad [29]$$

$$\gamma = \sqrt{\cos^2\theta + \frac{1}{\alpha^2}\sin^2\theta} \quad [30]$$

An easy mean to remember the transformation is that horizontal distances are divided by $\alpha^{1/3}$, while vertical distances are multiplied by $\alpha^{2/3}$. For a vertical well, $\gamma = 1$ and

$$r'_{wv} = \frac{r_w}{\alpha^{1/3}}.$$

For a horizontal well, $\gamma = \frac{1}{\alpha}$, $L' = \frac{L}{\alpha^{1/3}}$ and

$$r'_{wH} = \frac{r_w}{\alpha^{1/3}} \frac{1+\alpha}{2}$$

Equation [8] becomes,for a vertical well without skin damage :

$$P'_D = \frac{1}{2}(\ln t'_D + 0.809) \quad [31]$$

Since $k'H' = k_H H$ and $k'/r'^2_{wv} = k_H/r_w^2$,

$$P'_D = \frac{2\pi k_H H}{Q B \mu} \Delta P \quad [32]$$

$$t'_D = \frac{k_H t}{\phi \mu c_t r_w^2} \quad [33]$$

For a non-vertical well, equation [9] gives :

$$P'_D = \frac{1}{2}(\ln t'_D + 0.809) + \ln\frac{r'_{wv}}{r'_w} + S'_g \quad [34]$$

and the geometrical pseudoskin factor in anisotropic medium is :

$$S_g = \ln\frac{r'_{wv}}{r'_w} + S'_g \quad [35]$$

Use now equations [19] and [20] to get for a horizontal well :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{\alpha H}{L}\ln\left(\frac{H}{2\pi r_w}\frac{2\alpha}{1+\alpha}\frac{1}{1-(\frac{2e}{H})^2}\right) \quad \text{for } \frac{L}{\alpha H} \geq 2 \quad [36]$$

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{\alpha H}{L}\ln\left(\frac{H}{2\pi r_w}\frac{2\alpha}{1+\alpha}\frac{1}{\cos\frac{\pi e}{H}}\right) - \left(\frac{\alpha H}{L}\right)^2\left(\frac{1}{6} + 2(\frac{e}{H})^2\right) \quad \text{for } \frac{L}{\alpha H} \geq 0.4 \quad [37]$$

For a slanted well, $H/L = \cos\theta$ and :

$$\gamma = \sqrt{\frac{1}{\alpha^2} + \frac{H^2}{L^2}\left(1 - \frac{1}{\alpha^2}\right)} \quad [38]$$

Use now equation [23] and write, for a slanted well :

$$S_{gs} = \ln\left(\frac{4r_w}{L}\frac{1}{\alpha\gamma}\right) + \frac{H}{\gamma L}\ln\left(\frac{\sqrt{LH}}{4r_w}\frac{2\alpha\sqrt{\gamma}}{1+1/\gamma}\right) \quad [39]$$

Applications

Figures 5 to 8 present several charts obtained from equations [37] and [39], for values of $\frac{H}{r_w}$ = 100, 200, 500 and 1000.

On each figure, pseudoskin factors are plotted against the well length and for k_H/k_V = 1, 3, 10, 30 and 100.

Slanted wells are less affected by vertical anisotropy than horizontal wells : there is a critical length under which a slanted well will offer better performances than a horizontal well (see figure 9).

Take an example : a horizontal well with radius $r_w = 0.1$ m. has to be drilled into a 20 m thick reservoir, where the expected anisotropy is $0.01\ k_H \leq k_V \leq 0.1\ k_H$.

On figure 9, consider the curve $\frac{H}{r_w} = 200$

then find the critical lengths corresponding to $K_H/K_V = 10$ and $K_H/K_V = 100$, that is L/H = 5 and 17.

A horizontal well more than 340 m long will be better than a slanted one, and worse if less than 100 m. Between 100 and 340 m, it will depend on anisotropy but the risk of poor performance will be reduced with a slanted trajectory.

There are many ways to estimate then the productivity indexes, depending on the reservoir feeding conditions and the geometry of the drainage area. For steady-state flow and a large drainage area ($r_e > 2L$), write simply :

$$PI = \frac{2\pi H k_H / B\mu}{\ln (r_e/r_w) + S_g} \qquad [40]$$

where constant (2π) should be replaced by 0.00708 (U.S. field units) or 0.05358 (metric units).

For complex drainage areas or closed reservoirs (pseudo steady-state conditions), formulae used for horizontal wells can be adapted to slanted wells by substituting the proper components of the pseudoskin factor (eq. [39]), accounting either for horizontal flow or vertical convergence :

- $S_F = \ln\left(\frac{4r_w}{L\alpha\gamma}\right)$, pseudoskin factor for a vertical fracture of length $[L\alpha\gamma]$. Hence, the slanted well should be studied like a horizontal well of length $[L\alpha\gamma]$.

- $S_{vc} = \frac{H}{\gamma L} \ln\left(\frac{\sqrt{LH}}{4r_w}\frac{2\alpha\sqrt{\gamma}}{1+1/\gamma}\right)$, pseudoskin factor for convergence of the flow near the well, should replace the pseudoskin factor of vertical convergence for a horizontal well.

CONCLUSIONS

- The performance of horizontal and slanted wells can be studied through the definition of a geometrical pseudoskin factor : long-time performance is the same than for a fully penetrating vertical well with a wellbore skin factor.

- In anisotropic reservoirs, horizontal wells are affected by the anisotropy ratio $\alpha = \sqrt{k_H/k_V}$. The most accurate formula for the pseudoskin factor is :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{\alpha H}{L}\ln\left(\frac{H}{2\pi r_w}\frac{2\alpha}{1+\alpha}\frac{1}{\cos\frac{\pi e}{H}}\right) \ldots$$

$$\ldots - \left(\frac{\alpha H}{L}\right)^2\left(\frac{1}{6} + 2\left(\frac{e}{H}\right)^2\right) \quad \text{for } \frac{L}{\alpha H} \geq 0.4$$

In most cases, $\frac{L}{\alpha H} \geq 2$ and the following formula is easier to use :

$$S_{gH} = \ln\left(\frac{4r_w}{L}\right) + \frac{\alpha H}{L}\ln\left(\frac{H}{2\pi r_w}\frac{2\alpha}{1+\alpha}\frac{1}{1-\left(\frac{2e}{H}\right)^2}\right)$$

- Slanted wells are less influenced by anisotropy, and can be evaluated by :

$$S_{gs} = \ln\left(\frac{4r_w}{L}\frac{1}{\alpha\gamma}\right) + \frac{H}{\gamma L}\ln\left(\frac{\sqrt{LH}}{4r_w}\frac{2\alpha\sqrt{\gamma}}{1+1/\gamma}\right), \text{ valid}$$

for any angle of slant, with :

$$\gamma = \sqrt{\frac{1}{\alpha^2} + \frac{H^2}{L^2}\left(1-\frac{1}{\alpha^2}\right)}.$$

- In isotropic reservoirs, a horizontal well is slightly better than a slanted well of the same producing length. But in anisotropic reservoirs, there exists a critical length under which the slanted well becomes better than the horizontal ones. Graphic charts are provided to easy the comparison.

- The spatial transformation detailed in Appendix can be applied to convert any formula, correlation or flow equation from isotropic into anisotropic medium.

NOMENCLATURE

B	= formation volume factor
a,b,c	= coefficients of the spatial transformation in the x,y,z directions.
c_t	= total compressibility.
e	= eccentricity of the horizontal well (distance between middle of the reservoir and well axis).
H	= reservoir thickness.
H_D	= dimensionless thickness, H/r_w.
k	= permeability of the porous medium.
k_x,k_y,k_z	= permeabilities in the x,y,z directions.
L	= producing length of the well.
L_D	= dimensionless length, L/H.
P_D	= dimensionless pressure.
Q	= production flowrate.

r = distance to the source.

r_D = dimensionless distance to the well axis, r/r_w.

re = distance to the external boundary of the reservoir.

r_w = wellbore radius.

R_1, R_2 = minor and major radius of the wellbore (elliptic).

S = skin factor (see subscripts below).

t_D = dimensionless time.

x_{eq} = abscissa of the point of the wellbore where the pressure is measured for the uniform flux solution (equivalent point).

x,y,z = cartesian coordinates in the reference system of the reservoir (upper and lower boundaries parallel to Oxy plane).

u,v,w = cartesian coordinates in the reference system of the well (well axis parallel to O_w).

Greek symbols :

α = anisotropy ratio $\sqrt{k_H/k_V}$.

β, γ = geometrical factors of the well.

μ = fluid viscosity.

ϕ = rock porosity.

Φ = potential function.

φ = azimuth of the well.

θ = inclination of the well ($\theta = 0°$ for vertical well).

Subscripts (of pseudoskin factors) :

e = pseudoskin for eccentricity.

F = pseudoskin for vertical fracture.

g = geometrical skin.

H = horizontal well.

HC = horizontal, centered well.

S = Slanted well.

p = pseudoskin factor, after Rosa et al[4] (horizontal, centered well).

θ = pseudoskin factor, after Cinco et al[1] (slanted well, angle θ)

vc = pseudoskin for vertical convergence.

Superscripts :

* = values calculated for infinite-conductivity solution.

' = transformed distances in the equivalent isotropic medium.

AKNOWLEDGEMENTS :

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APPENDIX : SPATIAL TRANSFORMATION FROM ANISOTROPIC INTO EQUIVALENT ISOTROPIC MEDIUM

Purpose of transformation

Let a linear well, defined by its angle of slant, its length and its wellbore radius, produce in an anisotropic reservoir under conditions of monophasic flow and constant compressibility.

The analytical solution, suitable for an isotropic medium, is entirely defined by :

- the diffusivity equation
- a set of boundary conditions, both internal (at the wellbore) and external (outer limits).

The equivalent isotropic medium, transformed from the real medium through an operator T, should afford for a same flowrate history Q(t), a same pressure response P(t) at the transformed well.

Diffusivity equation

The diffusivity equation in the real medium is :

$$\mathrm{div}\left[\bar{k}\ \vec{\mathrm{grad}}\ [\Phi(M,t)]\right] = \phi\mu C_t \frac{\partial\Phi(M,t)}{\partial t} \quad \ldots [A\text{-}1]$$

where : Φ = flow potential
$\bar{k}$ = permeability tensor
M = (x,y,z)
μ = fluid viscosity
ϕ = porosity
C_t = total compressibility

Choosing the same axes for spatial coordinates and the main directions of the permeability tensor, we can write :

$$\bar{k} = \begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{pmatrix}$$

Let T be $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ and $M' = T(M)$;

$$a=\frac{\sqrt{k_yk_z}}{\sqrt[3]{k_xk_yk_z}},\quad b=\frac{\sqrt{k_xk_z}}{\sqrt[3]{k_xk_yk_z}}\quad \text{and}\quad c=\frac{\sqrt{k_xk_y}}{\sqrt[3]{k_xk_yk_z}} \quad \ldots [A\text{-}2]$$

Or with $k_H = \sqrt{k_xk_y}$ and $k_v = k_z$:

$$a=\sqrt[6]{\frac{k_v}{k_H}}\sqrt[4]{\frac{k_y}{k_x}},\quad b=\sqrt[6]{\frac{k_v}{k_H}}\sqrt[4]{\frac{k_y}{k_x}}\quad \text{and}\quad c=\sqrt[3]{\frac{k_H}{k_v}} \quad [A\text{-}2bis]$$

The equation [A-1] becomes then :

$$\sqrt[3]{k_xk_yk_z}\ \mathrm{div}\left[\vec{\mathrm{grad}}\ (\Phi(M,t))\right] = \phi\mu c_t \frac{\partial\Phi(M,t)}{\partial t} \ldots [A\text{-}3]$$

Being rewritten as : $\nabla^2\Phi'(M',t) = \frac{\phi\mu c_t}{k'}\frac{\partial\Phi'(M',t)}{\partial t}$,

it represents the diffusivity in an isotropic medium, with potential : $\Phi'(M',t) = \Phi(M,t)$

and permeability : $k' = \sqrt[3]{k_xk_yk_z}$

Boundary conditions :

For most analytical solutions, the boundary conditions are either surfaces with constant potential, or surfaces with constant flow. Both cases are examined thereafter.

An equipotential surface is obviously transformed into another equipotential surface, since $\Phi'(M',t) = \Phi(M,t)$.

The conservation of flow through a transformed surface requires a rigourous demonstration, although it can be admitted intuitively since T keeps the volumes (abc = 1). An elementary surface $\vec{ds}$ is transformed into $\vec{ds}'$ through the operator :

$$\begin{pmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix} = T^{-1}$$

Write then the elementary flow :

$$dq' = -\frac{k'}{\mu}\ \vec{\mathrm{grad}}\ \Phi'(M',t)\cdot\vec{ds}'$$

Substitute $\bar{k}' = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}\bar{k}$ and $\vec{\mathrm{grad}}\ \Phi' = T^{-1}(\vec{\mathrm{grad}}\ \Phi)$

to get : $dq' = -\frac{\bar{k}}{\mu}\ \vec{\mathrm{grad}}\ \Phi(M,t)\cdot\vec{ds} = dq$,

which is the elementary flow through $\vec{ds}$ in the real medium.

Transformation of outer limits :

The problem is tricky when dealing with non-planes, like domes and anticlines. In most cases, the reservoir can be modelled as an isopach layer, limited vertically by plane adjacent beds. The reservoir thickness is then transformed into H' = cH.

Laterally, a square or rectangular drainage area will be transformed into a rectangle (with different aspect ratio if $k_x \# k_y$)

or even a lozenge if the sides are not parallel with the main axes O_x and O_y.

Transformation of the well :

Let the real well be defined by :

- length L
- radius r_w
- azimuth φ, $0 \le \varphi \le \frac{\pi}{2}$ (relatively to O_x)
- deviation θ, $0 \le \theta \le \frac{\pi}{2}$ (relatively to O_z)

The transformed well is then associated with :

- length $L' = L\sqrt{c^2\cos^2\theta + (a^2\cos^2\varphi + b^2\sin^2\varphi)\sin^2\theta}$ ...... [A-4]

- azimuth $\operatorname{tg}\varphi' = \frac{b}{a}\operatorname{tg}\varphi$ .... [A-5]

- deviation $\operatorname{tg}\theta' = \operatorname{tg}\theta \dfrac{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}{c}$ [A-6]

While the real well has a circular section, the transformed well is elliptical. Here appears the problem of the equivalent well radius, already encountered in the litterature about horizontal wells in anisotropic reservoirs.

Brigham[9] has discussed the choice of the equivalent radius, and its influence on early-time well-test interpretation and late-time deliverability prediction. His conclusions are valid whatever the geometry of the well : the correct equivalent radius is the arithmetic average of the major and minor axes of the elliptic section. In a plane perpendicular to the well axis, equipotential lines are homofocal ellipses which rapidly degenerate into circles when moving away from the well. The arithmetically-averaged radius is the one for which equipotential circles match the equipotential ellipses obtained with the transformed well.

Define now the anisotropy ratio :

$$\alpha = \sqrt{\frac{k_H}{k_V}} = c^{3/2} \quad \text{..........} \quad [A\text{-}7]$$

And two geometrical factors dependant on the angles θ and φ :

$$\beta = \sqrt{\frac{a}{b}\cos^2\varphi + \frac{b}{a}\sin^2\varphi} \quad \text{..........} \quad [A\text{-}8]$$

$$\gamma = \sqrt{\cos^2\theta + \left(\frac{a^2}{c^2}\cos^2\varphi + \frac{b^2}{c^2}\sin^2\varphi\right)\sin^2\theta} \quad [A\text{-}9]$$

Coordinates of the real well and transformed well in the reference systems Oxyz and Ox'y'z' are related by $x' = ax$, $y' = by$ and $z' = az$; two other reference systems will be defined, Ouvw and Ou'v'w', with Ow and Ow' parallel respectively to the axes of the real well and the transformed well.

The following calculations aim at defining the transformed-well section perpendicular to axis Ow', i.e. located in the plane Ou'v'.

Write first :

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\varphi & -\cos\theta\sin\varphi & -\sin\theta \\ \sin\varphi & \cos\varphi & 0 \\ \sin\theta\cos\varphi & -\sin\theta\sin\varphi & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad [A\text{-}10]$$

and :

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\varphi'\cos\theta' & \sin\varphi' & \cos\varphi'\sin\theta' \\ -\sin\varphi'\cos\theta' & \cos\varphi' & -\sin\varphi'\sin\theta' \\ -\sin\theta' & 0 & \cos\theta' \end{pmatrix} \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} \quad [A\text{-}11]$$

Combine [A-10] and [A-11] after simplification and substitution of a,b,c by α,β,γ (eq. [A-7], [A-8], [A-9]), note that u and v do not depend on w :

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha^{1/3}\frac{\gamma}{\beta} & \frac{\alpha}{\beta}\cos\theta\cos\varphi\sin\varphi(b^2-a^2) \\ 0 & \alpha^{1/3}\beta \end{pmatrix} \begin{pmatrix} u' \\ v' \end{pmatrix} \quad [A\text{-}12]$$

The real well is a cylinder generated by a circle defined in the plane Ouv :

$$u^2 + v^2 = r_w^2 \quad \text{..........} \quad [A\text{-}13]$$

Its image is an ellipse, whose projection on the plane Ou'v' generates the transformed well :

$$\alpha^{2/3}\frac{\gamma^2}{\beta^2}u'^2 + \left[\frac{\alpha^2}{\beta^2}\cos^2\theta\cos^2\varphi\sin^2\varphi(b^2-a^2)^2 + \alpha^{2/3}\beta^2\right]v'^2$$
$$+ 2\alpha^{4/3}\frac{\gamma}{\beta^2}\cos\theta\cos\varphi\sin\varphi(b^2-a^2)u'v' = r_w^2 \ . \quad [A\text{-}14]$$

Call R_1 and R_2 the major and minor radii of the ellipse, then write :

$$\left(\frac{r_w}{R_1}\right)^2 + \left(\frac{r_w}{R_2}\right)^2 = \alpha^{2/3}\frac{\gamma^2}{\beta^2} + \frac{\alpha^2}{\beta^2}\cos^2\theta\cos^2\varphi\sin^2\varphi(b^2-a^2)^2$$
$$+ \alpha^{2/3}\beta^2 \quad \text{..........} \quad [A\text{-}15]$$

Moreover, the transformation T keeps the volumes unchanged :

$$\pi r_w^2 L = \pi R_1R_2L',$$

$$\text{or :} \quad R_1R_2 = \frac{r_w^2}{c\gamma} = \frac{r_w^2}{\alpha^{2/3}\gamma} \quad \text{..........} \quad [A\text{-}16]$$

Substitute [A-15] and [A-16] into :

$$\frac{R_1+R_2}{2} = \frac{R_1R_2}{2}\sqrt{\left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{R_2}\right)^2 + \frac{2}{R_1R_2}}$$

to get :

$$\frac{R_1+R_2}{2} = \frac{r_w}{2\alpha^{1/3}}\frac{1}{\beta}\sqrt{\left(1+\frac{\beta^2}{\gamma}\right)^2+\left[(b^2-a^2)\frac{\alpha^{2/3}}{\gamma}\cos\theta\cos\varphi\sin\varphi\right]^2} \quad \text{[A-17]}$$

$r_w' = \frac{R_1+R_2}{2}$ is the equivalent radius of the transformed well.

The transformation of the well is summarized by equations [A-7], [A-8] and [A-9] rewritten as :

$$\alpha = \sqrt{\frac{k_H}{k_V}} \quad \text{[A-7]}$$

$$\beta = \sqrt{\sqrt{\frac{k_y}{k_x}}\cos^2\varphi + \sqrt{\frac{k_x}{k_y}}\sin^2\varphi} \quad \text{[A-18]}$$

$$\gamma = \sqrt{\cos^2\theta + \frac{k_V}{k_H}\beta^2\sin^2\theta} \quad \text{[A-19]}$$

and the transformed-well characteristics from equations [A-4], [A-5], [A-6] and [A-17] :

$$L' = \alpha^{2/3}\gamma L \quad \text{[A-20]}$$

$$tg\varphi' = \sqrt{\frac{k_x}{k_y}}\, tg\varphi \quad \text{[A-21]}$$

$$tg\theta' = \frac{\beta}{\alpha}\, tg\theta \quad \text{[A-22]}$$

$$r_w' = \frac{r_w}{\alpha^{1/3}}\frac{1}{2\beta} \times \ldots$$

$$\sqrt{\left(1+\frac{\beta^2}{\gamma}\right)^2 + \left[\left(\sqrt{\frac{k_x}{k_y}} - \sqrt{\frac{k_y}{k_x}}\right)\frac{\cos\theta\cos\varphi\sin\varphi}{\gamma}\right]^2}. \quad \text{[A-23]}$$

Application : vertical-to-horizontal anisotropy $k_V \neq k_H$:

In most practical cases, $k_x = k_y$ and the permeability tensor reduces to :

$$\bar{k} = \begin{pmatrix} k_H & 0 & 0 \\ 0 & k_H & 0 \\ 0 & 0 & k_V \end{pmatrix}$$

Since the geometrical factor β equals 1, equations [A-19] to [A-23] simplify into (keeping $\alpha = \sqrt{k_H/k_V}$) :

$$\gamma = \sqrt{\cos^2\theta + \frac{1}{\alpha^2}\sin^2\theta} \quad \text{[A-24]}$$

$$L' = \alpha^{2/3}\gamma L \quad \text{[A-25]}$$

$$tg\varphi' = tg\varphi \quad \text{[A-26]}$$

$$tg\theta' = \frac{1}{\alpha} tg\theta \quad \text{[A-27]}$$

$$r_w' = \frac{r_w}{\alpha^{1/3}}\frac{1}{2}\left(1+\frac{1}{\gamma}\right) \quad \text{[A-28]}$$

For a vertical well ($\theta = 0°$), $\gamma = 1$ and :

$$L_V' = \alpha^{2/3} L_V = \sqrt[3]{\frac{k_H}{k_V}}\, L_V \quad \text{[A-29]}$$

$$\theta' = 0°$$

$$r'_{w_V} = \frac{r_{w_V}}{\alpha^{1/3}} = \sqrt[3]{\frac{k_V}{k_H}}\, r_{w_V} \quad \text{[A-30]}$$

For a horizontal well ($\theta = 90°$), $\gamma = \frac{1}{\alpha}$ and :

$$L_H' = \frac{L_H}{\alpha^{1/3}} = \sqrt[6]{\frac{k_V}{k_H}}\, L_H \quad \text{[A-31]}$$

$$\theta' = 90°$$

$$r'_{w_H} = \frac{r_{w_H}}{\alpha^{1/3}}\frac{1+\alpha}{2} = \sqrt[6]{\frac{k_V}{k_H}}\left(\frac{1+\sqrt{\frac{k_H}{k_V}}}{2}\right) r_w \quad \text{[A-32]}$$

Finally, horizontal distances D and vertical thicknesses H are transformed into :

$$D' = \frac{D}{\alpha^{1/3}} = \sqrt[6]{\frac{k_V}{k_H}}\, D, \quad \text{[A-33]}$$

and $H' = \alpha^{2/3}H = \sqrt[3]{\frac{k_H}{k_V}}\, H$ .... [A-34]

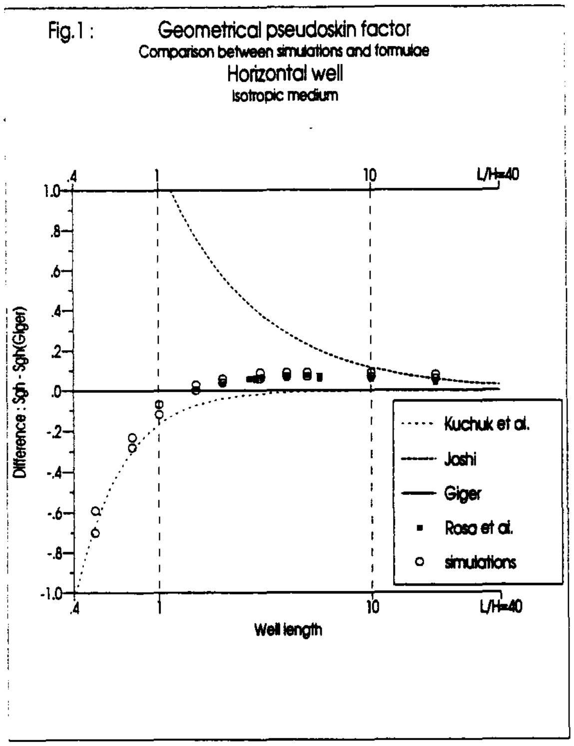
Fig.1 :
Geometrical pseudoskin factor
Comparison between simulations and formulae
Horizontal well
Isotropic medium
Difference : Sgh - Sgh(Giger)
Well length
L/H=40
Kuchuk et al.
Joshi
Giger
Rosa et al.
simulations

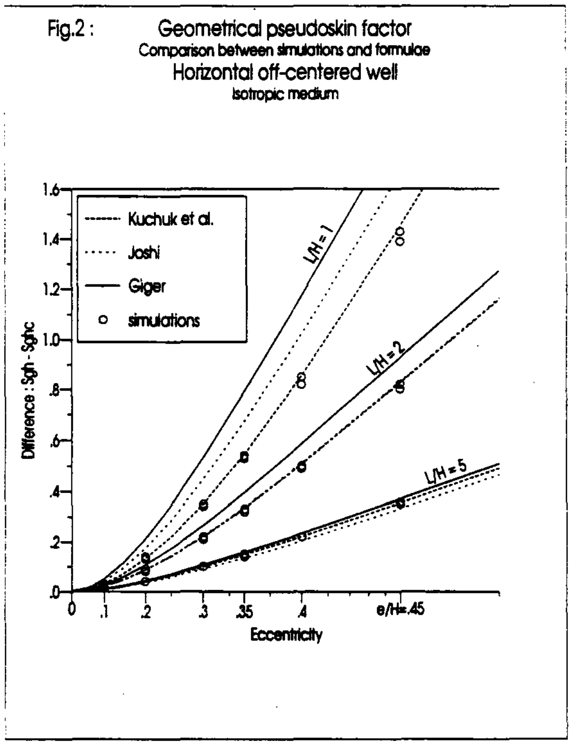
Fig.2 :
Geometrical pseudoskin factor
Comparison between simulations and formulae
Horizontal off-centered well
Isotropic medium
Difference : Sgh - Sghc
Eccentricity
e/H=.45
L/H=1
L/H=2
L/H=5
Kuchuk et al.
Joshi
Giger
simulations

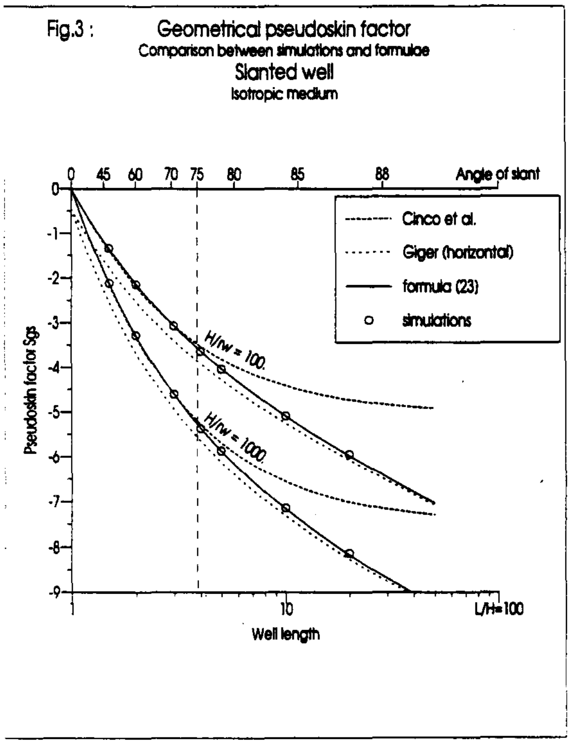
Fig.3 :
Geometrical pseudoskin factor
Comparison between simulations and formulae
Slanted well
Isotropic medium
Angle of slant
Pseudoskin factor Sgs
Well length
L/H=100
H/rw = 100.
H/rw = 1000.
Cinco et al.
Giger (horizontal)
formula (23)
simulations

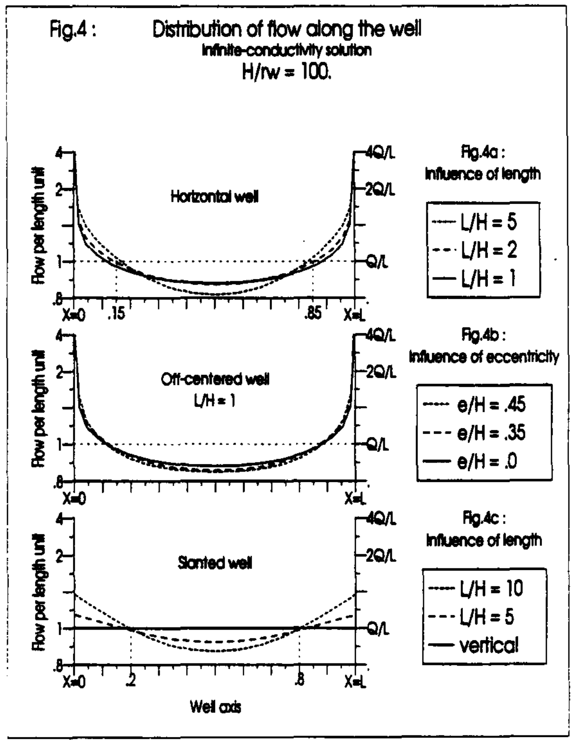
Fig.4 :
Distribution of flow along the well
Infinite-conductivity solution
H/rw = 100.
Flow per length unit
Horizontal well
Fig.4a :
Influence of length
L/H = 5
L/H = 2
L/H = 1
Off-centered well
L/H = 1
Fig.4b :
Influence of eccentricity
e/H = .45
e/H = .35
e/H = .0
Slanted well
Fig.4c :
Influence of length
L/H = 10
L/H = 5
vertical
4Q/L
2Q/L
Q/L
X=0
X=L
Well axis

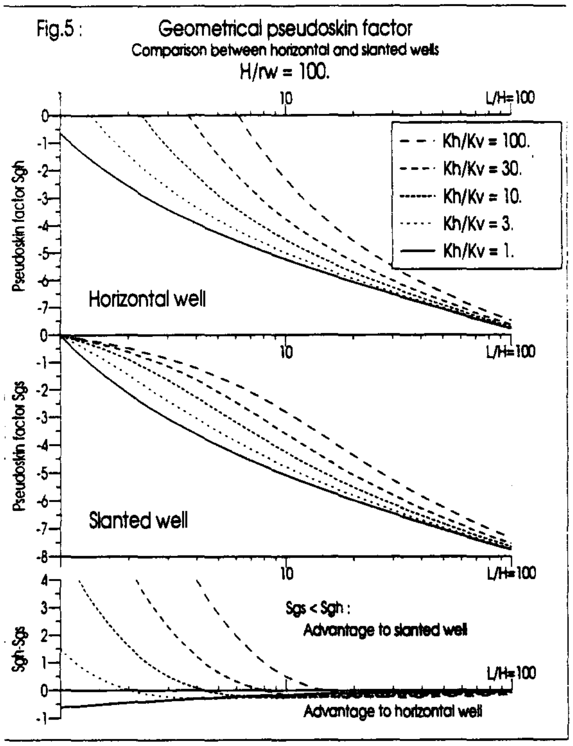
Fig.5 :
Geometrical pseudoskin factor
Comparison between horizontal and slanted wells
H/rw = 100.
Kh/Kv = 100.
Kh/Kv = 30.
Kh/Kv = 10.
Kh/Kv = 3.
Kh/Kv = 1.
Pseudoskin factor Sgh
Horizontal well
Pseudoskin factor Sgs
Slanted well
Sgh-Sgs
Sgs < Sgh :
Advantage to slanted well
Advantage to horizontal well
L/H=100

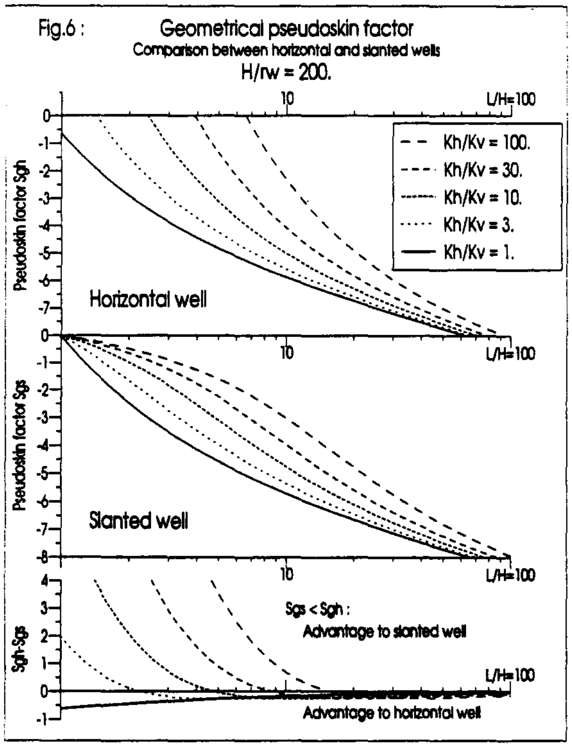
Fig.6 :
Geometrical pseudoskin factor
Comparison between horizontal and slanted wells
H/rw = 200.
Kh/Kv = 100.
Kh/Kv = 30.
Kh/Kv = 10.
Kh/Kv = 3.
Kh/Kv = 1.
Pseudoskin factor Sgh
Horizontal well
Pseudoskin factor Sgs
Slanted well
Sgh-Sgs
Sgs < Sgh :
Advantage to slanted well
Advantage to horizontal well
L/H=100

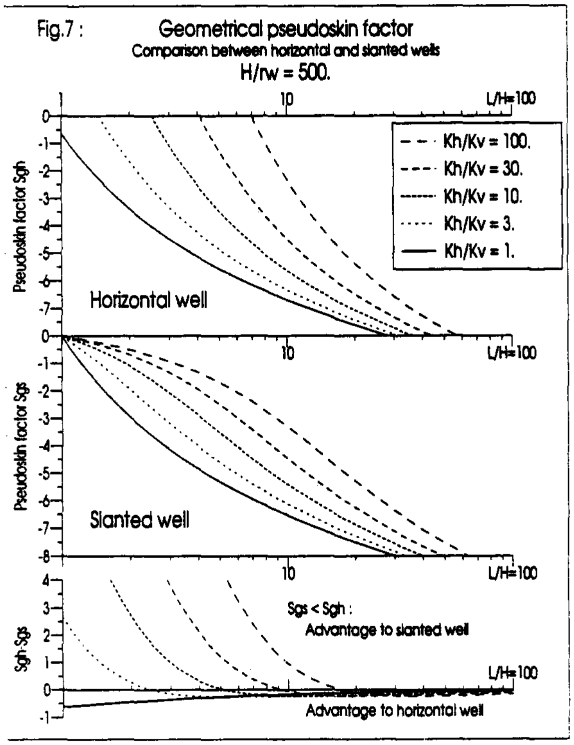
Fig.7 :
Geometrical pseudoskin factor
Comparison between horizontal and slanted wells
H/rw = 500.
Kh/Kv = 100.
Kh/Kv = 30.
Kh/Kv = 10.
Kh/Kv = 3.
Kh/Kv = 1.
Pseudoskin factor Sgh
Horizontal well
Pseudoskin factor Sgs
Slanted well
Sgh-Sgs
Sgs < Sgh :
Advantage to slanted well
Advantage to horizontal well
L/H=100

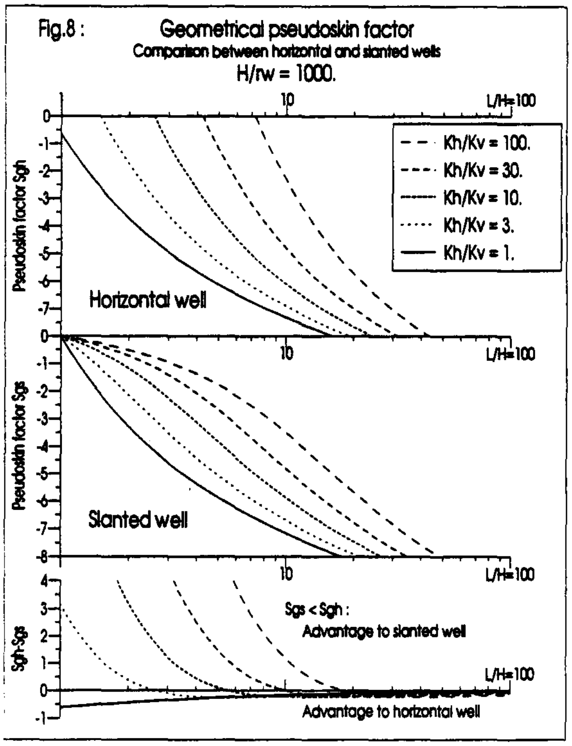
Fig.8 :
Geometrical pseudoskin factor
Comparison between horizontal and slanted wells
H/rw = 1000.
Kh/Kv = 100.
Kh/Kv = 30.
Kh/Kv = 10.
Kh/Kv = 3.
Kh/Kv = 1.
Pseudoskin factor Sgh
Horizontal well
Pseudoskin factor Sgs
Slanted well
Sgh-Sgs
Sgs < Sgh :
Advantage to slanted well
Advantage to horizontal well
L/H=100

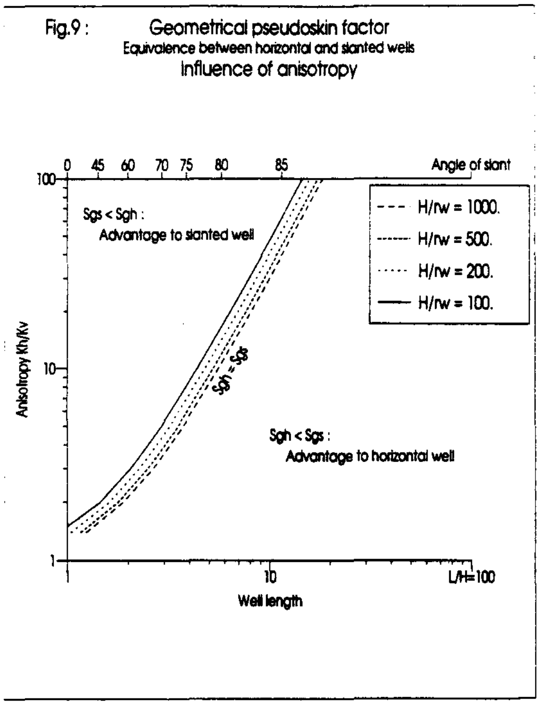

Table 1a — Horizontal centered well

L/H	t_D	P_D	P_D^*	S_{gH}^*	S_{gH} GIGER
1	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6	3.60 4.47 5.45 6.45 7.45	6.43	-0.57	-0.45
1.5	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6	2.51 3.28 4.23 5.23 6.23	5.22	-1.78	-1.78
2	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6	1.93 2.59 3.51 4.50 5.50	4.51	-2.49	-2.53
3	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6	1.32 1.83 2.68 3.65 4.65	3.67	-3.33	-3.39
4	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6	1.00 1.42 2.18 3.14 4.13	4.16	-3.84	-3.91
5	1327 9808 7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7	0.81 1.16 1.85 2.78 3.77 4.77	3.80	-4.20	-4.27
10	9808 7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7	0.61 1.06 1.85 2.81 3.80	3.82	-5.18	-5.25
10	9808 7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	.31 .58 1.13 2.00 2.98 3.97	2.99	-6.01	-6.08

Table 1b — Slanted well (fully penetrating)

tg θ	P_D	P_D^*	S_{gs}^*
0	4.00 5.00 6.00 7.00 8.00	4 5 6 7 8	0
1.118	2.83 3.69 4.67 5.66 6.66	5.66	-1.34
1.732	2.20 2.93 3.87 4.86 5.86	4.86	-2.14
2.828	1.52 2.08 2.94 3.92 4.92	3.93	-3.07
3.873	1.16 1.61 2.39 3.35 4.34	4.36	-3.64
4.90	.93 1.32 2.01 2.95 3.94 4.94	3.97	-4.03
9.95	.69 1.14 1.93 2.89 3.89	3.92	-5.08
19.98	.35 .62 1.17 2.04 3.02 4.01	3.05	-5.95

Table 1a **Table 1b**

Tables 1 a/b : Calculated Pseudoskin factor for horizontal and slanted wells : uniform-flux and infinite-conductivity solutions for dimensionless thickness $H/r_w = 100$

Table 1c — Horizontal centered well

L/H	t_D	P_D	P_D^*	S_{gH}^*	S_{gH} GIGER
1	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	5.72 6.53 7.50 8.49 9.49	8.48	-0.52	-0.45
1.5	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	3.90 4.57 5.49 6.48 7.48	8.48	-2.52	-2.55
2	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	2.96 3.52 4.39 5.37 6.37	5.38	-3.62	-3.68
3	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	1.99 2.42 3.18 4.14 5.13	4.16	-4.84	-4.93
4	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8	1.50 1.84 2.51 3.44 4.43	4.45	-5.55	-5.64
5	7.247 10^4 5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8 1.596 10^9	1.20 1.49 2.08 2.97 3.95 4.95	3.97	-6.03	-6.12
10	5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8 1.596 10^9	.76 1.12 1.82 2.76 3.75	3.77	-7.23	-7.32
20	5.355 10^5 3.957 10^6 2.924 10^7 2.16 10^8 1.596 10^9 1.187 10^{10}	.38 60 1.05 1.85 2.81 3.81	2.82	-8.18	-8.26

Table 1d — Slanted well (fully penetrating)

tg θ	P_D	P_D^*	S_{gs}^*
0	6.00 7.00 8.00 9.00 10.00	6 7 8 9 10	0
1.118	4.12 4.95 5.90 6.89 7.89	6.89	-2.11
1.732	3.14 3.83 4.73 5.72 6.72	5.72	-3.28
2.828	2.12 2.64 3.43 4.39 5.39	4.40	-4.60
3.873	1.60 2.02 2.70 3.63 4.62	4.64	-5.36
4.90	1.28 1.64 2.23 3.13 4.11 5.11	4.13	-5.87
9.95	.84 1.20 1.90 2.84 3.83	3.86	-7.14
19.98	.42 .63 1.08 1.88 2.85 3.85	2.87	-8.13

Table 1c **Table 1d**

Tables 1 c/d : Calculated Pseudoskin factor for horizontal and slanted wells : uniform-flux and infinite-conductivity solutions for dimensionless thickness $H/r_w = 1000$

H_D	L/H	t_D		Offsetting e/H: 0	0.1	0.2	0.3	0.35	0.4	0.45
1000	1	2.924 10^7	P_D^*	8.48	8.51	8.62	8.83	9.02	9.33	9.91
			$S_{gH}^* - S_{gHc}^*$		0.03	0.14	0.35	0.54	0.85	1.43
	2	2.924 10^7	P_D^*	5.38	5.40	5.47	5.60	5.71	5.88	6.20
			$S_{gH}^* - S_{gHc}^*$		0.02	0.09	0.22	0.33	0.50	0.82
	5	2.16 10^8	P_D^*	3.97	3.98	4.01	4.07	4.12	4.19	4.33
			$S_{gH}^* - S_{gHc}^*$		0.01	0.04	0.10	0.15	0.22	0.36
100	1	5.355 10^5	P_D^*	6.43	6.46	6.56	6.77	6.96	7.25	7.82
			$S_{gH}^* - S_{gHc}^*$		0.03	0.13	0.34	0.53	0.82	1.39
	2	5.355 10^5	P_D^*	4.51	4.53	4.59	4.72	4.83	5.00	5.31
			$S_{gH}^* - S_{gHc}^*$		0.02	0.08	0.21	0.32	0.49	0.80
	5	3.957 10^6	P_D^*	3.80	3.81	3.84	3.90	3.94	4.02	4.15
			$S_{gH}^* - S_{gHc}^*$		0.01	0.04	0.10	0.14	0.22	0.35

Table 1e · Calculated Pseudoskin factor for horizontal off-centered well (Infinite-conductivity solution)

Table 2a — Horizontal well (centered)

$L_D = \frac{L}{H}$	$H_D = \frac{H}{r_w}$	S_{pc}^*	S_{gH}^*	S_{gH}GIGER
2	333	2.711	-3.098	-3.131
2.667	500	2.392	-4.110	-4.168
2.857	700	2.406	-4.502	-4.565
	233	2.011	-3.798	-3.851
3.077	928	2.385	-4.879	-4.947
	433	2.131	-4.371	-4.433
4	1 000	2.035	-5.566	-5.640
	714	1.949	-5.315	-5.388
	500	1.857	-5.051	-5.120
	333	1.753	-4.749	-4.816
	167	1.572	-4.237	-4.297
5	1 000	1.784	-6.040	-6.117
	800	1.737	-5.864	-5.938
5.714	875	1.632	-6.192	-6.267
	500	1.531	-5.733	-5.805
	350	1.466	-5.442	-5.511
	233	1.391	-5.111	-5.176
	117	1.262	-4.547	-4.605
10	400	1.173	-6.428	-6.492
	286	1.136	-6.128	-6.190
	200	1.098	-5.810	-5.869
	133	1.054	-5.468	-5.504
20	250	0.925	-6.900	-6.947
	200	0.912	-6.689	-6.735
	143	0.893	-6.371	-6.415
	100	0.872	-6.036	-6.076

Table 2b — Horizontal well (off-centered)

S_p^*	$S_p^* - S_{pc}^*$	$\frac{H}{L}\ln\left[1 - \left(\frac{2e}{H}\right)^2\right]$
2.849	0.138	0.144
2.501	0.109	0.108
2.510	0.104	0.101
2.114	0.103	0.101
2.482	0.097	0.093
2.228	0.097	0.093
2.113	0.078	0.072
2.026	0.077	0.072
1.934	0.077	0.072
1.829	0.076	0.072
1.649	0.077	0.072
1.846	0.062	0.057
1.800	0.063	0.057
1.688	0.056	0.050
1.587	0.056	0.050
1.521	0.055	0.050
1.447	0.056	0.050
1.318	0.054	0.050

Table 2b
(Offsetting e/H = 0.25)

Table 2a

Tables 2 a/b : Pseudoskin factor for horizontal well comparison against Rosa et al's calculated values (Infinite conductivity solution)

$H_D = \frac{H}{r_w}$	L/H	S_{gs}^*	S_{gs}GIGER	S_{gs}	S_θ CINCO
100	1	0	-0.45	0	0
	1.5	-1.34	-1.78	-1.34	-1.39
	2	-2.14	-2.53	-2.13	-2.19
	3	-3.07	-3.39	-3.06	-3.06
	4	-3.64	-3.91	-3.62	-3.52
	5	-4.03	-4.27	-4.02	-3.81
	10	-5.08	-5.25	-5.09	-4.41
	20	-5.95	-6.08	-5.98	-4.73
1 000	1	0	-0.45	0	0
	1.5	-2.11	-2.55	-2.11	-2.15
	2	-3.28	-3.68	-3.28	-3.33
	3	-4.60	-4.93	-4.60	-4.59
	4	-5.36	-5.64	-5.35	-5.26
	5	-5.87	-6.12	-5.87	-5.68
	10	-7.14	-7.32	-7.16	-6.55
	20	-8.13	-8.26	-8.16	-7.00

Table 3

Slanted well (Fully penetrating) geometrical Pseudoskin factor S_{gs}

θ	L/H	H_D	S_θ calc	S_{gs}
15°	1.035	100	-0.123	-0.127
		200	-0.152	-0.150
		500	-0.183	-0.181
		1 000	-0.207	-0.205
		5 000	-0.261	-0.259
30°	1.155	100	-0.516	-0.514
		~~200~~	-0.608	-0.607
		500	-0.731	-0.730
		1 000	-0.824	-0.823
		5 000	-1.040	-1.039
45°	1.414	100	-1.175	-1.166
		200	-1.378	-1.369
		500	-1.647	-1.638
		1 000	-1.850	-1.841
		5 000	-2.321	-2.312
60°	2.0	100	-2.148	-2.129
		200	-2.494	-2.476
		500	-2.953	-2.934
		1 000	-3.299	-3.281
		5 000	-4.104	-4.085
75°	3.864	100	-3.586	-3.562
		200	-4.099	-4.076
		500	-4.778	-4.756
		1 000	-5.292	-5.269
		5 000	-6.485	-6.462

Table 4

Pseudoskin factor for slanted well comparison against CINCO et al's calculated values (Uniform-Flux solution)