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New Shape Factors for Wells Produced at Constant Pressure

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Abstract

New shape factors are developed for bounded reservoirs produced by wells operating at constant bottomhole pressure. The new shape factors should replace Dietz[1] shape factors (constant rate shape factors) when calculating the productivity of wells producing at constant bottomhole pressure; thus obtain a more reliable long term production forecast. Although the constant pressure and constant rate shape factors are conceptually different, they are close in value for regular shapes (square) with well in the center. However, for irregular shapes and off-centered wells, the difference is significant. *The use of constant rate shape factors for predicting the performance of wells producing at constant bottomhole pressure results in over-estimation of well productivity.*

Introduction

A large number of bounded reservoirs are produced form wells operating at ***constant flowing bottomhole pressure***. To calculate the production performance of such wells under boundary dominated flow conditions, the following Productivity Index (PI) equation is currently used in the industry:

$$J = \frac{kh}{141.2B\mu\left(\frac{1}{2}\ln\frac{4A}{\gamma C_A r_w^2} + s\right)}, \quad\text{..................(1)}$$

where C_A is a shape factor dependent on the shape of the drainage area and well location. Eq. (1) is also often used along with a material balance equation to forecast the long term well performance and ultimate recovery. Such forecasts are usually made assuming constant flowing bottomhole pressure at the well.

However, the shape factors used in Eq. (1) were introduced by Dietz[1] (in 1965) for wells producing at ***constant rate***. It has been the belief in the oil industry that the same shape factors can apply to wells operating at ***constant bottomhole pressure***. The subject study shows that using the constant rate shape factors for well producing at constant pressure introduces an error as high as 10% in the calculated production forecast and ultimate recovery.

The purpose of this paper is (a) to show that the shape factors are function of not only the reservoir shape and well location, but also the inner boundary condition (constant rate versus constant pressure); and (b) to calculate the proper shape factors for use in production forecast of wells producing at constant bottomhole pressure.

Background

Dietz[1] developed the constant rate shape factors based on the original work of Mathews, Brons and Hazebroek.[2] They used the Ei approximation of the constant rate solution of the diffusivity equation and the method of images to calculate the wellbore pressure under pseudo-steady-state conditions:[3]

$$p_D = \tfrac{1}{2}\ln\frac{4A}{\gamma C_A r_w^2} + s + 2\pi t_{DA} \quad\text{......................(2)}$$

Eq. (2) gives a straight line when P_D is plotted versus t_{DA} on Cartesian coordinates. The constant rate shape factors, C_A, of different drainage areas and well locations are calculated from the y-axis intercept of this straight line.

Constant Pressure Shape Factors

Appendix A shows the derivation of a semi-analytical solution for the diffusivity equation (closed rectangular reservoir) under constant flowing bottomhole-pressure condition. The semi-analytical solution was verified using an in-house numerical simulation model which showed an excellent match, Fig. 1. The solution is given in terms of rate and cumulative production. Under boundary dominated flow conditions, the rate is related to the cumulative production by the following equation:

$$\frac{1}{q_D} = \tfrac{1}{2}\ln\frac{4A}{\gamma\, C_{ACP} r_w^2} + s + 2\pi\frac{r_w^2}{A}\frac{N_{pD}}{q_D} \quad\text{.........(3)}$$

In this equation, C_{ACP} is the constant pressure shape factor. A plot of $1/q_D$ versus N_{pD}/q_D on Cartesian coordinates gives a straight line. Similar to Eq. (2), the constant pressure shape shape factor is calculated from the intercept of the line with the y-axis. (Note that Eq. (2) and Eq. (3) are analogous.) Table 1 gives a listing of the constant pressure shape factors along with the constant rate shape factors.

Productivity Index

By definition, the productivity index, J, is given by:

$$J = \frac{q}{\bar{p} - p_{wf}} \quad\text{...(4)}$$

Recognizing the definitions at the end of the paper for dimensionless pressure, dimensionless rate and dimensionless cumulative production, the dimensionless productivity index for the constant rate case can be given by:

$$J_D\big|_{CR} = \frac{1}{p_D - 2\pi t_{DA}} = \frac{1}{\tfrac{1}{2}\ln\dfrac{4A}{\gamma\, C_A r_w^2} + s} \quad\text{....(5)}$$

And for the constant pressure case:

$$J_D\big|_{CP} = \frac{1}{\dfrac{1}{q_D} - 2\pi\dfrac{r_w^2}{A}\dfrac{N_{pD}}{q_D}}, \quad\text{.......................(6)}$$

which can be written as:

$$J_D\big|_{CP} = \frac{1}{\tfrac{1}{2}\ln\dfrac{4A}{\gamma\, C_{ACP} r_w^2} + s}. \quad\text{.......................(7)}$$

Fig. 2 is a comparison plot of the dimensionless productivity indexes of the constant rate and constant pressure cases for a well producing from a rectangle. The constant rate productivity index is higher than the constant pressure productivity index. This indicates that *using the constant rate shape factors for predicting the performance of wells producing at constant bottomhole pressure results in over-estimation of well productivity.*

Example Application

The following numerical example shows the overestimation of productivity when using the constant rate shape factors to forecast the production of a well producing at constant pressure. The following reservoir and well data is used:

k	=	1.0	md;	ϕ	=	0.25	
h	=	100	ft;	B_{oi}	=	2.0	RB/STB
μ	=	1.0	cp;	r_w	=	0.25	ft
c_o	=	5.E-6	psi^{-1};	p_i	=	5,000	psi
b	=	4,000	ft;	a	=	1,000	ft
p_{wf}	=	3,000	psi = constant.				

The drainage area has a 4:1 ratio and the well is located near to the edge as in case 14 of Table 1. The following parameters then apply:

C_{ACP} = 0.029; C_A = 0.232

Two production forecasts are made using the productivity index equation, Eq. (1), and the material balance equation for undersaturated reservoirs. The first forecast uses the constant rate shape factor, and the second uses the constant pressure shape factor. The well is kept producing at constant bottomhole pressure of 3,000 psi for both forecasts. As shown in Fig. 3, the forecast with the constant rate shape factor is 10% higher than that with the constant pressure shape factor.

Discussion

In the above analysis, it has been shown that the shape factors are function of the inner boundary condition (constant rate versus constant pressure). In addition, the constant pressure shape factors are function of the dominant flow regime (radial versus linear). A comparison between the shape factors for Case 3 of Table 1 (more radial) and the shape factors for Case 7 of Table 1 (more linear) shows that although constant rate shape factors are the same for both cases, the constant pressure shape factors are different. The difference between constant rate and constant pressure shape factors becomes larger as the flow regime tends more towards linear flow (Case 7 in Table 1). Therefore, a complete linear flow regime would represent the largest difference between constant rate and constant pressure shape factors. Although the constant pressure and constant rate shape factors are conceptually different, they are close in value for regular shapes (square) with well in the center. However, for irregular shapes and off-centered wells, the difference is significant.

Conclusions

New shape factors are developed for bounded reservoirs produced by wells operating at constant bottomhole pressure. The new shape factors should replace Dietz[1] shape factors (constant rate shape factors) when calculating productivity of wells producing at constant bottomhole pressure. *The use of constant rate shape factors for predicting the performance of wells producing at constant bottomhole pressure results in over-estimation of well productivity.* The difference between constant rate and constant pressure shape factors becomes larger as the flow regime becomes linear.

Nomenclature

A = well drainage area, L^2, ft^2
a = drainage area width, L, ft
B = formation volume factor, RB/STB
b = drainage area length, L, ft
C = shape factor, dimensionless
c = compressibility, Lt^2/m, $psia^{-1}$
h = net formation thickness, L, ft
J = Productivity index, L^4t/m, STB/D/psi
j = summation counter
K_0 = Bessel function of the second kind, order zero
K_1 = Bessel function of the second kind, order one
k = reservoir permeability, L^2, md
N_p = cumulative production, L^3, STB
$\overline{N}_{pD}$ = Laplace transform of cumulative production
m = summation counter
n = summation counter
p = pressure, m/Lt^2, psia
$\overline{p}$ = average reservoir pressure, m/Lt^2, psia
$\overline{p}_D$ = Laplace transform of pressure solution
q = production rate, L^3/t, STB/D
$\overline{q}_D$ = Laplace transform of rate solution
r = radius, L, ft
s = damage skin factor, dimensionless
t = producing time, t, days
u = Laplace operator
β = relative well location, fraction
δ = relative well location, fraction
ϕ = porosity, fraction
μ = viscosity, m/Lt, cp
τ = integration variable

Subscripts

A = based on area
CP = constant pressure
CR = constant rate
D = dimensionless
f = flowing
i = initial
o = oil
t = total
w = well

Constants

γ = 1.781 (exponential of Euler's constant)

Dimensionless Variables

$$t_D = \frac{0.00633kt}{\phi\mu c_t r_w^2}$$

$$t_{DA} = \frac{0.00633kt}{\phi\mu c_t A}$$

$$J_D = \frac{141.2B\mu}{kh} J$$

Constant Rate Case:

$$p_D = \frac{kh(p_i - p)}{141.2qB\mu}$$

Constant Pressure Case:

$$p_D = \frac{(p_i - p)}{(p_i - p_{wf})}$$

$$q_D = \frac{141.2qB\mu}{kh(p_i - p_{wf})}$$

$$N_{pD} = \frac{0.8938BN_p}{\phi c_t r_w^2 h(p_i - p_{wf})}$$

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Appendix A

The constant bottomhole pressure solution of the diffusivity equation is deduced from the constant rate solution using the fundamental relationship between the two solutions in Laplace space.[4] Using the dimensionless variables described earlier, the diffusivity equation can be written as follows:

$$\frac{\partial^2 p_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_D}{\partial r_D}=\frac{\partial p_D}{\partial t_D} \quad \text{(A-1)}$$

The initial and boundary conditions for a well producing at constant rate in an infinite reservoir are given by:

Initial condition $p_D(r_D,t_D)\big|_{t_D=0}=0,$

Inner boundary $\frac{\partial p_D}{\partial r_D}(r_D,t_D)\Big|_{r_D=1}=-1,$

Outer boundary $p_D(r_D,t_D)\big|_{r_D\to\infty}=0.$

The Laplace space solution for the pressure drop at the well was presented by van-Everdingen and Hurst (cylindrical source solution):[4]

$$\bar{p}_D(r_D,u)=\frac{K_0\left(r_D\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)} \quad \text{(A-2)}$$

Using the method of images, the constant rate solution for the pressure drop at a well located anywhere in a bounded reservoir is given by:

$$\bar{p}_D(r_D,u)=\frac{K_0\left(\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}+\sum_{j=1}^{\infty}\frac{K_0\left(r_{Dj}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)} \quad \text{(A-3)}$$

Details of the imaging process are given in appendix B. The constant pressure solution at the well is deduced from Eq. (A-3) using the fundamental equation derived by van-Everdingen and Hurst[4]:

$$\bar{q}_D=\frac{1}{u^2\bar{p}_D(r_D,u)}, \quad \text{(A-4)}$$

and the cumulative production:

$$\bar{N}_{pD}=\frac{1}{u^3\bar{p}_D(r_D,u)}, \quad \text{(A-5)}$$

The Stehfest algorithm[5] is used to generate the real time solution (rate and cumulative production) for the constant bottomhole pressure case.

During boundary dominated flow of a reservoir producing from a well with constant flowing bottomhole pressure, the following material balance equation applies:

$$dp=-\frac{5.615Bq\,dt}{Ah\phi c_t}=-2\pi\frac{0.8938B}{Ah\phi c_t}q\,dt. \quad \text{(A-6)}$$

Integrating Eq. (A-6):

$$\int_{p_i}^{\bar{p}}dp=-2\pi\frac{0.8938B}{Ah\phi c_t}\int_0^t q\,d\tau, \quad \text{(A-7)}$$

leading to:

$$p_i-\bar{p}=2\pi\frac{0.8938B}{Ah\phi c_t}N_p. \quad \text{(A-8)}$$

Recognizing the definition of dimensionless cumulative production, Eq. (A-8) becomes:

$$p_i-\bar{p}=2\pi\left(p_i-p_{wf}\right)\frac{r_w^2}{A}N_{pD} \quad \text{(A-9)}$$

Adding and subtracting p_{wf} in the right hand side and rearranging:

$$\left(\bar{p}-p_{wf}\right)=\left(p_i-p_{wf}\right)\left(1-2\pi\frac{r_w^2}{A}N_{pD}\right) \quad ..(A\text{-}10)$$

Also during boundary dominated flow, the productivity equation is given by:

$$\left(\bar{p}-p_{wf}\right)=\frac{141.2qB\mu}{kh}\left[\tfrac{1}{2}\ln\frac{4A}{\gamma C_{ACP}r_w^2}+s\right], \quad (A\text{-}11)$$

where, C_{ACP} is the constant pressure shape factor. Recognizing the definition of the dimensionless production rate, Eq. (A-11) becomes:

$$\left(\bar{p}-p_{wf}\right)=\left(p_i-p_{wf}\right)q_D\left[\tfrac{1}{2}\ln\frac{4A}{\gamma C_{ACP}r_w^2}+s\right] \quad (A\text{-}12)$$

Combining Eq. (A-10) and Eq. (A-12):

$$\frac{1}{q_D}=\tfrac{1}{2}\ln\frac{4A}{\gamma C_{ACP}r_w^2}+s+2\pi\frac{r_w^2}{A}\frac{N_{pD}}{q_D} \quad ....(A\text{-}13)$$

Values of q_D and N_{pD} are calculated for different drainage area shapes and well locations from the inversion of the Laplace space solution, Eq. (A-4) and Eq (A-5). The constant pressure shape factors can then be calculated from Eq. (A-13).

Appendix B

Very similar to the approach taken by Mathews, Brons and Hazebroek[2], the method of images is used to calculate the constant rate solution for the pressure drop at the wellbore. The infinite reservoir solution (in Laplace space) is superposed in the following manner, Fig. 4:

$$\bar{p}_D\left(r_D,u\right)=\frac{K_0\left(\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}+\sum_{j=1}^{\infty}\frac{K_0\left(r_{Dj}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)} \quad ,.....(B\text{-}1)$$

where, r_{Dj} is the distance from the image well j to the actual well. In order to allow for a variety of well locations, the summation term takes the following form:

$$\sum_{j=1}^{\infty}\frac{K_0\left(r_{Dj}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}=\sum_{\substack{m=-\infty\\n=-\infty}}^{\substack{m=\infty\\n=\infty}}\frac{K_0\left(r_{D1}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}+\frac{K_0\left(r_{D2}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}+\frac{K_0\left(r_{D3}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}+\frac{K_0\left(r_{D4}\sqrt{u}\right)}{u^{3/2}K_1\left(\sqrt{u}\right)}, \quad .(B\text{-}2)$$

where (m and n) simultaneously $\neq 0$, and

$$r_{D1}=\frac{\sqrt{[2ma]^2+[2nb]^2}}{r_w}, \quad ......(B\text{-}3)$$

$$r_{D2}=\frac{\sqrt{\left[2(m+\delta)a\right]^2+\left[2nb\right]^2}}{r_w}, \quad ......(B\text{-}4)$$

$$r_{D3}=\frac{\sqrt{\left[2ma\right]^2+\left[2(n+\beta)b\right]^2}}{r_w}, \quad ......(B\text{-}5)$$

$$r_{D4}=\frac{\sqrt{\left[2(m+\delta)a\right]^2+\left[2(n+\beta)b\right]^2}}{r_w} \quad ......(B\text{-}6)$$

SI Metric Conversion Factors

cp	×1.0	E-03	= Pa • s
ft	×3.048	E-01	= m
ft^2	×9.290 304	E-02	= m^2
ft^3	×2.831 685	E-02	= m^3
in.	×2.54	E+00	= cm
lbf	×4.448 222	E+00	= N
md	×9.869 233	E-04	= μm^2
psi	×6.894 757	E+00	= kPa

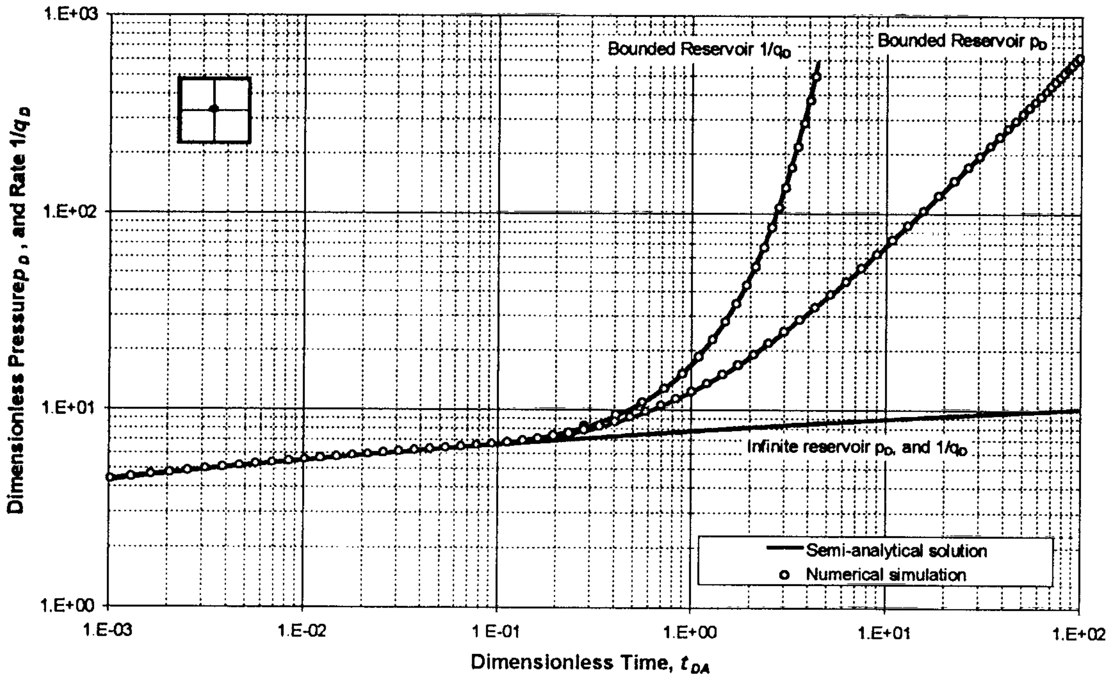


Fig. 1 - Verification of semi-analytical solution using numerical simulation

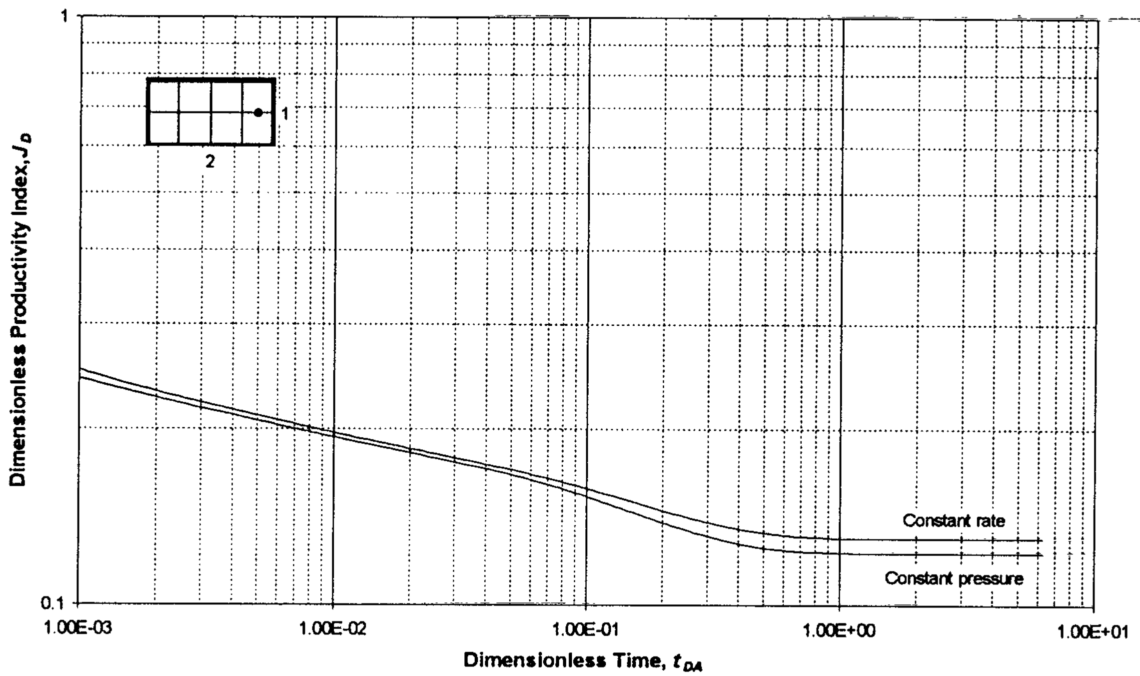


Fig. 2 - Dimensionless productivity index for constant rate and constant pressure cases

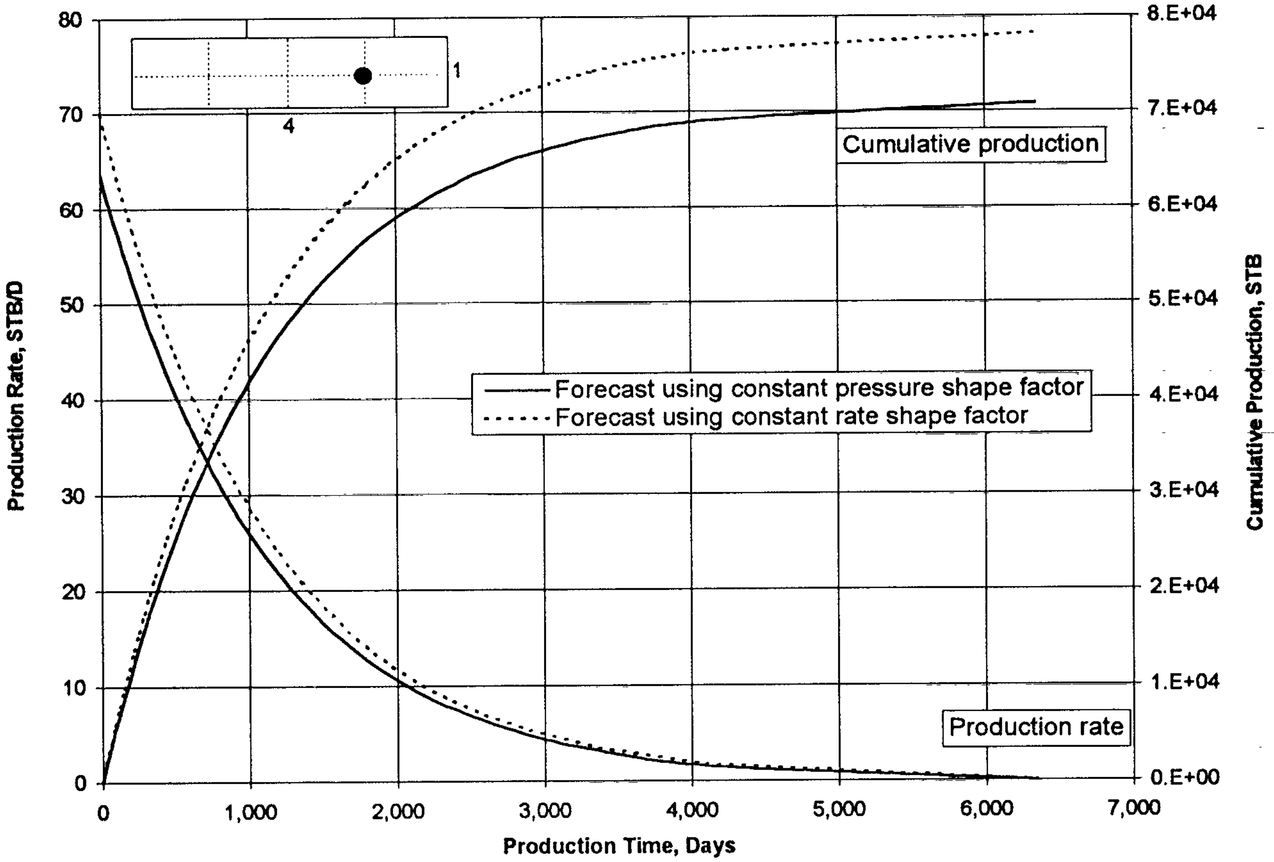


Fig. 3 - Over estimating well productivity when using constant rate shape factors to forecast the production of wells producing at constant pressure.

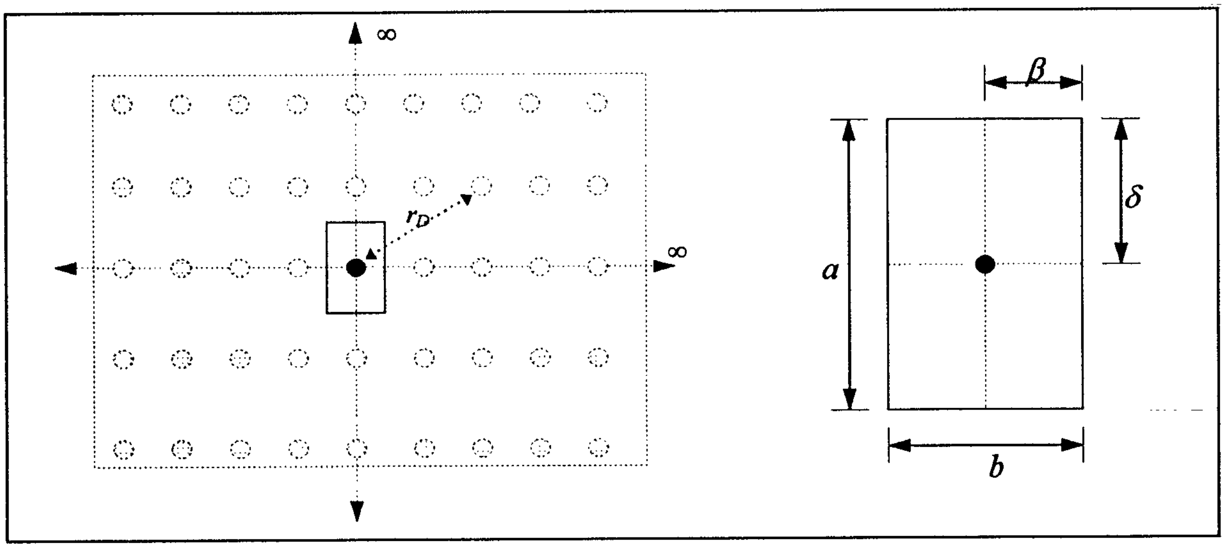
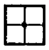
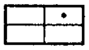
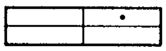


Fig. 4 - Images of a well inside a rectangular drainage area.

TABLE 1 - CONSTANT RATE AND CONSTANT PRESSURE SHAPE FACTORS

		Constant Rate		Constant Pressure		Constant Rate & Constant Pressure		
Case	Drainage Area Shape	C_A	$\frac{1}{2}\ln\frac{4}{\gamma C_A}$	C_{ACP}	$\frac{1}{2}\ln\frac{4}{\gamma C_{ACP}}$	Exact for $t_{DA}>$	Less than 1% error $t_{DA}>$	Use infinite solution with less than 1% error for $t_{DA}<$
1 1x1		30.88	-1.311	29.34	-1.285	0.1	0.05	0.09
2 1x1		12.99	-0.877	10.92	-0.791	0.7	0.25	0.03
3 1x1		4.51	-0.349	3.38	-0.204	0.6	0.30	0.025
4 1x1		3.34	-0.198	2.59	-0.071	0.7	0.25	0.01
5 1x2		21.84	-1.137	19.88	-1.090	0.3	0.15	0.025
6 1x2		10.84	-0.787	9.50	-0.721	0.4	0.15	0.025
7 1x2		4.51	-0.349	2.50	-0.054	1.5	0.50	0.06
8 1x2		2.08	0.039	1.14	0.339	1.7	0.50	0.02
9 1x2		3.16	-0.170	2.70	-0.092	0.4	0.15	0.005
10 1x2		0.581	0.676	0.249	1.100	2.0	0.60	0.02
11 1x2		0.111	1.504	0.047	1.939	3.0	0.60	0.005
12 1x4		5.38	-0.437	3.95	-0.282	0.8	0.30	0.01
13 1x4		2.69	-0.090	1.97	0.066	0.8	0.30	0.01
14 1x4		0.232	1.136	0.029	2.175	4.0	2.00	0.03
15 1x4		0.116	1.484	0.016	2.485	4.0	2.00	0.01
16 1x5		2.36	-0.025	1.49	0.205	1.0	0.40	0.025