Augmentation of Well Productivity With Slant and Horizontal Wells

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Summary. This paper presents an equation to calculate the productivity of horizontal wells and a derivation of that equation using potential-fluid theory. This equation may also be used to account for reservoir anisotropy and well eccentricity (i.e., horizontal well location other than midheight of a reservoir). The theoretical predictions were used to calculate the effective wellbore radius and the effective skin factors of horizontal wells. Laboratory experiments with an electrical analog were also conducted. These laboratory experimental data and also the laboratory data available in the literature show good agreement with the theoretical equation, indicating its accuracy.

The paper also compares vertical-, slant-, and horizontal-well productivity indices, assuming an equal drainage area. In addition, the comparison also assumes an equal reservoir contact area for slant and horizontal wells. The results show that in a 100-ft [30.48-m] -thick reservoir, horizontal-well productivities are two to five times greater than unstimulated vertical- or slant-well productivities, depending on reservoir anisotropy. Conversely, in a 400-ft [122-m] -thick reservoir, slant wells perform better than horizontal wells if vertical permeability is less than horizontal permeability.

Horizontal wells perform significantly better than vertical wells in reservoirs with gas cap and/or bottomwater. This study reports an equation to compare horizontal- and vertical-well gas-coning tendencies.

The results indicate that horizontal wells are suitable for reservoirs that are thin, show high vertical permeability, or exhibit gas- and water-coning problems. The equations reported should be useful in initial evaluation of a horizontal-well drilling proposal.

Introduction

Currently, about 30 horizontal wells are producing oil successfully worldwide.[1] The wells have been drilled in Prudhoe Bay in Alaska,[2] Empire Abo Unit in New Mexico,[3] France, and offshore Italy.[4-6] Because of a large reservoir contact area, horizontal-well oil-production rates are two to five times greater than unstimulated vertical-well rates. In addition, horizontal wells may intersect several fractures and help drain them effectively. Horizontal wells have also been known to reduce water- and gas-coning tendencies. The disadvantages of horizontal wells are that (1) they are ineffective in thick (~500 to 600 ft [~150 to 180 m]), low-vertical-permeability reservoirs; (2) reservoirs with several oil zones, separated by impermeable shale barriers, may require drilling of a horizontal hole in each reservoir layer to be drained; (3) some limitations currently exist in well-completion and stimulation technology; and (4) drilling costs are 1.4 to 2 times more than those for vertical wells.

Objectives

The main goal of this work is to develop necessary mathematical equations for an initial evaluation of horizontal-well drilling prospects. This included the following objectives.

1. To develop a mathematical equation to calculate steady-state oil production with horizontal wells.
2. To determine the influence of reservoir anisotropy, height, well drainage area, and eccentricity (well location other than the reservoir midheight) on horizontal-well productivity.
3. To devise laboratory electrical analog experiments to measure horizontal-well productivities and to compare them with the theoretical equation.
4. To compare vertical-, slant-, and horizontal-well productivities.
5. To determine gas- and water-coning tendencies of horizontal wells and to compare them with those of vertical wells.

Literature Review

Borisov[7] reported a theoretical equation to calculate steady-state oil production from a horizontal well; however, the report does not include the derivation of the equation. Later, using Borisov's equation, Giger *et al.*[8,9] and Giger[10-12] reported reservoir engineering aspects of horizontal drilling. Giger[10,11] developed a concept of replacement ratio, F_R, which indicates the number of vertical wells required to produce at the same rate as that of a single horizontal well. The replacement-ratio calculation assumes an equal drawdown for the horizontal and vertical wells. In addition, Giger studied fracturing of a horizontal well[8] and provided a graphical solution to calculate reduction of water coning using horizontal wells.[11,12] Giger *et al.*[9] reported that horizontal wells are suitable for thin reservoirs, fractured reservoirs, and reservoirs with gas- and water-coning problems.

Recently, Reiss[6] reported a productivity-index equation for horizontal wells, but his equation is little different from that reported by Borisov[7] and others.[8,11] To clarify these differences, it was decided to derive the basic steady-state equation from fundamental fluid-flow theory. Such a derivation is reported here.

Daviau *et al.*[13] and others[14,15] recently reported time-dependent theoretical analyses for horizontal wells. Their results, as well as our time-dependent horizontal-well theoretical results[16] (not included here), indicate that if the length of a horizontal well is significantly larger than the reservoir height ($L/h \gg 1$), then in the long time, horizontal-well production is the same as that obtained from a fully penetrating, infinite-conductivity vertical fracture. This is also shown by the steady-state equation derived in this paper. It is important to note that implications of these results are restricted to a single-phase flow.

Horizontal-Well Oil-Production Equation

Fig. 1 shows that a horizontal well of length L drains an ellipsoid, while a conventional vertical well drains a right circular cylindrical volume. Both of these wells drain a reservoir of height h, but their drainage volumes are different. To calculate oil production from a horizontal well mathematically, the three-dimensional (3D) equation ($\nabla^2 p=0$) needs to be solved first. If constant pressure at the drainage boundary and at the wellbore is assumed, the solution would give a pressure distribution within a reservoir. Once the pressure distribution is known, oil production rates can be calculated by Darcy's law.

To simplify the mathematical solution, the 3D problem is subdivided into two two-dimensional (2D) problems. Fig. 2 shows the following subdivision of the ellipsoid drainage problem: (1) oil flow into a horizontal well in a horizontal plane and (2) oil flow into a horizontal well in a vertical plane. Appendices A and B describe mathematical solutions to these two problems with potential-fluid-flow theory. The solutions are added to calculate oil production

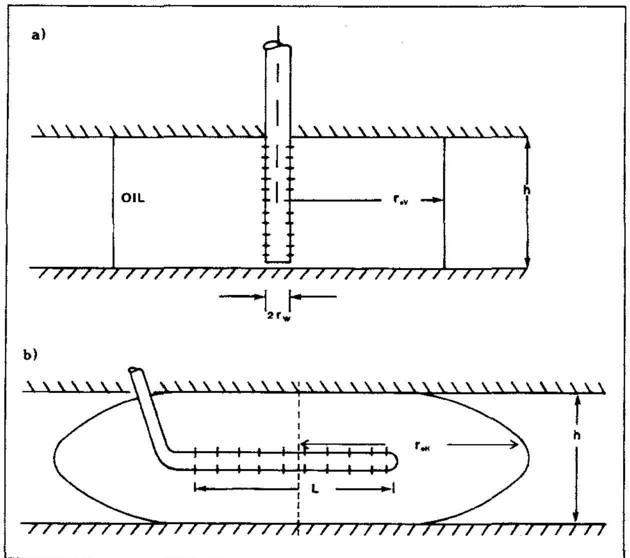

Fig. 1—Schematics of vertical- and horizontal-well drainage volume.

from a horizontal well. Eq. 1, derived in Appendix A, gives oil flow to a horizontal well in a horizontal plane:

$$q_1=\frac{2\pi k_o \Delta p/\mu B_o}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]}. \qquad (1)$$

Multiplying Eq. 1 by reservoir height, h, gives oil production from a number of horizontal wells stacked from the reservoir bottom to the top:

$$q_1^*=q_1 h. \qquad (2)$$

Using an electrical analog concept, flow resistance in a horizontal direction is given as

$$R_{fH}=(\Delta p/q_1^*)=\frac{\mu B_o}{2\pi k_o h}\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]. \qquad (3)$$

In addition to the above horizontal-direction resistance, R_{fH}, a horizontal well of height $2r_w$ experiences a vertical-flow resistance. Eq. 4, derived in Appendix B, gives the flow into a horizontal well of a unit length located at the reservoir midheight.

$$q_2=\frac{2\pi k_o \Delta p/\mu B_o}{\ln[h/(2r_w)]}. \qquad (4)$$

The oil flow to a horizontal well of length L, q_2^*, and the corresponding flow resistance in a vertical direction, R_{fV}, are calculated as

$$q_2^*=q_2 L \qquad (5)$$

and

$$R_{fV}=(\Delta p/q_2^*)=[\mu B_o/(2\pi k_o L)]\ln[h/(2r_w)]. \qquad (6)$$

The vertical-resistance term in Eq. 6 represents resistance in a vertical plane in a circular area of radius $h/2$ around the wellbore. Part of this resistance is already accounted for in the horizontal-resistance term. As shown later, this duplication did not severely affect the accuracy of the solution.

Several different methods of combining R_{fH} and R_{fV} were considered to calculate effective flow resistance. The addition of R_{fH} and R_{fV} not only gave mathematically simple results but also showed good agreement with the laboratory experimental data. Therefore, horizontal and vertical resistances were added to calculate horizontal-well oil production:

$$R_{fH}+R_{fV}=\Delta p\left(\frac{1}{q_1^*}+\frac{1}{q_2^*}\right)=\Delta p/q_H \qquad (7a)$$

and

$$q_H=\frac{2\pi k_o h \Delta p/(\mu B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]+\dfrac{h}{L}\ln\left(\dfrac{h}{2r_w}\right)}$$

for $L>h$ and $(L/2)<0.9r_{eH}$, (7b)

where a, half the major axis of a drainage ellipse in a horizontal plane in which the well is located (Fig. 2), is obtained by reformulating Eq. A-10 as shown below:

$$a=(L/2)\left[\tfrac{1}{2}+\sqrt{\tfrac{1}{4}+\frac{1}{(0.5L/r_{eH})^4}}\right]^{0.5}. \qquad (8)$$

Table 1 lists the correspondence between $L/(2a)$ and $L/(2r_{eH})$ values. Eq. 7b shows that if the horizontal-well length significantly exceeds the reservoir height ($L/h \gg 1$), then the second term in the denominator of Eq. 7b representing the vertical-flow resistance tends to be very small as compared with the first term (horizontal resistance). Moreover, if $L/(2a) \ll 1$, then, as Table 1 shows, $a \cong r_{eH}$. Substituting this into Eq. 7b results in the following classic infinite-conductivity vertical-fracture solution[17]:

$$q=\frac{2\pi k_o h \Delta p/(\mu B_o)}{\ln[r_{eH}/(L/4)]}. \qquad (9)$$

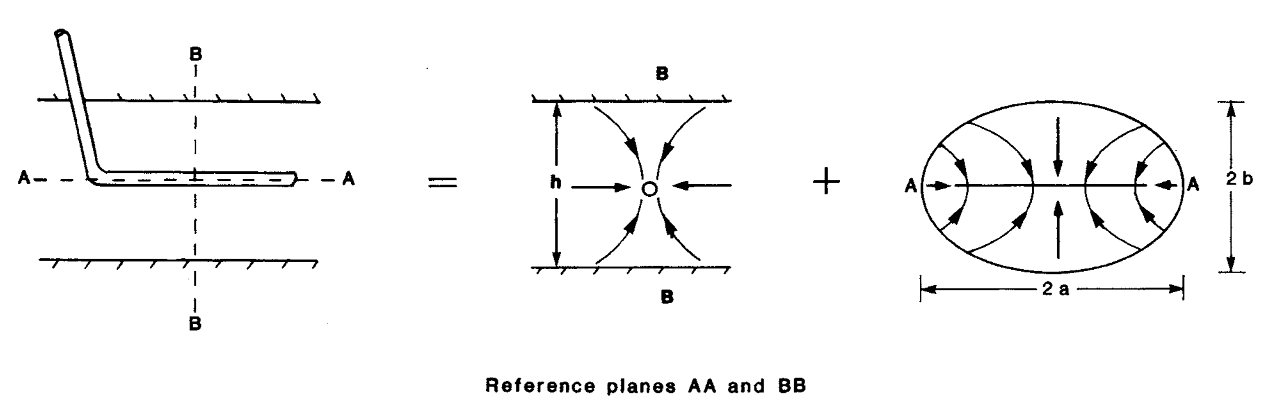

Fig. 2—Division of 3D horizontal-well problem into two 2D problems.

Eq. 9 will yield results identical to that given by Eq. 7b if $L/h \geq 6$. Thus, if $L/h \geq 6$, then horizontal-well production can be approximated as production from a fully penetrating vertical fracture. This conclusion confirms similar findings from the transient well-test solutions.[15,16] As noted earlier, this is valid only for single-phase flow. This observation is important, for it provides a ready means to compare horizontal-well productivity with that of stimulated wells.

Eq. 7b is different from that reported in the literature.[7-12] The equation used by these authors has an additional π term in the denominator of the vertical-resistance term. Because these references do not include the derivation of their equation, it was not possible to investigate the reasons for this difference. As noted later, Eq. 7b shows fairly good agreement with the laboratory data, indicating its usefulness to predict oil production from horizontal wells.

Influence of Anisotropy

In many reservoirs, the vertical permeability is less than the horizontal permeability. For a horizontal well, a decrease in vertical permeability results in an increase in vertical-flow resistance and a decrease in oil production rates. As Muskat[18] showed, the reservoir anisotropy could be accounted for by modifying the vertical axis as $z'=h\sqrt{k_H/k_V}$ and the average reservoir permeability as $\sqrt{k_H k_V}$. The modification of the z axis makes the wellbore elliptic. If the elliptic wellbore effects are assumed to be negligible, Eq. 7b is modified to account for the reservoir anisotropy:

$$q_H=\frac{2\pi k_H h\Delta p/(\mu B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]+\dfrac{\beta h}{L}\ln\left(\dfrac{\beta h}{2r_w}\right)},$$

$$\text{for } L>\beta h, \quad \text{(10a)}$$

where $\beta=\sqrt{k_H/k_V}$.

As shown in Appendix C, an alternative equation to account for reservoir anisotropy (Eq. C-7) could also be developed and rewritten as

$$q_H=\frac{2\pi k_H h\Delta p/(\mu B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]+\dfrac{\beta^2 h}{L}\ln\left(\dfrac{h}{2r_w}\right)}. \quad \text{(10b)}$$

Although Eq. 10b is derived more rigorously than Eq. 10a, there is less than 14% difference in the productivity indices ($q/\Delta p$) calculated with these two equations for $L>0.4\beta h$. Moreover, these productivity indices show less than 10% difference from the productivity index calculated with the skin-factor correlation developed by transient pressure analysis (Eq. 20 of Ref. 16). In general, Eq. 10b gives a slightly higher productivity index than Eq. 10a. Although either Eq. 10a or 10b could be used for engineering calculation purposes, Eq. 10a is recommended for a conservative production forecast. Therefore, in the rest of the paper, anisotropic calculations are based on Eq. 10a.

Horizontal-Well Eccentricity

Eqs. 7 and 10 assume that a horizontal well is located at the reservoir center in a vertical plane—i.e., at a distance $h/2$ from the reservoir top and bottom. With Muskat's[18] formulation for off-centered wells, oil production in a vertical plane from a horizontal well placed a distance ℓ_δ from the reservoir midheight could be shown as

$$q_2=\frac{2\pi k_o L\Delta p h/(\mu B_o h)}{\ln\left[\dfrac{(h/2)^2-\ell_\delta^2}{(hr_w/2)}\right]}, \text{ for } \ell_\delta<(h/2), \quad \text{(11)}$$

resulting in the following expressions for R_{fV} and q_H:

$$R_{fV}=\frac{\mu B_o h}{2\pi k_o L h}\ln\left[\frac{(h/2)^2-\ell_\delta^2}{(hr_w/2)}\right] \quad \text{(12)}$$

TABLE 1—RELATIONSHIP BETWEEN VARIOUS GEOMETRIC FACTORS

$\dfrac{L}{2r_{eH}}$	$\dfrac{L}{2a}$	$\dfrac{L}{r_{eH}}$
0.1	0.0998	1.002
0.2	0.198	1.010
0.3	0.293	1.024
0.4	0.384	1.042
0.5	0.470	1.064
0.6	0.549	1.093
0.7	0.620	1.129
0.8	0.683	1.171
0.9	0.739	1.218

and

$$q_H=\frac{2\pi k_o h\Delta p/(\mu B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right]+\dfrac{h}{L}\ln\left[\dfrac{(h/2)^2-\ell_\delta^2}{hr_w/2}\right]},$$

$$\text{for } \ell_\delta<h/2. \quad \text{(13)}$$

The calculations demonstrate that a horizontal well located $\pm h/4$ from the reservoir midheight exhibits <10% deviation in productivity. This indicates that during drilling, deviations in the well location from the reservoir midheight should not significantly affect well productivity. However, if $L/h<2$ and/or $2\ell_\delta/h>0.5$, then well eccentricity exhibits some influence on well productivity. These results were also confirmed experimentally with an electrical analog, which is described later.

Effective Wellbore Radius and Skin Factor

For a given drainage area, the vertical-well production rate increases with an increase in wellbore radius. Hence, the higher oil production of slant and horizontal wells could also be represented by a vertical well of a large wellbore diameter. In a conventional vertical well, increased well production resulting from well stimulation is represented as a decrease in a negative skin factor or an increase in effective wellbore diameter. The relationship between skin factor and effective wellbore radius is defined as

$$r_{we}=r_w \exp(-s). \quad \text{(14)}$$

To calculate the required vertical-wellbore diameter to produce oil at the same rate as that from a horizontal well, equal drainage volumes, $r_{eH}=r_{eV}$, and equal productivity indices, $(q/\Delta p)_H=(q/\Delta p)_V$ were assumed. With these assumptions,

$$\left[\frac{2\pi k_o h/(\mu B_o)}{\ln(r_e/r_{we})}\right]_V$$

$$=\left\{\frac{2\pi k_o h/(\mu B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{(L/2)}\right]+\dfrac{h}{L}\ln\left(\dfrac{h}{2r_w}\right)}\right\}_H \quad \text{(15)}$$

and

$$r_{we}=\frac{r_{eH}(L/2)}{a[1+\sqrt{1-(L/2a)^2}\,][h/(2r_w)]^{h/L}}, \quad \text{(16)}$$

with a calculated from either Eq. 8 or Table 1. Eqs. 16 and 14 could be used to calculate effective wellbore radius, r_{we}, and skin factor, s, respectively. Eq. 17 yields the ratio of horizontal- and

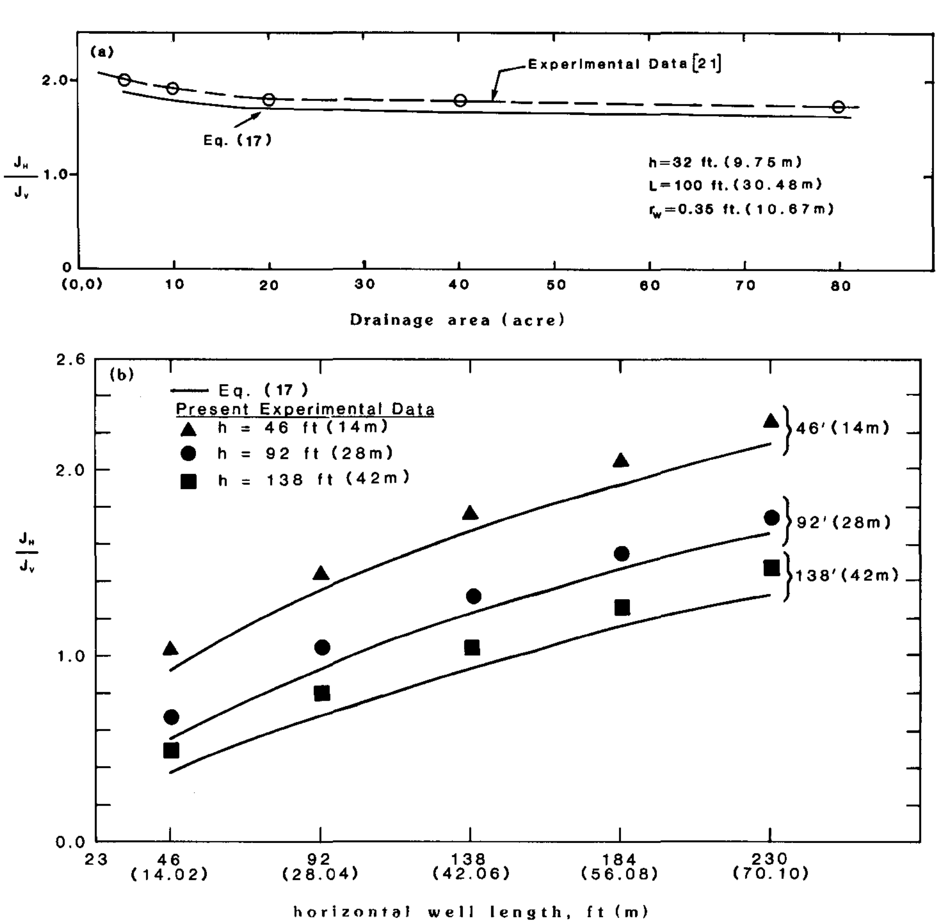

Fig. 3—Comparison of the laboratory data with the theoretical prediction.

vertical-well productivity indices:

$$J_H/J_V=[\ln(r_{eV}/r_w)]/[\ln(r_{eH}/r_{we})],\ \text{for } L>h\sqrt{k_H/k_V}\ \text{and}$$

$$(L/2)<0.9r_{eH}. \quad \text{.............................} (17)$$

It is important to note that the above comparison assumes an *unstimulated* vertical well. Because vertical-well stimulation varies from region to region, only unstimulated vertical-well productivities are used for general comparison. The productivity increases calculated from Eq. 17 will have to be adjusted, depending on local experience with the vertical-well stimulation treatments. Eq. 17 is valid only for reservoirs operating above the bubblepoint. Nevertheless, in a solution-gas-drive reservoir, the productivity index is a first derivative of an inflow-performance-relationship curve. Therefore, in a solution-gas-drive reservoir, Eq. 17 would give a fair estimation of productivity improvements with horizontal wells.

Slant-Well Productivity

A horizontal well represents a limiting case of a slant well when slant angle becomes 90° [1.6 rad]. Therefore, this study includes comparisons of slant- and horizontal-well productivities. For a well with a slant angle <75° [<1.3 rad], Cinco *et al.*[19] and Van Der Vlis *et al.*[20] reported equations to calculate well productivities. Cinco *et al.* proposed an equation to calculate the skin factor caused by a slant well, while Van Der Vlis *et al.* proposed an equation to calculate the effective wellbore radius of a slant well. The slant-well productivities calculated with Cinco *et al.*'s and Van Der Vlis *et al.*'s equations were in excellent agreement. Hence, either correlation could be used to calculate slant-well productivities.

Measurements of Well-Productivity Augmentation

The skin factor and the ratio of productivity indices measure the augmentation of well productivity for slant and horizontal wells. These parameters could be calculated from Eqs. 14, 16, and 17 by assuming an equal drainage volume for vertical, slant, and horizontal wells. For horizontal wells, an additional productivity-improvement measurement is the replacement ratio, which accounts for different drainage areas for horizontal and vertical wells. As noted earlier, the replacement ratio indicates the number of vertical wells required to produce at the same rate as that of a single horizontal well.

Giger[10] introduced the concept of areal productivity index to calculate the replacement ratio, F_R. This equation is modified here to facilitate calculation of F_R. In addition, equations presented include the influence of reservoir anisotropy, which was not included in the original work.[10] Appendix D describes the derivation of equations to calculate F_R. Eq. D-6 suggests that F_R depends on horizontal-well drainage radius, r_{eH}. Therefore, F_R values were calculated by varying $L/(2r_{eH})$ from 0.1 to 0.9. This resulted in less than 7% variation in F_R values; the results demonstrate that F_R depends on horizontal-well length L rather than on the ratio $L/(2r_{eH})$. Therefore, all calculations presented in this paper assume that $r_{eH}=L$.

It is important to avoid confusion between the productivity-index ratio and the replacement ratio, F_R. A productivity-index ratio assumes an equal drainage area for horizontal and vertical wells. Conversely, a replacement ratio accounts for the different drainage areas of horizontal and vertical wells.

Comparison of Theoretical Predictions and Experimental Data

Perrine[21] and others[22,23] conducted electrical analog experiments

TABLE 2—PRODUCTIVITY-INDEX RATIOS OF HORIZONTAL AND VERTICAL WELLS, 10-ACRE WELL SPACING*

h (ft)	L (ft)	J_H/J_V $(k_V/k_H)=0.1$	J_H/J_V $(k_V/k_H)=0.5$	J_H/J_V $(k_V/k_H)=1.0$
25	100	1.08	1.70	1.93
	200	1.79	2.57	2.83
	400	3.09	4.17	4.50
	500	3.77	5.05	5.44
	650	4.89	6.55	7.05
50	100	—	1.17	1.44
	200	1.11	1.91	2.26
	400	2.01	3.26	3.75
	500	2.48	3.97	4.56
	650	3.22	5.16	5.91
100	200	—	1.21	1.55
	400	1.13	2.18	2.72
	500	1.40	2.68	3.33
	650	1.82	3.48	4.32
200	400	—	1.25	1.68
	500	—	1.55	2.07
	650	—	2.01	2.69
400	500	—	—	1.13
	650	—	1.04	1.47

*$(L/2)<0.9\, r_{eH}$, $r_w=0.365$ ft, and $L>h\sqrt{k_H/k_V}$ are assumed.

TABLE 3—PRODUCTIVITY-INDEX RATIOS OF HORIZONTAL AND VERTICAL WELLS, 30-ACRE WELL SPACING*

h (ft)	L (ft)	J_H/J_V $(k_V/k_H)=0.1$	J_H/J_V $(k_V/k_H)=0.5$	J_H/J_V $(k_V/k_H)=1.0$
25	100	1.08	1.62	1.81
	200	1.70	2.31	2.49
	400	2.68	3.39	3.59
	800	4.56	5.55	5.82
	1,000	5.63	6.82	7.14
	1,150	6.51	7.91	8.28
50	100	—	1.15	1.39
	200	1.10	1.79	2.07
	400	1.87	2.80	3.13
	800	3.34	4.73	5.19
	1,000	4.13	5.83	6.39
	1,150	4.77	6.75	7.40
100	200	—	1.19	1.49
	400	1.12	2.01	2.42
	800	2.09	3.55	4.18
	1,000	2.60	4.39	5.15
	1,150	3.00	5.07	5.96
200	400	—	1.23	1.60
	600	—	1.75	2.24
	800	1.15	2.28	2.90
	1,000	1.43	2.83	3.59
	1,150	1.64	3.26	4.15
400	600	—	—	1.32
	800	—	1.27	1.73
	1,000	—	1.58	2.15
	1,150	—	1.82	2.48

*$(L/2)<0.9\, r_{eH}$, $r_w=0.365$ ft, and $L>h\sqrt{k_H/k_V}$ are assumed.

to measure productivity improvements with horizontal drainholes. Similar experiments have been conducted with a 46-in. [117-cm] -diameter tank filled with brine solution. A 1/32-in. [0.08-cm] -diameter copper wire was used to simulate the wellbore; a copper ring mounted along the tank periphery simulated the drainage boundary. The wire dimensions gave a scaling ratio of 23 between the experimental apparatus and field dimensions. A fixed voltage drop (6 V) was imposed between the wellbore and the copper peripheral ring. The similarity of Ohm's and Darcy's laws was used to estimate horizontal-well productivity index, $q/\Delta p$. The productivity indices were measured for different well lengths and water heights in the tank. Figs. 3a and 3b show comparisons of Perrine's[21] laboratory data and the present laboratory data with the theoretical Eq. 17. As shown, theoretical predictions are about 7 to 10% lower than the experimental data. Although Eq. 17 assumes that $L>h$, Fig. 3b shows that even for $L<h$, the theoretical predictions are only 8 to 10% lower than the experimental data. This indicates that Eq. 17 could also be used if $L<h$.

The conservative model predictions could be a result of the overestimation of flow resistance around the wellbore. As noted earlier, the vertical-resistance term accounts for the flow resistance 360° [6.3 rad] around the wellbore. Part of this resistance is also accounted for in a horizontal-resistance term. Despite this duplication, the theoretical equation, Eq. 17, shows good agreement with the experimental data, indicating the usefulness of the equation to predict horizontal-well oil-production rates.

Electrical analog experiments were also conducted to verify Eq. 13, which accounts for well eccentricity (well location other than the midheight of a reservoir). The experimental results showed <10% variation with the results of Eq. 13.

Results and Discussion

Well-productivity enhancements from horizontal wells were calculated for the following variables: (1) reservoir heights of 25 to 400 ft [8 to 122 m]; (2) horizontal-well lengths of 100 to 2,000 ft [30 to 610 m]; (3) vertical-well drainage areas of 10, 20, 30, 40, and 80 acres [4, 8, 12, 16, and 32 ha]; and (4) vertical-to-horizontal permeability ratios of 0.1, 0.5, and 1.0.

Tables 2 through 4 list typical productivity improvements (Eq. 17) for 10-, 30-, and 80-acre [4-, 12-, and 32-ha] well spacings, respectively. These tables, based on the assumption of $r_{eH}=r_{eV}$, list results for various horizontal-well lengths and k_V/k_H ratios. The productivity-improvement results from Table 3 (30-acre [12-ha] well spacing) are plotted in Figs. 4 through 6. Fig. 4 shows the influence of reservoir height on the horizontal- and vertical-well productivity-index ratio, while Fig. 5 shows the results plotted as negative skin factors. Fig. 4 shows that in a uniform-permeability reservoir, a 1,000-ft [305-m] -long horizontal well augments unstimulated vertical-well productivity by factors of 7.1 and 2.15 in 25- and 400-ft [8- and 122-m] -thick reservoirs, respectively. In other words, as shown in Fig. 5, a 1,000-ft [305-m] -long horizontal well could be represented by a vertical well with negative skin factors of −6.43 and −4.0 for 25- and 400-ft [8- and 122-m] -thick reservoirs, respectively. In thin reservoirs, contact areas of horizontal wells are significantly larger than vertical wells ($L \gg h$), resulting in substantially higher productivities. If $L/h \gg 1$, the vertical-flow resistance is very small compared with the horizontal-flow resistance; therefore, vertical-flow resistance can be neglected (see Eqs. 7b and 9). The problem then reduces mathematically to flow into a fully penetrating, infinite-conductivity vertical fracture. As noted earlier, this is consistent with the other theoretical predictions.[15,16]

Fig. 6 shows the influence of reservoir anisotropy on the productivity augmentation through horizontal wells. The low vertical permeability significantly reduces horizontal-well productivity. Conversely, the high vertical permeability enhances horizontal-well productivity.

Figs. 7 through 9 depict typical replacement-ratio plots for horizontal wells. The calculations assume that $r_{eH}=L$. As shown

TABLE 4—PRODUCTIVITY-INDEX RATIOS OF HORIZONTAL AND VERTICAL WELLS, 80-ACRE WELL SPACING*

		J_H/J_V		
h (ft)	L (ft)	$(k_V/k_H)=0.1$	$(k_V/k_H)=0.5$	$(k_V/k_H)=1.0$
25	100	1.07	1.56	1.72
	200	1.63	2.13	2.28
	400	2.43	2.95	3.09
	800	3.75	4.35	4.50
	1,000	4.40	5.05	5.21
	1,500	6.19	7.04	7.25
	1,900	7.88	8.97	9.24
50	100	—	1.14	1.36
	200	1.09	1.71	1.94
	400	1.77	2.52	2.76
	800	2.92	3.86	4.14
	1,000	3.48	4.52	4.83
	1,500	4.96	6.34	6.74
	1,900	6.31	8.08	8.60
100	200	—	1.18	1.45
	400	1.11	1.89	2.22
	800	1.96	3.07	3.50
	1,000	2.37	3.65	4.12
	1,500	3.43	5.18	5.82
	1,900	4.36	6.60	7.41
200	400	—	1.21	1.54
	800	1.14	2.11	2.60
	1,000	1.39	2.55	3.11
	1,500	2.05	3.68	4.45
	1,900	2.60	4.68	5.66
400	800	—	1.25	1.65
	1,000	—	1.53	2.01
	1,500	1.08	2.24	2.93
	1,900	1.37	2.85	3.72

*$(L/2)<0.9\,r_{eH}$, $r_w=0.365$ ft, and $L>h\sqrt{k_H/k_V}$ are assumed.

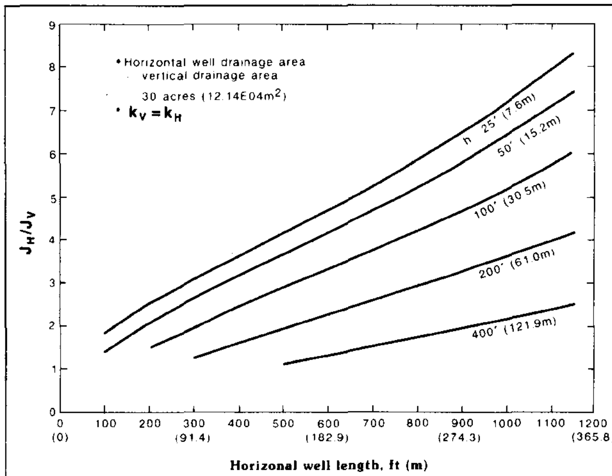

Fig. 4—Influence of reservoir height on horizontal- and vertical-well productivity-index ratio.

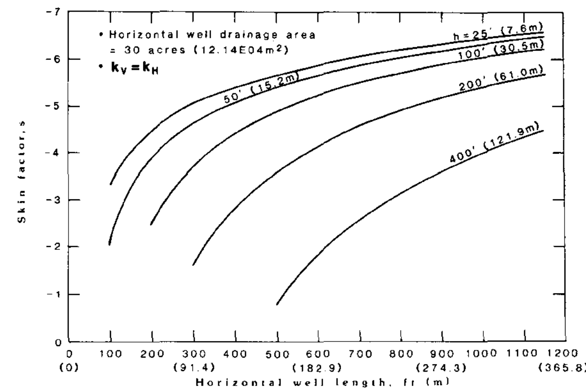

Fig. 5—Influence of reservoir height on horizontal-well skin factors.

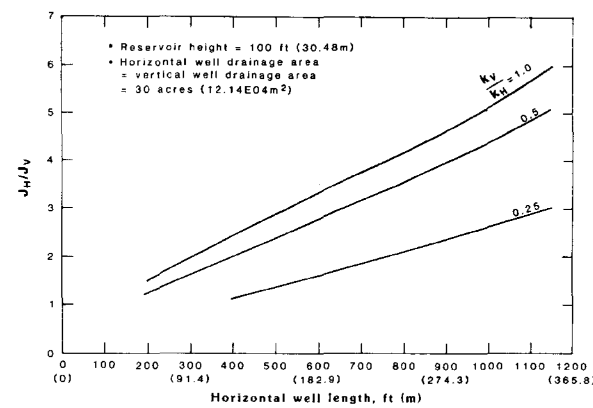

Fig. 6—Influence of reservoir anisotropy on horizontal- and vertical-well productivity-index ratio.

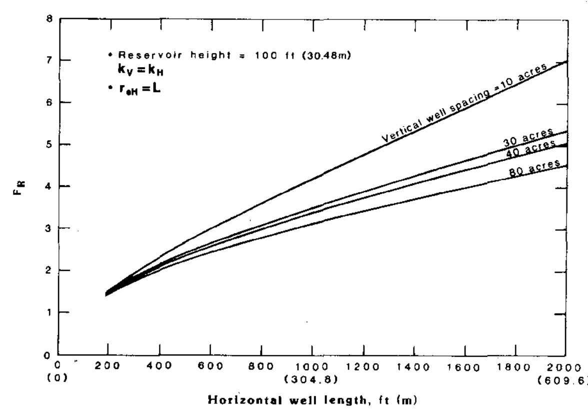

Fig. 7—Influence of well spacing on replacement ratio, F_R.

in Fig. 7, in a 100-ft [30-m] -thick reservoir, a 1,000-ft [305-m] -long horizontal well replaces 4.2 and 3.1 conventional vertical wells drilled at 10- and 80-acre [4- and 32-ha] spacing, respectively. Fig. 8 depicts the influence of reservoir height on the replacement ratio. A horizontal well replaces more vertical wells in a thin-bed reservoir than in a thick-bed reservoir, demonstrating the effectiveness of horizontal wells in thin-bed reservoirs. Fig. 9 shows the typical influence of reservoir anisotropy on the replacement ratios.

Fig. 10 depicts the influence of reservoir height on slant-well productivity. The figure shows that in an isotropic reservoir, a minimum of 35° [0.6-rad] slant is required to realize 20% productivity improvements. These requirements increase as the ratio of vertical and horizontal permeability decreases.[19,20] In contrast to horizontal wells, slant wells are more productive in thick reservoirs than in thin reservoirs. Hence, depending on reservoir thickness and anisotropy, one should be able to maximize oil production by selecting between vertical, slant, and horizontal wells. Figs. 11

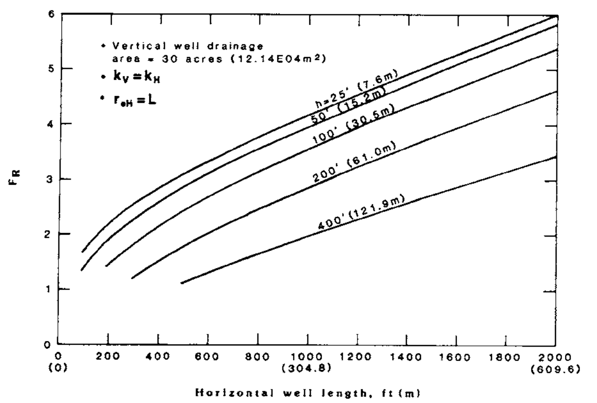

Fig. 8—Influence of reservoir height on replacement ratio, F_R.

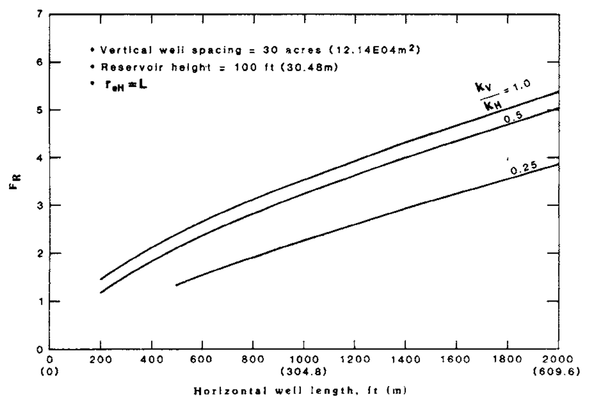

Fig. 9—Influence of reservoir anisotropy on replacement ratio, F_R.

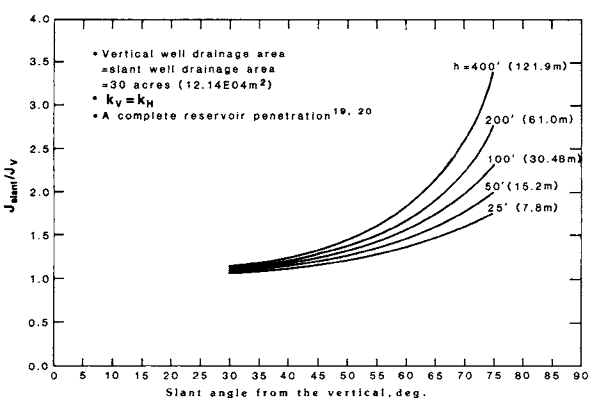

Fig. 10—Influence of reservoir height on slant- and vertical-well productivity-index ratio.

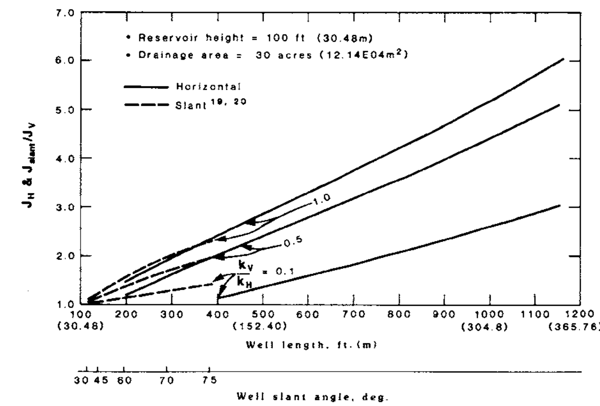

Fig. 11—Comparison of slant- and horizontal-well performance ($h = 100$ ft).

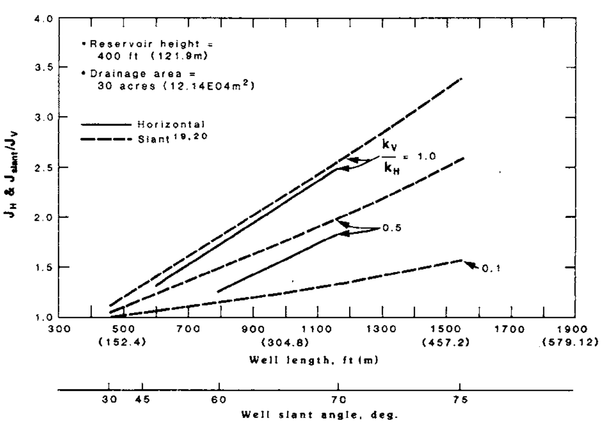

Fig. 12—Comparison of slant- and horizontal-well performance ($h = 400$ ft).

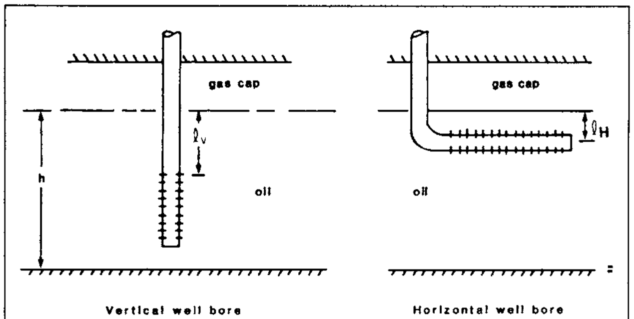

Fig. 13—Gas-coning schematic using vertical and horizontal wells.

and 12 show productivity comparisons of the three well geometries for 100- and 400-ft [30- and 122-m] -thick reservoirs. The comparisons were made for a 30-acre [12-ha] drainage area and for *an equal reservoir contact area* for horizontal and slant wells. The slant wells were assumed to give complete reservoir penetration. Fig. 11 demonstrates that in a 100-ft [30-m] -thick reservoir, a horizontal well gives better productivity than either a slant or a vertical well if $k_V/k_H \geq 0.1$. Conversely, Fig. 12 shows that in a 400-ft [122-m] -thick reservoir, the horizontal- and slant-well productivities are almost equal for $k_V/k_H = 1.0$. However, for $k_V/k_H < 1$, a slant well performs better than a horizontal well.

In conclusion, horizontal wells are suitable for thin reservoirs (200 ft [61 m]) and in high-vertical-permeability reservoirs ($k_V \cong k_H$). Slant wells are suitable for thick reservoirs or in reservoirs with several impermeable shale streaks.

Gas- and Water-Coning Characteristics

Field experience[2-6] indicates that horizontal wells not only increase oil production but also reduce gas- and water-coning tendencies. Figs. 13 and 14 show schematics of gas and water coning for horizontal and vertical wells. A horizontal well exhibits better performance than vertical wells in resisting gas and water coning because

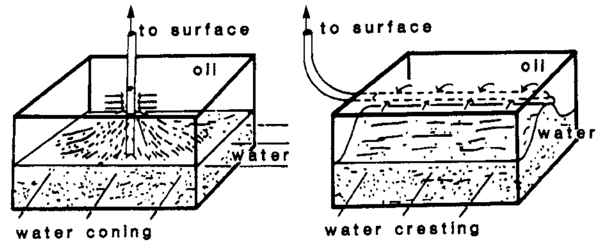

Fig. 14—Water-coning schematic using vertical and horizontal wells.

it requires much smaller pressure drawdown than a vertical well to produce oil at the same rate and because it has almost a linear pressure gradient from the wellbore to the drainage radius.[9] This results in a steady gas dip or water rise over the long producing horizontal-well length. In contrast, in a vertical well, a log-linear pressure gradient from the wellbore to drainage radius, along with a conical flow at a single point (Fig. 14), accelerates coning problems. As noted earlier, Refs. 11 and 12 developed a chart to calculate water coning for horizontal wells. Another method, probably simpler, is described below. Eq. 18 can be used to calculate oil production without gas coning from a vertical well[24]:

$$(q_{\max})=\frac{1.535(\rho_o-\rho_g)k_o[h^2-(h-\ell_V)^2]}{\ln(r_e/r_w)}, \quad \ldots\ldots\ldots\ldots (18)$$

where

h = reservoir height, ft,
ℓ_V = distance between the gas/oil interface and perforation top, ft, and
$q_{\max}$ = maximum oil production without gas coning (B/D).

To calculate $q_{\max}$ for a horizontal well, we can substitute horizontal-well effective wellbore radius, r_{we}, for r_w in Eq. 18. Thus, substituting Eq. 16 into Eq. 18, assuming $r_{eV}=r_{eH}$, and taking a ratio of maximum oil production using horizontal and vertical wells, we have

$$\frac{(q_{\max})_H}{(q_{\max})_V}=\frac{[h^2-(h-\ell_H)^2]\ln(r_e/r_w)}{[h^2-(h-\ell_V)^2]\ln(r_e/r_{we})}, \quad \ldots\ldots\ldots\ldots (19)$$

where ℓ_H=distance between the horizontal well and the gas/oil interface (Fig. 13). Horizontal wells provide the following two operational options.

1. Drill the horizontal well at the top of the vertical well perforations—i.e., $\ell_H=\ell_V$. Substituting this into Eq. 19 we have

$$\frac{(q_{\max})_H}{(q_{\max})_V}=\frac{\ln(r_e/r_w)}{\ln(r_e/r_{we})}. \quad \ldots\ldots\ldots\ldots (20)$$

In practical field applications, r_{we} of a horizontal well is always greater than r_w of a vertical well; hence, we always have the relationship

$$(q_{\max})_H>(q_{\max})_V.$$

Thus, without gas coning, a horizontal well produces oil at a rate higher than a vertical well even though both are perforated at the same distance from the gas/oil interface.

2. Produce the horizontal well at the same rate as the vertical well—i.e., $(q_{\max})_H=(q_{\max})_V$. Using Eq. 19, we have

$$(h-\ell_H)^2=h^2-\frac{[h^2-(h-\ell_V)^2]\ln(r_e/r_{we})}{\ln(r_e/r_w)}. \quad \ldots\ldots\ldots\ldots (21)$$

Solving Eq. 21, we obtain $\ell_H<\ell_V$. Thus, to produce at the same rate as a vertical well, a horizontal well could be located closer to the gas/oil interface than the top of the vertical-well perforations. Although a horizontal-well location is closer to the gas/oil interface than a vertical well, it still resists gas coning.

Because Option 1 gives higher production rates than Option 2, it is the preferred operational choice. A similar analysis could be made for the water-coning problem.

Conclusions

1. Mathematical equations that include the influence of reservoir anisotropy are presented to calculate steady-state oil production from a horizontal well. An equation is given to account for a change in oil-production rate as a result of well placement in a vertical plane other than at the midheight of a reservoir.
2. The theoretical predictions show good agreement with the present electrical analog experimental data and the laboratory data available in the literature.
3. A horizontal well could produce oil at a rate two to six times greater than an unstimulated vertical well, assuming the same pressure drawdown. The well-productivity improvements depend on reservoir height, horizontal-well length, and reservoir anisotropy.
4. A slant well with a 60 to 70° [1- to 1.2-rad] angle to a vertical through the pay zone produces oil at a rate two to three times faster than a vertical well under the similar pressure-drawdown conditions. A minimum slant of 35° [0.6 rad] is required to increase oil production by at least 1.2 times that of a vertical well.
5. Vertical, slant, and horizontal wells were compared assuming equal drainage areas. In addition, the reservoir contact areas of horizontal and slant wells were assumed to be the same. The comparison shows that in a thick (400-ft [122-m]) reservoir, slant- and horizontal-well production rates are comparable if $k_V/k_H=1$. However, if $k_V/k_H<1$, slant wells perform better than horizontal and vertical wells. In a thin (100-ft [30-m]) reservoir, horizontal-well production rates are significantly better than slant and vertical wells if $k_V/k_H\geq 0.1$.
6. Horizontal wells reduce gas- and water-coning tendencies. Equations are described to estimate the reduction in gas and water coning with horizontal wells.
7. Horizontal wells are suitable for thin reservoirs, high-vertical-permeability reservoirs, and reservoirs with gas- and water-coning problems.

Nomenclature

a = half the major axis of drainage ellipse, ft [m]
A = drainage area, acres [m^2]
B = FVF, dimensionless
F_R = replacement ratio
h = reservoir height, ft [m]
i = complex number variable
J = productivity index, B/D-psi [$m^3/d\cdot kPa$]
J_a = areal productivity index, B/D-psi-ft^2 [$m^3/d\cdot kPa\cdot m^2$]
k = permeability, darcies
ℓ_H = distance between gas/oil interface and horizontal well, ft [m]
ℓ_V = distance between gas-oil interface and top of vertical-well perforations, ft [m]
ℓ_δ = vertical distance between reservoir center and horizontal-well location, ft [m]
L = horizontal- or slant-well length, ft [m]
n = number of wells
p = pressure, psia [kPa]
Δp = pressure drop, psia [kPa]
q = flow rate, liquid vol-unit/D
q_H = flow rate into horizontal well, liquid vol-unit/D
q_1 = flow rate defined in Eq. 1
q_1^* = flow rate defined in Eq. 2
q_2 = flow rate defined in Eq. 4
q_2^* = flow rate defined in Eq. 5

r = radius, ft [m]
r_e = drainage radius, ft [m]
r_w = wellbore radius, ft [m]
r_{we} = effective vertical wellbore radius, ft [m]
Δr = half horizontal well length, ft [m]
R_f = flow resistance
s = skin factor, dimensionless
z = distance along vertical axis, ft [m]
α = slant angle with vertical axis, degrees [rad]
β = square root of permeability ratio, $\sqrt{k_H/k_V}$
θ = coordinate in cylindrical coordinate system, degrees [rad]
μ = fluid viscosity, cp [mPa·s]
ρ = density, g/cm^3
ϕ = potential function
Ψ = stream function

Subscripts

d = drainage
g = gas
H = horizontal well
max = maximum
o = oil
V = vertical well
w = well

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Appendix A—Calculation of Flow in a Horizontal Plane

Fig. A-1 shows a schematic of flow to a horizontal well in a horizontal plane. The ellipses represent constant-pressure (constant-ϕ) curves while the hyperbolas represent constant streamlines (constant Ψ). Slichter[25] showed that such a system of confocal ellipses and hyperbolas could be represented as

$$w(z)=\phi+i\Psi=\cosh^{-1}(z/\Delta r). \qquad \text{(A-1)}$$

By definition, $z=x+iy$. Substituting this into Eq. A-1 and then equating real and imaginary parts, we have

$$x=\Delta r \cosh \phi \cos \Psi \qquad \text{(A-2)}$$

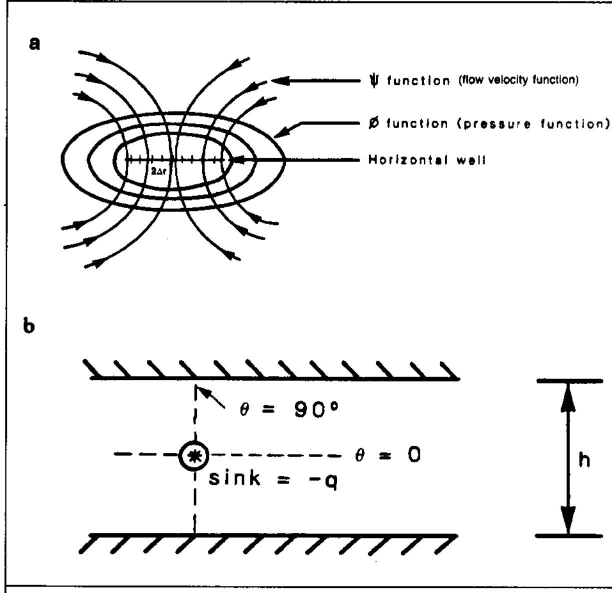

Fig. A-1—Schematic of potential flow to a horizontal well: (a) horizontal plane and (b) vertical plane.

and

$$y=\Delta r \sinh \phi \sin \Psi . \qquad \text{(A-3)}$$

The equation with the hyperbolic function represents a classic equation of an ellipse, while the equation with the trigonometric functions represents a classic equation of a hyperbola. Eqs. A-2 and A-3 could be reformulated as

$$\phi=\cosh^{-1} H^* \qquad \text{(A-4)}$$

and

$$\Psi=\cos^{-1} H^*, \qquad \text{(A-5)}$$

where

$$H^*=\left[\frac{x^2+y^2+\Delta r^2 \pm \sqrt{(x^2+y^2+\Delta r^2)^2-4\Delta r^2 x^2}}{2\Delta r^2}\right]^{1/2}. \qquad \text{(A-6)}$$

The plus sign refers to ϕ; the minus sign refers to Ψ. Boundary conditions for the horizontal well located along the x axis are

$\phi = 0$, for $|x| \leq \Delta r$, $p_w=0$,

$\phi = \cosh^{-1}\left|\frac{a(x.y)}{\Delta r}\right|$, for $|x| \geq \Delta r$; reservoir pressure,

$\Psi = \cos^{-1}\left|\frac{a(x,y)}{\Delta r}\right|$, for $|x| < \Delta r$; wellbore flow,

$\Psi = 0$, for $x > \Delta r$,

and

$\Psi = \pi$, for $x < -\Delta r$.

The potential function, ϕ, is the same as the pressure, p. At drainage radius, r_{eH}, half the major and minor axes of the ellipse of constant pressure are a and b (Fig. 2). Hence, pressure at the drainage boundary, p_e, is

$$p_e=\cosh^{-1}(a/\Delta r)=\ln\frac{(a+\sqrt{a^2-\Delta r^2})}{\Delta r}. \qquad \text{(A-7)}$$

The pressure drop between the drainage boundary and well, Δp, is the same as p_e defined in Eq. A-7 because wellbore pressure is assumed to be zero. Substituting this into Darcy's porous-medium equation, we can show it to be[25]

$$q_1=\frac{2\pi k_o \Delta p/\mu}{\ln\left(\dfrac{a+\sqrt{a^2-\Delta r^2}}{\Delta r}\right)},$$

where Δr=well half-length=$L/2$. Eq. 1 represents flow to a horizontal well from a horizontal plane.

We normally use drainage radius in calculations. To calculate horizontal-well drainage radius, r_{eH}, areas of a circle and ellipse (in a horizontal plane, Fig. A-1a) are equated. This reduces to

$$r_{eH}=\sqrt{ab}, \qquad \text{(A-8)}$$

where a and b are major and minor axes of a drainage ellipse. Moreover, $+L/2$ and $-L/2$ represent foci of a drainage ellipse. Hence, using properties of an ellipse, we can show that

$$b=\sqrt{a^2-(L/2)^2}. \qquad \text{(A-9)}$$

Substituting Eq. A-9 into Eq. A-8, we have

$$r_{eH}=a[1-(L/2a)^2]^{1/4}. \qquad \text{(A-10)}$$

Eq. A-10 shows that if $L/2a \leq 0.5$, $r_{eH} \cong a$. Table 1 lists the geometrical relationships between L, r_{eH}, and a.

Appendix B—Calculation of Flow in a Vertical Plane

As shown in Fig. A-1b, a horizontal well is represented as a sink in a parallel-plate channel. Hence, using the Schwarz-Christoffel mapping function,[26] we can show that

$$w(z)=\phi+i\Psi=-q \ln[\sinh(\pi z/h)], \qquad \text{(B-1)}$$

where $-q$ represents a sink strength—i.e., flow rate per unit length of a horizontal well. The right side of Eq. B-1 is split into real and imaginary parts, and these parts are then equated with ϕ and Ψ, respectively. To achieve this, $\sinh(\pi z/h)$ is first rewritten in its exponential form. The exponential terms are then expanded as a power series. Neglecting high-order series terms reduces Eq. B-1 to

$$-q \ln[\sinh(\pi z/h)]=q[(\pi z/h)-\ln(\pi z/h)]. \qquad \text{(B-2)}$$

By definition, $z=r \exp(i\theta)$.

Substituting Eq. B-2 into Eq. B-1 and expanding $\exp(i\theta)$ in terms of $\sin\theta$ and $\cos\theta$, we can show that

$$\phi+i\Psi=q[(\pi r/h)\cos\theta-\ln(\pi r/h)]+iq[(\pi r/h)\sin\theta-\theta]. \qquad \text{(B-3)}$$

The real part of Eq. B-3 represents the potential function (pressure function), ϕ, and the complex part represents the stream function (flow rate), Ψ.

At the channel wall, $r=h/2$ and $\theta=90°$ [1.6 rad]. Here, the Ψ function calculated from Eq. B-3 has zero value, indicating no flow across the reservoir boundary. Similarly, the channel-wall boundary conditions are substituted into the real part of Eq. B-3 to obtain pressure at the wall. The presssure at the wellbore is assumed constant. Hence, by substituting $r=r_w$ and $\theta=90°$ [1.6 rad] into the real part of Eq. B-3, we can obtain pressure at the wellbore. The pressure difference between the channel wall and the wellbore, Δp, can be shown to be

$$\Delta p=-q \ln(h/2r_w). \qquad \text{(B-4)}$$

Darcy's equation for flow through a porous medium is

$$q_w=q_2=\int_0^{2\pi}(-qk/\mu)d\theta. \qquad \text{(B-5)}$$

Substituting q from Eq. B-4 into Eq. B-5 and integrating reduces to (Eq. 4):

$$q_2=\frac{(2\pi k_o \Delta p/\mu)}{\ln(h/2r_w)}.$$

Eq. 4 represents flow in a vertical plane to a horizontal well of unit length.

Appendix C—Influence of Anisotropy

Let us assume that k_H represents horizontal permeability and k_V represents vertical permeability in a reservoir of thickness h. Muskat[18] showed that the influence of anisotropy could be accounted for by modifying the z axis as $z\sqrt{k_H/k_V}$ and using effective reservoir permeability as $\sqrt{k_V k_H}$. Thus, the horizontal-flow equations, Eqs. 1 through 3, can be modified as

$$q_1=\frac{2\pi\sqrt{k_V k_H}\,\Delta p/(\mu/B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]}, \quad \text{(C-1)}$$

$$q_1^*=q_1 h\sqrt{k_H/k_V}, \quad \text{(C-2)}$$

and

$$R_{fH}=\frac{\mu B_o}{2\pi k_H h}\ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]. \quad \text{(C-3)}$$

For the vertical flow in a horizontal well of length L, Eqs. 4 through 6 can be modified as

$$q_2=\frac{2\pi\sqrt{k_V k_H}\,\Delta p/(\mu B_o)}{\ln(h/2r_w)}, \quad \text{(C-4)}$$

$$q_2^*=q_2 L\sqrt{k_V/k_H}, \quad \text{(C-5)}$$

and

$$R_{fV}=\left(\frac{\mu B_o}{2\pi k_V L}\right)(h/h)\ln(h/2r_w). \quad \text{(C-6)}$$

As shown in Eq. 7, adding R_{fH} and R_{fV}, we have

$$R_{fH}+R_{fV}=\Delta p/q_H \quad \text{(C-7)}$$

and

$$q_H=\frac{2\pi k_H h\Delta p/(\mu/B_o)}{\ln\left[\dfrac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]+\dfrac{\beta^2 h}{L}\ln\left(\dfrac{h}{2r_w}\right)},$$

where

$$\beta=\sqrt{k_H/k_V}.$$

Appendix D—Calculation of Replacement Ratio

The replacement ratio, F_R, represents the number of vertical wells required to produce at the same rate as that of a single horizontal well. The replacement ratio takes into account the differences in drainage areas of horizontal and vertical wells.

Assume that it is desired to drain a reservoir with area A. Let n_V and n_H represent the number of vertical and horizontal wells, respectively, required to drain area A. If A_V and A_H represent vertical- and horizontal-well drainage areas, then

$$A=n_V A_V=n_H A_H=n_V\pi(r_{eV})^2=n_H(\pi r_{eH})^2. \quad \text{(D-1)}$$

F_R is defined as

$$F_R=n_V/n_H=(r_{eH}/r_{eV})^2. \quad \text{(D-2)}$$

Giger[10,11] introduced the concept of areal productivity index (in B/D-psi-ft^2), which is defined below:

$$J_a=J/A_d. \quad \text{(D-3)}$$

Assuming equal J_a for the vertical and horizontal wells, we obtain

$$\left[\frac{2k_o h/\mu_o}{B_o\ \ln(r_{eV}/r_w)r_{eV}^2}\right]_V=\left[\frac{2k_o h/\mu_o}{B_o\ \ln(r_{eH}/r_{we})r_{eH}^2}\right]_H.$$

This reduces to

$$(r_{eH}/r_{we})^{r_{eH}^2/r_{eV}^2}=r_{eV}/r_w. \quad \text{(D-4)}$$

Combining Eqs. D-2 and D-4 yields

$$r_{we}=\sqrt{F_R}\,r_w(r_w/r_{eV})^{(1/F_R)-1}. \quad \text{(D-5)}$$

Eq. 14 for calculating r_{we} of a horizontal well could be reformulated to include reservoir anisotropy:

$$r_{we}=\frac{\left(\dfrac{Lr_{eH}}{2}\right)\left(\dfrac{2r_w}{\beta h}\right)^{\beta h/L}}{a\left\{1+\sqrt{1-\dfrac{1}{F_R}\left[\left(\dfrac{L}{2a}\right)\left(\dfrac{r_{eH}}{r_{eV}}\right)\right]^2}\right\}}. \quad \text{(D-6)}$$

F_R is calculated by solving Eqs. D-5 and D-6 simultaneously. The solution procedure involves the following.

1. Know h, r_w, r_{eV}, and $\beta^2=k_H/k_V$.
2. Choose L and select r_{eH} so that $0.1<L(2r_{eH})<0.9$.
3. Try different F_R values ($F_R>1$) until for a given F_R, r_{we} values calculated with Eqs. D-5 and D-6 are equal.

Newton's midpoint numerical method was used to calculate the appropriate F_R values, which gave $<1\%$ difference in calculated r_{we} with Eqs. D-5 and D-6.

The replacement ratio, F_R, shows a limited dependence on the ratio $L/2r_{eH}$. If $L/2r_{eH}$ is assumed to be 0.1, a maximum value of F_R is obtained. In contrast, if $L/2r_{eH}$ is assumed to be 0.9, a minimum value of F_R is obtained. However, the difference between the two extreme F_R values is $<7\%$. Therefore, the calculations presented in this paper assume that $L/2r_{eH}=0.5$.

SI Metric Conversion Factors

acres	× 4.046 873	E+03	=	m^2
degrees	× 1.745 329	E−02	=	rad
ft	× 3.048*	E−01	=	m

*Conversion factor is exact.

JPT

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Discussion of Augmentation of Well Productivity With Slant and Horizontal Wells

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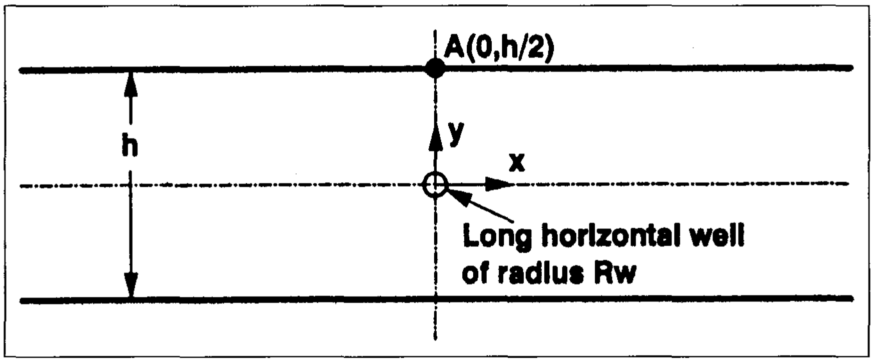

Fig. D-1—Vertical cross setion through reservoir.

In "Augmentation of Well Productivity With Slant and Horizontal Wells" (June 1988 *JPT,* Pages 729–39; *Trans.,* AIME, **285**), Joshi derived an equation for the additional pressure difference required for fluid to converge to a long horizontal well centered in a uniform reservoir with horizontal upper and lower boundaries.

This note is to point out that this equation is significantly in error, even though correct equations appeared earlier in the literature and were also referenced by Joshi. **Fig. D-1** shows the geometry and variables Joshi uses in his derivation. He starts with the following correct equation for the complex fluid potential around the well.

$$F=\Phi+i\Psi=-q\ \ln[\sinh(\pi z/h)], \quad \text{(D-1)}$$

where $z=x+iy(x,y$ as in Fig. D-1) and $-q=Q\mu/2\pi kL$.

Joshi then derives the potential that exists between Point A and the well (Fig. D-1) by an approximate method that produces errors. This difference in potential can be calculated by writing the complex potential for Point A and the well with Eq. D-1 and subtracting as follows.

$$\begin{aligned}\Phi_A-\Phi_w&=R_e(F_{z=ih/2}-F_{z=r_w})\\&=-q\ \ln[1/\sin(\pi r_w/h)]\\&\cong -q\ \ln(h/\pi r_w)\text{ if } r_w\ll h\\&=\frac{Q\mu}{2\pi kL}\ln(h/\pi r_w). \quad \text{(D-2)}\end{aligned}$$

Joshi's expression is the same as Eq. D-2 except that he replaces the π in the logarithmic term by 2. However, even with this correction, a second error still remains in Joshi's treatment. He assumes that, if the flow delivered to a horizontal well and a vertical, infinite-conductivity fracture is the same, then the well pressure must be reduced below that in the fracture by the amount corresponding to Eq. D-2. This is not the case. The actual difference in pressure required can be calculated by considering the potentials along the x axis of Fig. D-1. The difference in potential between a point located at $(x,0)$ and the well is given by

$$\begin{aligned}\Phi_x-\Phi_w&=R_e(F_{z=x}-F_{z=r_w})\\&=-q\{\ln[\sinh(\pi|x|/h)]-\ln(\pi r_w/h)\}\\&\cong -q[(\pi x/h)+\ln(h/2\pi r_w)]\text{ if }|x|\gg h\\&=\frac{Q\mu|x|}{2khL}+\frac{Q\mu}{2\pi kL}\ln(h/2\pi r_w). \quad \text{(D-3)}\end{aligned}$$

This shows that the potential difference between a point located at x (assuming that x is sufficiently distant so that the approximation for the sinh function in Eq. D-3 is true) is given by a term linear in $|x|$ plus a second term involving the ratio h/r_w. The factor 2 in the logarithmic term of Eq. D-3 is from the substitution

$$\sinh(\pi x/h)=[\exp(\pi x/h)-\exp(-\pi x/h)]/2.$$

For the case of a fracture at $x=0$, the equation corresponding to Eq. D-3 has only the linear term. It follows then that the additional pressure differential that we seek is given by the second term in Eq. D-3; the argument in the logarithm of this term is $h/2\pi r_w$, whereas in Joshi's paper the term was $h/2r_w$. This error makes a significant difference in cases where the pressure drop resulting from convergence is appreciable.

The incorrect formula described above has been widely accepted in the literature.

Nomenclature

- F = complex potential defined by Eq. D-1, $ML^{-1}T^{-2}$
- h = reservoir height, ft
- i = $\sqrt{-1}$
- k = permeability, L^2
- L = well length, L
- q = flow as used by Joshi, $ML^{-1}T^{-2}$
- Q = production rate, L^3T^{-1}
- r_w = wellbore radius, L
- R_e = real function
- x,y = Cartesian coordinates, L
- $|x|$ = absolute value of x, L
- μ = viscosity, $ML^{-1}T^{-1}$
- Φ = fluid potential, $ML^{-1}T^{-2}$
- Φ_A = fluid potential at Point A, $ML^{-1}T^{-2}$
- Φ_w = fluid potential at well, $ML^{-1}T^{-2}$
- Ψ = stream function, $ML^{-1}T^{-2}$

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Author's Reply to Discussion of Augmentation of Well Productivity With Slant and Horizontal Wells

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I do not agree with Butler's comments. The equation that I presented in my paper will provide reliable horizontal-well-rate estimates for practical oilfield applications.

The equation for steady-state horizontal-well productivity that I derived in my original paper[1] is a simplified mathematical solution that is sufficiently accurate for engineering purposes and highly useful for field practicing engineers such as myself. The assumptions and limits of the equation were stated clearly in the paper.

The equation I derived was checked for accuracy using laboratory data. I also compared it with an exact 3D mathematical solution, which was published in 1988.[2] On the basis of these comparisons, under proper boundary conditions and with the assumptions stated in the paper, I expect the equation I derived to give answers within an accuracy of 5% to 10%. This is well within the accuracy of data available for normal oilfield estimates used by practicing engineers.

After applying the formula to many wells around the world, my observations have met this expectation. In some cases, I have observed that the field-measured values compare within 2% to 3% of the calculated values. Recently, I have seen this in some unpublished proprietary data. For example, in a recently drilled oil well, the productivity calculated with the equation for a 1,800-ft-long well was 1.62 (STB/D)/psi, while the measured productivity was 1.67 (STB/D)/psi. In a recently completed gas-storage well, the theoretically calculated flow rate for a 1,583-ft-long horizontal well was within 5% of the measured value. These field examples clearly show good agreement with the equation and field data when used with appropriate boundary conditions and the stated assumptions.

The formula I derived assumes that a flow problem for a 3D horizontal well can be solved as an addition of two different 2D problems. The first problem is flow in the horizontal plane, and the second is flow in the vertical plane. This solution is appropriate if and only if the flow convergence to a horizontal well occurs in the vicinity of the wellbore (i.e., when $x \leq h/2$, using Butler's terminology).

If flow from the horizontal well does not converge in the vicinity of the wellbore, then one has to solve a complete 3D problem. Such solutions are readily available in the literature. In fact, my colleagues and I have published such an exact 3D solution,[2] which specifies the precise mathematical regions where each solution can be used.

Butler's formulation is also approximate because he uses a 2D approach to solve a 3D problem for all convergence points. When the convergence point is sufficiently away from the horizontal wellbore, one should not use Eq. D-3. Instead, the results of 3D solutions should be used, especially when they are readily available in the literature.

For small convergence values, the first term in Eq. D-3 can be ignored, and this equation becomes similar to the one I proposed, except for the $[(-h/L)\ln \pi]$ term in the denominator. I explained the consequences of this term in Ref. 1. Moreover, a practicing engineer can calculate this term and demonstrate that the impact on the oil-rate calculation is normally insignificant, especially when $L > \beta h$. (Please note that my equation is for $L > \beta h$.) Therefore, inclusion of this term in the denominator of the rate equation rarely enhances the accuracy of rate calculations for normal oilfield estimates.

Butler's proposed Eq. D-3 is not complete as presented. Horizontal well rates cannot be calculated without defining how to calculate the term "x" in this equation.

Finally, Butler offers no numerical comparisons to substantiate his statements that my equation is "significantly in error" for the range of parameters encountered in practical situations.

In conclusion, on the basis of the field histories and the advanced mathematical analyses that have been published since 1988, I find no technical or practical merit in Butler's comments or his derivations.

Nomenclature

h = reservoir height, ft
k_H = horizontal permeability, md
k_V = vertical permeability, md
L = well length, ft
$\beta = \sqrt{k_H/k_V}$

References

1. Joshi, S.D.: "Augmentation of Well Productivity Using Slant and Horizontal Wells," paper SPE 15375 presented at the 1986 SPE Annual Technical Conference and Exhibition, New Orleans, Oct. 5-8.
2. Ozkan, E., Raghavan, R., and Joshi, S.D.: "Horizontal-Well Pressure Analysis," *SPEFE* (Dec. 1989) 567-75; *Trans.*, AIME, **287.**

SI Metric Conversion Factors

bbl × 1.589 873	E−01	=	m^3
ft × 3.048*	E−01	=	m
psi × 6.894 757	E+00	=	kPa

*Conversion factor is exact.

(SPE 25308)

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