A Simple Approximation to the Pseudoskin Factor Resulting From Restricted Entry

Jan Vrbik, Brock U.

Summary. Several procedures for computing the pseudoskin factor have been reported in the past, but they are either insufficiently accurate or require the use of a computer to perform the actual evaluation. This paper derives a simple formula that can achieve an accuracy quite sufficient for any practical application by means of a few approximations and use of an ordinary calculator.

Introduction and Results

This paper discusses how to evaluate the pseudoskin factor that arises by allowing the flow of oil into the wellbore in only a portion of the pay zone (the so-called partial well completion). This is done under the assumptions that the pay zone consists of only one homogeneous layer (of uniform porosity and uniform horizontal and vertical permeability) and that gravity effects can be neglected.

The restricted-entry pseudoskin factor, S_L, is a function of three parameters, H, D, and R, and is used to compute the flow rate:

$$q=2\pi k_H e(p_e-p_{sf})/\mu[\ln(r_e/r_w)+S]. \quad (1)$$

The computation of S_L has undergone a sequence of consecutive improvements and simplifications since Nisle[1] introduced it. Brons and Marting[2] presented the first approximations, and Odeh[3] presented a more extensive treatment that requires a rather complex computer program. In 1986, Vrbik[4] used a similar approach to express the final answer in the form of a single infinite series that can be easily evaluated with a minicomputer. The main difference between the last two techniques is the method of evaluation; the results are practically identical.

The purpose of this paper is to advance this line of development a step further and, with the help of a few well-justified approximations, convert Vrbik's equation into a form suitable for an ordinary calculator. The derivation of the new equation rests on an improved approximation to the $K_0(x)/K_1(x)$ ratio. Vrbik[4] proposed that $x[\ln(2/x)-0.577]$ be used [even though $x\ln(1/x)$ would have been quite adequate] on the basis of Dwight's[5] expansion of $K_0(x)$ and $K_1(x)$, which is valid only for small values of x. For large values, the same ratio approaches unity. However, $x\ln(1+1/x)$ is a function that has the correct behavior for both small and large x, and luckily enough, an error $<3.5\%$ everywhere else! With this new approximation (see the Appendix), one obtains

$$S_L=[(1-H)/H][1.2704-\ln(R)]-$$
$$-\{f(0)-f(H)+f(1-2D)-[f(1-2D+H)/2]$$
$$-[f(1-2D-H)/2]\}/H^2, \quad (2)$$

where $f(y)=y\ln(y)+(2-y)\ln(2-y)+R\ln[\sin^2(\pi y/2)+0.1053R^2]/\pi$, with $0\ln(0)=0$.

Table 1 compares the values obtained by means of Eq. 2 with the exact results of Ref. 4 (Eq. 5) quoted in parentheses. The agreement is excellent as long as $R<0.1$ and $H>R$ (this would be met in practically all situations), and remains reasonable even outside these limits. The new equation fully supersedes Eq. 6 in Ref. 4. Note that Eq. 2 bears a distant resemblance to the semiempirical Eqs. 32 and 34 in Ref. 6. Eq. 2 will provide more accurate results with a wider range of possibilities, especially when R is relatively large and H is small; the equations in Ref. 6 are of little practical use when $R>0.1$.

A Practical Example

Suppose that a 4-m pay zone has been opened to flow only from 1 to 2 m, the horizontal and vertical permeabilities have a 3:1 ratio, and the wellbore has a 28-cm diameter. First, we calculate $H=0.25$ (the opening is 1 m long), $D=0.125$ (its center is 0.5 m above the middle of the pay zone), and $R=0.0202$, in which case,

$$f(0)=2\ln(2)+R\ln(0.1053R^2)/\pi=1.3216,$$

$$f(H)=0.25\ln(0.25)+1.75\ln(1.75)+R\ln[\sin^2(0.125\pi)+0.1053R^2]/\pi=0.6204,$$

$$f(1-2D)=0.75\ln(0.75)+1.25\ln(1.25)+R\ln[\sin^2(0.375\pi)+0.1053R^2]/\pi=0.0621,$$

$$f(1-2D+H)=R\ln[\sin^2(\pi/2)+0.1053R^2]/\pi=0.0000,$$

and $f(1-2D-H)=0.5\ln(0.5)+1.5\ln(1.5)+R\ln[\sin^2(\pi/4)+0.1053R^2]/\pi=0.2572.$

The resulting pseudoskin factor is

$$S_L=3[1.2704-\ln(R)]-(1.3216-0.6204+0.0621-0.2572/2)/0.25^2=5.36.$$

Nomenclature

- d = distance between center of perforations and middle of pay zone, m
- D = relative offset of opening from pay-zone center, d/e, dimensionless
- e = pay-zone thickness, m
- h = thickness of perforated interval, m
- H = proportion of pay zone opened to flow, h/e, dimensionless
- k_H = horizontal permeability, m^2
- k_V = vertical permeability, m^2
- $K_0(x)$ = modified Bessel function of second kind, zero order
- $K_1(x)$ = modified Bessel function of second kind, first order
- p_e = reservoir pressure at r_e, Pa
- p_{sf} = sandface pressure, Pa
- q = reservoir oil flow rate, m^3/s
- r_e = drainage radius, m
- r_w = wellbore radius, m
- R = $r_w(k_V/k_H)^{1/2}/L$
- S = $S_d+S_L+S_p+S_t+S_f+S_{sw}+S_l$
- S_d = skin factor from permeability alteration or well damage
- S_f = skin factor from fractures
- S_l = pseudoskin from liquid or gas saturation in vicinity of wellbore
- S_L = pseudoskin from restricted entry
- S_p = pseudoskin from perforations
- S_{sw} = pseudoskin from slanted wellbore
- S_t = pseudoskin from non-Darcy flow
- t = variable
- x = variable
- y = variable
- $\Gamma(x)$ = gamma function
- μ = oil viscosity, Pa·s

References

1. Nisle, R.G.: "Effect of Partial Penetration on Pressure Build-Up," *Trans.*, AIME (1958) **213**, 85-90.
2. Brons, F. and Marting, V.E.: "The Effect of Restricted Fluid Entry on Well Productivity," *JPT* (Feb. 1961) 172-74; *Trans.*, AIME, **222**.
3. Odeh, A.S.: "Steady-State Flow Capacity of Wells With Limited Entry To Flow," *SPEJ* (March 1968) 43-51; *Trans.*, AIME, **243**.
4. Vrbik, J.: "Calculating the Pseudo-Skin Factor Due to Partial Well Completion," *J. Cdn. Pet. Tech.* (Sept.-Oct. 1986) **25**, No. 5, 57-61.
5. Dwight, H.B.: *Tables of Integrals and Other Mathematical Data*, fourth edition, MacMillan Publishing Co., New York City (1961).
6. Yeh, N. and Reynolds, A.C.: "Computation of the Pseudoskin Factor Caused by a Restricted-Entry Well Completion in a Multiple Reservoir," *SPEFE* (June 1989) 253-63.
7. Prudnikov, A.P., Brychkov, Yu.A., and Marichev, O.I.: *Integrals and Series, Elementary Functions*, Gordon and Breach Science Publishers, New York City (1986) **1**.
8. Gradshteyn, I.S. and Ryzhik, I.M.: *Table of Integrals, Series, and Products*, Academic Press Inc., Washington, DC (1980).

Appendix—New S_L Approximation

The starting point of the derivation is the equation for the pseudoskin factor S_L derived in Vrbik[4]:

$$S_L = 8/(H^2R\pi^3)\sum_{n=1}^{\infty} n^{-3}\sin^2(n\pi H/2)\cos^2[n\pi(D-½)] \times K_0(n\pi R)/K_1(n\pi R). \quad \text{(A-1)}$$

Eq. A-1 can be simplified with the approximation

$$K_0(x)/K_1(x) \approx x\ln(1+1/x), \quad \text{(A-2)}$$

which has a relative accuracy $>3.5\%$ for all values of x. Thus, for all practical purposes, the results will remain virtually unchanged by such replacement.

We also use the trivial identities

$$\sin^2(x) = [1-\cos(2x)]/2, \quad \text{(A-3a)}$$

$$\cos^2(x) = [1+\cos(2x)]/2, \quad \text{(A-3b)}$$

and $\cos(x)\cos(y) = [\cos(x+y)+\cos(x-y)]/2$ .... (A-3c)

to obtain

$$S_L = (H\pi)^{-2}\sum_{n=1}^{\infty}\{2n^{-2}\cos[n\pi(1-2D)] - n^{-2}\cos[n\pi(1-2D+H)] - n^{-2}\cos[n\pi(1-2D-H)] + 2n^{-2} - 2n^{-2}\cos(n\pi H)\} \times \ln[1+(n\pi R)^{-1}]. \quad \text{(A-4)}$$

To simplify Eq. A-4 further, one needs only to approximate the infinite series

$$\sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln[1+(n\pi R)^{-1}] = \sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln(n+a) - \sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln(n), \quad \text{(A-5)}$$

where $a=(\pi R)^{-1}$.

To estimate the contribution of the first term in Eq. A-5, one may write

$$\sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln(n+a) = \int_x^{\pi}\sum_{n=1}^{\infty}(n+a)^{-1}\sin(nx') \times \ln(n+a)dx' + a\int_x^{\pi}\int_0^{x'}\sum_{n=1}^{\infty}(n+a)^{-1}\cos(nx'')\ln(n+a)dx''dx'. \quad \text{(A-6)}$$

By differentiating Eq. 5.4.3.1 in Ref. 7 with respect to their variable s, setting $s=1$, and adjusting the summation interval to start at $n=1$ (Ref. 7 uses $n=0$), Eq. A-6 can be modified to

TABLE 1—COMPARISON OF NEW EQ. 2 WITH VRBIK's[4] EQ. 5 GIVEN IN PARENTHESES

R	$H=0.1$	$H=0.2$	$H=0.4$	$H=0.6$	$H=0.8$
	$D=0$ (Pay Zone Perforated at Center)				
0.1	3.52	2.56	1.20	0.53	0.16
	(4.15)	(2.48)	(1.10)	(0.46)	(0.12)
0.03	9.57	5.49	2.40	1.07	0.34
	(9.27)	(5.27)	(2.27)	(0.97)	(0.29)
0.01	16.64	8.97	3.79	1.68	0.56
	(16.28)	(8.79)	(3.68)	(1.61)	(0.51)
0.003	25.98	13.34	5.47	2.43	0.83
	(25.81)	(13.29)	(5.43)	(2.39)	(0.80)
0.001	35.29	17.57	7.07	3.14	1.10
	(35.39)	(17.65)	(7.09)	(3.13)	(1.08)
	$D=(1-H)/2$ (Opened at Top or Bottom of Pay Zone)				
0.1	6.84	4.09	1.83	0.81	0.26
	(6.76)	(3.93)	(1.71)	(0.73)	(0.20)
0.03	13.79	7.58	3.23	1.44	0.47
	(13.46)	(7.40)	(3.13)	(1.36)	(0.42)
0.01	21.82	11.41	4.72	2.10	0.71
	(21.59)	(11.33)	(4.68)	(2.05)	(0.67)
0.003	31.76	15.96	6.45	2.87	1.00
	(31.79)	(16.02)	(6.47)	(2.86)	(0.97)
0.001	41.31	20.26	8.08	3.59	1.27
	(41.61)	(20.44)	(8.14)	(3.60)	(1.26)

$$-\int_x^{\pi}\int_0^{\infty}\frac{[\ln(t)+C]\exp(-at)\sin(x')\exp(-t)}{1-2\exp(-t)\cos(x)+\exp(-2t)}dt\,dx' - a\int_x^{\pi}\int_0^{x'}\int_0^{\infty}\frac{[\ln(t)+C]\exp(-at)[\exp(-t)\cos(x'')-\exp(-2t)]}{1-2\exp(-t)\cos(x)+\exp(-2t)} \times dt\,dx''\,dx' + m, \quad \text{(A-7)}$$

where $C=0.57722$ (Euler's number) and m is a constant that is independent of x, and thus, is inconsequential. (m will cancel out of Eq. A-4).

Interchanging the order of integration, one first evaluates

$$-\int_0^{\pi}\frac{\sin(x')\exp(-t)dx'}{1-2\exp(-t)\cos(x)+\exp(-2t)} = \ln[1-2\exp(-t)\cos(x)+\exp(-2t)]/2 - \ln[1+\exp(-t)]$$

$$= \begin{cases} \ln[\sin^2(x/2)]/2 + \ldots 0(t^2) & \text{for } x>0 \\ -\ln(2)+\ln(t)+\ldots 0(t^2) & \text{for } x=0 \end{cases} \quad \text{(A-8)}$$

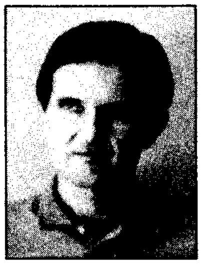
and
$$-\int_x^{\pi}\int_0^{x'}\frac{\exp(-t)\cos(x'')-\exp(-2t)}{1-2\exp(-t)\cos(x)+\exp(-2t)}dx''dx' = \int_x^{\pi}\left\{x'/2 - \arctan\left[\frac{1+\exp(-t)}{1-\exp(-t)}\tan(x/2)\right]\right\}dx'$$

$$= \begin{cases} -(x-\pi)^2/4 - t\ln[\sin(x/2)] + \ldots 0(t^3) & \text{for } x>0 \\ -\pi^2/4 - t\ln(t) + t[1+\ln(2)] + \ldots 0(t^3) & \text{for } x=0 \end{cases} \quad \text{(A-9)}$$

Eq. A-8 is a derivative with respect to t of Eq. A-9. Thus, beyond the $-(x-\pi)^2/4$ term, Eq. A-9 can be obtained by integrating Eq. A-8. $0(t^2)$ and $0(t^3)$ represent terms of the order of t^2 and t^3, respectively.

Regarding the integration of t in Eq. A-7, note that when R is small and $a=(\pi R)^{-1}$ is large, $\exp(-at)$ decreases rapidly to zero with increasing t. This means that one needs only the first few terms in the expansion of Eqs. A-8 and A-9 to achieve adequate accuracy.

Author



Jan Vrbik is professor of mathematics and chemistry at Brock U. in St. Catharines, Ont. Previously, he was a mathematics and statistics consultant at the U. of Calgary and a senior scientist at the Petroleum Recovery Inst. in Calgary. He holds a BS degree in physics from Charles U., Prague, and an MS degree in physics and a PhD degree in mathematics from the U. of Calgary.

The integration (performed with the help of Eqs. 3.381.4, 4.352.1, and 4.358.2 of Ref. 8) yields

$$\sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln(n+a)=(x-\pi)^2\ln(a)/4$$

$$+\begin{cases} -(2a)^{-1}\ln[\sin^2(x/2)]+\ldots 0(a^{-3}) & \text{for } x>0 \\ +[1+\ln(2)+C]/a+\ln(a)/a+\ldots 0(a^{-3}) & \text{for } x=0 \end{cases} \quad \text{(A-10)}$$

This means that we are discarding terms of the third and higher order in R (which will have little effect when $R<0.1$, but it will make our results somehow less accurate in the rather unusual situation when $R>1.0$).

Now it is essential to translate the previous result into a single expression (covering both the $x>0$ and the $x=0$ cases) to ensure its continuity at $x=0$. A possible solution is

$$(x-\pi)^2\ln(a)/4-(2a)^{-1}\ln[\sin^2(x/2)+0.01067/a^2]+\ldots 0(a^{-3}). \quad \text{(A-11)}$$

Similarly, the contribution of the second term in Eq. A-5 is obtained by integrating Eq. 5.5.1.24 of Ref. 7:

$$-(½)\int_x^{\pi}\{(x-\pi)[C+\ln(2\pi)]+\pi\ln[\sin(x/2)\Gamma(x/2\pi)^2/\pi]\}dx. \quad \text{(A-12)}$$

Another approximation, accurate to within 1.5% for all possible values of x, is used to simplify Eq. A-12:

$$\sin(x/2)\Gamma(x/2\pi)^2/\pi\approx(2\pi-x)/x. \quad \text{(A-13)}$$

Again, this can hardly affect the practical accuracy of our results.

Substitution into Eq. A-12 and one more integration yields

$$(x-\pi)^2[C+\ln(2\pi)]/4-\pi[x\ln(x)+(2\pi-x)\ln(2\pi-x)]/2$$

$$+m. \quad \text{(A-14)}$$

With all these approximations, Eq. A-5 can be simplified in the following manner:

$$\sum_{n=1}^{\infty} n^{-2}\cos(nx)\ln[1+(n\pi R)^{-1}]=(x-\pi)^2[C+\ln(2)-\ln(R)]/4$$

$$-\pi\{x\ln(x)+(2\pi-x)\ln(2\pi-x)+R\ln[\sin^2(x/2)+0.1053R^2]\}/2$$

$$+m, \quad \text{(A-15)}$$

which, when substituted into Eq. A-4, yields Eq. 2.

SI Metric Conversion Factors

ft	× 3.048*	E−01	= m
in.	× 2.54*	E+00	= cm

*Conversion factor is exact.

SPEFE

Original SPE manuscript received for review June 21, 1990. Paper (SPE 21469) accepted for publication July 18, 1991. Revised manuscript received May 13, 1991.