SPE 135820

Reservoir Pressure and Well Performance in Multi-well System

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This paper was prepared for presentation at the 2010 SPE Russian Oil & Gas Technical Conference and Exhibition held in Moscow, Russia, 26–28 October 2010.

Abstract

Novel simplified method of well performance calculation in multi-well system is considered. Differences between engineering content of various pressures in multi-well system traditionally denoted by one term “reservoir pressure” are shown. A new terminology to designate these pressures and estimation methods based on production and well test data are proposed.

In practice due to the complexity of well productivity analysis in multi-well system single well is often “extracted” from the environment. The influence of neighboring wells is taken into account by assigning for each well a certain value of reservoir pressure at the certain drainage area. At first glance, this simplification procedure seems so natural that does not require any justification. However, unlike the unambiguous and well-defined single well concept of "reservoir pressure", for wells in multi-well system this approach depends strongly on the context. In order to distinguish the concepts traditionally designated a single term "reservoir pressure" the definition of three types of pressure is introduced. Instantaneous and static reservoir pressures are the local parameters describing well performance subject to interference effects. The average (integral) reservoir pressure characterizes the energy state of the reservoir in whole. For the case of a single well system all these pressures are equal that creates the semantic difficulties in multi-well system consideration.

Reservoir pressures proposed can be used to provide well productivity calculations at different time periods after the change of well operation conditions. Instantaneous reservoir pressure enables to determine production rate changes at the end of transient flow regime and static reservoir pressure gives an estimation of production rate reached at the new steady state regime. The impact of well operation condition changes to the performance of surrounding wells can be also analyzed.

Introduction

Two approaches can be distinguished in the modern petroleum engineering: single-well approach (when inflow to the single well with certain external boundery conditions is considered) and multi-well approach (when neighboring production and injection wells interference effect is taken into account).

Single well inflow theory is well developed but its direct application is limited in practice and envelopes mostly the performance of exploratory wells or small fields recovered by several wells. In spite of this a great attention is paid in all reservoirs engineering courses to the consideration of single well performance model because an important basic conceptions are usually introduced using single well approach such as flow regime (transient, pseudosteady state, steady state), skin factor, productivity index, average reservoir pressure.

Equations that determine the performance of multi-well systems are also known1 but in general formulation they are quite complicated for practical application complicated. Simple equations can be obtained only for regular well systems (i.e. the systems formed by repetitive symmetry elements or patterns: five-spot, seven-spot, nine-spot, line-drive pattern, etc) producing at constant bottomhole pressure or constant rate condition2. In reality pattern regularity often can not be sustained due to wells start produce (inject) at different times or due to asynchronous changes in downhole pressure at different wells.

Therefore when single well approach is applied to the multi-well system well considered is often “pulled out” from its environment and simple equations mentioned above are used. In this case influence of neighboring wells is taken into account by assigning to each well in the system a certain value of reservoir pressure. At first glance this simplification procedure seems to be so natural that it does not require any justification. However concept of reservoir pressure is accurate defined only for a single well approach, meanwhile for a multi-well system definition of reservoir pressure can be essentially depend on the issue context. In order to distinguish the concepts traditionally denoted by the single term “reservoir pressure” definitions of three types of reservoir pressure (instantaneous, static and average reservoir pressure) are introduced for multi-well system performance analysis. Instant and static reservoir pressures are local indicators of the nearest environment influence to the analyzed well. Average (integral) reservoir pressure characterizes the energy state of the reservoir in general. For the case of a single well in the closed drainage area these three pressures coincide that creates some semantic difficulties in their description and application during transition from single well approach to multi-well systems analysis.

Necessity of using different types of reservoir pressure emerges during transition period examination occurring after each change in operating condition of the well (for example after bottomhole pressure decrease). Instantaneous reservoir pressure allows to estimate well production at the end of transient flow regime and static reservoir pressure enables to calculate well productivity with accounting of neighboring wells impact at the time when new stready state regime is reached.

Object of our work is to to analyze the impact of interference between wells to the production growth due to drawdown increase at certain producing well during waterflooding. Such arrangements aimed to the production intensification are often carried out in practice. Darcy's law (and single well approach) is usually applied to assess the effectiveness of these operations considering the reservoir pressure (which describes neighboring well influence) unchanged. It is shown below in our study that this approximation corresponds to the application of "instantaneous" inflow performance relationship (IPR) curve (and instantaneous reservoir pressure) to well productivity prediction. The issue is that this procedure is valid only during the time interval determined by the end of transient flow regime and beginning of pseudosteady sate regime at the well considered. The final (stationary) value of well productivity subject to the interference effect can be obtained by "static" IPR curve (and the static reservoir pressure). The transition from "instantaneous" IPR to the "staic" IPR occurs during pseudosteady state regime when impact of neighboring wells is dominated. As a result of interference effect production rate at other wells in the system decreases so the overall production increment of the well intensification is less than the effect estimated by single well technique when only the well exposed is considered.

The paper presents quantitative estimates of these effects obtained using the equations from some previous works[1] in which pseudosteady state regime in multi-well systems is described. Comparison with numerical model results is also presented.

Reservoir pressure definition in multi-well system

Consider the concept of "reservoir pressure" for developed field with waterflooding. In most cases two aspects of practical application of the reservoir pressure value in field development management can be distinguished.

First reservoir pressure is used for well potential production rate prediction and particularly for assessing the effect of well operating parameters optimization. Optimization can be carried out in two ways: 1) without changing of well productivity index (PI), for example by reducing the bottomhole pressure (p_{wf}) by changing the pump or 2) through well stimulatiion activities that increase productivity index (hydraulic fracturing, bottomhole treatment, etc.). In both cases potential increment production is usually calculated by nodal analysis method with using inflow and outflow well performance curves[3]. Technial effect from the optimization and its economical feasibility is estimated by the difference between IPR curves plotted before and after operating activity (Fig. 1). Note that in this case single well approach is commonly applied. Meanwhile the bottomhole pressure corresponding to zero flow rate is the only point which does not change its position at IPR curve during downhole pressure or productivity index changing. So this pressure is called "reservoir pressure" (p_r in Fig. 1).

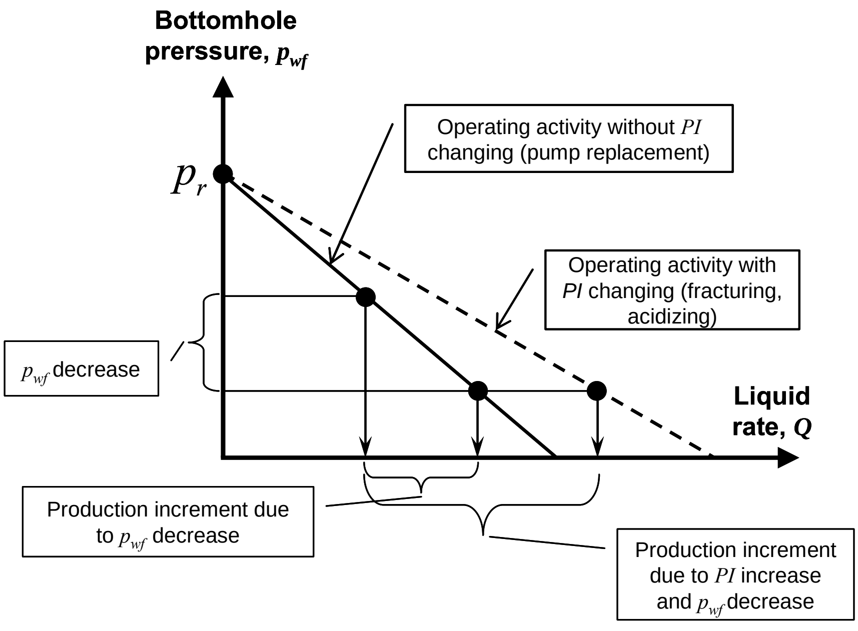


Fig.1. Single well optimization effect due to bottomhole pressure decrease and production index increase estimated by IPR method. p_r – average reserivor pressure at the vicinity of single well.

Secondly the term "reservoir pressure" defines an integral energy state of the reservoir or of the part of the reservoir. Practical issue in this case is the problem of localization of waterflooding system bottleneck. Low value of reservoir pressure usually means that there are problems with the injection wells whereas a large value of resevoir pressure signals that producing wells can recover more fluid and should be stimulated. In this case material balance method is often used to calculate the reservoir pressure.

Methods of reservoir pressure determination in multi-well system

Following methods can be utilized to reservoir pressure determination:

- drawdown or buildup tests (using various pressure transient analysis method);
- step rate test and IPR curve construction (steady state production analysis);
- methods based on material balance.

Let consider the reservoir pressure values that we obtain from these methods when they are applied in multi-well system.

It is supposed that Horner method with interpretation results correction by Matthews-Brons-Hazebroek method (MBH) as well as Dietz method combined with Miller-Dyes-Hutchinson method (MDH)[4,5] are the most common techniques of reservoir pressure determining from pressure transient analysis. Horner-MBH method is suitable for buildup test of exploratory wells and wells with short production time before shut-in. Data of bottomhole pressure response is plotted in this method versus the special time function called Horner time. Reservoir pressure is estimated from the extrapolation of linear trend on a Horner plot to infinite Horner time with subsequent accounting reservoir boundaries (that simulate the influence of neighboring wells in the case of multi-well system) by using MBH correction function. Dietz-MDH method is applicable for the wells which have reached pseudosteady or steady state regime before shut-in. It is based on the logarithmic approximation of diffusivity equation solution and extrapolation of linear trend on semilog pressure plot to the certain closed-in time which we will call Dietz time. These methods allow us to obtain the average pressure inside the well drainage zone which shape is assumed to be known.

Consider the example of producing well buildup test in multi-well waterflooded system which consists of a single injection well **I1** and four producing wells **P1**, **P2**, **P3** and **P4** (see Fig.2).Assume that all wells are working in a closed rectangular reservoir with constant downhole pressures at the steady state regime before shut-in. For better clarity single phase flow and homogeneous reservoir is assumed in all models considered below.

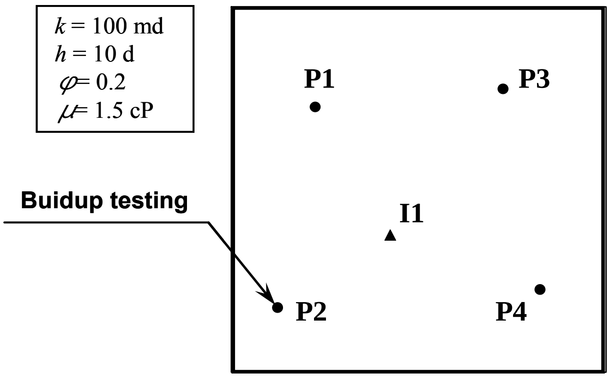


Fig.2. Considering multi-well system consisted of four producing wells **(P1**, **P2**, **P3**, **P4)** and one injection well **(I1)** with constant downhole pressures

Results of syntethic buildup test at the producing well **P2** obtained by numerical simulation are shown in Fig.3 as a semi-log pressure plot (MDH plot). Diagnostic log-log plot of pressure changing and logarithmic derivative is also shown in Fig.4 in order to clarify flow regimes identification.

Figures show that after the end of transient flow regime (IA) with duration of approximately 3 days pressure growth at shut-in well continues but its growth rate changes due to beginning of neighboring wells influence and increasing of average reservoir pressure in the drainage zone. This regime we call pseudosteady flow regime (PSS) as for the single well approach[1].

Note that pressure growth is terminated only at a very long shut-in time (about 100 days) even for the quite large value of considered reservoir permeability of 100 md. Finally bottomhole pressure reaches a threshold value $p_{r,stat} = 220$ atm which we call static reservoir pressure at steady stateflow regime (SS).

In this case shut-in time at which reservoir pressure should be calculation by Deitz-MDH method is equal $\Delta T_{Dietz} \approx 20$ days. It exceeds the beginning time of pseudosteady flow regime (T_{PSS}) but is less than the beginning time of steady flow regime (T_{SS}). Reservoir pressure can be estimated using Deitz method by extrapolation of linear trend at semi-log plot to the time ΔT_{Dietz} (see Fig.3). For the considered example we obtain $p_{r,Deitz} = 190$ atm that is less than static reservoir pressure. We call this pressure instantaneous reservoir pressure.

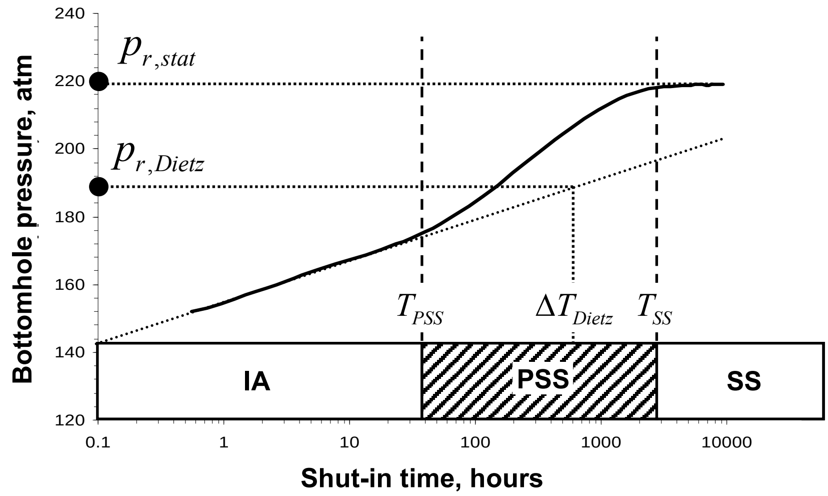


Fig.3. MDH semi-log plot for the synthetic buildup test at well **P2** obtained by numerical simulation. IA – infinite-acting radial flow regime; PSS – pseudosteady state flow regime; SS – steady state flow regime.

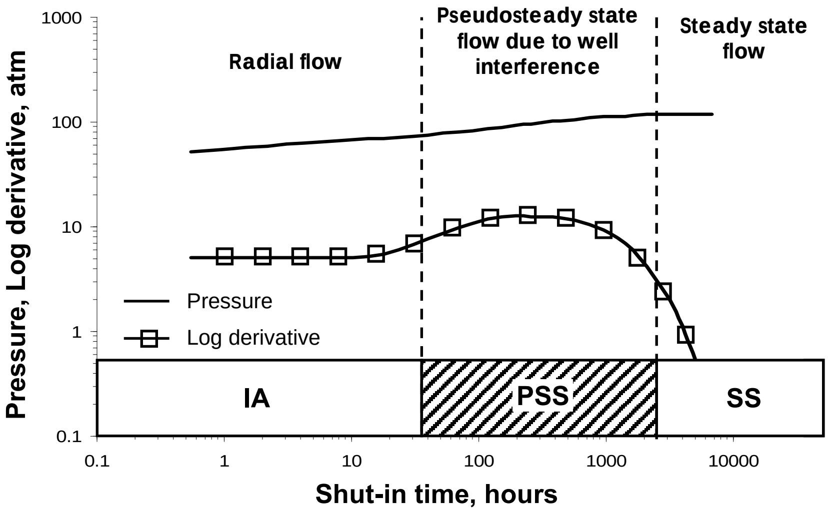


Fig.4. Diagnostic log-log plot for the synthetic buildup test at well **P2** obtained by numerical simulation. IA – infinite-acting radial flow regime; PSS – pseudosteady state flow regime; SS – steady state flow regime.

Consider now the correlation between determined from the buildup reservoir pressure (instantaneous or static) and reservoir pressure that can be obtained from IPR curve. Numerical simulation was carried out to derive IPR curve and define reservoir pressure at well **P2**. Calculation results are presented in Fig.5 as a plot of flow rate and bottomhole pressure dynamics for well **P2**.

Assume that after steady state regime has been reached operating conditions at well **P2** ware optimized by decreasing downhole pressure from 100 atm to 50 atm. Point 0 in Fig.5 corresponds to the situation before optimization (we call it basic mode). Starting flow rate at well **P2** after bottomhole pressure optimization is depicted in Fig.5 by point 1. During transient flow regime a rapid flow rate decrease is occurred and at the beginning of pseudosteady state flow regime well production corresponds to point 2. Then flow rate continues to fall but its decline rate becomes smaller as it approaches the value corresponding to the new steady state flow regime in the system (point 3).

Variation of well **P2** operating point described above is shown in Fig.6 using IPR curves concept. Points corresponding to the differents flow regimes depicted in Fig.5 are also presented in Fig.6 with the same labels. Figure shows that changing of bottomhole pressure causes the sharply increase of the production rate and operating point shifts from stabilized point 0 to point 1. Flow rate decline from point 1 to point 2 corresponds to the transient flow regime in the resevoir. During this time reservoir pressure in the system is unable to change substantially and the neighboring wells impact does not affect to the considered well production. Further rate decrease from point 2 to point 3 is occurred during pseudosteady state flow regime conditions and lasts much longer than the transient flow period. In reality this change of flow rate can be hidden by cooperating conditions changing at other wells in the system. In this case downhole pressure at other wells remains constant thus a new steady state regime corresponding to point 3 is reached.

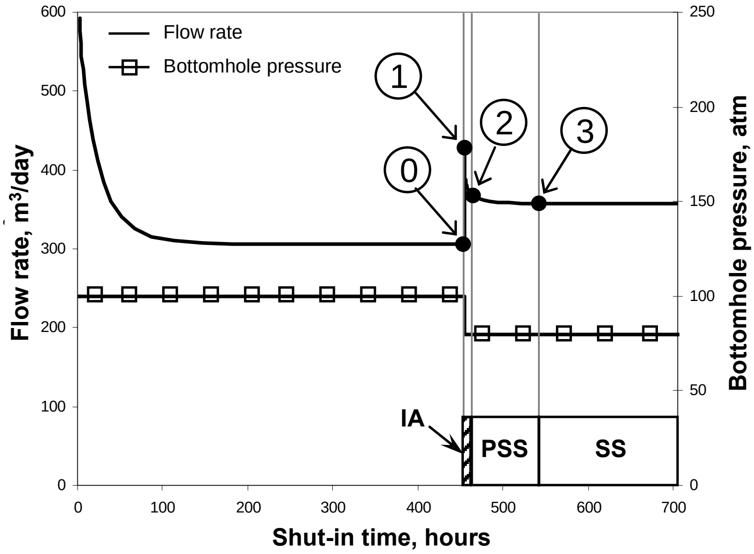


Fig.5. Flow rate and bottomhole pressure dynamics at well **P2** before and after optimization obtained by numerical simulation

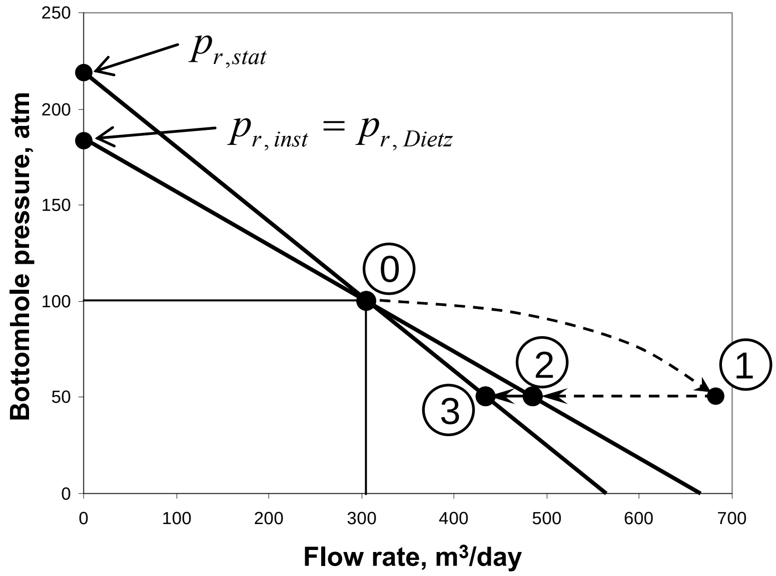


Fig.6. Variation of producing well **P2** operating point on IPR plot caused by bottomhole pressure decrease

From figures presented it is clear that the operating point considered at different moments of time has its own IPR curve and differents points of zero flow rate corresponding to different "reservoir pressures" can be determined from these curves. From comparison with results previously obtained it was found out that reservoir pressure from IPR curve is equal to the instantaneous reservoir pressure from transient pressure analysis if IPR curve is considered at the end of transient flow regime (point 2 in Fig.5 and Fig.6). Another reservoir pressure which can be derived from IPR curve plotted through the point 3 corresponding to the new stabilized pressures and flow rates (new steady state period) coincides with the static reservoir pressure defined earlier from buildup test interpretation. In the first case IPR curve we will call it "instantaneous IPR curve" and "static IPR curve" in the second case.

Instantaneous and static reservoir pressure and IPR curves

To investigate instantaneous IPR curve behavior several operating regimes at well P2 in the system shown in Fig. 2 were considered by using numerical simulator. Stepwise changing of bottomhole pressure was supposed after period of time when all wells in the system have been reached steady state flow regime (called basic mode). Two cases were considered:

when duration of well production with each bottomhole pressure value (ΔT) is equal to the duration of the transient flow regime (Fig.7a);

when duration of each regime is larger than the ending time of transient (T_{PSS}) and pseudosteady state (T_{SS}) flow regimes (Fig.7b).

In both cases appropriate production rate values for IPR plot were calculated at the ending time of unsteady (transient) flow regime. Results obtained are presented in Fig. 8. Bottomhole pressure decrease and increase cases were studied.

Note that only at the first case of pressure changing well operating points correspond to the instantaneous IPR. Meanwhile only those points are situated right at the IPR curve that immediately follow the basic mode after a time T_{PSS} (points A and B in Fig.8). Other points do not lie exactly at the instantaneous IPR curve. The deviation increases with the time needed to reach the certain point. The reason for these deviations is that the instantaneous pressure itself depends on the flow regime at the well obsereved. Average reservoir pressure inside the drainage zone is continually changed when we change well operating conditions. Thus the closest points to the instantaneous IPR curve are conformed to the first case of bottomhole pressure changes (they are marked in Fig.7a by triangles) because instantaneous reservoir pressure doesn't vary considerably during each of the operating period. Points corresponding to the second case of pressure changing (marked by circles in Fig.7b) deviate from instantaneous IPR curve significantly that is associated with prolonged well production period between regime changes and is caused by instantaneous reservoir pressure changing.

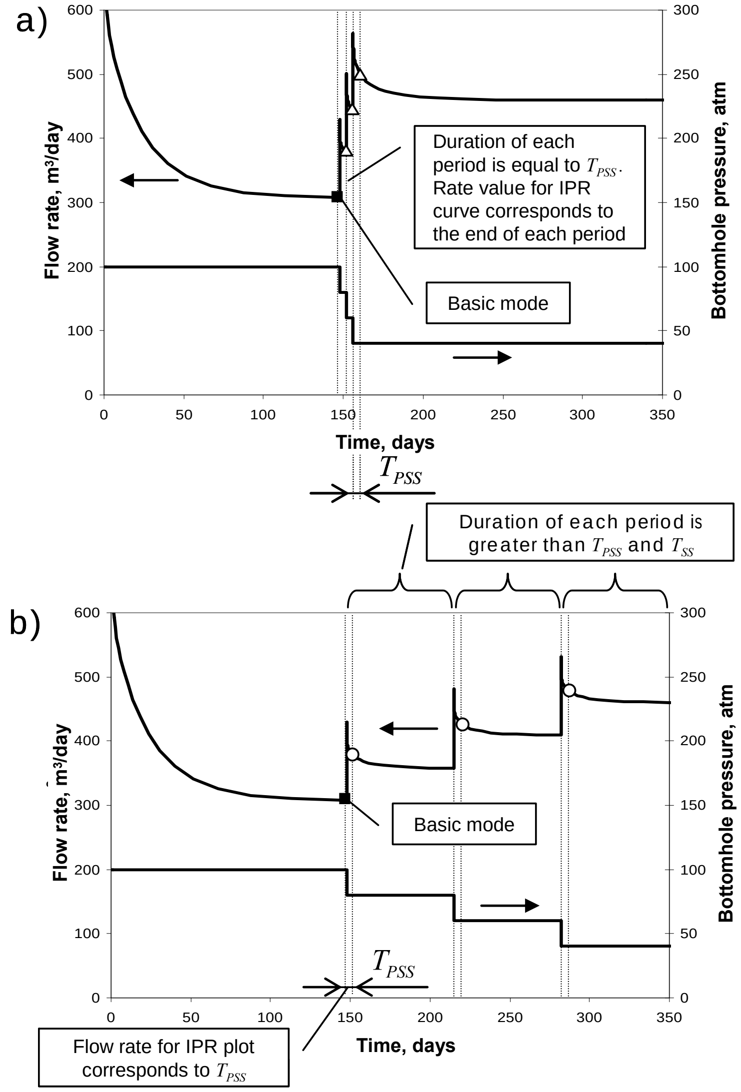


Fig.7. Simulated production rate and bottomhole pressure at well **P2** with two cases of well operating conditions changing for instantaneous IPR curve behavior study

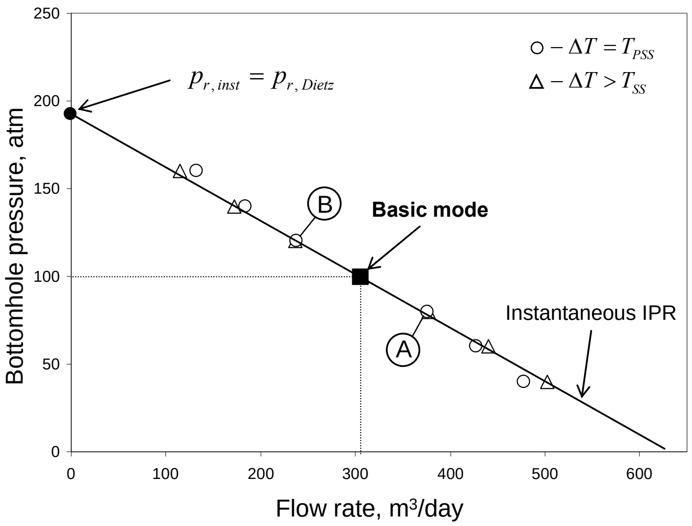


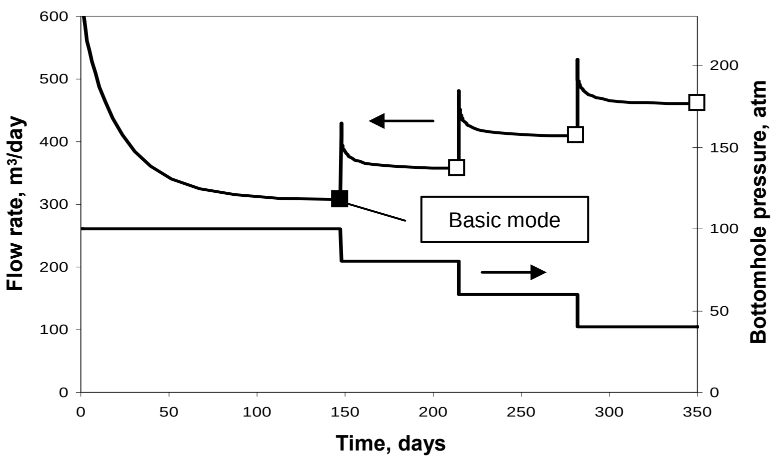
Fig.8. Instantaneous IPR curve for two cases of well operating conditions changing at well **P2**

Similar study was carried out for the static IPR curve. By changing bottomhole pressure at well P2 in two different ways (Fig. 9) IPR curve was plotted based on stabilized steady state rate and pressure values at each operating regime (Fig. 10). As for the case of instantaneous IPR steady state flow regime (basic mode) has been reached at all wells in the system before downhole pressure was changed.

Fig.10 shows that all points in this case lie exactly at static IPR curve. It can be observed from the plots in Fig.9 that points of static IPR curve didn't change regardless of the period duration when values of production rate and bottomhole pressure for the IPR plot were derived. Note that in this case duration of each period $\Delta T > T_{SS}$. Thereby one can conclude that the static reservoir pressure is not dependent on the operating regime of the well and characteristic of its changing.

Summarizing results presented above it can be noted that the instantaneous IPR (and instantaneous reservoir pressure) provides a well behavior describtion during the short-term time period. Deviation of operating point from instantaneous IPR curve grows when well production time after operating conditions changing is increased. Thus the instantaneous reservoir pressure is only estimated, unobserved value and can not be measured in practice. Each change of well's work parameters to determine the instantaneous reservoir pressure causes the change of reservoir pressure itself. Instantaneous IPR curve is a tool of approximate calculations which using requires remembering of time limits of its applicability.

Static IPR (and the static reservoir pressure) describes accurately behavior of wells during long-term time period (with any number of steady state regimes regardless of their sequence and duration). Static pressure is not dependent on the analyzed well operating regime but is influenced by operating conditions of neighboring wells (see Appendix A). Static pressure can be observed at the well after a long shut-in time period significantly exceeding the time of transient flow regime. Inter alia this time is defined by the well production rate in contrast to the duration time of unsteady flow regime. In practice the static reservoir pressure is usually not determined because it requires a very long well shut-in.



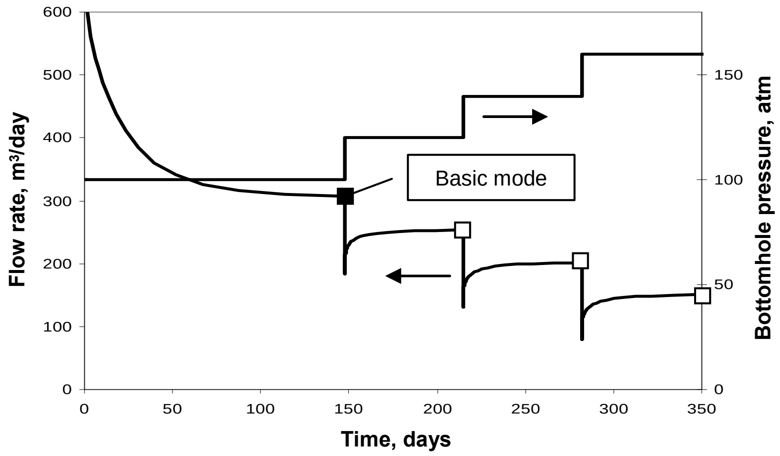


Fig.9. Simulated production rate and bottomhole pressure at well **P2** with two cases of well operating conditions changing for static IPR curve behavior study

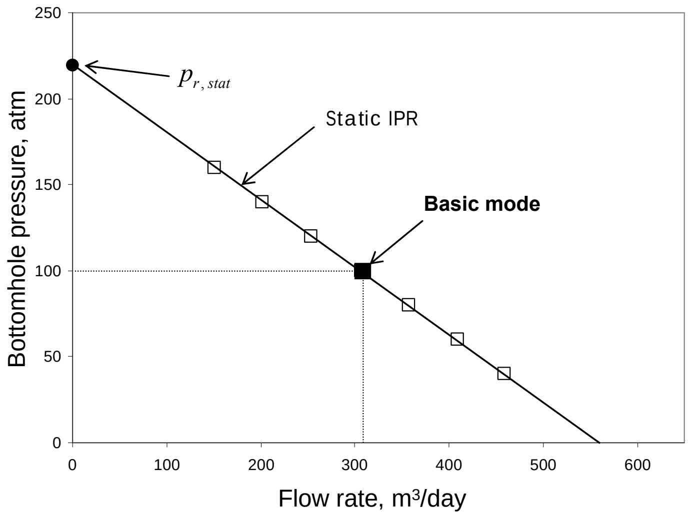


Fig.10. Static IPR curve for two cases of well operating conditions changing at well **P2**

Instantaneous and static reservoir pressure can be calculated using a simple analytical model[2]. Details are given in Appendix A. It is also shown that the integral (average) reservoir pressure at the steady state regime can be defined as an average value between bottomhole pressures at each well in the system with weights coefficients derived from the producing wells productivity index and injection wells injectivity index. This integral reservoir pressure differs from the instantaneous reservoir pressure at each well and the static reserbvoir pressure. Generally speaking all three reservpor pressures (instantaneous, static and integral) have different values but they can coincide in some cases determined by the symmetry of well pattern. Operating conditions changing at one well affects to the neighboring wells performance characteristics through the variation of the integral reservoir pressure.

As it is shown in Appendix A instantaneous IPR is a tool of the factor analysis which allows to estimate well productivity increase (or decrease) caused just by changing bottomhole pressure at this well itself and to separate the well interference effect. Static IPR takes into account the cumulative effect from well operating regime changing and subsequent reservoir pressure change caused by this operating conditions fluctuation. Instantaneous IPR can be applied to assess the effect of operating activity at the short-term period whereas the static IPR provides estimation of well performance changes when steady state regime is reached (i.e. at the long-term period).

Dynamics of well operating point location due to bottomhole pressure changing is shown in Fig.11. Note that initial instantaneous reservoir pressure ($p_{r,inst1}$) decreased after downhole pressure was decreased and a new steady-state flow regime (labeled by point 3 at static IPR curve in Fig. 11) was reached. Instantaneous IPR curve shifted down parallel (as shown by dotted curve in Fig. 11) and a new value of the instantaneous reservoir pressure ($p_{r,inst2}$) was established.

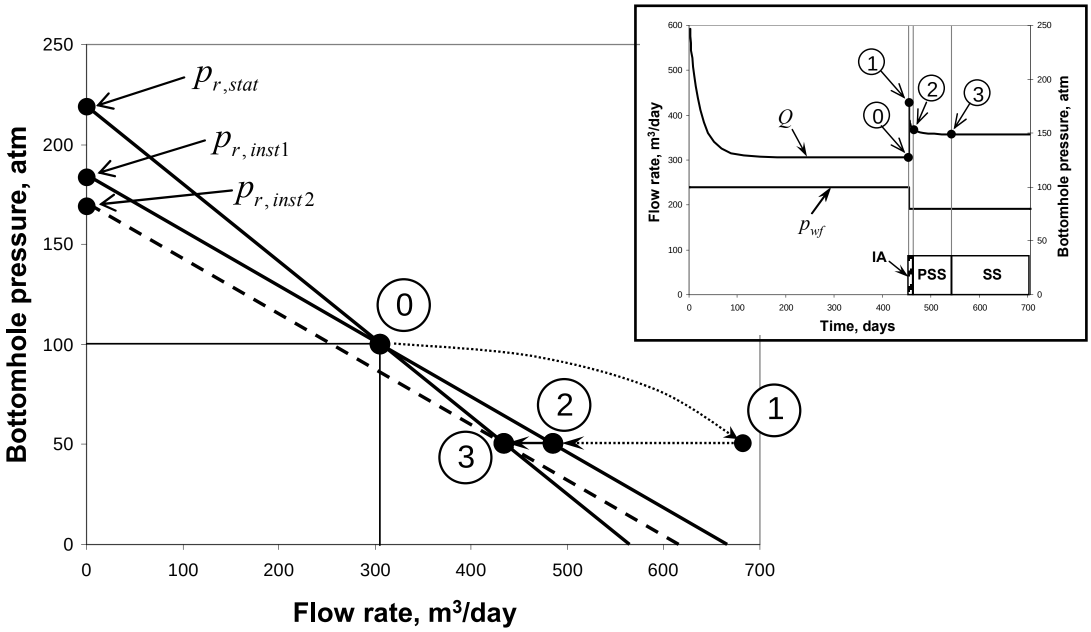


Fig.11. Instantaneous reservoir pressure decrease due to bottomhole pressure changing at well **P2**

Effect of well operating conditons optimization with accounting of well interference

Changing of operating conditions at one well in multi-well system to increase production affects not only to the well stimulated but also to the neighboring wells. For example flow rate dynamics before and after bottomhole pressure change at well **P2** obtained by numerical calculations for all four producing wells in the system depicted in Fig.2 is presented in Fig.12.

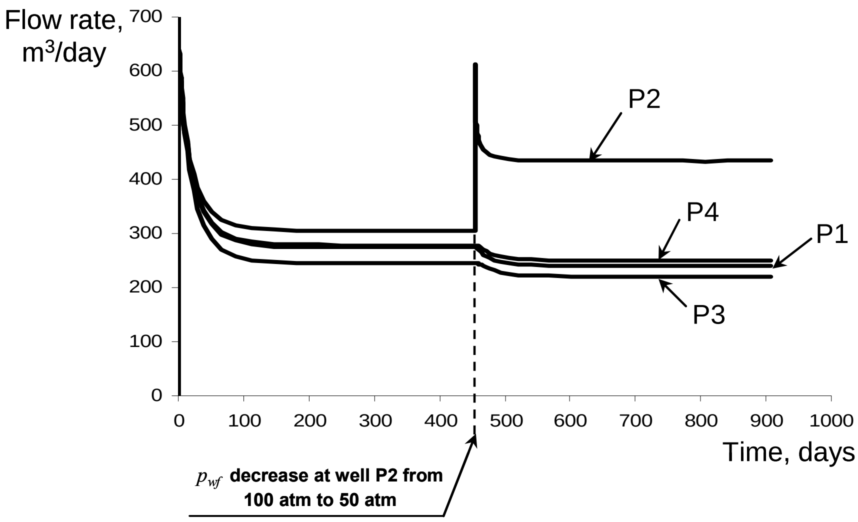


Fig.12. Simulated production rate curves for the wells shown in Fig.2 before anf after operating conditions optimization at well **P2**

Production rate increase at well P2 is accompanied by a gradual production decline at the surrounding wells occured due to average reservoir pressure change (note that bottomhole pressure at injection well has not been changed). Comparison of the total production rate from the reservoir between the case with explicit accounting productivity decline due to interference effect and the case without considering of production change at neighboring wells is shown in Fig.13. It can be seen that effect of operating conditions optimization at well **P2** in the fist case which is based on multi-well system performance estimation approach can be significant less than in the secomd case based on single well performance approach. In addition it is clear that time limits always need to be captured when future production is assessed after well stimulation (production intensification) and when thus well workover activity efficiency is predicted.

Points 0 and 1 in Fig. 13 correspond to the time moments immediately before and after bottomhole pressure change at well **P2**. Working point 2 corresponds to the end of the transient flow regime. Instantaneous IPR curve described above can be used to estimate increment production at this point. Then average (integral) reservoir pressure change begins to affect to the flow rates of surrounding wells that leads to the reduction of increment production. Total production rate in long-term period from single well performance analysis is depicted in Fig.13 by point 3. Actual total production rate from the multi-well system is shown by point 4. Static IPR can be applied to production icrease calculation when a new steady state regime is occurred.

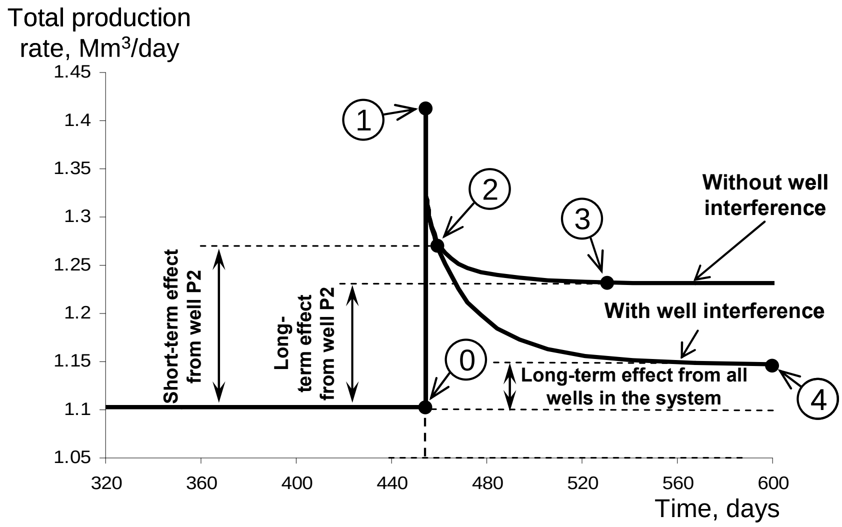


Fig.13. Total production from the resevoir before and after well **P2** operating conditions optimization for the case with accounting well interference effect and for the single well performance approach

Simple equations for increment production calculating after well stimulation are given in Appendix B. It is shown that increment production decline coefficient ε can be introduced as a ratio between static production increase effect (occurred in long-term period during new steady state flow regime) and instantaneous intensification effect (occurred in short-term period after the ending of transient flow regime). This coefficient can be found by following equation in the case when in the multi-well system producing wells productivity index is equal to injection wells injectivity index:

$$\varepsilon = \frac{M_T}{P/I + M_T},$$

where P/I is producer/injector ratio in the well pattern, M_T – total mobility ratio during waterflooding[2]. Relationship between ε and M_T for different flooding patterns is shown in Fig. 14. From the figure one can determine that for $M_T = 1$ instantaneous rate reduction will be 50% for inverted five-spot pattern, 33% for seven-spot pattern and 25% for nine-spot pattern. This well stimulation efficiency decrease is occurred due to the decrease of the integral reservoir pressure caused by operating conditions change at stimulated well and further changing of surrounding well performance.

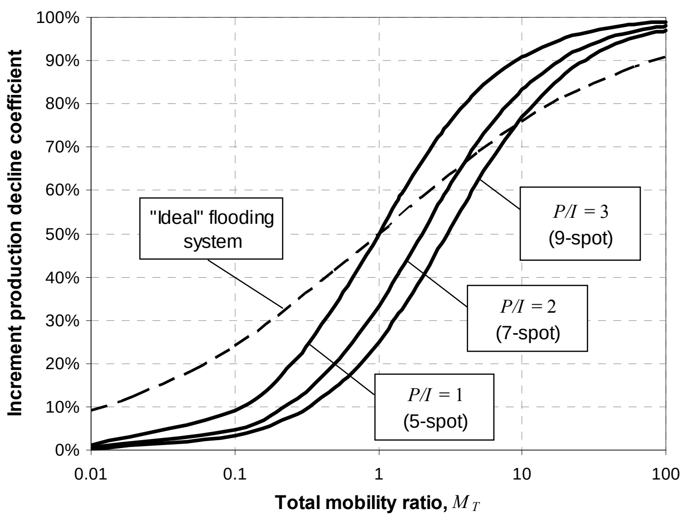


Fig.14. Relationship between increment production decline coefficient and total mobility ratio for different flooding pattern and for the “ideal” flooding system

It can be also shown that the the following condition is true for the waterflood pattern with maximum flow capacity:

$$P/I = \sqrt{M_T} \ .$$

Thus for the “ideal” flooding system increment production decline coefficient will be equal to

$$\varepsilon = \frac{1}{1+\frac{1}{\sqrt{M_T}}} .$$

Dependence of the increment production decline coefficient from total mobility ratio for the “ideal” flooding system is shown in Fig.14 by dotted line. Note that in this case ε depends only on M_T .

Conclusions

Estimation of well production in the flooding pattern including the calculation of well treatment effect requires using of multi-well approach. It is shown that three types of reservoir pressure: instantaneous, static and integral can be considered for well performance evaluation in multi-well system. Several conclusions can be made:

- Calculation based on the instantaneous reservoir pressure application gives an increment production effect after the well performance optimization during the short-term period. Estimated value can be achieved at the end of the transient flow regime. Instantaneous reservoir pressure is determined as a result of well test interpretation. Instantaneous reservoir pressure is the average pressure inside the drainage area for given well but it is not equal to the integral reservoir pressure in the multi-well system. Instantaneous reservoir pressure corresponds to an instantaneous IPR curve passing through the well operating point at the end of the transient flow regime.
- Instantaneous reservoir pressure changes with any change of well operating con ditions thus it is the conditional, unobserved characteristic. Any attempt to measure the instantaneous reservoir pressure through the well shut-in leads to its change. Calculations carried out using the instantaneous reservoir pressure can be applied during the time interval that is approximately comparable with the transient flow regime duration. As pdeudosteady state flow regime will be reached well fkow rate decline rate continues until a new steady state flow regime will occur with a new value of instantaneous reservoir pressure.
- Static reservoir pressure corresponds to the bottomhole pressure observed at tested well after a long shut-in period (as compared to transient flow regime duration). Static reservoir pressure is always higher than the instantaneous reservoir pressure. All points corresponding to the steady state flow regimes lie on a static IPR curve passing through the static reservoir pressure. The slope of the static IPR is greater than the slope of the instantaneous IPR. Well performance analysis using the static IPR curve takes into account both the effect of bottomhole pressure change at given well and the associated changes of the instantaneous reservoir pressure. Thus the static IPR curve enables to evaluate the effect of well treatment in the long-term period with taking into account production decline due to well interference.
- During well operating conditions optimization both short-term (before pseudosteady state flow regime) and long-term prediction of production icrease effect with prolonged rate decline due to instantaneous reservoir pressure change should be estimated. The increment production calculated subject to wells interaction (well interference) effect can be significantly lower than results of the single well approach application.
- A significant production increase can be achieved in waterflood systems only through the simultaneous stimulation of producing and injection wells.

Nomenclature

B_o – oil formation volume factor
B_w – water formation volume factor
h – formation thickness, m
J_D^I – dimensionless injectivity index
J_D^P – dimensionless productivity index
k_o – relative permeability of oil
k_w – relative permeability of water
K_{inst} – instantaneous productivity index, m^3/day/atm
K_{stat} – static productivity index, m^3/day/atm
M_T – total mobility ratio
N – total number of wells

N_I	– number of injectors
N_P	– number of producers
p_r	– reservoir pressure, atm
$p_{r,inst}$	– instantaneous reservoir pressure, atm
$p_{r,stat}$	– static reservoir pressure, atm
$\overline{p}$	– integral reservoir pressure, atm
$\Delta\overline{p}$	– integral reservoir pressure change due to changes in bottomhole pressure at the producing well, atm
p_{wf}	– bottomhole pressure, atm
p_{wf}^I	– injection bottomhole pressure, atm
p_{wf}^P	– production bottomhole pressure, atm
Δp_{wf}	– bottomhole pressure change at the producing well, atm
Q^I	– injection flow rate, m^3/day
Q^P	– production flow rate, m^3/day
Q_Σ^I	– total injection flow rate, m^3/day
Q_Σ^P	– total production flow rate, m^3/day
$\Delta_{inst}Q$	– instantaneous production increase after the operating conditions change at producing well, m^3/day
$\Delta_{stat}Q$	– static production increase after the operating conditions change at producing well, m^3/day
α	– weighting coefficient for the integral reservoir pressure calculation
ε	– increment production decline coefficient
μ_o	– oil viscosity, cp
μ_w	– water viscosity,cp

References

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Appendix A. Calculation of the integral, instantaneous and the static reservoir pressure in the flooding pattern

Consider the flooding pattern consisting of N wells with N_P producers and N_I injectors. All wells are assumed to be operated in homogenious reservoir at steady state flow regime with constant bottomhole pressures $p_{wf,i}^P$, $i=1..N_P$ and $p_{wf,j}^I$, $j=1..N_I$.

It can be shown[2] that for the symmetric flooding patterns production flow rate at *i*-th producer Q_i^P and injection flow rate at *j*-th injector Q_j^I can be defined as follows:

$$Q_i^P = \frac{k_o h}{18.42\mu_o B_o} J_{D,i}^P \left(\overline{p} - p_{wf,i}^P\right), \qquad \text{(A-1)}$$

$$Q_j^I = \frac{k_w h}{18.42 \mu_w B_w} J_{D,j}^I \left(p_{wf,j}^I - \overline{p} \right), \quad \text{(A-2)}$$

where k_o, k_w – relative permeability of oil and water, h – formation thickness, m; μ_o, μ_w – oil and water viscosity, cp; B_o, B_w – oil and water formation volume factor; $J_{D,i}^P$ – dimensionless productivity index of *i*-th producer; $J_{D,j}^I$ – dimensionless injectivity index of *j*-th injector; $\overline{p}$ - integral reservoir pressure in the flooding pattern, atm.

Total production and injection rate of oil Q_Σ^P and water Q_Σ^I in such a system will expressed as follows:

$$Q_\Sigma^P = \frac{k_o h}{18.42 \mu_o B_o} \sum_{i=1}^{N_P} J_{D,i}^P \cdot \left(\overline{p} - p_{wf,i}^P \right), \quad \text{(A-3)}$$

$$Q_\Sigma^I = \frac{k_w h}{18.42 \mu_w B_w} \sum_{j=1}^{N_I} J_{D,j}^I \cdot \left(p_{wf,j}^I - \overline{p} \right). \quad \text{(A-4)}$$

Suppose that total production rate of the system is equal to the total injection rate. Then from Eq.A-3 Eq.A-4 integral reservoir pressure can be defined:

$$\overline{p} = \frac{\sum_{i=1}^{N_P} J_{D,i}^P \cdot p_{wf,i}^P + M_T \sum_{j=1}^{N_I} \cdot J_{D,j}^I p_{wf,j}^I}{\sum_{i=1}^{N_P} J_{D,i}^P + M_T \sum_{j=1}^{N_I} J_{D,j}^I}, \quad \text{(A-5)}$$

where $M_T = \frac{k_o \mu_w B_w}{k_w \mu_o B_o}$ – total mobility ratio of water and oil.

From the Eq.A-1 follows that the production rate of *i*-th producing well can be considered as a function of bottomhole pressure and integral reservoir pressure also depended on the bottomhole pressure:

$$Q_i^P = f\left(\overline{p}\left(p_{wf,i}^P \right), p_{wf,i}^P \right). \quad \text{(A-6)}$$

By definition productivity index of *i*-th well can be presented as bottomhole pressure derivative of well production rate with opposite sign. Thus instantaneous productivity index $K_{inst,i}$ of the *i*-th well can be introduced in the form of partial derivative of Eq.A-6. It will be true if flow rate depends only on the well bottomhole pressure, i.e. when the integral reservoir pressure is considered to be constant (and when the fact that it also depends on bottomhole pressure is ignored). Derivation of Eq.A-6 with taking into account Eq.A-1 gives:

$$K_{inst,i} = -\frac{\partial Q_i^P}{\partial p_{wf,i}^P} = \frac{k_o h}{18.42 \mu_o B_o} J_{D,i}^P. \quad \text{(A-7)}$$

Instantaneous reservoir pressure in the vicinity of *i*-th well can be determined by plotting the IPR curve passing through the well operating poing (i.e. current flow rate and bottomhole pressure) with the slope $K_{inst,i}$ and defining of intersection point. Thus we obtain:

$$p_{r,\,inst\,i} = p_{wf,i}^P + \frac{Q_i^P}{K_{inst,i}} = \overline{p} \quad \text{(A-8)}$$

In this case of symmetric flooding pattern the instantaneous reservoir pressure is equal to the integral reservoir pressure that means that the average pressure inside the well drainage zone (instantaneous reservoir pressure) is equal to the integral average reservoir pressure throughout the system (because the system consists of identical pattern elements). The instantaneous reservoir pressure in non-regular flooding patterns will differ from the integral one and can be calculated by using more sophisticated approach based on multi-well productivity index matrix calculation[1].

Let's define a static productivity index of *i*-th well $K_{stat,i}$ as a total bottomhole pressure derivative of production rate considering the last one as a multivariable function of bottomhole pressure and integral reservoir pressure (see Eq.A-6). Using the total derivative definition for multivariable function from derivation of Eq. (A-1) we obtain:

$$K_{stat,i} = -\frac{dQ_i}{dp_{wf,i}} = \frac{k_o h}{18.42 \mu_o B_o} J_{D,i}^P \left(1 - \frac{J_{D,i}^P}{\sum_{n=1}^{N_P} J_{D,n}^P + M_T \sum_{j=1}^{N_I} J_{D,j}^I} \right). \quad \text{(A-9)}$$

Plotting the IPR curve at the current operating point with the slope equaled to the static productivity index ($K_{stat,i}$) and identifying the intersection value we determine the static reservoir pressure for the i-th well in the multi-well system:

$$p_{r,stat\ i} = p^P_{wf,i} + \frac{Q^P_i}{K_{stat,i}} = \frac{\sum\limits_{\substack{n=1\\n\neq i}}^{N_P-1} J^P_{D,n} \cdot p^P_{wf,n} + M_T \sum\limits_{j=1}^{N_I} \cdot J^I_{D,j} p^I_{wf,j}}{\sum\limits_{\substack{n=1\\n\neq i}}^{N_P-1} J^P_{D,n} + M_T \sum\limits_{j=1}^{N_I} J^I_{D,j}}. \qquad \text{(A-10)}$$

From comparing Eq.A-10 and Eq.A-5 physical meaning of static reservoir pressure becomes clear. It is equal to the value of the integral reservoir pressure which would be observed in the flooding pattern if we exclude i-th well from the system or it would be stoped for a long period of time.

Appendix B Approximate equations for the increment production estimation after optimization of producing well operating parameters in flooding pattern

Suppose that operating conditions at one of the producers in the waterflooding pattern (denoted by the subscript k) were optimized by change of the bottomhole pressure by value $\Delta p_{wf,k}$.

According to Eq.A-1 instantaneous flow rate increase at k-th well (and in the whole system) $\Delta_{inst} Q$ during short-term period with ignoring the integral reservoir pressure change can be epressed as follows:

$$\Delta_{inst} Q = \frac{\partial Q^P_k}{\partial p^P_{wf,k}} \Delta p^P_{wf,k} = -\frac{k_o h}{18.42 \mu_o B_o} J^P_{D,k}\, \Delta p_{wf,k}. \qquad \text{(B-1)}$$

Static flow rate increase for this well $\Delta_{stat} Q$ during long-term period with taking into account the integral reservoir pressure change will be equal to:

$$\Delta_{stat} Q = \frac{dQ^P_k}{dp^P_{wf,k}} \Delta p^P_{wf,k} = -\frac{k_o h}{18.42 \mu_o B_o} J^P_{D,k}\, \Delta p_{wf,k} \left(1 - \frac{\Delta \bar{p}}{\Delta p_{wf,k}}\right). \qquad \text{(B-2)}$$

Assume that all production wells have the same bottomhole pressure p^P_{wf} and all injection wells have the same bottomhole pressure p^I_{wf}. Then Eq.A-5 for the integral reservoir pressure can be written in the following form:

$$\bar{p} = \alpha p^I_{wf} + (1-\alpha) p^P_{wf}, \qquad \text{(B-3)}$$

where

$$\alpha = \frac{1}{1 + \frac{1}{M_T} \frac{\sum\limits_{i=1}^{N_P} J^P_{D,i}}{\sum\limits_{j=1}^{N_I} J^I_{D,j}}}. \qquad \text{(B-4)}$$

Eq. B-2 with accounting Eq.B-3 can be written as follows:

$$\Delta_{stat} Q = -\frac{k_o h}{18.42 \mu_o B_o} J^P_{D,k}\, \Delta p_{wf,k} \alpha. \qquad \text{(B-5)}$$

Increment production decline coefficient, ε, can be introduced to show the ratio between the static production increase (that describes the optimization effect during long-term period) and the instantaneous production increase (corresponded to the well treatment efficiency during short-term period). By definition it will be equal to

$$\varepsilon = \frac{\Delta_{stat} Q}{\Delta_{inst} Q} = \alpha. \qquad \text{(B-6)}$$

If all the producers and injectors in the flooding pattern have the same productivity index (i.e. $J^P_{D,i} = J^I_{D,j}$, $i = 1..N_P$, $j = 1..N_I$) then Eq.B-6 subject to definition given by Eq.B-4 becomes:

$$\varepsilon = \frac{M_T}{P/I + M_T}, \qquad \text{(B-7)}$$

where $P/I \equiv \frac{N_P}{N_I}$ - producer/injector ratio in the pattern.