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Low-Permeability Reservoirs Development Using Horizontal Wells

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ABSTRACT

In recent years, horizontal wells have played an increasing role as alternate production tools to vertical or slanted wells. Do they provide anything new to exploitation strategies for low permeability reservoirs ?

In this paper, we will contribute to answering this question by presenting results of analytical derivations pertaining to cases favorable to development with horizontal wells.

The effects of anisotropy related to shaly interbedding and natural fissuring will be discussed. The solutions and criteria are derived independently from the nature of the drive mechanisms because of their strong influence on the production profile of horizontal wells.

As some basic concepts usually applied in classical reservoir engineering to production with vertical wells have to be drastically re-evaluated when dealing with horizontal wells, the economic assessment of their spudding must be evaluated with new criteria. The new criteria, such as areal productivity and replacement ratio of vertical wells by horizontal wells, are introduced in order to help the decision-maker responsible for the exploitation of a low permeability reservoir.

The chief technical contributions of the paper are :

- Criteria for comparing horizontal and vertical well productive efficiency.
- Spatial depletion profiles around the horizontal well.
- Areal productivity index and replacement ratio equations.

References and illustrations at end of paper.

INTRODUCTION

Horizontal wells have become a subject of renewed interest over the last ten years. More than thirty horizontal wells have now been drilled [1-4]. The increase in productivity comes up to expectations based on theoretical calculations. Excellent results have also been obtained in restricting the inflow of undesirable fluids when producing reservoirs involving a gas cap or an aquifer. Theoretical tools have been proposed to explain this favorable behavior [5].

Since low permeability reservoirs contain large reserves of oil that are difficult to extract, it is worth examining whether horizontal wells can contribute to increasing the recovery rate in an economical manner.

OBJECTIVES

This paper aims firstly at providing a basis for estimating horizontal well production in low permeability reservoirs.

Steady state flow to a horizontal well is described first in a homogeneous environment then in a fractured or anisotropic environment. The influence of the eccentricity of the well is analyzed.

On the basis of the results obtained, the useful contribution of horizontal wells is discussed with reference to low permeability reservoirs, in particular by means of the evaluations method based on areal productivity.

PRESSURE PROFILE CREATED BY A STEADY STATE FLOW TO A HORIZONTAL WELL

Several authors have devoted themselves to calculating the pressure drops in a steady state flow to a horizontal well [8,9]. Although their results differ slightly, they all use the same method which consists in dividing the flow area into two zones so as to break the 3D flow problem down into series of 2D problems. In the first zone, in the immediate vicinity of the well, the flow is studied in vertical planes perpendicular to the well axis. In the second zone, located further from the well, the flow is studied in horizontal planes as if it were a flow to a vertical fracture of the same length as the horizontal well. The total pressure drop is the sum of the pressure drops in the first and second zones.

$$P(x,y) - P_H(r_w) = P_F - P_H(r_w) + P(x,y) - P_F \quad (1)$$

Appendix A shows the calculation of the flow in the first zone for a length unit of a well located in the middle of the producing layer. This gives the following equation :

$$P_F - P_H(r_w) = \frac{\mu}{k} \frac{Q}{2\pi L} \ln\left(\frac{h}{2\pi r_w}\right) \quad (2)$$

If the well is off-centered, the result given in Appendix B should be used :

$$P_F - P_H(r_w) = \frac{\mu Q}{2\pi kL} \ln\left(\frac{h}{2\pi r_w \sin\left(\frac{\pi b}{h}\right)}\right) \quad (3)$$

Most authors are in agreement on the description of the flow in the second zone.

$$P(x,y) - P_F = \frac{\mu}{k} \frac{Q}{2\pi h} \operatorname{Re}\left(\operatorname{arccosh} \frac{2(x+iy)}{L}\right) \quad (4)$$

This expression is simplified along the x - axis collinear with the fracture and along the y - axis perpendicular to it [10].

$$P(x) - P_F = \frac{\mu}{k} \frac{Q}{2\pi h} \ln\left[\frac{2x}{L} + \left(\frac{4x^2}{L^2} - 1\right)^{\frac{1}{2}}\right], \quad x > \frac{L}{2} \quad (5)$$

$$P(y) - P_F = \frac{\mu}{k} \frac{Q}{2\pi h} \ln\left[\frac{2y}{L} + \left(\frac{4y^2}{L^2} + 1\right)^{\frac{1}{2}}\right] \quad (6)$$

where $L = 2\ x_F$ (7)

Fig.1 shows the pressure profile along the x - and the y - axis compared to the pressure profile created by the same steady-state flow rate to a vertical well. Under this flow regime, the smaller drawdown needed to produce with the fracture is clearly evidenced in the picture.

For distance values of 2 x_F and over, the three pressure profiles do not differ significantly. With exception of the vicinity of the fracture, the pressure profile created by the flow to it is similar to that created by a vertical well with a wellbore radius equal to a quarter of the fracture extension $x_F/2$ as can be seen on the picture [8,9,11].

As expressed in formula (1), for the horizontal well, the pressure drop created in the immediate vicinity of the well should be added. The expression of the total pressure drop for an off-centered horizontal well is then :

$$P(x,y) - P_H(r_w) = \frac{\mu Q}{2\pi k}\left[\frac{1}{L} \ln\left(\frac{h}{2\pi r_w \sin\left(\frac{\pi b}{h}\right)}\right) + \frac{1}{h} \operatorname{Re}\left(\operatorname{arccosh} \frac{2(x+iy)}{L}\right)\right] \quad (8)$$

If the horizontal well is centered in the layer and for distances from it greater than 2L, this expression yields :

$$P(r) - P_H(r_w) = \frac{\mu Q}{2\pi k}\left(\frac{1}{L} \ln\left(\frac{h}{2\pi r_w}\right) + \frac{1}{h} \ln\left(\frac{4r}{L}\right)\right) \quad (9)$$

CRITERION FOR ASSESING THE ECONOMIC ADVANTAGE OF A HORIZONTAL WELL.

A quick look at the pressure profiles in the vicinity of a horizontal well provides a useful criterion for comparing its productive efficiency with that of a vertical well.

For a given steady production flowrate, equations (2) and (3) provide the additional drawdown necessary to produce with a horizontal well in comparison with a fracture extending over the same length.

Referring to the pressure profile in Fig. 1, for the same steady production rate, the additional drawdown necessary to produce with a vertical well in comparison with the fracture is :

$$P_F - P_V(r_w) = \frac{\mu}{k} \frac{Q}{2\pi h} \ln\left(\frac{L}{4 r_w}\right) \quad (10)$$

Therefore, as far as productivity is concerned, the horizontal well performs better than the vertical well if the additional drawdown $P_F - P_H(r_{wH})$ needed to produce it is lower than the additional drawdown $P_F - P_V(r_{wV})$ needed to produce the vertical well :

$$P_F - P_H(r_{wH}) < P_F - P_V(r_{wV}) \quad (11)$$

This relation yields the very simple criterion :

$$\left(\frac{h}{2\pi r_{WH}}\right)^{h} < \left(\frac{L}{4 r_{WV}}\right)^{L} \qquad (12)$$

The criterion states that a horizontal well with a length L equal to the thickness of the layer h produces more than the vertical well as soon as its well bore radius r_{WH} is greater than 0.64 r_{WV}. This ratio of radii expresses the advantage provided by a horizontal well over a vertical well with the same productive length ; the horizontal well behaves more like a fracture than the vertical well. This is consistent with the fact that the flow entering the horizontal well per unit of length is larger at its ends than in the middle part. This end effect is similar for vertical fractures.

If the ratio of the well bore radius of the vertical well r_{WV} to the well bore radius of the horizontal well r_{WH} is equal to $\pi/2$, then the criterion states that a horizontal well may prove superior to a fully penetrating vertical well in respect to productivity only if its productive length L is greater than the thickness of the layer h. This second result is very consistent with the intuitive feeling that the well with the longest open section should be the most productive.

EFFECT OF PERMEABILITY ANISOTROPY

The two main causes of reservoir anisotropy are the process of sedimentation and the fracturing which may follow.

Interbedding of horizontal shaly laminations during sedimentation creates an increase in vertical flow resistance. This effect is only noticeable in the vicinity of a partially penetrating vertical well or in the vicinity of a horizontal well, because it appears in the area where the flowlines cross the horizontal layers. Often a vertical permeability k_V taken smaller than the horizontal permeability k_H represents this type of anisotropy.

The unfavorable effect of lower vertical permeability on horizontal well productivity has been discussed earlier [7-9,11]. As established by Muskat [13], the reduction of a one-phase flow problem in an anisotropic porous medium to flow in "an equivalent isotropic medium" uses the transformation dictated by dimensional analysis. This transformation consists in multiplying the dimensions along each axis by the square root of the ratio of the equivalent permeability to the permeability in that direction. The application of this transformation to formula (12) is somehow difficult as the radius of the horizontal well bore appears in both horizontal and vertical directions. Anyway, by assuming that the elliptical well bore effects are negligible, we obtain :

$$k_H^{\frac{1}{2}}\; h \ln\left(\frac{h}{2\pi r_{WH}}\right) < k_V^{\frac{1}{2}}\; L \ln\left(\frac{L}{4 r_{WV}}\right) \qquad (13)$$

This inequation is a simple criterion for the evaluation of the superiority of horizontal wells over vertical wells in anisotropic reservoirs. It enables a comparison of the thickness of payzones opened with horizontal and vertical wells. If vertical fracturing is to be encountered by a horizontal well, the vertical permeability created by the fractures is higher [14] and the criterion of formula (13) will often show the advantage of the horizontal well.

EQUIVALENT VERTICAL WELL

Another approach is to reckon the well bore radius of an equivalent vertical well that would create at large distances the same pressure profile as the horizontal well [8,12]. If the equivalent vertical well radius r'_{WV} is larger than the radius of the actual vertical well r_{WV}, then the horizontal well is a better producing well. Otherwise it is not interesting under productivity criteria. The expression of the equivalent vertical well bore radius derives from relation (13) :

$$r'_{WV} = \frac{L}{4}\left(\frac{2\pi r_{WH}}{h}\right)^{\left(\frac{k_H}{k_V}\right)^{\frac{1}{2}}\frac{h}{L}} \qquad (14)$$

EQUIVALENT AREAL PRODUCTIVITY

The calculations given above focus on flowline shapes and evaluations of the productivity index. What is needed within the rapidly changing economic, political and tax context, is an evaluation method that is relatively independent of economic data associated with the costing of a producing field.

The purpose of this section is to suggest analytical procedures that may help the decision-maker responsible for the development of a field to decide which system is more economical : a conventional system using vertical wells (taken as a reference point) or an alternative system using horizontal wells.

From one view point, if horizontal wells were spudded at the same spacing as vertical wells, they would, for instance in a given reservoir, produce initially 6 times more. Their drilling cost could be the double the cost of vertical wells, but, due to an inadequate spacing, the long term recovery might not be significantly higher for the horizontal well system compared to the vertical well system. The economic interest of one solution compared to the other might not be clearly assessed under changing hypotheses for the future value of the produced barrel and different discounting rates.

From another viewpoint the higher productivity of the horizontal wells makes it possible to spud them with a larger spacing than vertical wells. The proposed criterion is that overall production from the entire reservoir is the same for the same bottom hole pressure drawdown. However the following problem arises. The productivity of wells depends on their drainage radius. Increasing the spacing means, for a given type of wells, a decrease in their productivity because the external radius to be used in formulae is greater. In our example, when drilling 6 times fewer horizontal wells one will not obtain the same overall production as with vertical wells.

The answer proposed earlier [12] and discussed by Joshi [8] is to calculate the replacement ratio RR using the concept of equivalent areal productivity resulting from the proposed criterion. The replacement ratio will indicate the spacing and number of horizontal wells able to produce as much as the vertical wells.

Calculations are provided for permanent flows obtained in the steady state with lateral feed. These theoretical results may be extended to sweep analysis for enhanced recovery by waterflooding, using a five-spot pattern if the mobility ratio is close to 1.

Let us take API_i to be the areal productivity index of well "i" draining area A_i as follows :

$$API_i = \frac{PI_i}{A_i} \qquad (15)$$

We will now apply this concept defined for a single well to an entire reservoir.

In a field drained by a system of wells having the same productivity index (PI) and regularly spaced so that the areas drained by each are equal, the areal productivity indices are equal and their value is :

$$API = \frac{PI}{\pi r_e^2} \qquad (16)$$

This same value for all the wells can be used to define the areal productivity index of the system of wells in the field.

More generally, let us define the areal productivity index (API) of a system of wells D draining a field with area A as follows :

$$API_D = \frac{\sum_{i \in D} PI_i}{A} \qquad (17)$$

Just as two wells with the same productivity index have the same rate of production under equal drawdown, two systems of drainage wells draining the same field and having the same areal productivity index have the same overall rate of production when the drawdown is assumed to be equal at the bottom of all the wells.

Therefore, two systems of drainage wells D and D', containing n and n' wells respectively, will be considered equivalent with respect to productivity if they drain the same field and have the same areal productivity index. Likewise, if the wells within each system are assumed to have the same productivity index and each to drain an equal area, the following equation is obtained for any two wells "i" and "j", with one coming from each of the systems :

$$API_i = PI_j \qquad i \in D, \ j \in D' \qquad (18)$$

Although the wells from both systems have the same areal productivity index, they do not necessarily drain the same area in both systems and do not have the same productivity index. According to these hypotheses, if the drainage radius $r_{e,i}$ of a well "i" is defined from the area drained by the well A_i as :

$$r_{e,i} = \left(\frac{A_i}{\pi}\right)^{\frac{1}{2}} \qquad (19)$$

then, for the two wells "i" and "j" defined above as having the same areal productivity index, we obtain :

$$\frac{PI_i}{r_{e,i}^2} = \frac{PI_j}{r_{e,j}^2} \qquad (20)$$

REDUCTION OF WELL DENSITY WITH EQUIVALENT AREAL PRODUCTIVITY

Let us consider two systems of drainage wells, D and D', capable of draining the same field and equivalent with respect to productivity. The wells within each system are assumed to have the same productivity index PI and to drain the same area A. The field is also assumed to be large enough so that any boundary effects may be disregarded.

The reduction in well density RWD with system D' containing n' wells as compared to system D containing fewer productive wells n is defined as :

$$RWD = \frac{n - n'}{n} \qquad (21)$$

RWD expresses the percentage of wells that are not needed in system D' as compared to system D, through an increase in the areal productivity index.

If A represents the area of the field, then :

$$n = \frac{A}{\pi r_e^2} \qquad (22)$$

$$RWD = 1 - \left(\frac{r_e}{r'_e}\right)^2 \qquad (23)$$

The same principle may also be used to define the replacement ratio RR of systems D' by system D as the number of wells in D that are replaced by one well in D'. This is expressed as

$$RR = \left(\frac{r'_e}{r_e}\right)^2 \qquad (24)$$

The replacement ratio RR is clearly different in its definition from the ratio of productivity indices.

REDUCTION IN WELL DENSITY BY REPLACING VERTICAL WELLS WITH RADIUS "r_W" BY VERTICAL WELLS WITH RADIUS "r'_W"

The same areal productivity index is obtained as follows :

$$\frac{2\pi kh}{\mu} \frac{1}{\ln\left(\frac{r_e}{r_W}\right)} \frac{1}{r_e^2} = \frac{2\pi kh}{\mu} \frac{1}{\ln\left(\frac{r'_e}{r'_W}\right)} \frac{1}{r'^2_e} \qquad (25)$$

$$\left(\frac{r_W}{r_e}\right)^{-r_e^2} = \left(\frac{r'_W}{r'_e}\right)^{-r'^2_e} \qquad (26)$$

$$r'_W = \frac{r_W}{\sqrt{1 - RWD}} \left(\frac{r_W}{r_e}\right)^{-RWD} \qquad (27)$$

The chart in Figure 5 shows the last equation which is implicit in RWD. The curves for the RWD (r'_W/r_W) function are drawn for logarithmic values (r_e/r_W) that are typical of the ones found in industrial applications.

The way to use the chart is as follows :

- r_e and r_W may be the external radius and the well bore radius of the vertical wells. Their ratio defines the curve to be used, for instance $r_e/r_W = 1100$.

- r'_W may be the equivalent well bore radius of the horizontal wells which will replace the vertical wells. On the horizontal scale you enter the ratio r'_W/r_W, for instance 120.

- The reduction in well density RWD is obtained on the left hand vertical scale, that is 0.61 in the example.

- The replacement ratio RR is obtained on the right hand vertical scale, that is 2.6 in the example.

The example shows that an augmentation in the well bore radius by a factor of 120, obtained for instance in replacing vertical wells by horizontal wells in a given reservoir, allows to reduce the number of wells to be spudded by a factor of 2.6.

The chart may also be used to evaluate the advantage of stimulation or fracturing processes for vertical wells when the apparent radius of the well is known.

CONCLUSIONS

- Horizontal wells appear to be alternate production tools to vertical wells for low permeability reservoirs.

- A simple criterion has been described for assessing the advantage of a horizontal well in comparison to a vertical well. This criterion refers to productivity and applies for homogeneous as well as for heterogeneous reservoirs. It shows that horizontal wells are highly suitable for thin reservoirs and reservoirs with good vertical permeability.

- A method aimed at determining the optimum spacing of horizontal wells is presented.

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NOMENCLATURE

a'	=	half of the thickness of the layer
A	=	area
API	=	areal productivity index
arccosh	=	inverse hyperbolic cosine function $\text{arccosh}(x) = \ln (x + (x^2 - 1)^{1/2})$ $(x \geq 1)$
b	=	vertical height of the horizontal well
cosh	=	hyperbolic cosine function
D	=	drainage system (one or several wells)
e	=	eccentricity of the quasi-circular curve
h	=	thickness of the layer
i	=	solution of the equation $x^2 + 1 = 0$
i,j	=	index
k	=	permeability
k	=	integer dummy variable
L	=	length of the horizontal well
ln	=	Napierian logarithm
n	=	number of wells
P	=	pressure
PI	=	productivity index
q	=	linear production rate of a horizontal well
Q	=	production rate
r	=	radius
Re	=	real part of
RR	=	replacement ratio
RWD	=	reduction in well density
sh	=	hyperbolic sine function
sin	=	sine function
tg	=	tangent function
th	=	hyperbolic tangent function
x	=	abscissa
y	=	ordinate
z	=	affix in the complex plane
Z	=	set of the positive and negative integers

Greek symbols

Φ	=	real part of the complex flow potential
Ψ	=	value of the imaginary part of the complex flow potential
μ	=	viscosity
Ω	=	complexe flow potential

Suscripts and superscripts

e	=	external (drainage)
F	=	fracture
H	=	horizontal
W	=	well
X	=	along the x axis
Y	=	along the y axis

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APPENDIX A

HORIZONTAL WELL OF INFINITE LENGTH EQUIDISTANT FROM TWO PARALLEL IMPERMEABLE PLANES

In the simplest configuration, the horizontal well is in the center of the reservoir and equidistant from its upper and lower boundaries. (Fig.2)

By using the image method, the image-wells are obtained by successive symmetries in relation to the two walls of the reservoir, which gives the following complex potential of the flow :

$$\Omega(z) = \frac{\mu}{k}\,\frac{q}{2\pi}\,\ln\left[\sin\left(\frac{\pi}{2}\,\frac{iz}{a'}\right)\right] \qquad (1)$$

By separating the real part and the imaginary part, the result is :

$$\Phi = \frac{\mu}{k}\,\frac{q}{4\pi}\,\ln\left[\frac{1}{2}\left(\cosh\frac{\pi x}{a'} - \cos\frac{\pi y}{a'}\right)\right] \qquad (2)$$

$$\Psi = -\frac{\mu}{k}\,\frac{q}{2\pi}\,\mathrm{Arctg}\left[\frac{\mathrm{th}\left(\frac{\pi x}{2a'}\right)}{\mathrm{tg}\left(\frac{\pi y}{2a'}\right)}\right] \qquad (3)$$

The pressure is zero at A and A' since for $z = a'i$:

$$\Omega(a'i) = \frac{\mu}{k}\,\frac{q}{2\pi}\,\ln\left(\sin\frac{\pi}{2}\right) = 0 \qquad (4)$$

In the vicinity of the well, for $\left\|\frac{Z}{a'}\right\| < 0{,}2$

$$\Omega(z) = \frac{\mu}{k}\,\frac{q}{2\pi}\left[\ln\left(\frac{z}{a'}\right) + \ln\left(\frac{\pi}{2}\right) + \frac{\pi i}{2} + \frac{\pi^2}{24}\left(\frac{z}{a'}\right)^2 + o\left(\frac{z}{a'}\right)^3\right] \qquad (5)$$

Hence the pressure :

$$P(z) = \frac{\mu}{k}\,\frac{q}{2\pi}\left[\frac{1}{2}\ln\left(\frac{x^2+y^2}{a'^2}\right) + \ln\left(\frac{\pi}{2}\right) + \frac{\pi^2}{24}\,\frac{x^2-y^2}{a'^2} + o\left(\frac{z}{a'}\right)^3\right] \qquad (6)$$

In this representation, the section of the well that coincides with an isobar is not exactly circular, but its approximate corresponding equation is :

$$\ln\left(\frac{x^2+y^2}{a'^2}\right) + \frac{\pi^2}{12}\,\frac{x^2-y^2}{a'^2} = P_i \qquad (7)$$

and the radii r_x and r_y satisfy the relation :

$$\ln\left(\frac{r_x^2}{a'^2}\right) + \frac{\pi^2}{12}\frac{r_x^2}{a'^2} = \ln\left(\frac{r_y^2}{a'^2}\right) - \frac{\pi^2}{12}\frac{r_y^2}{a'^2} \quad (8)$$

Therefore the eccentricity e_r of the curve, where $e_r = \frac{r_y}{r_x} - 1$, is

$$e_r = \frac{\pi^2}{12}\left(\frac{r}{a'}\right)^2 \quad (9)$$

The eccentricity of the curve representing the well is lower than the square of the ratio of the wellbore diameter to the thickness of the layer. The representation of a circular section wellbore is therefore excellent. The well pressure is given by the relation :

$$P(r_w) = \frac{\mu}{k}\frac{q}{2\pi}\ln\left(\frac{\pi r_w}{2a'}\right) \quad (10)$$

Further away from the well :

$$P(z) = \frac{\mu}{k}\frac{q}{2\pi}\mathrm{Re}\left(\ln\left[\sin\left(\frac{\pi i}{2}\frac{z}{a'}\right)\right]\right) \quad (11)$$

$$P(z) = \frac{\mu}{k}\frac{q}{4\pi}\ln\left(sh^2\left(\frac{\pi}{2}\frac{x}{a'}\right) + \sin^2\left(\frac{\pi}{2}\frac{y}{a'}\right)\right) \quad (12)$$

Since the absolute value of the sine term is smaller than 1, it is negligible (less than 1 %) with regard to the term $\exp\left(\frac{\pi x}{a'}\right)$ of the development of the hyperbolic sine when :

$$\left|\frac{x}{a'}\right| > 3$$

The expression of the pressure then becomes :

$$P(x) = \frac{\mu}{k}\frac{q}{2}\left(\frac{x}{2a'} - \frac{\ln 2}{\pi}\right) \quad (13)$$

The difference in pressure between a point located at a distance x from the wellbore with a radius r_w and the latter is therefore equivalent to :

$$P(x) - P(r_w) = \frac{\mu}{k}q\left[\frac{x}{4a'} - \frac{1}{2\pi}\ln\left(\frac{\pi r_w}{a'}\right)\right] \quad (14)$$

The flow to a horizontal well can be compared to the flow to a vertical fracture with the same length as the horizontal well. If the flow were to a vertical fracture reaching the top and bottom of the producing layer, the result would be :

$$P(x) - P_F = \frac{\mu}{k}q\frac{x}{4a'} \quad (15)$$

Therefore, draining a layer with thickness 2 a' by a rectilinear horizontal well with radius r_w, small in relation to a' and located in the middle of the layer, leads to a pressure drop which is greater than in the case of drainage by a vertical fracture through the layer. At a large distance from the well, the pressure drop is equivalent to :

$$P_F - P(r_w) = \frac{\mu}{k}\frac{q}{2\pi}\ln\left(\frac{a'}{\pi r_w}\right) \quad (16)$$

APPENDIX B

OFF-CENTERED HORIZONTAL WELL IN A LAYER BORDERED BY TWO PARALLEL IMPERMEABLE PLANES

The case of a horizontal well equidistant from two parallel planes forming boundaries at the top and bottom of the reservoir has been discussed above (Appendix A).

Draining a layer with thickness 2 a' by an infinite horizontal well with radius r_w, small in relation to a' and located in the middle of the layer, leads to a pressure drop greater than in the case of drainage by a vertical fracture with infinite length intersecting the layer from top to bottom. The difference in pressure drop is equivalent to :

$$P_F - P(r_w) = \frac{\mu}{k}\frac{q}{2\pi}\ln\left(\frac{a'}{\pi r_w}\right) \quad (1)$$

The case of an off-centered infinite horizontal well can also be treated in two dimensions (Fig. 3). Using the image and source point method, the source point distribution pattern in Figure 4 is applied. The points are aligned along the imaginary axis in two series of respective affixes :

$$z = ib + 4kia' \quad (k \in Z) \tag{2}$$

$$z = -ib + 4kia' \quad (k \in Z) \tag{3}$$

According to the principle of superposition, the function that describes the flow is the sum of the functions describing each of the above two flow areas :

$$\Omega(z) = \frac{\mu}{k}\frac{q}{2\pi}\left[\ln\left(\sin\left[\frac{\pi}{4}\frac{iz+b}{a'}\right]\right) + \ln\left(\sin\left[\frac{\pi}{4}\frac{iz-b}{a'}\right]\right)\right] \tag{4}$$

By separating the real and the imaginary parts, we obtain :

$$P(z) = \frac{\mu}{k}\frac{q}{4\pi}\left[\ln\frac{1}{2}\left(\cosh\frac{\pi x}{2a'} - \cos\frac{\pi(y-b)}{2a'}\right) + \ln\frac{1}{2}\left(\cosh\frac{\pi x}{2a'} - \cos\frac{\pi(y+b)}{2a'}\right)\right] \tag{5}$$

$$\Psi(z) = -\frac{\mu}{k}\frac{q}{2\pi}\left[\mathrm{Arctg}\,\frac{\mathrm{th}\,\frac{\pi(y-b)}{2a'}}{\mathrm{tg}\,\frac{\pi x}{2a'}} + \mathrm{Arctg}\,\frac{\mathrm{th}\,\frac{\pi(y+b)}{2a'}}{\mathrm{tg}\,\frac{\pi x}{2a'}}\right] \tag{6}$$

The flow thus defined can include, among its flow lines, horizontal lines intersecting the affix points :

$$z = 2kia' \quad (k \in Z) \tag{7}$$

These straight lines represent the horizontal parallel planes limiting the different layers.

In the vicinity of the well at the affix point ib, that is for small $\left\| \frac{z - ib}{a'} \right\|$:

$$\text{With } r = \|z - ib\| \tag{8}$$

$$P(r) = \frac{\mu}{k}\frac{q}{2\pi}\ln\left(\frac{\pi r}{4a'}\sin\frac{\pi b}{2a'}\right) + o(r) \tag{9}$$

Further away from the well, the absolute value cosine terms become negligible in relation to the hyperbolic cosine expressions. The latter can be represented by their exponential term and :

$$P(x) = \frac{\mu}{k}\frac{q}{2}\left(\frac{x}{2a'} - \frac{\ln 2}{\pi}\right) \tag{10}$$

This expression does not depend on the eccentricity of the well since the term b is not involved.

The difference between a point located at a distance x from the well with radius r_w and the well itself is therefore equivalent to :

$$P(x) - P(r_w) = \frac{\mu q}{k}\left[\frac{x}{4a'} - \frac{1}{2\pi}\ln\left(\frac{\pi r_w}{a'}\sin\frac{\pi b}{2a'}\right)\right] \tag{11}$$

The flow to the off-centered horizontal well can be compared to the flow to a vertical fracture of the same length as the horizontal well.

If the flow were to a vertical fracture reaching the top and the bottom of the producing layer, the result would be :

$$P(x) - P_F = \frac{\mu}{k}\, q\, \frac{x}{4a'} \tag{12}$$

Therefore, draining a layer with thickness 2 a' by a rectilinear horizontal well with radius r_w, small in relation to a' and located at a distance b from the nearest wall, leads to a pressure drop which is greater than in the case of drainage by a vertical fracture fully intersecting the layer and which is equal to :

$$P_F - P(r_w) = -\frac{\mu q}{2\pi k}\ln\left(\frac{\pi r_w}{a'}\sin\frac{\pi b}{2a'}\right) \tag{13}$$

It can be seen that for b a' we have the same expression as that given in Appendix A for the centered well.

The eccentricity of the well creates an effect equivalent to the reduction of its radius by multiplying it by expression $\sin\frac{\pi b}{2a'}$.

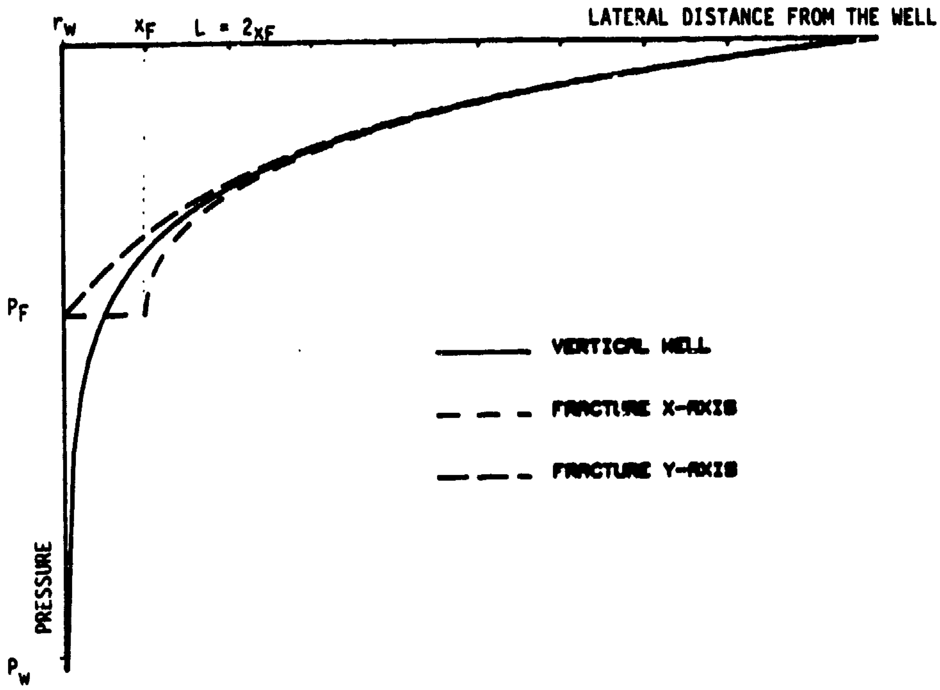

Fig. 1—Pressure profiles.

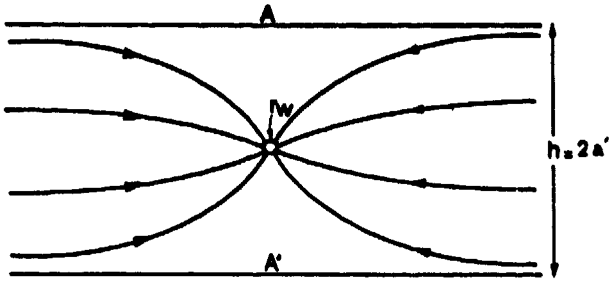

Fig. 2—Cross section of a centered horizontal well.

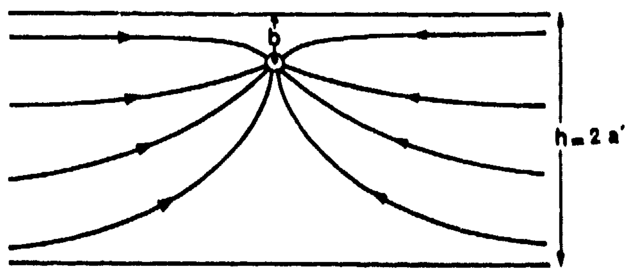

Fig. 3—Cross section o. ·n off-centered horizontal well.

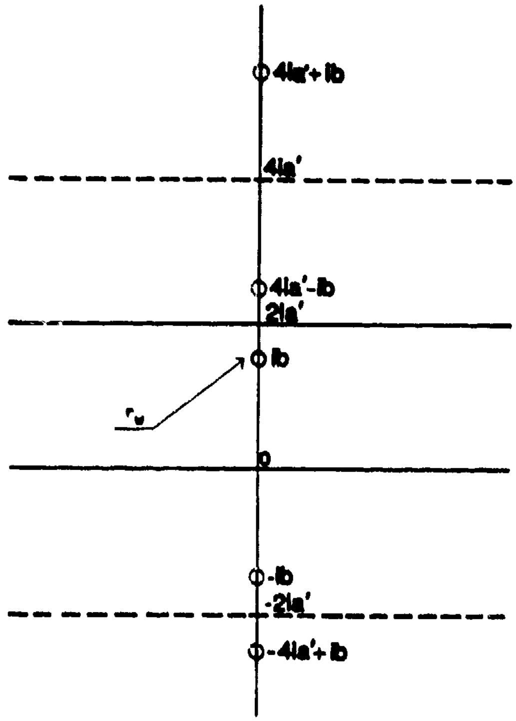

Fig. 4—Source points distribution.

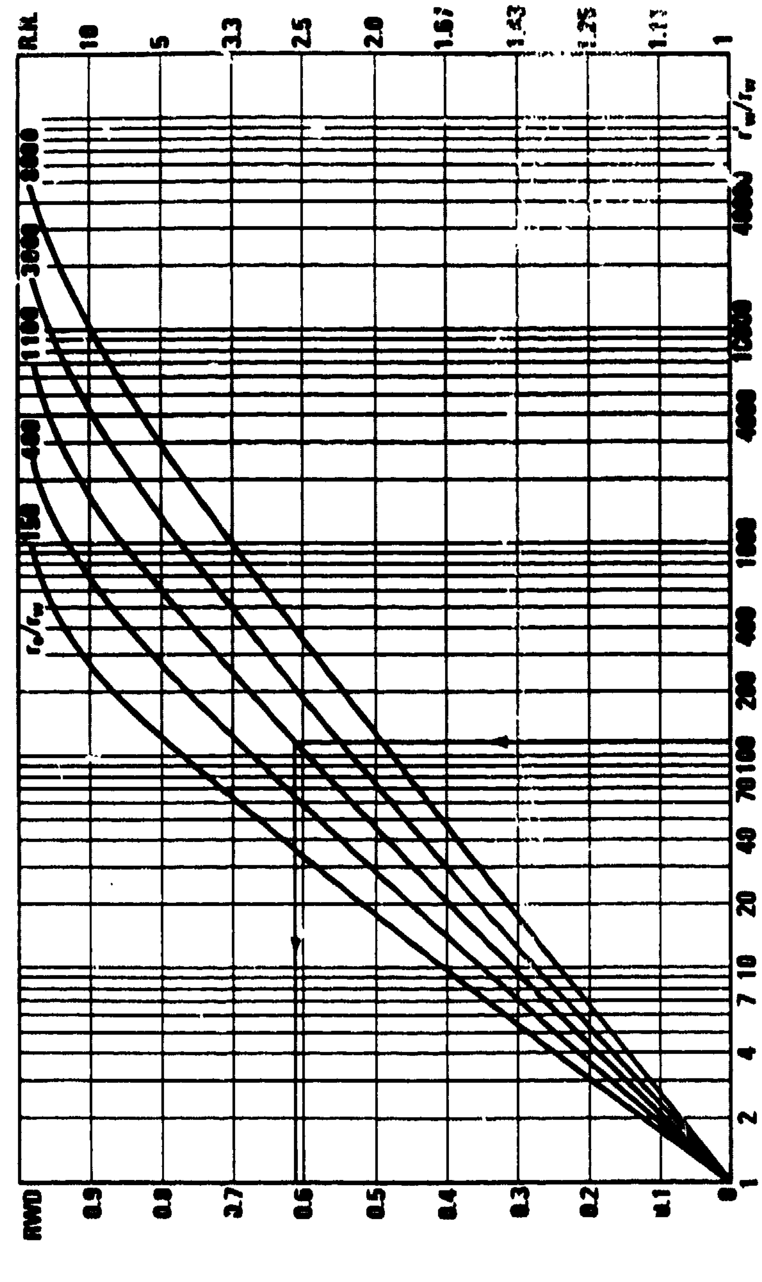

Fig. 5—Equivalent density of wells.