Wellbore Pressures in Reservoirs With Constant-Pressure or Mixed No-Flow/Constant-Pressure Outer Boundary

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Abstract

This paper reports on a general relationship existing between wellbore pressures in reservoirs with constant pressure or mixed no-flow/constant-pressure outer boundary and in related closed reservoirs, and uses this to derive expressions for shape factors, wellbore pressures, and Matthews-Brons-Hazebroek (MBH)[1] functions for static reservoir pressure.

Introduction

Effects of reservoir shape and well location on wellbore pressures have been studied extensively for closed reservoirs, but only a few cases have been presented for reservoirs with constant pressure or mixed no-flow/constant-pressure outer boundary. However, the latter group can be increased considerably by the methods presented in this paper, since the wellbore pressures for a wide variety of such reservoirs can be expressed by differences of wellbore pressures from related closed reservoirs. As a consequence, the shape factor, C_A, and the dimensionless wellbore pressure in the steady-state period, can be determined readily from existing tables for a large number of cases. Moreover, the end of the infinite-acting period, t_{DAeia}, can be estimated, and a rough upper bound can be given for the start of the steady-state period, t_{DAss}. Modifications of standard methods to determine the static reservoir pressure from buildup data also are obtained readily.

The results of this paper are related closely to those of Earlougher *et al.*[2] and Ramey *et al.*[3] Tables from those papers also are used as a basis for this paper. All cases that can be handled by direct use of these tables are included in Fig. 1. Tables presented by Earlougher[4] also are used. By using the approach of Ramey and Cobb,[5] it is possible to work with either dimensionless wellbore pressures or with MBH functions. These functions also are used to determine the static pressure from Horner[6] analysis.

Theory

Only single-well reservoirs that can be generated by regular patterns of producing and injecting (image) wells, with each well having the same rate, q, in absolute value, are considered. Storage and skin effects are not included. The dimensionless pressure drop for the actual well,

$$p_{wD}(t_D, A)=\frac{kh}{141.2qB\mu}[p_i-p_{wf}(t)], \quad \ldots\ldots\ldots (1)$$

can therefore be expressed as a sum of exponential integrals. Here, A denotes drainage area, p_i initial fully static pressure, p_{wf} flowing wellbore pressure, and

$$t_D=\frac{0.000264kt}{\phi\mu c_t r_w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

dimensionless time based on the wellbore radius. Dimensionless time based on drainage area, $t_{DA}=r_w^2 t_D/A$, also is used.

Consider p_{wD} for such a reservoir with constant pressure or mixed no-flow/constant-pressure outer boundary. Since the reservoir can be generated by a regular pattern of producing and injecting wells, a related closed reservoir is obtained by letting all wells be producers. By superimposing the two patterns, each injecting well cancels a producing well, and a new pattern of only producing wells is obtained, with each well having rate $2q$ and draining an area $2A$. The superscript "new" refers to the new closed reservoir and "old" refers to the first closed reservoir. It follows that

$$p_{wD}(t_D,A)=2p_{wD}^{\text{new}}(t_D,2A)-p_{wD}^{\text{old}}(t_D,A). \quad \ldots\ldots (3)$$

Moreover, since

$$p_{wD}(t_D,A)=\frac{1}{2}(\ln t_D+0.80907)+2\pi t_{DA} - \frac{1}{2}p_{D\text{MBH}}(t_{DA}), \quad \ldots\ldots\ldots\ldots (4)$$

where $p_{D\text{MBH}}$ denotes the MBH function, and $t_{D(2A)}=(\frac{1}{2})t_{DA}$, it follows that

$$p_{wD}(t_D,2A)=\frac{1}{2}\ln 2+p_{wD}\left(\frac{1}{2}t_D,A\right). \quad \ldots\ldots (5)$$

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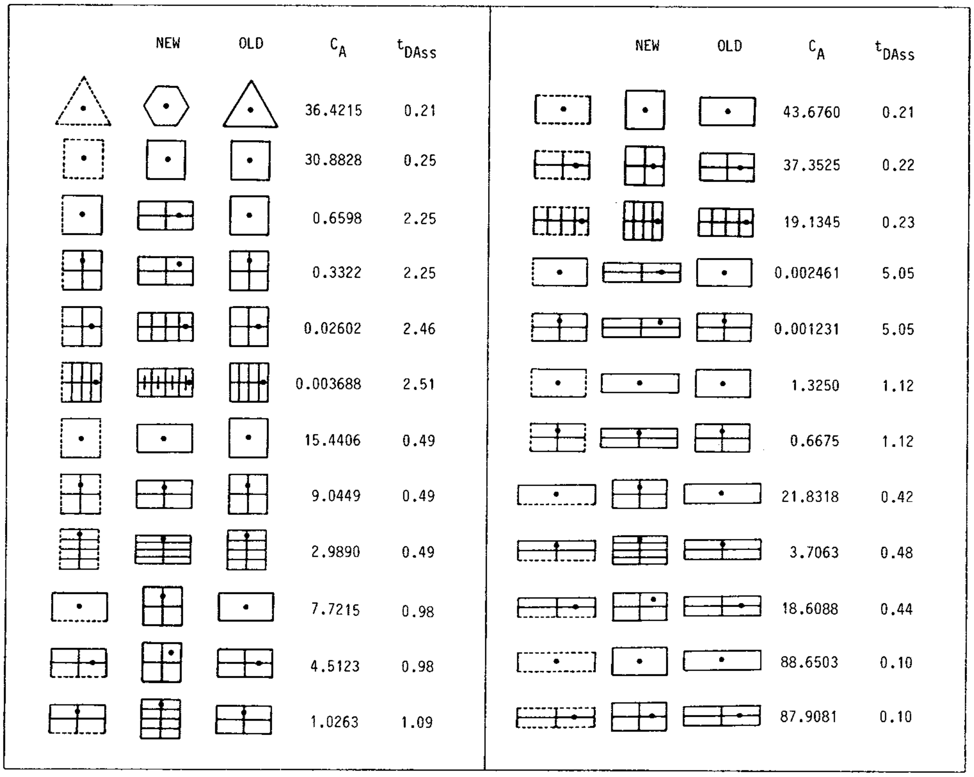

Fig. 1—New and old no-flow outer boundary equivalents; shape factors, C_A; and times to steady state, t_{DAss} (based on $\Delta p_{wD}=0.01$).

Eq. 3 can therefore be rewritten in the form

$$p_{wD}(t_D,A)=\ln 2+2p_{wD}^{\text{new}}\left(\frac{1}{2}t_D,A\right)-p_{wD}^{\text{old}}(t_D,A). \qquad (6)$$

Eq. 6 takes on a particularly simple form when both p_{wD}^{new} and p_{wD}^{old} have reached pseudosteady state, in which case

$$p_{wD}^{\text{new}}(t_D,A)=2\pi t_{DA}+\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A^{\text{new}}r_w^2}, \qquad (7)$$

and likewise for p_{wD}^{old}, where γ denotes Euler's constant (0.5772...). It follows that

$$p_{wD}(t_D,A)=\frac{1}{2}\ln\frac{16A}{e^{\gamma}C_A r_w^2} \qquad (8)$$

in the steady state period, with the shape factor, C_A, given by

$$C_A=(C_A^{\text{new}})^2/C_A^{\text{old}}. \qquad (9)$$

This definition of shape factor for reservoirs with constant pressure or mixed no-flow/constant-pressure outer boundary follows the convention in Ref. 3 and *not* that used by Dietz[7] for such reservoirs. In particular, note that $C_A=C_A^{\text{old}}$ only if $C_A^{\text{new}}=C_A^{\text{old}}$, which turns out to be the case for a square with constant-pressure outer boundary, as was shown in Ref. 3. Note also that $t_{DAeia}\approx t_{DAeia}^{\text{old}}$, and by Eq. 6 that $t_{DAss}\leqslant\max(2t_{DApss}^{\text{new}}, t_{DApss}^{\text{old}})$. This inequality gives only a rough upper bound since it does not take cancellations into account. Values of t_{DA} at the start of the steady-state period have therefore been included in Fig. 1, with these values determined numerically and corresponding to a dimensionless deviation of 0.01 (0.08 to 0.15%). If a dimensionless deviation of 0.1 is used as the criterion, then these times should roughly be divided by two.

To work with tables expressed in terms of MBH functions, as in Ref. 2, note from Eqs. 3 and 4 that

$$p_{wD}(t_D,A)=\frac{1}{2}(\ln t_D+0.80907)-\frac{1}{2}\cdot\left[2p_{D\text{MBH}}^{\text{new}}\left(\frac{1}{2}t_{DA}\right)-p_{D\text{MBH}}^{\text{old}}(t_{DA})\right]. \qquad (10)$$

The main practical problem with the procedures in this paper is to determine the shape and well location of the new closed reservoir. The old and new closed reservoirs therefore have been listed in Fig. 1 for the cases that can be handled by tables in Ref. 2 and 3. Fig. 2 illustrates the process of finding the new reservoir shape for three of the entries in Fig. 1, together with one that includes a nonstandard new shape. The procedure is simply to delete the injection well(s) and to locate no-flow boundaries between the remaining producing wells within the area considered. Rectangular reservoirs with three sides or two adjacent sides at constant pressure have not been included in Fig. 1 since these induce nonstandard new shapes, as shown in Fig. 2.

Determination of Static Pressure

The results of this paper can be used not only to determine the flowing wellbore pressure, p_{wf}, provided the static pressure, p_i, and skin factor are both known but also to determine the static pressure from buildup data.

If steady state has been reached prior to shut-in, then the dimensionless shut-in pressure,

$$p_{Ds}(\Delta t_{DA})=\frac{kh}{141.2qB\mu}[p_i-p_{ws}(\Delta t)], \quad\ldots\ldots\ldots(11)$$

will be given by

$$p_{Ds}(\Delta t_{DA})=\frac{1}{2}\ln\frac{16A}{e^{\gamma}C_A r_w^2}-\frac{1}{2}\ln\frac{4\Delta t_D}{e^{\gamma}}$$

$$=\frac{1}{2}\ln\frac{4}{C_A\Delta t_{DA}} \quad\ldots\ldots\ldots\ldots\ldots(12)$$

in the infinite-acting period, since

$$p_{Ds}(\Delta t_{DA})=p_{wD}(t_D+\Delta t_D,A)-p_{wD}(\Delta t_D,A). \quad\ldots(13)$$

It follows that p_i can be determined from the semilog straight line on a Miller-Dyes-Hutchinson[8] (MDH) plot by reading off the pressure at

$$\Delta t_{DA}=\frac{4}{C_A}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Next, if steady state has not been reached prior to shut-in, then Horner analysis can be combined with Eq. 10 to determine the static pressure. The point is that

$$p_{Ds}(\Delta t_{DA})=p_{wD}(t_D,A)+\frac{1}{2}\ln\frac{t+\Delta t}{t}-p_{wD}(\Delta t_D,A)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

with negligible error for small Δt's. Hence, if Δt is in the infinite-acting period, then Eq. 15 can be rewritten in the form

$$p_{Ds}(\Delta t_{DA})=p_{wD}(t_d,A)$$

$$+\frac{1}{2}\ln\frac{t+\Delta t}{\Delta t}-\frac{1}{2}(\ln t_D+0.80907). \quad\ldots\ldots\ldots\ldots(16)$$

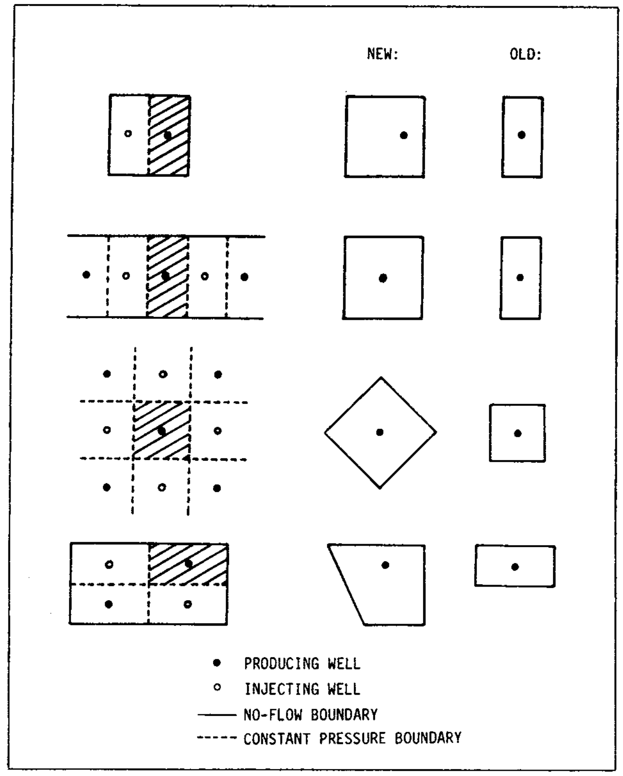

Fig. 2—Determining new (area doubled) and old no-flow outer boundary equivalents.

Therefore, by extrapolation one gets

$$p^*_{Ds}=p_{wD}(t_D,A)-\frac{1}{2}(\ln\ t_D+0.80907), \quad\ldots\ldots(17)$$

where

$$p^*_{Ds}=\frac{kh}{141.2qB\mu}(p_i-p^*). \quad\ldots\ldots\ldots\ldots\ldots(18)$$

From Eq. 10 it now follows that

$$p^*_{Ds}=\frac{1}{2}\left[p^{\text{old}}_{DMBH}(t_{DA})-2p^{\text{new}}_{DMBH}\left(\frac{1}{2}t_{DA}\right)\right]. \quad .(19)$$

The last two equations form the basis for determining the static pressure from Horner analysis.

Example Computations

To illustrate the methods of this paper, consider a square with the well at the center and all four sides at constant pressure or with one side at constant pressure and the remaining sides closed.

For the first case, which can be found in Ref. 3 or in a paper by Kumar and Ramey,[9] it follows from Fig. 1, results in Ref. 4, and Eq. 9 that $C_A=30.8828$. This agrees with Ref. 3. By using data from Ref. 3 for p_{wD}^{new} $(0.05, A)$ and p_{wD}^{old} $(0.1, A)$, with $\sqrt{A}/r_w=2{,}000$, the value p_{wD} $(0.1, A)=6.8071$ is obtained from Eq. 6. The value 6.8066 was reported in Refs. 2 and 3. If Eq. 10 is used with MBH data from Ref. 2, then the value 6.8066 is obtained.

For the second case, which also can be found in Ref. 3, it follows from Fig. 1 and Ref. 4 that $C_A^{new}=4.5141$ and $C_A^{old}=30.8828$, and hence that $C_A=0.6598$ by Eq. 9. From this shape factor it follows that $p_{wD}=8.9065$ in the steady-state period. The value 8.9045 was reported in Ref. 3 for $t_{DA}=10.0$. Note from Fig. 1 that $t_{DAss}=2.25$. Next, by using data from Ref. 3 in Eq. 6, the value p_{wD} $(1.0, A)=8.6882$ is obtained, while MBH data from Ref. 2 used with Eq. 10 gives the value 8.6906. The value reported in Ref. 3 for $t_{DA}=1.0$ is 8.6886.

The numerical discrepancies are caused by roundoff and truncation errors in the basic tables, since the identities in this paper are mathematically exact.

Nomenclature

A = area, sq ft [m^2]
B = formation volume factor, RB/STB [res m^3/stock-tank m^3]
c_t = total compressibility, psi^{-1} [kPa^{-1}]
C_A = Dietz shape factor
e = base of natural logarithm [2.71828...]
h = thickness, ft [m]
k = permeability, md
p = pressure, psi/a [kPa]
q = rate, STB/D [stock-tank m^3/d]
r = wellbore radius, ft [m]
t = time, hours
γ = Euler's constant [0.5772...]
μ = viscosity, cp [Pa·s]
ϕ = porosity

Subscripts

A = area
D = dimensionless
eia = end infinite acting
f = flowing
i = initial
pss = pseudosteady state
s = shut-in
ss = steady state
w = wellbore

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