Determination of Average Reservoir Pressure From Build-Up Surveys

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ABSTRACT

A method for determining average reservoir pressure is presented, which is simpler to apply than that devised by Matthews, Brons and Hazebroek. For bounded reservoirs, identical results are obtained if stabilized-flow conditions prevail. The present method yields inferior results in the transient state. The method can, with a slight modification, also be used for water-drive reservoirs.

INTRODUCTION

In the method proposed by Matthews, Brons and Hazebroek[1] for determining average reservoir pressure in a multi-well reservoir, the cumulative production (the production time of each well) enters twice: once when the build-up is plotted against $\ln\,(t + \Delta t)/\Delta t$ to arrive at p^*, and a second time when the correction, $p^* - \bar{p}$ is determined with one of the several formulas for differently shaped drainage areas.

Once a steady state has been attained, however, the previous production history should be immaterial. The same pressure distribution could have been arrived at after different cumulative productions of the individual wells. In principle, therefore, it should be possible to determine average pressures without referring to cumulative productions.

In the following paragraphs an expression is presented for the difference between the pressure in a producing well and the average pressure of its drainage area. Then the build-up time needed to overcome this difference is indicated.

CASE OF A CIRCULAR DRAINAGE AREA AND A CENTRAL WELL

PRESSURE DISTRIBUTION BEFORE SHUT-IN

The general differential equation of radial flow (see Muskat,[2] Eq. 10.2) may be written

$$\frac{k}{\mu} 2\pi h \frac{\partial}{\partial r}\left(r \frac{\partial p}{\partial r} \right) = 2\pi h \phi c \, r \frac{\partial p}{\partial t} . \quad . \quad (1)$$

In the steady state, the rate of production of the well is equal to the rate of expansion of the fluid contained in the the drainage area; thus

$$q = -\,\pi r_b^2\, h\phi c \left(\frac{\partial p}{\partial t} \right)_g \quad , \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and therefore, combining Eqs. 1 and 2 we have

$$\frac{\partial}{\partial r}\left(r \frac{\partial p}{\partial r} \right) = \frac{-\,q\mu r}{\pi k h\; r_b^2} \quad , \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which can be integrated to

$$\frac{\partial p}{\partial r} = -\frac{q\mu}{2\pi k h\; r_b^2}\, r + \frac{C_1}{r} . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The boundary condition can be introduced as

$$\frac{\partial p}{\partial r} = 0 \text{ at } r = r_b , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

so that Eq. 4 can be rewritten

$$\frac{\partial p}{\partial r} = \frac{q\mu}{2\pi k h}\left(\frac{1}{r} - \frac{r}{r_b^2} \right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A second integration gives

$$p = \frac{q\mu}{2\pi k h}\left(\ln\, r - \frac{r^2}{2 r_b^2} \right) + C_2 . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

AVERAGE PRESSURE OF THE DRAINAGE AREA

The average pressure can be expressed as

$$\bar{p} = \frac{1}{\pi r_b^2} \int_o^{r_b} p \; 2\pi r dr = \frac{2}{r_b^2} \int_o^{r_b} p r dr \quad . \quad . \quad . \quad . \quad (8)$$

Substitution of Eq. 7 and integration yields

$$\bar{p} = \frac{q\mu}{\pi k h}\left\{ \frac{\ln\, r_b}{2} - \frac{3}{8} \right\} + C_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

BUILD-UP

If Eq. 7 is applied to the wellbore at r_w, and if C_2 is eliminated by the combination with Eq. 9, we have for the pressure at the well under steady-state conditions

$$p_w = \bar{p} - \frac{q\mu}{2\pi k h}\left(\ln \frac{r_b}{r_w} - \frac{3}{4} \right) . \quad . \quad . \quad . \quad . \quad (10)$$

After shut-in, and as long as the physical boundaries of the entire reservoir have no influence, the pressure increases according to the well-known expression

$$\Delta p_w = -\frac{q\mu}{4\pi k h}\, Ei \left(-\frac{\phi \mu c r_w^2}{4k\Delta t} \right) \quad , \quad . \quad . \quad . \quad . \quad (11)$$

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[1]References given at end of paper.

Discussion of this and all following technical papers is invited. Discussion in writing (three copies) may be sent to the office of the *Journal of Petroleum Technology*. Any discussion offered after Dec. 31, 1965, should be in the form of a new paper. No discussion should exceed 10 per cent of the manuscript being discussed.

which, for $\phi\mu c r_w^2/4k\Delta t < 0.01$, can be approximated by

$$\Delta p_w = -\frac{q\mu}{4\pi kh}\left(0.5772 + \ln\frac{\phi\mu c r_w^2}{4k\Delta t}\right). \quad . \quad (12)$$

Addition of Eqs. 10 and 12 yields an expression for the straight-line part of the build-up curve if pressure is plotted against $\ln\Delta t$, thus

$$p_w(\Delta t) = \bar{p} - \frac{q\mu}{4\pi kh}\left[\ln\frac{r_b^2}{r_w^2} - \frac{3}{2} + 0.5772 + \ln\frac{\phi\mu c r_w^2}{4k\Delta t}\right] \quad . \quad (13)$$

The well radius cancels out, as a skin-factor would have done if it had been introduced; therefore

$$p_w(\Delta t) = \bar{p} - \frac{q\mu}{4\pi kh}\left[\ln\frac{\phi\mu c r_b^2}{4k\Delta t} - 0.9228\right]. \quad (14)$$

On this straight line, or on its extrapolation, the value of $\bar{p}$ is found at the point where the form in brackets vanishes. For this purpose, the line should be read at

$$\Delta t_{\bar{p}} = \frac{\phi\mu c r_b^2}{4k\, e^{0.9228}} = \frac{\phi\mu c r_b^2}{10.07k} = \frac{\phi\mu c A}{31.6k} \quad . \quad . \quad . \quad (15)$$

k/μ is known from the slope of the build-up curve, and ϕ, c and A can be determined by known methods.

DRAINAGE AREAS HAVING DIVERSE SHAPES AND WELL POSITIONS

Eq. 15 is applicable only to a circular drainage area and a central well. Although similar expressions could be obtained in a like manner for other drainage areas, it is much more convenient to use some of the figures presented in Ref. 1. The curves of Figs. 2 through 8 in Ref. 1 become straight lines for sufficiently large values of t. These straight-line parts can be represented by equations such as

$$\frac{p^* - \bar{p}}{q\mu/4\pi kh} = \ln\frac{C_A kt}{\phi\mu c A}, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where C_A is a constant dependent on the shape of the area and on the well position.

p^* is a point at $\Delta t = \infty$ on a straight-line extrapolation of the build-up plot vs $\ln\,(t+\Delta t)/\Delta t$. $\bar{p}$ can be considered as a point at $\Delta t_{\bar{p}}$ on a straight-line extrapolation of the build-up plot vs $\ln\, t/\Delta t$, since the early part of the build-up can be approximated by

$$p_w(\Delta t) = p^* - \frac{q\mu}{4\pi kh}\ln\frac{t+\Delta t}{\Delta t} \approx p^* - \frac{q\mu}{4\pi kh}\ln\frac{t}{\Delta t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Extrapolating the straight-line part of this type of plot is equivalent to using the latter expression beyond the validity of the above approximation. By definition, the point $(p, \Delta t_{\bar{p}})$ is on the straight-line extrapolation, and this pair of values should satisfy Eq. 17, which leads to

$$\bar{p} = p^* - \frac{q\mu}{4\pi kh}\ln\frac{t}{\Delta t_{\bar{p}}} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Combination of Eqs. 16 and 18 yields

$$\Delta t_{\bar{p}} = \frac{\phi\mu c A}{C_A k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is a generalized form of Eq. 15.

The shape factor C_A can be obtained from Ref. 1 by the consideration that Eq. 16 is reduced to

$$\frac{p^* - \bar{p}}{q\mu/4\pi kh} = \ln C_A \text{ for } \frac{kt}{\phi\mu c A} = 1 \quad . \quad . \quad . \quad . \quad (20)$$

Therefore, $\ln C_A$ can be read from the straight-line parts of the curves in Figs. 2 through 8 of the reference, or from their extrapolations, at the abscissa value of $kt/\phi\mu c A = 1$. The results are listed in Table 1.

For smaller values of t, where the curves of Figs. 2 through 8 of Ref. 1 are not straight lines, Eq. 16 does not indicate an actual curve. In those cases, the $\bar{p}$ following from Eq. 19, which depends on Eq. 16 representing an actual curve, cannot be corrected. The range of validity of the present method can therefore be established by observing where the graphs in Figs. 2 through 8 of Ref. 1 start to deviate from straight lines. The limits of validity thus found are presented in the last column of Table 1.

APPLICATION OF METHOD

1. Divide the reservoir on a map into drainage areas proportional to offtake rates per unit sand thickness of wells, as prescribed in Ref. 1.
2. Plot pressure build-ups against $\ln\,\Delta t$, and determine k/μ from slopes.
3. Determine t, ϕ, c and A by generally known methods.
4. Check applicability of method by comparing $kt/\phi\mu c$A with required values in the table.
5. Select C_A from the table and read straight-line parts of pressure build-ups or their extrapolations at $\Delta t_{\bar{p}} = \phi\mu c A/C_A k$ for p.

DISCUSSION

In a discussion with Matthews, Brons and Hazebroek and with other build-up experts, it was pointed out that the Matthews-Brons-Hazebroek method worked accurately also in the transient state, although the assumption of steady state is used in the division into drainage areas. The present method relies more heavily on this assumption, and, in the transient state, becomes increasingly inaccurate for smaller production times. Identical results are obtained in the steady state, and in this region the present method may be preferred for its simplicity.

ADAPTATION TO WATER-DRIVE RESERVOIRS

COMPLETE WATER DRIVE

Under complete water drive the pressure at any point tends to become constant. Drainage areas, defined in the usual sense, have very irregular shapes, each one having to be in contact with the advancing water front. In this case it is preferable to divide the reservoir as regularly as possible into what, in accordance with D. R. Horner,[3] can be called associated reservoir areas allocated to the wells.

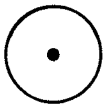
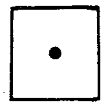
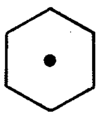
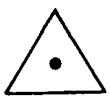
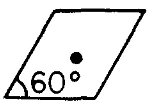
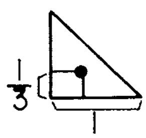
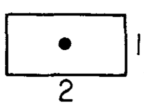
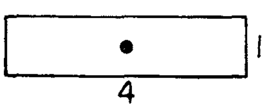
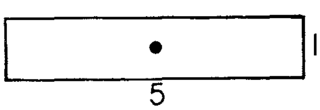
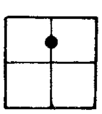
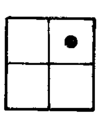
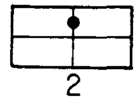
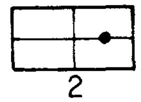
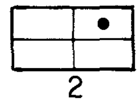
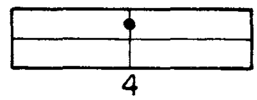
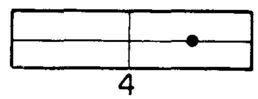
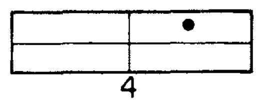
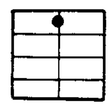
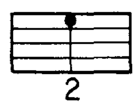
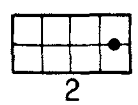
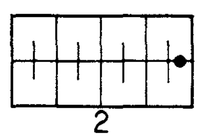
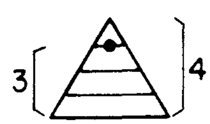
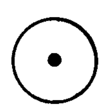
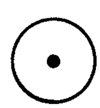
In a regularly drilled field the associated areas can be approximated by circles. The pressure distribution can be irregular along the boundary of such an area, but it should be constant with time. The pressure at any point in the area will be the sum of a pressure p_1, due to the pressure distribution at the boundary if the well were not producing, and a negative pressure p_2 due to the withdrawal by the well, zero pressure being assumed along the boundary.

According to Frank and v. Mises,[4] (Eq. 12),

$$p_{1w} = \bar{p}_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

As the total flow originates from the boundary, the sec-

TABLE 1

	$\ln C_A$	C_A	Stabilized conditions for $\frac{kt}{\phi \mu c A} >$
In bounded reservoirs			
	3.45	31.6	0.1
	3.43	30.9	0.1
	3.45	31.6	0.1
	3.32	27.6	0.2
	3.30	27.1	0.2
	3.09	21.9	0.4
	3.12	22.6	0.2
	1.68	5.38	0.7
	0.86	2.36	0.7
	2.56	12.9	0.6
	1.52	4.57	0.5
	2.38	10.8	0.3
	1.58	4.86	1.0
	0.73	2.07	0.8
	1.00	2.72	0.8
	−1.46	0.232	2.5
	−2.16	0.115	3.0
	1.22	3.39	0.6
	1.14	3.13	0.3
	−0.50	0.607	1.0
	−2.20	0.111	1.2
	−2.32	0.098	0.9
In water-drive reservoirs			
	2.95	19.1	0.1
In reservoirs of unknown production character			
	3.22	25	0.1

ond pressure field is described by

$$p_2 = \frac{q\mu}{2\pi kh} \ln \frac{r}{r_b} . \quad (22)$$

Integration and division by πr_b^2 lead to

$$\bar{p}_2 = -\frac{q\mu}{4\pi kh} . \quad (23)$$

Application of Eq. 22 to the well radius gives

$$p_{2w} = \frac{q\mu}{2\pi kh} \ln \frac{r_w}{r_b} \quad (24)$$

The well pressure before closing in can be expressed as

$$p_w = \bar{p} + \frac{q\mu}{4\pi kh}\left(1 + 2 \ln \frac{r_w}{r_b}\right), \quad (25)$$

and after closing in as

$$p_w(\Delta t) = \bar{p} + \frac{q\mu}{4\pi kh}\left\{ 1 + 2 \ln \frac{r_w}{r_b} - 0.5772 - \ln \frac{\phi\mu c r_w^2}{4k\Delta t}\right\} , \quad (26)$$

from which it follows that

$$\Delta t_{\bar{p}} = \frac{\phi\mu c r_b^2}{4ke^{0.4228}} = \frac{\phi\mu c r_b^2}{6.1\ k} = \frac{\phi\mu c A}{19.1\ k} . \quad (27)$$

APPLICATION OF PROPOSED METHOD TO RESERVOIRS OF UNKNOWN PRODUCTION CHARACTER

The usual purpose of average pressure determinations is to calculate by material balance the strength of the water drive. It may therefore appear that the problem has entered a vicious circle. As shown in the Appendix, bounded reservoirs also can be analyzed after division into associated reservoir areas, rather than into drainage areas. The same constant $C_A = 31.6$ for a circle is applicable in the latter case. When there is doubt about the amount of water drive, it is recommended that the reservoir be divided regularly into associated reservoir areas and that the intermediate value $C_A = 25$ be used for the circularized area. The inevitable range of uncertainty from 19.1 to 31.6 should lead to errors no worse than those from the uncertainty in the compressibility. These errors in $\Delta t_{\bar{p}}$ have but little influence on p, because $\bar{p}$ is read from a semi-logarithmic plot.

NOMENCLATURE

The formulas are suitable for any consistent system of units. The units indicated below will serve as an example.

A = drainage area or associated reservoir area, sq cm

C_A = constant dependent on shape of area, position of well and on production character

C_1, C_2 = integration constants, atm

Ei = exponential integral defined by

$$Ei(-x) = -\int_x^\infty \frac{e^u}{u} du$$

c = effective compressibility of reservoir fluid, atm^{-1}

h = sand thickness, cm

k = permeability, darcies

p = reservoir pressure dependent on place and time, atm

p_w = well pressure, atm

$\bar{p}$ = average pressure in drainage area or associated reservoir area, atm

p^* = closed-in pressure linearly extrapolated on plot against $\ln(t + \Delta t)/\Delta t$ for infinite closed-in time, atm

p_1 = pressure field due only to pressure distribution on boundary, atm

p_2 = pressure field due only to withdrawal by well, atm

p_3 = pressure field due only to oil expansion, atm

Δp_w = pressure increase at the well since closing in, atm

$(\partial p/\partial t)_g$ = general rate of pressure drop in stabilized reservoir, atm sec^{-1}

q = production rate of well before closing in, cc sec^{-1}

q_e = rate of oil expansion in associated reservoir area, cc sec^{-1}

r = distance to well center, cm

r_b = outer radius of circular drainage area or associated reservoir area, cm

r_w = well radius, cm

t = corrected production time, defined as cumulative well production divided by rate before closing-in, sec

Δt = closed-in time, sec

$\Delta t_{\bar{p}}$ = defined by $p_w\ (\Delta t_{\bar{p}}) = \bar{p}$ on linear extrapolation of plot against $\ln \Delta t$, sec

μ = viscosity of reservoir fluid, cp

ϕ = porosity, fraction

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REFERENCES

1. Matthews, C. S., Brons, F. and Hazebroek, P.: "A Method for Determination of Average Pressure in a Bounded Reservoir", *Trans.*, AIME (1954) **201,** 182.
2. Muskat, Morris: *The Flow of Homogeneous Fluid Through Porous Media,* McGraw-Hill Book Co., Inc., New York (1937).
3. Horner, D. R.: "Average Reservoir Pressure", *Proc.,* Fourth World Petroleum Congress, 131.
4. Frank and v. Mises: "Die Differential and Integralgleichungen", Teil Auflage, 691.

APPENDIX

A CIRCULAR ASSOCIATED RESERVOIR AREA IN A BOUNDED RESERVOIR

We introduce q_e as the rate at which oil is expanding within the associated area because of the general pressure decline $(\partial p/\partial t)_g$. This rate will differ from the production rate q of the well to the extent that the associated area differs from the drainage area. In the case of a non-producing (observation) well, the outward flow through the boundary of the associated area would be q_e.

The pressure field is now split into three simpler ones: p_1 due to the pressure distribution along the boundary at the time of closing in, while this distribution is considered constant with time; p_2 (negative) caused by the withdrawal by the well, constant zero pressure being assumed along the boundary; and p_3 as it would occur for a uniform rate of pressure drop, if the well were not producing and if the pressure everywhere along the boundary just reached zero.

The first two fields are identical to p_1 and p_2 discussed for water-drive reservoirs. Therefore,

$$p_{1_w} = p_1 \quad , \qquad (21)$$

$$\bar{p}_2 = -\frac{q\mu}{4\pi kh} \quad , \qquad (23)$$

$$p_{2_w} = \frac{q\mu}{2\pi kh} \ln \frac{r_w}{r_b} \qquad (24)$$

The third field is governed by

$$\frac{k}{\mu} 2\pi h \frac{\partial}{\partial r}\left(r \frac{\partial p_3}{\partial r} \right) = 2\pi h \phi c r \left(\frac{\partial p_3}{\partial t} \right)_a \qquad (28)$$

which can be rewritten as

$$\frac{\partial}{\partial r}\left(r \frac{\partial p_3}{\partial r} \right) = -\frac{q_e \mu r}{\pi kh\ r_b^2} \qquad (29)$$

Integration gives

$$\frac{\partial p_3}{\partial r} = -\frac{q_e \mu r}{2\pi k h r_b^2} + \frac{C_1}{r} \qquad (30)$$

The condition at the inner boundary

$$\frac{\partial p_3}{\partial r} = 0 \text{ for } r = 0 \quad , \qquad (31)$$

reduces Eq. 30 to

$$\frac{\partial p_3}{\partial r} = -\frac{q_e \mu r}{2\pi k h r_b^2} \qquad (32)$$

Further integration leads to

$$p_3 = -\frac{q_e \mu}{4\pi k h r_b^2} r^2 + C_2 \quad , \qquad (33)$$

which, after introduction of the outer boundary condition

$$p_3 = 0 \text{ for } r = r_b \quad , \qquad (34)$$

becomes

$$p_3 = \frac{q_e \mu}{4\pi kh} \frac{r_b^2 - r^2}{r_b^2} \qquad (35)$$

The average value is then found to be

$$\bar{p}_3 = \frac{1}{\pi r_b^2} \int_0^{r_b} p_3\, 2\pi r dr = \frac{q_e \mu}{8\pi kh} \quad , \qquad (36)$$

and the value at the well

$$p_{3_w} = \frac{q_e \mu}{4\pi kh} \qquad (37)$$

Addition of the three fields yields the average pressure

$$\bar{p} = \bar{p}_1 - \frac{q\mu}{4\pi kh} + \frac{q_e \mu}{8\pi kh} \quad , \qquad (38)$$

and the well pressure before shut-in

$$p_w = \bar{p}_1 + \frac{q\mu}{2\pi kh} \ln \frac{r_w}{r_b} + \frac{q_e \mu}{4\pi kh} \quad , \qquad (39)$$

or

$$p_w = \bar{p} + \frac{q\mu}{4\pi kh} \left(1 + 2 \ln \frac{r_w}{r_b} \right) + \frac{q_e \mu}{8\pi kh} . \qquad (40)$$

After closing in, the well pressure can now be expressed by

$$\begin{aligned} p_w(\Delta t) &= \bar{p} + \frac{q\mu}{4\pi kh} \left\{ 1 + 2 \ln \frac{r_w}{r_b} - 0.5772 \right. \\ &\quad \left. - \ln \frac{\phi \mu c r_w^2}{4k\Delta t} \right\} + \frac{q_e \mu}{8\pi kh} \\ &= \bar{p} + \frac{q\mu}{4\pi kh} \left\{ \frac{3}{2} + 2 \ln \frac{r_w}{r_b} - 0.5772 \right. \\ &\quad \left. - \ln \frac{\phi \mu c r_w^2}{4k\Delta t} \right\} + \frac{(q_e - q)\mu}{8\pi kh} \end{aligned} \qquad (41)$$

The bracket form again vanishes for

$$\Delta t_{\bar{p}_{\text{uncorrected}}} = \frac{\phi \mu c A}{31.6k} \quad , \qquad (42)$$

and the true average may be expressed as

$$\bar{p}_{\text{correct}} = \bar{p}_{\text{uncorrected}} - \frac{(q_e - q)\mu}{8\pi kh} . \qquad (43)$$

The indicated correction is but a minor one. Moreover, in the determination of the average pressure of the entire reservoir, it cancels out because

$$\Sigma q_e = \Sigma q \qquad (44)$$

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