

Figure 3: (Left-to-right) In the first figure we simulate the scoring rule where ρ is a dictatorship with a fixed mixing weight of 0.5, in the middle figure we simulate the scoring rule with min-max ρ (pessimistic decision maker) and in the last figure we simulate the scoring rule where the aggregation is a randomized dictatorship and the forecaster obtains a distribution $\theta = \mathcal{U}[0,1]$ over ρ . The lower half of the figure is not plotted since that corresponds to region $q_1 > q_2$, i.e. lower probability being greater than upper probability

E SIMULATIONS

To test the sanity of our proposed scoring rule, we simulate a scenario where an imprecise forecaster predicts a binary outcome (e.g., chance of rain tomorrow). We assume the forecaster has an imprecise forecast [0.4, 0.6] and uses an imprecise scoring rule s_{ρ} where ρ is a dictatorship or some other aggregation like min-max. We compare this to our randomized imprecise scoring rule s_{θ} . Given the binary outcome, the forecaster reports an interval $\mathcal{Q} := [q_1, q_2]$ where q_1 denotes the lower probability and q_2 the upper probability respectively. Figure 3 highlights that the randomized scoring rule s_{θ} is strictly proper for imprecise forecasts as it has the highest expected score for the forecaster only when the forecaster reports his true belief. While in other cases of using a deterministic imprecise scoring rule s_{ρ} , if DM provides a ρ such that it is a dictatorship, such as in the case of Figure 3(a), the scoring rule is proper; however, the forecaster can lie by reporting the dictator. This can be inferred from the contour that the point [0.5, 0.5], which corresponds to the precise forecast 0.5, also has the highest expected score. With ρ being a min-max rule, the scoring rule s_{ρ} is proper but not strictly as other imprecise forecasts allow the forecaster to obtain the same expected score. For our implementation we consider $\mathcal{A} = [0,1]$ and $u(a, o) := (o - a)^2$ to satisfy Lemma 4.10.