The following data are from an experiment comparing scores from the same people across 3 different time points before and after a treatment for depression. The means are the results of a depression inventory in which higher score indicate greater levels of depression.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1 month before treatment** | | | | **1 month after treatment** | | | | **6 months after treatment** | | | |  |  |
| **X** | **X2** | **X-M** | **(X-M)2** | **X** | **X2** | **X-M** | **(X-M)2** | **X** | **X2** | **X-M** | **(X-M)2** | P |  |
| 9 | 81 | 1 | 1 | 8 | 64 | 2 | 4 | 4 | 16 | 1 | 1 | 21 |  |
| 9 | 81 | 1 | 1 | 6 | 36 | 0 | 0 | 3 | 9 | 0 | 0 | 18 | *N*=18 |
| 7 | 49 | -1 | 1 | 6 | 36 | 0 | 0 | 3 | 9 | 0 | 0 | 16 | *n*=6 |
| 9 | 81 | 1 | 1 | 6 | 36 | 0 | 0 | 4 | 16 | 1 | 1 | 19 | *k*=3 |
| 6 | 36 | -2 | 4 | 4 | 16 | -2 | 4 | 2 | 4 | -1 | 1 | 12 |  |
| 8 | 64 | 0 | 0 | 6 | 36 | 0 | 0 | 2 | 4 | -1 | 1 | 16 |  |
| *M*1=8 |  |  | *SS1=8* | *M*2=6 |  |  | *SS2=8* | *M*3=3 |  |  | *SS3=4* |  |  |

G =102

∑ X2 = 674

REPEATED MEASURES ANOVA

Test whether there are any significant differences among the means/groups (α = .01)

Step 1. Define hypotheses

H0: μ1 = μ2 = μ3

H1: At least one μ differs from rest

Step 2. Find the critical value

DF total = N – 1 = 18 – 1 = 17

DF between treatments (numerator) = k-1 = 3-1 = 2

DF within = N – k = 18 – 3 = 15

DF between subjects = n -1 = 6 – 1 = 5

DF error (denominator) = DF within – DF between subjects = 15 – 5 = 10

Look up in F Table with DF numerator = 2, DF denominator = 10

Fcrit for α = .01 = 7.56

Step 3. Calculate the test statistic (F-statistic)

*Source SS df MS F\_\_\_\_*

Between 76 2 38 95

Treatments

Within 20 15

Between

Subject 16 5

Error 4 10 0.4\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Total 96 17

SS total = ∑ X2 – (G2/N) = 674 – (1022/18) = 674 – 578 = 96

SS within = SS1 + SS2 + SS3 = 20

SS between treatments = SS total – SS within = 76

SS between subject = ∑ P2/k – (G2/N) = (212/3) + (182/3) + (162/3) + (192/3) + (122/3) + (162/3) – (1022/18) = 16

SS error =SS within – SS between subjects = 20 - 16 = 4

MS between treatments =SS between treatments / DF between treatments = 76 / 2 = 38

MS error =SS error /DF error = 4 / 10 = 0.4

F=MS between treatments / MS error = 38 / 0.4 = 95

Step 3.5.1 Calculate the Effect Size

η2= SS between treatments / (SS between treatments + SS error) = 76 / (76 + 4) = 0.95

Step 3.5.2 Post-Hoc

Tukey’s HSD = q √(MSerror / n) q = 5.27

= 5.27 √(0.4/6) = 1.36

M1-M3 = 8 – 3 = 5 yes

M1-M2 = 8 – 6 = 2 yes

M2-M3 = 6 – 3 = 3 yes

Step 4. Make a decision and write a conclusion statement

Reject the null hypothesis (95 is greater than our critical value of 7.56)

There is evidence that depression symptoms were different between 1 month before (M1=8), 1 month after (M2=6), and 6 months after treatment (M3=3), F(2,10)=95, p<0.01. Post hoc tests using Tukey’s HSD revealed that the number of symptoms 1 month before treatment was significantly greater than both 1 month after treatment and 6 months after treatment. 6 months after treatment also had significantly less symptoms than 1 month after treatment, p <0.01.

The following data are from an experiment comparing 3 treatment groups on flu symptoms (higher numbers mean more symptoms). There was a separate sample of *n*=4 in each treatment. Below are the data.

|  |  |  |  |
| --- | --- | --- | --- |
| **Treatment Group 1** | **Treatment Group 2** | **Treatment Group 3** | *G*=60 ∑*x*2=344 |
| *M*1=4 | *M*2=5 | *M*3=6 | *N*=12 |
| *SS*1=16 | *SS*2=10 | *SS*3=10 | *k*=3 |

ONE-WAY ANOVA

Test whether there are any significant differences among the 3 treatments above.

Step 1: State the hypotheses (H0 and H1)

H0: H0: μ1 = μ2 = μ3

HA: At least one μ differs from rest

Step 2: Find the critical value/region for an α = .01

*df* total = N – 1 = 12 – 1 = 11

*df* between = k – 1 = (3-1) = 2

*df* within = N – k = (12-3) = 9

Critical value from F table (top number) = **8.02**

Step 3: Calculate the test statistic (F-statistic)

*Source SS df MS F\_\_\_\_*

Between **8 2 4 1**

Within\_\_\_\_\_\_**36\_\_\_\_\_\_\_\_\_\_9\_\_\_\_\_\_\_\_\_\_\_\_4**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Total **44 11**

SS between = SS total ***–*** SS within 🡪 44-36 =8

SS within = SS1+SS2+SS3….🡪 16 + 10 + 10 = 36

SS total = sum of x2 – (G)2/N 🡪 344 – (60)2/12 = 344 – (3600 / 12) = 344 – 300 = **44**

*MSbetween = SS between/Df between = 8* / 2 = **4**

*MSwithin=SS within/Df within =* 36 / 9 = **4**

F = MSbetween/MSwithin = 4 / 4 = **1**

Step 3.5.1 Calculate the Effect Size

ηp2= SS between treatments / SS total = 8 / 44 = 0.18

Step 3.5.2 Calculate Post-hoc tests

Should not calculate Tukey’s HSD because we retain.

Step 4: Make a decision and write a conclusion statement.

Retain the null hypothesis (1 is less than our critical value of 8.02)

There is no difference in flu symptoms for Treatment 1 (M=4), Treatment2 (M=5), and Treatment 3 (M=6), F(2,9)=1, p > 0.01.

Imagine you are a researcher working for the Smithsonian Museum in Washington D.C. You’ve recently launched a new exhibit about food from around the world and want to know if the exhibit is being enjoyed by all patrons equally. To test this, you’ve polled patrons as the entered the food exhibit to see if they’ve already eaten lunch and to record their gender. Below you will find the amount of time spent in the exhibit (in minutes), separated by gender and whether or not they have had lunch. You’re interested in knowing if these variables, or an interaction of these variables, affect the amount of time patrons spend in the exhibit.

|  |  |  |
| --- | --- | --- |
|  | Male | Female |
| Lunch | 14 | 16 |
| 14 | 17 |
| 18 | 15 |
| 17 | 15 |
| 19 | 14 |
|  | | |
| No Lunch | 15 | 20 |
| 18 | 18 |
| 20 | 15 |
| 22 | 15 |
| 22 | 24 |

Test whether there are any significant differences or interactions among the 2 factors above.

Step 1: State the hypotheses

* **Factor A**
  + **H0: μA1=μA2**
  + **H1: μA1≠μA2**
* **Factor B**
  + **H0: μB1=μB2**
  + **H1: μB1≠μB2**
* **Interaction**
  + **H0: The effect of either factor does not depend on the levels of the other factor**
  + **H1: The effect of either factor depends on the levels of the other factor**

Step 2: Find the critical value/region for an α = .05

DF total = N – 1 = 20 – 1 = 19

DF within = sum of df for each treatment = (5-1)+(5-1)+(5-1)+(5-1) = 16

DF between = DF total – DF within = 19 – 16 = 3

DF A = # rows – 1 = 2 – 1 = 1

DF B = # columns – 1 = 2 – 1 = 1

DF AxB = DF between – DF A – DF B = 3 – 1 – 1 = 1

Use df for Factor A, Factor B, and Interaction as your numerator. Use DF within as denominator.

F crit A = 4.49

F crit B = 4.49

F crit AxB = 4.49

Step 3: Calculate the test statistic (F-statistic)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *Source* | *SS* | *Df* | *MS* | *F* |
| *Between Treatments* | *50* | *3* |  |  |
| *Factor A* | *45* | *1* | *45* | *6.06* |
| *Factor B* | *5* | *1* | *5* | *.67* |
| *Interaction (A x B)* | *0* | *1* | *0* | *0* |
| *Within* | *118.8* | *16* | *7.43* |  |
| *Total* | *168.8* | *19* |  |  |

SS total = sum of x2 – (G)2/N = 6224 – (348)2 / 20 = 168.8

SS within = SS1 + SS2 + SS3 + SS4 = 21.2 + 5.2 + 35.2 + 57.2 = 118.8

SS between treatments = SS total – SS within = 168.8 – 118.8 = 50

SS A = sum (T2row / n row) – (G)2/N = (1592 / 10) + (1892 / 10) – (348)2 / 20 = 45

SS B = sum (T2column / n column) – (G)2/N = (1792 / 10) + (1692 / 10) – (348)2 / 20 = 5

SS AxB = SS between treatments – SS A – SS B = 50 – 45 – 5 = 0

MS A = SS A / df A = 45 / 1 = 45

MS B = SS B / df B = 5 / 1 = 5

MS AxB = SS AxB / df AxB = 0 / 1 = 0

MS With = SS with / df with = 118.8 / 16 = 7.425 = 7.43

F A = MS A / MS with = 45 / 7.43 = 6.06

F B = MS B / MS with = 5 / 7.43 = 0.67

F AxB = MS AxB / MS with = 0 / 7.43 = 0

Step 3.5 Calculate the effect size for each component of the ANOVA

ηp2 A = SS A / (SS A + SS within) = 45 / (45 + 118.8) = .2747

ηp2 B = SS B / (SS B + SS within) = 5 / (5 + 118.8) = .04038

ηp2 AxB = SS AxB / (SS AxB + SS within) = 0 / (0 + 118.8) = 0

Step 4: Make a decision and write a conclusion statement.

There was a significant main effect of eating on participants’ time spent in the exhibit, *F*(1,16) = 6.06, *p* < 0.05, ηp2= 0.27. The main effect of gender, *F*(1,16) = 0.67, *p* > 0.05, ηp2 = 0.04, and the interaction, *F*(1,16) = 0, *p* > 0.05, ηp2= 0, were not significant.

**Don’t forget to study for the Theoretical Questions!**

Be sure to review the questions form the homework assignments and the information from the top of the study guide.

A researcher conducts a study investigating the effects of product packaging on purchase behavior. Specifically, the researcher is interested in the effects of package color (blue, red, green, or yellow), package size (small, medium, and large), and package material (plastic, cardboard). How many factors was the researcher interested in? How many are in each factor? How would you communicate this information? (i.e. we talked about a 2 x 2 ANOVA in class…what would this study be?)

This would be a 4 x 3 x 2 ANOVA. There are three factors (package color, package size, and package material). The package color factor has 4 levels (blue, red, green, or yellow), the package size factor has 3 levels (small, medium, and large), and the package material factor has 2 levels (plastic, cardboard).

Describe the components of the F-distribution we discussed in class.

1. **Represents the ratio of two variance estimates**
2. **Denominator of F is called the “error term”**
3. **When there is no treatment effect, F = 1. When there is a treatment effect, F is > 1.**
4. **Shape of the distribution changes with df**
5. **F-values are always positive**
6. **Distribution of F piles up around 1.**

**Make sure you understand the “Why?” behind each of the points above.**