1.	A car traveled 281 miles in 4 hours 41 minutes. What was the average speed of the car in miles per hour?
2.	In a group of 120 people, 90 have an age of more 30 years, and the others have an age of less than 20 years. If a person is selected at random from this group, what is the probability the person's age is less than 20?
3.	The length of a rectangle is four times its width. If the area is 100 m2 what is the length of the rectangle?
4.	A six-sided die is rolled once. What is the probability that the number rolled is an even number greater than 2?
5.	Point A has the coordinates (2,2). What are the coordinates of its image point if it is translated 2 units up and 5 units to the left, and reflected in the x axis?
6.	The length of a rectangle is increased to 2 times its original size and its width is increased to 3 times its original size. If the area of the new rectangle is equal to 1800 square meters, what is the area of the original rectangle?
7.	Each dimension of a cube has been increased to twice its original size. If the new cube has a volume of 64,000 cubic centimeters, what is the area of one face of the original cube?
8.	Pump A can fill a tank of water in 5 hours. Pump B can fill the same tank in 8 hours. How long does it take the two pumps working together to fill the tank?(round your answer to the nearest minute).

- 9. A water tank, having the shape of a rectangular prism of base 100 square centimeters, is being filled at the rate of 1 liter per minute. Find the rate at which the height of the water in the water tank increases. Express your answer in centimeters per minute.
- 10. Dany bought a total of 20 game cards some of which cost \$0.25 each and some of which cost \$0.15 each. If Dany spent \$4.2 to buy these cards, how many cards of each type did he buy?
- 11. The size of the perimeter of the square ABCD is equal to 100 cm. The length of the segment MN is equal to 5 cm and the triangle MNC is isosceles. Find the area of the pentagon ABNMC.

12. Water is being pumped, at a constant rate, into an underground storage tank that has the shape of a rectangular prism. Which of the graphs below best represent the changes in the height of water in the tank as a function of the time?

13. Initially the rectangular prism on the left was full of water. Then water was poured in the right cylindrical container so that the heights of water in both containers are equal. Find the height h of water in both containers. (round your answer to the nearest tenth of a cm).

14. Peter drove at a constant speed for 2 hours. He then stopped for an hour to do some shopping and have a rest and then drove back home driving at a constant speed. Which graph best represents the changes in the distance from home as Peter was driving?

15.	Two balls A and B rotate along a circular track. Ball A makes 2 full rotations in 26 minutes. Ball B makes 5 full rotation in 35 minutes. If they start rotating now from the same point, when will they be at the same starting point again?
16.	In a certain college, 40% of the senior class students is taking Physics, 30% is taking calculus and 10% is taking both. If 40 students are enrolled in the senior class, how many students are taking neither Physics nor calculus?
17.	Joe drove at the speed of 45 miles per hour for a certain distance. He then drove at the speed of 55 miles per hour for the same distance. What is the average speed for the whole trip?
18.	If the radius of a cylindrical container is doubled, how do you change the height of the container so that the volume will stay the same?

19.	One leg of a right triangle is 18 cm and its area is 108 square cm. Find its primeter.
20.	What is the sum of the sizes of the interior angles of a polygon with 53 sides?
21.	Jack is taller than Sarah but shorter than both Malika and Tania. Malika is shorter than tania. Natasha is shorter than Sarah. Who is the shortest?
22.	What is the height (one of the legs) and the hypotenuse of an isosceles right triangle that has an area of 800 square feet?
23.	Find the circumference of a circle inscribed inside a square with a side of 20 meters.
24.	Two different schools (A and B) have the same number of pupils. The ratio of the boys in school A and the boys in school B is 2:1 and the ratio of the girls in school A and the girls in school B is 4:5. Find the ratio of the boys in school A to the girls in school A.
25.	A water tank has the shape of a rectangular prism of base 50 cm ² . This tank is being filled at the rate of 12 liters per minutes. Find the rate at which the height of the water in the water tank increases; express your answer in millimeters per second.

Answers to the Above Questions

- 60 miles per hour
 0.25
- 3. 20 meters

5. 6. 7. 9. 111 131 141 151 171 171 171 171 171 171 171 171 17	1/3 (-3,-4) 300 square meters 400 square cm 3 hours and 5 minutes 10 cm per minute .12 cards at \$0.25 and 8 cards at \$0.15 .618.75 square cm .graph at the bottom left .7.2 cm .graph at the bottom left .After 1 hour and 31 minutes .16 students .49.5 miles per hour .1/4 of the original height .51.6 cm .9180 degrees .Natasha .height (leg) = 40 feet , hypotenuse = 40 sqrt(2) feet .20pi meters .ratio of the boys in school A to the girls in school A is 1:2 .40 mm/second
1.	Two number N and 16 have LCM = 48 and GCF = 8. Find N.
2.	If the area of a circle is 81pi square feet, find its circumference.
3.	Find the greatest common factor of 24, 40 and 60.
4.	In a given school, there 240 boys and 260 girls. a) What is the ratio of the number of girls to the number of boys? b) What is the ratio of the number of boys to the total number of pupils in the school?

5. If Tim had lunch at \$50.50 and he gave 20% tip, how much did he

	spend?
6.	Find \mathbf{k} if $64 \div \mathbf{k} = 4$.
7.	Little John had \$8.50. He spent \$1.25 on sweets and gave to his two friends \$1.20 each. How much money was left?
8.	What is x if $x + 2y = 10$ and $y = 3$?
9.	A telephone company charges initially \$0.50 and then \$0.11 for every minute. Write an expression that gives the cost of a call that lasts N minutes.
10	A car gets 40 kilometers per gallon of gasoline. How many gallons of gasoline would the car need to travel 180 kilometers?
11.	A machine fills 150 bottles of water every 8 minutes. How many minutes it takes this machine to fill 675 bottles?
12.	A car travels at a speed of 65 miles per hour. How far will it travel in 5 hours?
13	A small square of side 2x is cut from the corner of a rectangle with a width of 10 centimeters and length of 20 centimeters. Write an expression

14. A rectangle A with length 10 centimeters and width 5 centimeters is similar to another rectangle B whose length is 30 centimeters. Find the

in terms of x for the area of the remaining shape.

	area of rectangle B.
15.	A school has 10 classes with the same number of students in each class. One day, the weather was bad and many students were absent. 5 classes were half full, 3 classes were 3/4 full and 2 classes were 1/8 empty. A total of 70 students were absent. How many students are in this school when no students are absent?
16.	A large square is made of 16 congruent squares. What is the total number of squares of different sizes are there?
17.	The perimeter of square A is 3 times the perimeter of square B. What is
	the ratio of the area of square A to the area of square B.
18.	John gave half of his stamps to Jim. Jim gave gave half of his stamps to Carla. Carla gave 1/4 of the stamps given to her to Thomas and kept the remaining 12. How many stamps did John start with?
19.	Two balls A and B rotate along a circular track. Ball A makes 4 full rotations in 120 seconds. Ball B makes 3 full rotation in 60 seconds. If they start rotating now from the same point, when will they be at the same starting point again?
20.	A segment is 3 units long. It is divided into 9 parts. What fraction of a unit

21. Mary wants to make a box. She starts with a piece of cardboard whose length is 15 centimeters and width is 10 centimeters. Then she cuts 4 congruent squares with sides of 3 centimeters at the four corners and folded at the broken lines to make the box. What is the volume of the box?

22. A car is traveling 75 kilometers per hour. How many meters does the car travel in one minute?

23. Carla is 5 years old and Jim is 13 years younger than Peter. One year ago, Peter's age was twice the sum of Carla's & Jim's age. Find the present age of each one of them.

24. Linda spent 3/4 of her savings on furniture. She then spent 1/2 of her remaining savings on a fridge. If the fridge cost her \$150, what were her original savings?

Answers to the Above Questions

- 1. 24
- 2. 9 feet
- 3. 4
- 4. a) 13:12 b)12:25
- 5. \$60.60
- 6. 16
- 7. 4.85
- 8. 4
- 9. 0.50 + N * 0.11
- 10. 4.5 gallons
- 11. 36 minutes
- 12. 325 miles
- 13. $200 4x^2$
- 14. 450 centimeters squared
- 15. 200 pupils
- 16. 30
- 17. 9:1
- 18. 64 stamps
- 19. 60 seconds
- 20. 2/3
- 21. 108 cubic centimeters
- 22. 1250 meters/minute
- 23. Carla:5 years, Jim: 6 years, Peter: 19 years.
- 24. \$1200

1. 30% of 30 =

- A. 900
- B. 9%
- C. 9
- D. 90,000

150% of 60 =

A. 9000

B. 9

C. 900

D. 90

3.

1/4 =

A. 4%

B. 1%

C. 0.25%

D. 25%

4.

0.05 =

A. 50%

B. 500%

C. 5%

D. 0.5%

5.	If 100% of a number is 15, what is 50% of the number? A. 7.5 B. 50 C. 5000 D. 50
6.	If 10% of a number is 7, what is 80% of the number? A. 70 B. 56 C. 5600 D. 0.7
7.	Which is the greatest? A. 90% of 10 B. 6% of 1000 C. 5% of 1400 D. 3% of 2500

8.	The original price of a toy was \$15. If the price is reduced by 20%, what is the new price of the toy?
	A. \$12 B. \$17 C. \$14.80 D. \$5
9.	George bought a car at \$5000 and sold it at \$5500. What benefit, in percent, did he make?
	A. 500% B. 10% C. 5000% D. 5%
10	If 20% of n is equal to 40, what is n ?
	A. 200 B. 2000 C. 800 D. 80

4	4	
7	7	

The price of a T-shirt was \$20. It was first increased by 20%. They did not sell well the shop owner decreased the price by 20%. What is the new price of the T-shirt?

- A. \$20
- B. \$22
- C. \$21
- D. \$19.20

12.

What percent of 1 hour is 15 minutes?

- A. 50%
- B. 15%
- C. 75%
- D. 25%

Answers to the Above Questions

- 1. C
- 2. D
- 3. D
- 4. C
- 5. A
- 6. B
- 7. D
- 8. A
- 9. B
- 10. A

- 11. D
- 12. D

Algebra Questions with Answers for Grade 10

Grade 10 algebra questions with <u>answers</u> are presented.

- 1. Which real numbers are equal to their cubes?
- 2. Write 4*10⁻² as a decimal.
- 3. Write 0.12*10⁻³ as a decimal.
- 4. Write $2 \log_3 x + \log_3 5$ as a single logarithmic expression.
- 5. Factor the algebraic expression $6x^2$ 21xy + 8xz 28yz.
- 6. Factor the algebraic expression $(x 1)^2 (y 2)^2$.
- 7. Factor the algebraic expression $x^2 z^4$.
- 8. Evaluate the algebraic expression |-2x y + 3| for x = 3 and y = 5
- 9. Simplify the algebraic expression -2(x 3) + 4(-2x + 8)

10. Expand and simplify the algebraic expression (x + 3)(x - 3) - (-x - 9)

11. Which property is used to write a(x + y) = ax + ay

12. Simplify 8 $x^3 / 2 x^{-3}$

13. Simplify $(-a^2b^3)^2(c^2)^0$

14. For what value of **k** is the point (-2, k) on the line with equation -3x + 3y = 4?

15. For what value of **a** will the system given below have no solutions?

$$2x + 6y = -2$$

 $-3x + ay = 4$

16. Which equation best describes the relationship between x and y in this table?

X	y
0	-4
4	-20
-4	12
8	-36

A. y = -x/4 - 4

B. y = -x/4 + 4

C. y = -4x - 4

D. y = -4x + 4

17. Which equation best represents the area of the rectangle below?

.

A. area =
$$2(x+1) + 2(x-1)$$

B. area =
$$4(x+1)(x-1)$$

C. area =
$$2x^2$$

D. area =
$$x^2 - 1$$

18. Which line given by its equation below contains the points (1, -1) and (3, 5)?

A.
$$-2y - 6x = 0$$

B.
$$2y = 6x - 8$$

C.
$$y = 3x + 4$$

D.
$$y = -3x + 4$$

19. Solve the equation 2|3x - 2| - 3 = 7.

20. Solve for x the equation $(1/2)x^2 + mx - 2 = 0$.

21. For what values of **k** the equation $-x^2 + 2kx - 4 = 0$ has one real solution?

- 22. For what values of **b** the equation $x^2 4x + 4b = 0$ has two real solutions?
- 23. Function f is described by the equation $f(x) = -x^2 + 7$. What is the set of values of f(x) corresponding to the set for the independent variable x given by $\{1, 5, 7, 12\}$?
- 24. Find the length and width of a rectangle whose perimeter is equal to 160 cm and its length is equal to triple its width.
- 25. Simplify: |-x| + |3x| |-2x| + 3|x|
- 26. If $(x^2 y^2) = 10$ and (x + y) = 2, find x and y.

Answers to the Above Questions

- 1. 0, 1, -1 are all equal to their respective cubes.
- 2. 0.04
- 3. 0.00012
- 4. = $\log_3 x^2 + \log_3 5 = \log_3 (5x^2)$
- 5. = 3x(2x 7y) + 4z(2x 7y) = (2x 7y)(3x + 4z)
- 6. = [(x-1)-(y-2)][(x-1)+(y-2)] = (x-y+1)(x+y-3)
- 7. = $(x + z^2)(x z^2) = (x + z^2)(x + z)(x z)$
- 8. = |-2(3) (5) + 3| = |-8| = 8
- 9. = -2x + 6 8x + 32 = -10x + 38
- $10. = x^2 9 + x + 9 = x^2 + x$
- 11. distributivity
- $12.4 x^6$

```
13. a^4b^6

14. k = -2/3

15. a = -9

16. C. y = -4x - 4

17. D. area = (x + 1)(x - 1) = x^2 - 1

18. B. 2y = 6x - 8

19. solution set : \{7/3, -1\}

20. solution set : \{-m + sqrt(m^2 + 4), -m - sqrt(m^2 + 4)\}

21. k = 2, k = -2

22. all values of b less than 1

23. \{6, -18, -42, -137\}

24. width = 20 cm and length = 60 cm

25. |-x| + |3x| - |-2x| + 3|x| = 5|x|
```

Math Word Problems with Solutions and Answers for Grade 10

Grade 10 math word problems with answers and solutions are presented.

- 1. A real estate agent received a 6% commission on the selling price of a house. If his commission was \$8,880, what was the selling price of the house?
- 2. An electric motor makes 3,000 revolutions per minutes. How many degrees does it rotate in one second?
- 3. The area of a rectangular field is equal to 300 square meters. Its perimeter is equal to 70 meters. Find the length and witdh of this rectangle.

•	The area of the trapezoid shown below is equal to 270 square units. Find its perimeter and round your answer to the nearest unit.
	If a tire rotates at 400 revolutions per minute when the car is travelling 72km/h, what is the circumference of the tire?
	In a shop, the cost of 4 shirts, 4 pairs of trousers and 2 hats is \$560. The cost of 9 shirts, 9 pairs of trousers and 6 hats is \$1,290. What is the total cost of 1 shirt, 1 pair of trousers and 1 hat?
	Four children have small toys. The first child has 1/10 of the toys, the second child has 12 more toys than the first, the third child has one more toy of what the first child has and the fourth child has double the third child. How many toys are there?
	A class average mark in an exam is 70. The average of students who scored below 60 is 50. The average of students who scored 60 or more is 75. If the total number of students in this class is 20, how many students scored below 60?

9. For what value of x will the function f(x) = -3(x - 10)(x - 4) have a maximum value? Find the maximum value. 10. Evaluate: (1 - 1/10)(1 - 1/11)(1 - 1/12)...(1 - 1/99)(1 - 1/100) 11. It takes a boat 3 hours to travel down a river from point A to point B, and 5 hours to travel up the river from B to A. How long would it take the same boat to go from A to B in still water? 12. An airplane flies against the wind from A to B in 8 hours. The same airplane returns from B to A, in the same direction as the wind, in 7 hours. Find the ratio of the speed of the airplane (in still air) to the speed of the wind. Solutions to the Above Problems 1. A. 6% x = 8,880: x = selling price of house.B. x = \$148,000: solve for x. 2. A. 3000 revolutions / minute = 3000*360 degrees / 60 seconds = 18,000 degrees / second 3. A. L * W = 300: area, L is the length and W is the width. B. 2 L + 2 W = 70: perimeter C. L = 35 - w: solve for L D. (35 - W) * W = 300: substitute in the area equation E. W = 15 and L = 20: solve for W and find L using L = 35 - w.

- A. Let h be the height of the trapezoid.
- B. area = (1/2) * h * (10 + 10 + 3 + 4) = 270
- C. h = 20: solve for h
- D. $20^2 + 3^2 = L^2$: Pythagora's theorem applied to the right triangle on the left.
- E. L = sqrt(409)
- F. $20^2 + 4^2 = R^2$: Pythagora's theorem applied to the right triangle on the right.
- G. R = sqrt(416)
- H. perimeter = sqrt(409) + 10 + sqrt(416) + 17 = 27 + sqrt(409) + sqrt(416)

- A. 400 rev / minute = 400 * 60 rev / 60 minutes = 24,000 rev / hour
- B. 24,000 * C = 72,000 m : C is the circumference
- C. C = 3 meters

6.

- A. Let x be the price of one shirt, y be the price of one pair of trousers and z be the price of one hat.
- B. 4x + 4y + 2z = 560:
- C. 9x + 9y + 6z = 1,290
- D. 3x + 3y + 2z = 430: divide all terms of equation C by 3
- E. x + y = 130: subtract equation D from equation B
- F. 3(x + y) + 2z = 430: equation D with factored terms.
- G. 3*130 + 2z = 430
- H. z = 20: solve for z
- I. x + y + z = 130 + 20 = \$150

- A. x: the total number of toys
- B. x/10: the number of toys for first child
- C. x/10 + 12: the number of toys for second child
- D. x/10 + 1: the number of toys for the third child
- E. 2(x/10 + 1): the number of toys for the fourth child
- F. x/10 + x/10 + 12 + x/10 + 1 + 2(x/10 + 1) = x
- G. x = 30 toys : solve for x

- A. Let n the number of students who scored below 60 and N the number of students who scored 60 or more. Xi the grades below 60 and Yi the grades 60 or above.
- B. [sum(Xi) + sum(Yi)] / 20 = 70: class average
- C. sum(Xi) / n = 50: average for less that 60
- D. sum(Yi) / N = 75: average for 60 or more
- E. 50n + 75N = 1400: combine the above equations
- F. n + N = 20: total number of students
- G. n = 4 and N = 16: solve the above system

9.

- A. $f(x) = -3(x 10)(x 4) = -3x^2 + 42x 120$: expand and obtain a quadratic function
- B. h = -b/2a = -42/-6 = 7: h is the value of x for which f has a maximum value
- C. f(h) = f(7) = 27: maximum value of f.

10.

- A. (1 1/10)(1 1/11)(1 1/12)...(1 1/99)(1 1/100)
- B. = (9/10)(10/11)(11/12)...(98/99)(99/100)
- C. = 9/100: simplify

11.

- A. Let: S be the speed of the boat in still water, r be the rate of the water current and d the distance between A and B.
- B. d = 3(S + r): boat traveling down river
- C. d = 5(S r): boat traveling up river
- D. 3(S + r) = 5(S r)
- E. r = S / 4: solve above equation for r
- F. d = 3(S + S/4): substitute r by S/4 in equation B
- G. d/S = 3.75 hours = 3 hours and 45 minutes.

12.

A. Let: S be the speed of the airplane in still air, r be the speed of the wind and d the distance between A and B.

- B. d = 8(S r): airplane flies against the wind
- C. d = 7(S + r): airplane flies in the same direction as the wind
- D. 8(S r) = 7(S + r)
- E. S/r = 15

Algebra Questions with Solutions and Answers for Grade 11

Grade 11 math algebra questions with answers and <u>solutions</u> are presented.

1. Complete the square in the quadratic function f given by

$$f(x) = 2x^2 - 6x + 4$$

- 2. Find the point(s) of intersection of the parabola with equation $y = x^2 5x + 4$ and the line with equation y = 2x 2
- 3. Find the constant k so that : $-x^2 (k + 7)x + 8 = -(x 2)(x 4)$
- 4. Find the center and radius of the circle with equation $x^2 + y^2 2x + 4y 11 = 0$
- 5. Find the constant k so that the quadratic equation $2x^2 + 5x k = 0$ has two real solutions.
- 6. Find the constant k so that the system of the two equations: 2x + ky = 2 and 5x 3y = 7 has no solutions.

- 7. Factor the expression $6x^2 13x + 5$
- 8. Simplify i^{231} where i is the imaginary unit and is defined as: i = sqrt(-1).
- 9. What is the remainder when $f(x) = (x 2)^{54}$ is divided by x 1?
- 10. Find b and c so that the parabola with equation $y = 4x^2 bx c$ has a vertex at (2, 4)?
- 11. Find all zeros of the polynomial $P(x) = x^3 3x^2 10x + 24$ knowing that x = 2 is a zero of the polynomial.
- 12. If x is an integer, what is the greatest value of x which satisfies 5 < 2x + 2 < 9?
- 13. Sets A and B are given by: $A = \{2, 3, 6, 8, 10\}$, $B = \{3, 5, 7, 9\}$.
 - a) Find the intersection of sets A and B.
 - b) Find the union of sets A and B.
- 14. Simplify $|-x^2 + 4x 4|$.
- 15. Find the constant k so that the line with equation y = kx is tangent to the circle with equation $(x 3)^2 + (y 5)^2 = 4$.

Solutions to the Above Problems

A.
$$f(x) = 2(x^2 - 3x) + 4$$
: factor 2 out in the first two terms

=
$$2(x^2 - 3x + (-3/2)^2 - (-3/2)^2) + 4$$
: add and subtract $(-3/2)^2$
= $2(x - 3/2))^2 + 17/2$: complete square and group like terms

A.
$$2x - 2 = x^2 - 5x + 4$$
: substitute y by $2x - 2$

B.
$$x = 1$$
 and $x = 6$: solution of quadratic equation

3.

A.
$$-x^2 - (k + 7)x + 8 = -(x - 2)(x - 4)$$
: given

B.
$$-x^2 - (k + 7)x + 8 = -x^2 + 6x + 8$$

k + 7 = 6: two polynomials are equal if their corresponding coefficients are equal.

C. k = -1: solve the above for k

4.

A.
$$x^2 - 2x + y^2 + 4y = 11$$
: Put terms in x together and terms in y together

B.
$$(x-1)^2 + (y+2)^2 - 1 - 4 = 11$$

B.
$$(x-1)^2 + (y+2)^2 - 1 - 4 = 11$$

C. $(x-1)^2 + (y+2)^2 = 4^2$: write equation of circle in standard form

5.

A.
$$2x^2 + 5x - k = 0$$
: given

B. discriminant =
$$25 - 4(2)(-k) = 25 + 8k$$

- C. 25 + 8k > 0: quadratic equations has 2 real solutions when discriminant is positive
- D. k > -25/8

A. Determinant =
$$-6 - 5k$$

- B. -6 5k = 0: when determinant is equal to zero (and equations independent) the system has no solution
- C. k = -6/5: solve for k

A.
$$6x^2 - 13x + 5 = (3x - 5)(2x - 1)$$

8.

- A. Note that i < 4 > = 1
- B. Note also that 231 = 4 * 57 + 3C. Hence $i^{231} = (i^4)^{57} * i^3$ D. $= 1^{57} * -i = -i$

9.

A. remainder =
$$f(1) = (1 - 2)^{54} = 1$$
: remainder theorem

10.

- A. h = b / 8 = 2: formula for x coordinate of vertex
- B. b = 16: solve for b
- C. y = 4 for x = 2: the vertex point is a solution to the equation of the parabola
- D. $4(2)^2 16(2) c = 4$
- E. c = -20: solve for c

11.

- A. divide P(x) by (x 2) to obtain $x^2 x + 12$
- B. $P(x) = (x^2 x + 12)(x 2)$
- C. = (x 4)(x + 3)(x 2): factor the quadratic term
- D. the zeros are: 4, -3 and 2

- A. 5 < 2x + 2 < 9: given
- B. 3/2 < x < 7/2
- C. the greatest integer value of is 3 (the integer less than 7/2)

- A. A intersection $B = \{3\}$: common element to both A and B is 3
- B. A union $B = \{2, 3, 6, 8, 10, 5, 7, 9\}$: all elements of A and B are in the union. Elements common to both A and B are listed once only since it is a set.

A.
$$|-x^2 + 4x - 4|$$
: given

B. =
$$|-(x^2 + 4x - 4)|$$

C. =
$$|-(x-2)^2|$$

D. =
$$(x - 2)^2$$

15.

A.
$$(x-3)^2 + (y-5)^2 = 4$$
: given

B.
$$(x - 3)^2 + (kx - 5)^2 = 4$$
: substitute y by kx

- C. $x^2(1 + k^2) x(6 + 10k) + 21 = 0$: expand and write quadratic equation in standard form.
- D. $(6 + 10k)^2 4(1 + k^2)(21) = 0$: For the circle and the line y = kx to be tangent, the discriminant of the above quadratic equation must be equal to zero.
- E. $16k^2 + 120k 48 = 0$: expand above equation
- F. k = (-15 + sqrt(273)/4, k = (-15 sqrt(273)/4 : solve the abovequadratic equation.

Algebra Questions with Answers and **Solutions - Grade 12**

Grade 12 algebra questions with answers and solutions are presented.

- 1. Order from greatest to least
 - a) 25¹⁰⁰
 - b) 2³⁰⁰
 - c) 3^{400}
 - d) 4²⁰⁰
 - e) 2⁶⁰⁰

2. Find all rational zeros of $P(x) = x^3 - 7x + 6$.

3. Round all real zeros in the graph to the nearest integer and find a polynomial function P of lowest degree, with the absolute value of the leading coefficient equal to 1, that has the indicated graph.

4. 2 - i, where i is the imaginary unit, is a zero of $P(x) = x^4 - 4x^3 + 3x^2 + 8x - 10$. Find all zeros of P.

5. Find a, b and c so that the graph of the quadratic function $f(x) = ax^2 + bx + c$ has a vertex at (-2, 1) and passes through the point (0, -3).

6. f(x) is a quadratic function such that f(1) = 3 and f(5) = 3. Find the x coordinate of the vertex of the graph of f.

- 7. Find a and b so that the rational function $f(x) = (ax^4 + bx^3 + 3) / (x^3 2)$ has an oblique asymptote given by y = 2x 3
- 8. Solve for x the equation $log_9(x^3) = log_2(8)$
- 9. Find the value of $log_y(x^4)$ if $log_x(y^3) = 2$
- 10. Solve for x the equation $log_x(8e^3) = 3$
- 11. If $8^x + 8^{x-1} = 10$, find 2^{2x} .
- 12. If $a^2 b^2 = 8$ and $a^*b = 2$, find $a^4 + b^4$.
- 13. What are the maximum value and minimum values of $f(x) = |2\sin(2x pi/3) 5| + 3$
- 14. If x < -7, simplify |4 |3 + x||
- 15. A car travels from A to B at an average speed of 50 km/hour. At what average speed would it have to travel from B to A to average 60 km/hour for the whole trip?
- 16. If $x^2 y^2 = -12$ and x + y = 6, find x and y.
- 17. f(x) is a function such that f(x) + 3f(8 x) = x for all real numbers x. Find the value of f(2).

- 18. f(x) is a function such that f(2x + 1) = 2f(x) + 1 for all real numbers x and f(0) = 2. Find the value of f(3).
- 19. Find b so that the line y = 2x + b is tangent to the circle $x^2 + y^2 = 4$.
- 20. What is the remainder of the division $(x^{100} x^{99} x + 1) / (x^2 3x + 2)$
- 21. Evaluate the number represented by the infinite series sqrt(1/3 + sqrt(1/3 + sqrt(1/3 + ...)).
- 22. Show that the 3 by 3 system of equations given below has no solutions.

$$2x + y - 3z = 5$$

$$-5x + 3y + 2z = 7$$

$$3x - 4y + z = 8$$

Solutions to the Above Problems

1.

A.
$$25^{100}$$

$$\mathbf{P} = \mathbf{2}^{300} - (\mathbf{2}^{3})^{100} - \mathbf{2}^{100}$$

C
$$3^{400} - (2^4)^{100} - 91^{100}$$

B.
$$2^{300} = (2^3)^{100} = 8^{100}$$

C. $3^{400} = (3^4)^{100} = 81^{100}$
D. $4^{200} = (4^2)^{100} = 16^{100}$

E.
$$2^{600} = (2^6)^{100} = 64^{100}$$

F. from greatest to least: 2^{600} , 3^{400} , 25^{100} , 4^{200} , 2^{300}

A.
$$P(x) = x^3 - 7x + 6$$
: given leading coefficient 1 and its factors are : +1,-1

- B. constant term is 6 and its factors are: +1,-1,+2,-2,+3,-3,+6,-6
- C. possible rational zeros: +1,-1,+2,-2,+3,-3,+6,-6

D. test:
$$P(1) = 0$$
, $P(2) = 0$ and $P(-3) = 0$

E.
$$x = 1$$
, $x = 2$ and $x = -3$ are the zeros of $P(x)$.

- A. x = -3 is a zero of multiplicity 2, x = 0 is a zero of multiplicity 1 and x = 2 is a zero of multiplicity 2.
- B. $P(x) = -x(x + 3)^2(x 2)^2$: polynomial with real zeros hence with lowest degree.

- A. if 2 i is a zero and the coefficients of the polynomial are real then 2 + i (the conjugate) is also a solution.
- B. $P(x) = (x (2 i))(x (2 + i))*q(x) = ((x 2)^2 + 1)*q(x)$
- C. $q(x) = P(x)/((x-2)^2 + 1) = (x^2 2)$
- D. x = 2 i, x = 2 + i, x = sqrt(2) and x = -sqrt(2) are the 4 zeros of P(x).

5.

- A. $f(x) = a(x + 2)^2 + 1$: equation of parabola in vertex form
- B. f(0) = -3 = 4a + 1
- C. a = -1: solve for a
- D. $f(x) = -(x + 2)^2 + 1 = -x^2 4x 3$
- E. a = -1, b = -4 and c = -3: identify coefficients

6.

- A. $f(x) = ax^2 + bx + c$
- B. f(1) = 3 which give 3 = a + b + c
- C. f(5) = 3 which gives 3 = 25a + 5b + c
- D. 24a + 4b = 0: subtract equation B from equation C
- E. x coordinate of vertex = -b/2a = 3: from above equation

- A. The oblique asymptote is the quotient resulting from the long division of $ax^4 + bx^3 + 3 by x^3 2$
- B. The quotient obtained is ax + b
- C. ax + b = 2x 3
- D. a = 2 and b = -3: for two polynomials to be equal, the corresponding coefficients has to be equal.

A.
$$\log_9(x^3) = \log_2(8)$$
 : given

B. $log_2(2^3) = 3$: simplify right hand side of given equation.

C. $log_9(x^3) = 3$: rewrite the above equation

D. $\log_9(x^3) = \log_9(9^3)$: rewite 3 as a log base 9.

E. $x^3 = 9^3$: obtain algebraic equation from equation D.

F. x = 9: solve above for x.

9.

A.
$$log_x(y^3) = 2$$
: given

A. $log_x(y^3) = 2$: given B. $x^2 = y^3$: rewrite in exponential form C. $x^4 = y^6$: square both sides

D. $x^4 = y^6$: rewrite the above using the log base y E. $log_y(x^4) = log_y(y^6) = 6$

10.

A.
$$log_x(8e^3) = 3$$
 : given

B.
$$x^3 = 8e^3 = (2e)^3$$

C.
$$x = 2e$$

11.

A.
$$8^x + 8^{x-1} = 10$$
: given

B.
$$4^{2x} + 4^{2x} / 8 = 10$$

C.
$$4^{2x} = 80/9$$
: solve for 4^{2x}

D.
$$4^x = \text{sqrt}(80)/3$$
: extract the square root

E.
$$2^{2x} = 4^x = sqrt(80)/3$$

A.
$$a^2 - b^2 = 8$$
: given

B.
$$a^4 + b^4 - 2a^2b^2 = 8^2$$
: square both sides and expand.

C.
$$a*b = 2$$
: given

D.
$$a^2b^2 = 2^2$$
: square both sides.

E.
$$a^4 + b^4 - 2(4) = 8^2$$
: substitute

F.
$$a^4 + b^4 = 72$$

- A. $-1 \le \sin(2x pi/3) \le 1$: range of a sine function
- B. $-2 \le 2\sin(2x pi/3) \le 2$: mulitply all terms of the double inequality by 2
- C. $-2 5 \le 2\sin(2x pi/3) 5 \le 2 5$: add -5 to all terms of the inequality.
- D. $-7 \le 2\sin(2x pi/3) 5 \le -3$
- E. $3 \le |2\sin(2x pi/3) 5| \le 7$: change the above using absolute value.
- F. $3 + 3 \le |2\sin(2x pi/3) 5| + 3 \le 7 + 3$: add 3 to all terms of the double inequality.
- G. The maximum value of f(x) is equal to 10 and the minimum value of f(x) is equal to 6.

14.

- A. If x < -7 then x < -3 and x + 3 < 0 and |3 + x| = -(3 + x)
- B. |4 |3 + x|| = |4 + 3 + x| = |x + 7| = -x 7: since $x + 7 \le 0$

15.

- A. Let d be the distance bewteen A and B
- B. T1 = d / 50: travel time from A to B
- C. Let S be the speed from B to A
- D. T2 = d/S: travel time from B to A
- E. 60 = 2d/(T1 + T2): average speed for the whole trip
- F. 60 = 2d/(d/50 + d/S): substitute T1 and T2
- G. S = 75 km/hour: solve the above equation for S.

16.

- A. $x^2 y^2 = (x y)(x + y) = -12$: given
- B. 6(x y) = -12: substitute x + y by 6
- C. (x y) = -2: solve for x y
- D. (x y) = -2 and x + y = 6 : 2 by 2 system.
- E. x = 2, y = 4: solve above system.

- A. f(x) + 3f(8 x) = x: given
- B. f(2) + 3f(6) = 2 : x = 2 above
- C. f(6) + 3f(2) = 6: x = 6 above

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D. f(6) = 6 - 3f(2): solve equation C for f(6)
E. f(2) + 3(6 - 3f(2)) = 2: substitute
F. f(2) = 2: solve above equation.
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A. f(2x + 1) = 2f(x) + 1: given
B. f(3) = 2f(1) + 1: x = 1 in A
C. f(1) = 2f(0) + 1: x = 0 in A
D. f(3) = 11: substitute
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19.

A.
$$x^2 + y^2 = 4$$
: given
B. $x^2 + (2x + b)^2 = 4$: substitute y by $2x + b$
C. $5x^2 + 4bx + b^2 - 4 = 0$

- D. The number of points of intersection is given by the number of solutions of the above equation. The line and circle are tangent if the above quadratic equation has only one solution which means that the disciminant is equal to zero. Find the discriminant as a function of b and solve.
- E. b = sqrt(2) and b = -sqrt(2): 2 solutions.

Grade 12 Math Word Problems with Solutions and Answers

Grade 12 math word problems with detailed solutions are presented.

- 1. Two large and 1 small pumps can fill a swimming pool in 4 hours. One large and 3 small pumps can also fill the same swimming pool in 4 hours. How many hours will it take 4 large and 2 small pumps to fill the swimming pool.(We assume that all large pumps are similar and all small pumps are also similar.)
- 2. Find all sides of a right triangle whose perimeter is equal to 60 cm and its area is equal to 150 square cm.

- 3. A circle of center (-3, -2) passes through the points (0, -6) and (a, 0). Find a.
- 4. Find the equation of the tangent at (0, 2) to the circle with equation

$$(x + 2)^2 + (y + 1)^2 = 13$$

- 5. An examination consists of three parts. In part A, a student must answer 2 of 3 questions. In part B, a student must answer 6 of 8 questions and in part C, a student must answer all questions. How many choices of questions does the student have?
- 6. Solve for x

$$x^2 - 3|x - 2| - 4x = -6$$

7. The right triangle ABC shown below is inscribed inside a parabola. Point B is also the maximum point of the parabola (vertex) and point C is the x intercept of the parabola. If the equation of the parabola is given by $y = -x^2 + 4x + C$, find C so that the area of the triangle ABC is equal to 32 square units.

- 8. The triangle bounded by the lines y = 0, y = 2x and y = -0.5x + k, with k positive, is equal to 80 square units. Find k.
- 9. A parabola has two x intercepts at (-2, 0) and (3, 0) and passes through the point (5, 10). Find the equation of this parabola.
- 10. When the polynomial $P(x) = x^3 + 3x^2 2Ax + 3$, where A is a constant, is divided by $x^2 + 1$ we get a remainder equal to -5x. Find A.
- 11. When divided by x 1, the polynomial $P(x) = x^5 + 2x^3 + Ax + B$, where A and B are constants, the remainder is equal to 2. When P(x) is divided by x + 3, the remainder is equal -314. Find A and B.
- 12. Find all points of intersections of the 2 circles defined by the equations

$$(x-2)^2 + (y-2)^2 = 4$$

$$(x-1)^2 + (y-1)^2 = 4$$

- 13. If 200 is added to a positive integer I, the result is a square number. If 276 is added to to the same integer I, another square number is obtained. Find I.
- 14. The sum of the first three terms of a geometric sequence is equal to 42. The sum of the squares of the same terms is equal to 1092. Find the three terms of the sequence.
- 15. A rock is dropped into a water well and it travels approximately 16t² in t seconds. If the splash is heard 3.5 seconds later and the speed of sound is 1087 feet/second, what is the height of the well?

- 16. Two boats on opposite banks of a river start moving towards each other. They first pass each other 1400 meters from one bank. They each continue to the opposite bank, immediately turn around and start back to the other bank. When they pass each other a second time, they are 600 meters from the other bank. We assume that each boat travels at a constant speed all along the journey. Find the width of the river?
- 17. Find the constants a and b so that all the 4 lines whose equation are given by

$$x + y = -1$$
$$-x + 3y = -11$$
$$ax + by = 4$$
$$2ax - by = 2$$

- 18. pass through the same point.

- 21. It takes pump A 2 hours less time that pump B to empty a swimming pool. Pump A is started at 8:00 a.m. and pump B is started at 10:00 a.m. At 12:00 p.m. 60% of the pool is empty when pump B broke down. How much time after 12:00 p.m. would it take pump A to empty the pool?
- 22. The number of pupils in school A is equal to half the number of pupils in school B. The ratio of the boys in school A and the boys in school B is 1:3 and the ratio of the girls in school A and the girls in school B is 3:5. The number of boys in school B is 200 higher than the number of boys in school A. Find the number of boys and girls in each school.

1.

- A. Let R and r the rate of work of the large and the small pumps respectively
- B. 4(2R + r) = 1 : 2 large and 1 small work for 4 hours to do 1 job
- C. 4(R + 3r) = 1 : 1 large and 3 small work for 4 hours to do 1 job
- D. T(4R + 4r) = 1: Find time T if 4 large and 4 small are to do one job.
- E. Solve for R and r the system of first two equations then sustitute in the third and solve for T to find the time. T = 5/3 hours = 1 hour 40 minutes.

- A. x + y + H = 60: perimeter, x, y and H be the two legs and the hypotenuse of the right triangle
- B. (1/2)xy = 150: area
- C. $x^2 + y^2 = H^2$: Pythagora's theorem.
- D. 3 equations with 3 unknowns.
- E. $(x + y)^2 2xy = H^2$: completing the square in the third equation.
- F. x + y = 60 H: express x + y using the first equation and use the second equation to find xy = 300 and substitute in equation 5.
- G. $(60 H)^2 600 = H^2$: one equation with one unknown.
- H. Solve for H to find H = 25 cm. Substitute and solve for x and y to find x = 15 cm and y = 20 cm.

- A. $\operatorname{sqrt}((-6+2)^2+(0+3)^2)=(a+3)^2+(0+2)^2$: distances from center to any point on the circle are equal to the radius.
- B. a = -3 + sqrt(21), a = -3 sqrt(21): solve for a and find two solutions.

- A. (-2, -1): center of circle
- B. m = (2 -1) / (0 -2) = 3 / 2: slope of line through the center and the point of tangency (0, 2)
- C. The line through the center and the point of tangency (0, 2) is perpendicular to the tangent.
- D. M = -2/3: slope of tangent
- E. y = -(2/3)x + 2: equation of tangent given its slope and point (0, 2).

5.

A. ${}_{3}\textbf{C}_{2} * {}_{8}\textbf{C}_{6} * 1 = 84$: Use of fundamental theorem of counting

6.

- A. $x^2 3|x 2| 4x = -6$: given
- B. Let Y = x 2 which gives x = Y + 2
- C. $(Y + 2)^2 3|Y| 4(Y + 2) = -6$: substitute in above equation
- D. $\dot{Y}^2 3|\dot{Y}| + 2 = 0$
- E. $Y^2 = |Y|^2$: note
- F. $|Y|^2 3|Y| + 2 = 0$: rewrite equation as
- G. (|Y| 2)(|Y| 1) = 0
- H. |Y| = 2, |Y| = 1: solve for |Y|
- I. Y = 2, -2, 1, -1: solve for Y
- J. x = 4, 0, 3, 1: solve for x using x = Y + 2.

- A. h = -b / 2a = 2: x coordinate of the vertex of the parabola
- B. $k = -(2)^2 + 4(2) + C = 4 + C$: y coordinate of vertex
- C. x = (2 + sqrt(4 + C)), x = (2 sqrt(4 + C)): the two x intercepts of the parabola.
- D. length of BA = k = 4 + C
- E. length of AC = 2 + sqrt(4 + C) 2 = sqrt(4 + C)
- F. area = (1/2)BA * AC = (1/2) (4 + C) * sqrt(4 + C)
- G. (1/2)(4 + C) * sqrt(4 + C) = 32: area is equal to 32

- A. A(0,0), B(2k/5, 4k/5), C(2k,0): points of intersection of the 3 points of intersection of the 3 lines
- B. (1/2) * (4k/5) * (2k) = 80: area given
- C. k = 10: solve the above equation for k, k positive is a given condition.

9.

- A. y = a(x + 2)(x 3): equation of the parbola in factored form
- B. 10 = a(5 + 2)(5 2): (5, 10) is a point on the graph of the parabola and therefore satisfies the equation of the parabola.
- C. a = 5/7: solve the above equation for a.

- A. Divide $x^{(3} + 3x^2 2Ax + 3)$ by $(x^2 + 1)$ to obtain a remainder = -x(1 + 2A)
- B. -x(1 + 2A) = 5x: remainder given
- C. -(1 + 2A) = 5 : polynomials are equal if they corresponding coefficient area equal.
- D. A = -3

- A. $P(1) = 1^5 + 2(1^3) + A^*(1) + B = 2$: remainder theorem
- B. $P(-3) = (-3)^5 + 2(-3)^3 + A^*(-3) + B = -314$
- C. A = 4 and B = -5: solve the above systems of equations.

12.

- A. $x^2 4x + 2 + y^2 4y + 2 = 4$: expand equation of first circle
- B. $x^2 2x + 1 + y^2 2y + 1 = 4$: expand equation of second circle
- C. -2x 2y 6 = 0: subtract the left and right terms of the above equations
- D. y = 3 x: solve the above for y.
- E. $2x^2 6x + 1 = 0$: substitute y by 3 x in the first equation, expand and group like terms.
- F. (3/2 + sqrt(7)/2, 3/2 sqrt(7)/2), (3/2 sqrt(7)/2, 3/2 + sqrt(7)/2): solve the above for x and use y = 3 x to find y.

13.

- A. $I + 200 = A^2$: 200 added to I (unknown integer) gives a square.
- B. I + 276 = B²: 276 added to I (unknown integer) gives another square.
- C. $B^2 = A^2 + 76$: eliminate I from the two equations.
- D. add squares A² (0, 1, 4, 9, 16, 25,...) to 76 till you obtain another square B².
- E. $76 + 18^2 = 400 = 20^2$
- F. $A^2 = 18^2$ and $B^2 = 20^2$
- G. $I = A^2 200 = 124$

- A. $sum1 = a + a r + a r^2 = 42$: the sum of the three terms given, r is the common ratio.
- B. $sum2 = a^2 + a^2r^2 + ar^2r^4 = 1092$: the sum of the squares of the three terms given .
- C. sum1 = $a + ar + ar^2 = a(r^3 1) / (r 1) = 42$: apply formula for a finite sum of geometric series.
- D. sum2 = $a^2 + a^2r^2 + ar^2r^4 = a^2(r^6 1) / (r^2 1) = 1092$: the sum of squares is a also a sum of geometric series.
- E. $sum2/sum1^2 = 1092 / 42^2 = [a^2(r^6 1)/(r^2 1)] / [a^2(r^3 1)^2 / (r 1)^2]$
- F. $(r^2 r + 1) / (r^2 + r + 1) = 1092 / 42^2$
- G. r = 4, r = 1/4: solve for r
- H. a = 2: substitute r = 4 and solve for a

- I. a = 32: substitute r = 1/4 and solve for a
- J. a = 2, ar = 8, $ar^2 = 32$: find the three terms for r = 4
- K. a = 32, ar = 8, $ar^2 = 2$: find the three terms for r = 1/4

- A. T1 + T2 = 3.5 : T1 time for the rock to reach bottom of well and T2 time for the sound to reach the top of the well.
- B. $16 * T1^2 = 1087 * T2$: same distance which the height of the well.
- C. T2 = 3.5 T1: solve for T2
- D. $16 * T1^2 = 1087 * (3.5 T1)$
- E. T1 = 3.34 seconds
- F. Height = $16 * (3.34)^2 = 178$ feet (to the nearest unit)

- A. S1*t1 = 1400 : S1 speed of boat 1, t1 : time to do 1400 meters(boat 1)
- B. 1400 + S2*t1 = X : S2 speed of boat 2
- C. S1*t2 = X + 600 : t2 time to do X + 600 (boat 2)
- D. S2*t2 = 2X 600
- E. S1 = 1400/t1
- F. S2 = (X-1400)/t1
- G. T = t2/t1: definition
- H. substitute S1, S2 and t2/t1 using the above expressions in equations 3 and 4 to obtain
- I. $1400^*T = X + 600$

- J. X*T 1400*T = 2X 600 : 2 equations 2 unknowns
- K. Eliminate T and solve for X to obtain X = 3600 meters.

- A. solve the system of the first two equations to obtain the solution (2, -3)
- B. The above solution is also a solution to the last two equations.
- C. a(2) + b(-3) = 4
- D. 2a(2) b(-3) = 2
- E. a = 1 and b = -2/3: solution to the above system of equations.