From Fiedman Equation, Equation for present time in terms of Ω_m and Ω_Λ can be written as:

$$t_{universe}(\Omega_m, \Omega_\Lambda) = \int_0^1 \frac{1}{H_0\sqrt{\frac{\Omega_m}{a} + \Omega_\Lambda a^2 + (1 - \Omega_m - \Omega_\Lambda)}} da$$

For $\Omega_\Lambda = 0$, we get

$$\begin{aligned} t_{universe}(\Omega_m, 0) &= H_0^{-1} \times [\frac{1}{1-\Omega_m} - \frac{\Omega_m \sinh^{-1}\left(\sqrt{\frac{1}{\Omega_m} - 1}\right)}{(1-\Omega_m)^{3/2}}], for[\Omega_m < 1] \\ &= H_0^{-1} \times (2/3), for[\Omega_m = 1] \\ &= H_0^{-1} \times (-\frac{\sqrt{x-1}\left(2x \arctan\left(\frac{1}{\sqrt{x-1}}\right) - \pi x\right)}{2x^2 - 4x + 2} - \frac{2x}{2x^2 - 4x + 2} + \frac{2}{2x^2 - 4x + 2}), for[\Omega_m > 1] \end{aligned}$$

For $\Omega_\Lambda = 1 - \Omega_m$, we get:

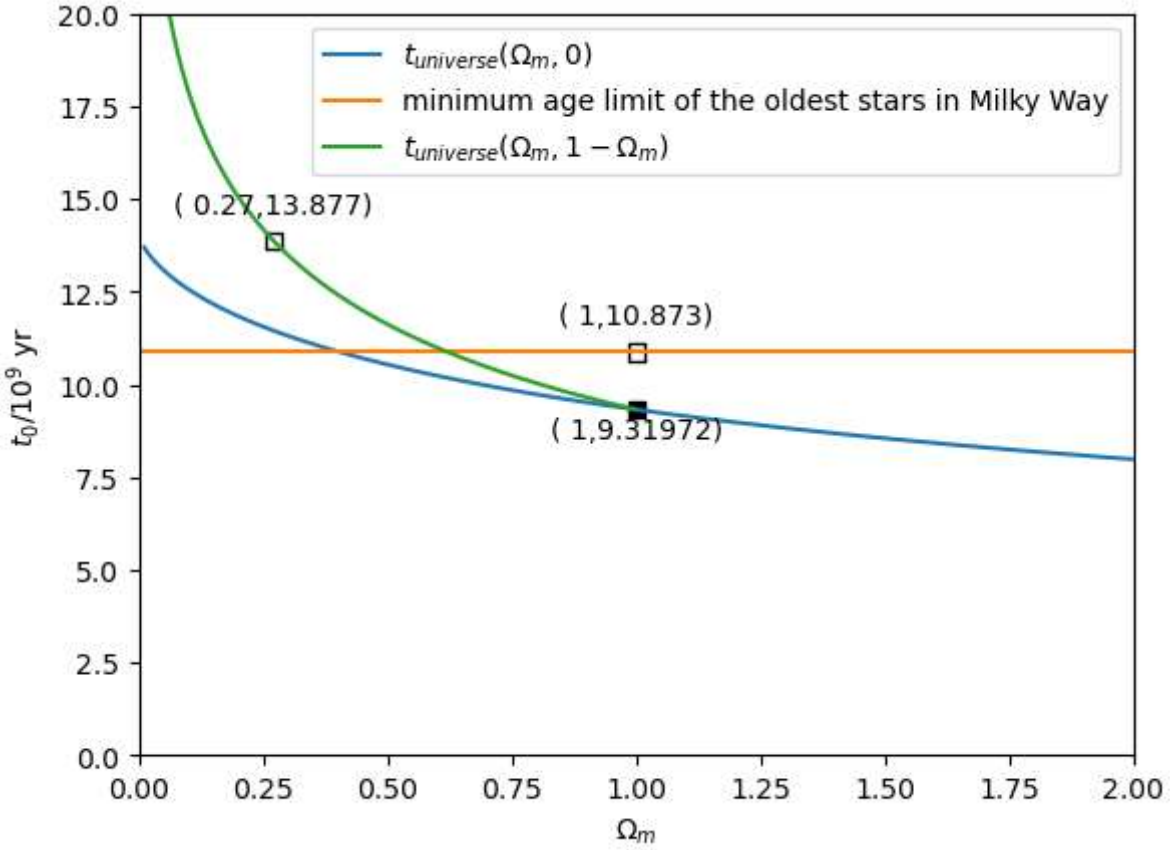
$$\begin{aligned} t_{universe}(\Omega_m, 1-\Omega_m) &= H_0^{-1} \times [\frac{2\sinh^{-1}\left(\sqrt{\frac{1}{\Omega_m} - 1}\right)}{3\sqrt{1-\Omega_m}}], for[\Omega_m < 1] \\ &= H_0^{-1} \times (2/3), for[\Omega_m = 1] \end{aligned}$$

In [44]:
```
t_H = 13.97958          #Gyrs(Hubbles time for H_0 = 70 km(sMpc)^{-1})
t_h = (7/6) * t_H       #Gyrs(Hubbles time for H_0 = 60 km(sMpc)^{-1})
import numpy as np
import matplotlib.pyplot as plt
import math

# for Ω_Λ = 0
import numpy as np
import math

def t_1(Omega_m):
    x = Omega_m
    numerator_1 = np.sqrt(x - 1) * (2 * x * np.arctan(1 / np.sqrt(x - 1)) - math.pi * x)
    denominator = 2 * x ** 2 - 4 * x + 2
    p_1 = t_H*(-(numerator_1 / denominator) - (2 * x / denominator) + (2 / denominator))
```

```
    p_2 = t_H * ((1 / (1 - x) - (x * np.arcsinh(np.sqrt(1 / x - 1))) / ((1 - x) ** (3/2))))
    if x <= 1:
        return p_2
    elif x == 1 :
        return 2*t_H/3
    else :
        return p_1
Omega_m = np.arange(0.01,2,0.003)
y_1 = []
for i in range (0,len(Omega_m)):
    y_1.append(t_1(Omega_m[i]))

# For  Ω_Λ=1-Ω_m
def t_2(Omega_m):
    x = Omega_m
    numerator = 2 * np.arcsinh(np.sqrt(-1 + 1 / x))
    denominator = 3 * np.sqrt(1 - x)
    #p_1 = t_H*(-(numerator_1 / denominator) - (2 * x / denominator) + (2 / denominator))
    p_2 = t_H * (numerator / denominator)
    if x <= 1:
        return p_2
    elif x == 1 :
        return 2*t_H/3
    else :
        return p_2
y_2 = []
for i in range (0,len(Omega_m)):
    y_2.append(t_2(Omega_m[i]))

#The horizontal line
def t_3(Omega_m):
    p = (2*t_h/3)* (Omega_m/Omega_m)
    return p
#plotting
fig , ax = plt.subplots()
ax.plot( Omega_m , y_1 ,label = r'$t_{universe}(\Omega_m,0)$')
ax.scatter( 1,2*t_H/3, color='black', marker='s')
plt.annotate(f'( 1,9.31972)', ( 1,2*t_H/3), textcoords='offset points', xytext=(0,-10), ha='center')

ax.plot(Omega_m,t_3(np.array(Omega_m)) ,label = r'minimum age limit of the oldest stars in Milky Way')
ax.scatter( 1,2*t_h/3, color='black', marker='s',facecolors='none')
plt.annotate(f'( 1,10.873)', ( 1,2*t_h/3), textcoords='offset points', xytext=(0,10), ha='center')

ax.plot(Omega_m , y_2 ,label = r'$t_{universe}(\Omega_m,1-\Omega_m)$')
ax.scatter( 0.27,0.992689*t_H, color='black', marker='s',facecolors='none')
```

```
plt.annotate(f'( 0.27,13.877)', ( 0.27,0.992689*t_H), textcoords='offset points', xytext=(0,10), ha='center')

ax.set_xlabel(r'$\Omega_m$')
ax.set_ylabel(r'$t_0/10^9$ yr')
ax.set_xlim([0,2])
ax.set_ylim([0,20])
#ax.set_title('plot of '+r'$d_A(z),d_L(z), d_C(z)$')
plt.legend()
plt.show()
```

```
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:13: RuntimeWarning: invalid value encountered in sqrt
  numerator_1 = np.sqrt(x - 1) * (2 * x * np.arctan(1 / np.sqrt(x - 1)) - math.pi * x)
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:15: RuntimeWarning: divide by zero encountered in double
_scalars
  p_1 = t_H*(-(numerator_1 / denominator) - (2 * x / denominator) + (2 / denominator))
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:15: RuntimeWarning: invalid value encountered in double_
scalars
  p_1 = t_H*(-(numerator_1 / denominator) - (2 * x / denominator) + (2 / denominator))
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:16: RuntimeWarning: invalid value encountered in sqrt
  p_2 = t_H * ((1 / (1 - x) - (x * np.arcsinh(np.sqrt(1 / x - 1))) / ((1 - x) ** (3/2))))
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:16: RuntimeWarning: invalid value encountered in double_
scalars
  p_2 = t_H * ((1 / (1 - x) - (x * np.arcsinh(np.sqrt(1 / x - 1))) / ((1 - x) ** (3/2))))
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:31: RuntimeWarning: invalid value encountered in sqrt
  numerator = 2 * np.arcsinh(np.sqrt(-1 + 1 / x))
C:\Users\mubas\AppData\Local\Temp\ipykernel_16148\600689433.py:32: RuntimeWarning: invalid value encountered in sqrt
  denominator = 3 * np.sqrt(1 - x)
```


$t_{universe}(\Omega_m, 0)$
minimum age limit of the oldest stars in Milky Way
$t_{universe}(\Omega_m, 1-\Omega_m)$
(0.27,13.877)
(1,10.873)
(1,9.31972)
$t_0/10^9$ yr
Ω_m
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

In [ ]:

In [ ]: