Due date: December 8, 2025 / 23:59

Course Project

Student ID: 2021400231

In this project, you are expected to formulate a multiple-layer transshipment problem as a linear programming model. Transshipment problem is a network flow problem in which a single commodity travels through the vertices. The vertices may have a supply capacity and/or a demand quantity. The objective is to satisfy the demand of the vertices with minimum cost. In your instances. There are four layers of vertices. The first layer contains the supply vertices and last contains the demand vertices. The two layers in between (transshipment layers) do not supply or demand the commodity, but they are used for transferring. Figure 1 is an example of such network.
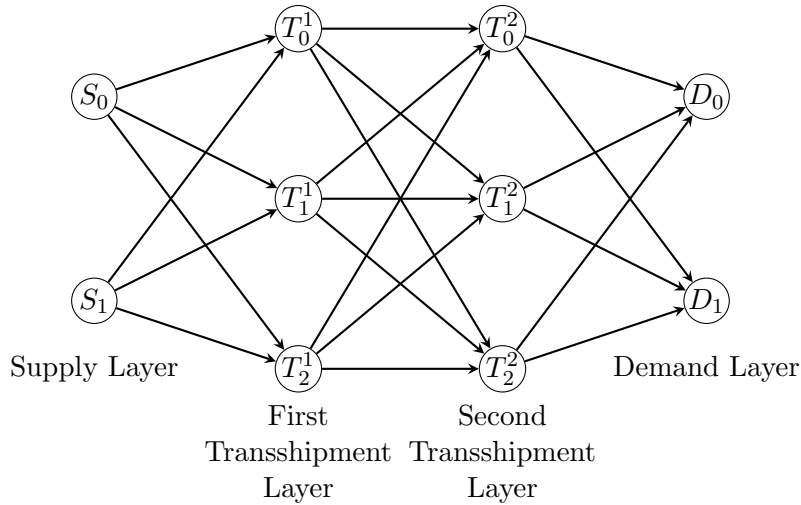

Figure 1: An Example Transportation Network

You may use Python and Pulp Package for programming. You are expected to upload your code and a report. Make sure your code is clear, free of repetitions and unnecessary outputs. Use descriptive comments in your code. Each student has a unique instance of the problem, provided at the end of this document. Make sure you use the dataset assigned to you. We expect a coherent report for the project. Grading will be based on both your report and code.

Question 1: Modeling

Formulate this problem as a parameterized **linear program**. Define your decision variables, problem parameters, objective function and constraints clearly. Make sure the decision variables are non-negative and continuous. In this question, you do not need the data.

Question 2: Implementation

Implement the model in Pulp. Output the model formulation (objective function and the constraints). Solve the model, then output the optimal solution **(only the basic variables)** and the optimal objective value. In your report, draw the solution on your network. You can draw by hand or use any drawing software (one example is the website draw.io).

Question 3: Economic Interpretation and Sensitivity Analysis

Answer the questions below according to the result of Question 2 and **separately** for each task. First, argue your expectations with reasoning using economic interpretation of the results without resolving the LP. For some tasks, you need to use reduced costs and shadow prices. After making your hand calculated results (make sure these are represented in your report clearly), solve the model with modified data and observe the results to confirm your expectations.

In Pulp, the shadow price of a constraint is accessed with `.pi`, the reduced cost of a variable is accessed with `.dj`.

3a. The capacity of supply node 3 increases by one unit. What is the expected change in the total cost?

3b. The capacity of supply node 0 increases by one unit. What is the expected change in the total cost?

3c. The demand quantity of demand node 0 increases by one unit. What is the expected change in the total cost?

3d. The cost of sending one unit from supply node 0 to first layer transshipment node 2 decreases by one. Do you expect a change in the basis? What is the expected change in the cost?

3e. The cost of sending one unit from supply node 0 to first layer transshipment node 0 decreases by one. Do you expect a change in the basis? What is the expected change in the cost?

3f. What if the decrease in Question 3.e. was 27? Do you expect a change in the basis? What is the change in the cost?

3g. Say that supply node 0 cannot ship to first layer transshipment node 2. Do you think that the total cost will increase or decrease in this case?

3h. Say that the demand quantity of first demand node increases. What is the maximum increase in the demand quantity that we can still satisfy?

Question 4: Integrality

Does your solution consist of integer values? If so, explain what is the reason for that, even though the decision variables are continuous variables.

Important Points: Make sure your project satisfies all the points below before submission.

[ ] You are formulating and solving a linear program. Make sure that you defined your decision variables as non-negative and continuous. If the decision variables are defined as integer, reduced costs and shadow prices do not work as intended.

[ ] For Question 3, if you keep the data globally and if you change it for each task (which we do not approve), do not forget to revert the data to its original state at each task. It is best to define functions that do not change the global dataset.

[ ] Note that the labels of nodes start with 0, just as the indices start with 0 in Python. Take this into consideration when answering the tasks in Question 3.

[ ] Do not provide any code or output in your report. Your project will be penalized, if you do so.

[ ] Avoid repetition of similar code. Your code should be minimalistic while also being readable. Again, you may define functions for similar use cases.

[ ] Definitely avoid massive outputs of your code if you use iPython Notebook (`.ipynb`). If you use script files (`.py`) for programming, provide a separate file (like a Markdown document) for your code outputs. But again, do not dump the code output into the file. Format it, clip the unnecessary parts, make it readable.

[ ] Do not introduce dummy supply/demand nodes. You do not need to introduce any new vertex to the network.

Problem Instance for Student ID: 2021400231

```
nSupplyNodes = 4
nT1Nodes = 3
nT2Nodes = 4
nDemandNodes = 5
costMatrixStoT1 = [ [28, 27, 11], [16, 21, 11], [11, 10, 24], [27, 17, 25]]
costMatrixT1toT2 = [ [47, 34, 43, 69], [61, 46, 32, 33], [47, 68, 31, 30]]
costMatrixT2toD = [ [15, 18, 19, 16, 18], [19, 15, 18, 13, 14], [17, 15, 19, 17,
18], [11, 12, 11, 12, 12]]
supplyCapacities = [191, 233, 276, 385]
demandQuantities = [102, 289, 127, 210, 352]
```