2D, One Group Neutron Transport Solver for a System with Vacuum Boundaries at Top and Left Faces and Reflective Boundaries at Bottom and Right Faces

1. Abstract

The neutron transport equation is used to find the neutron angular flux at each given point in space. Neutron flux is fundamental in nuclear engineering as it determines neutron absorption and fission, aids in material design, etc. The transport equation is complex with a lot of unknown variables and requires approximation and analytical methods to solve the equation numerically. This project deals with developing a 2D one group neutron transport using deterministic approach where the space and direction vectors are discretized. The angles are discretized using discrete ordinate method and space is discretized using diamond difference method.

Furthermore, the geometry would be divided into small cells to use diamond difference method for approximations. In this project, the code is only trying to resolve simple geometries of square and rectangle only. Irregular and curved geometry needs very fine meshing and is computationally intensive and harder to code.

2. Derivations and Methods Used

The energy independent 3D Boltzmann neutron transport equation can be written as [1]:

$$\Omega \cdot \nabla\psi(r,\Omega) + \Sigma t(r)\psi(\vec{r}, \hat{\Omega}) = q(\vec{r}, \hat{\Omega}) \tag{2.1}$$

This equation is modified for 2-Dimensional transport equation in x, y coordinates as:

$$\left(\frac{d}{\partial x}\mu + \frac{d}{\partial y}\eta\right).\psi(x,y,\hat{\Omega}) + \sigma(x,y).\psi(x,y,\hat{\Omega}) = q(x,y,\hat{\Omega}) \tag{2.2}$$

where $\sigma(x,y)$ is the macroscopic cross section and μ and η are the two angles of the direction vector $\hat{\Omega}$. $q(x,y,\mu,\eta)$ is the scattering and independent source term defined as:

$$q(x,y,\mu,\eta) = \sum_{l=0}^{L}\sum_{m=0}^{l}(2-\delta_{m0})\, Y_{lm}^{e}(\hat{\Omega})\sigma_l(x,y)\phi_l^m(x,y) + s(x,y,\hat{\Omega}) \tag{2.3}$$

where $\sigma_l(x,y)$ is the l^{th} scattering moment and $\phi_l^m(x,y)$ is the l^{th} flux moment [2].

2.1 Discrete Ordinate method

The discrete ordinate's method is used to solve eq 2.1 and is given by:

$$\left(\frac{d}{\partial x}\mu_n + \frac{d}{\partial y}\eta_n\right).\psi(x,y,\hat{\Omega}) + \sigma(x,y).\psi(\vec{r}, \hat{\Omega}) = q(\vec{r},\hat{\Omega}) \tag{2.4}$$

Where

$$q(\vec{r},\mu,\eta) = \sum_{l=0}^{L}\sum_{m=0}^{l}(2-\delta_{m0})\, Y_{lm}^{e}(\hat{\Omega})\sigma_l(\vec{r})\phi_l^m(\vec{r}) + s(\vec{r},\hat{\Omega}) \tag{2.5}$$

The discrete angles μ_n and η_n are selected using quadrature sets and spatial derivatives are discretized using either the finite difference or the finite volume method [2]. These selected angles which are relative to $\hat{\Omega}$ are used to solve the neutron transport equation.

2.2 Level Symmetric Quadrature Set

For this project, the level symmetric quadrature set is used. It use the same set of N/2 positive values of direction cosines with respect to each of the two axes, i.e., for each level n $\mu_n = \eta_n$ is set. We describe a level α as the ordinate set that has cosine μ_n with respect to the x-axis. There are 4 octants for @d geometries and have up and down symmetry. So, there are $N(N+2)/2$ levels for 2-D geometries. The weights of the quadrature set is normalized as:

$$\sum_{n=1}^{\frac{N(N+2)}{2}} w_n = 1 \tag{2.6}$$

Hence, the flux moments can be approximated as:

$$\phi_l^m(\vec{r}) = \frac{1}{4} \sum_{n=1}^{\frac{N(N+2)}{2}} w_n Y_{lm}^e\left(\widehat{\Omega_n}\right)\psi\left(\vec{r}, \widehat{\Omega}_n\right) \tag{2.7}$$

2.3 Spatial Discretization

The diamond difference method is used to create a cell centered mesh where the values of the variables only change at the cell-edge boundaries and cross sections values remain constant. The spatial balance equation is thus obtained as:

$$\frac{\mu_n}{\triangle x_i}\left(\psi_{n,i+\frac{1}{2},j} - \psi_{n,i-\frac{1}{2},j}\right) + \frac{\eta_n}{\triangle y_j}\left(\psi_{n,i,j+\frac{1}{2}} - \psi_{n,i,j-\frac{1}{2}}\right) + \sigma_{ij}\psi_{nij} = q_{nij} \tag{2.8}$$

The outgoing fluxes (flux at the edge of each cell) are:

$$\psi_{n,i\mp\frac{1}{2},j} = \psi_{nij} - \psi_{n,i\mp\frac{1}{2},j} \tag{2.9}$$

$$\psi_{n,i,j\mp\frac{1}{2}} = \psi_{nij} - \psi_{n,i,j\mp\frac{1}{2}} \tag{2.10}$$

3. Progress and Timeline

As per the project plan, the deliverables of this week were to start the coding process; looking at techniques and geometry choices. This has been partially completed with days remaining. For going forward, the timeline has been changed to accommodate the thanks giving week, upcoming midterms and other projects, and is shown below:

- Add grid sweep for cell centered angular flux (11/23 – 11/24)
- Implement Boundary Conditions (11/24 – 11/25)
- complete coding (11/26 – 12/4)
- Run Tests (12/4 – 12/5)
- Prepare for presentation and final report (12/6 – 12/9)

4. References

[1] Chen, Q., Wu, H., & Cao, L. (2008). Auto MOC—A 2D neutron transport code for arbitrary geometry based on the method of characteristics and customization of AutoCAD. *Nuclear Engineering and Design*, *238*(10), 2828-2833.

[2] Ortega, M.I., *Two-Dimensional Neutron Transport Equation Solver with Diffusion-Synthetic and Transport-Synthetic Acceleration*, UC Berkeley