

$$1. \det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{vmatrix} 0 - \lambda & 3 \\ 4 & 4 - \lambda \end{vmatrix}$$

$$\bullet (-\lambda)(4 - \lambda) - 12 = 0$$

$$-4\lambda + \lambda^2 - 12 = 0$$

$$\lambda = 6, -2$$

For $\lambda = 6,$

$$(A - 6I)x = 0$$

$$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + 3x_2 = 0$$

$$4x_1 - 2x_2 = 0.$$

solving, we get, $x_1 = \frac{x_2}{2}$

$$x = x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For $\lambda = -2,$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 3x_2 = 0$$

$$4x_1 + 6x_2 = 0, \quad x = \frac{-3y}{2}$$

$$x = x_1 \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

~~2/5~~

2+
+9
12
+5

~~2/5~~

7.

~~1000~~

3.46

(b)

spectral radius = 6.
We care about spectral radius because it has an impact on the convergence of matrix power sequences and series.

In nuclear engineering, spectral radius = $\frac{\Sigma_s}{\Sigma_t}$

Since Σ_s can not be greater than Σ_t , the max spectral radius will be 1.

Spectral radius of $\Sigma_s = 0.2$

spectral radii of the systems:

(i) $0.3/1.5 = 0.2$ (ii) $0.8/1.0 = 0.8$

(iii) $5.0/10.0 = 0.5$

(i) will converge most quickly and (ii) will converge most slowly. Smaller the spectral radius faster the convergence.

(c)

In the k-eigenvalue problem, k is a multiplication factor. Multiplication factor is the ratio of neutrons produced in one generation to the neutrons produced in the previous generation. For a self sustaining critical system the multiplication factor must be greater than or equal to one.

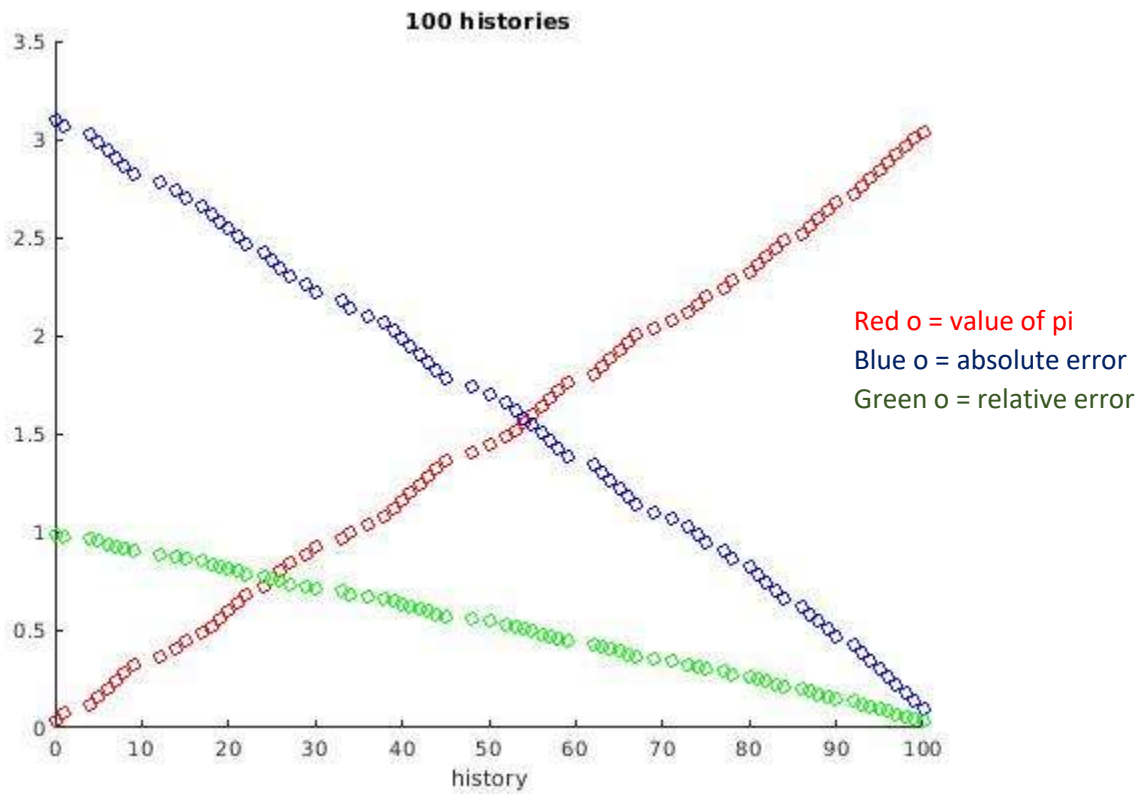
QUESTION 2

For 100 histories:

Final value of $\pi=3.04$

Absolute error= $|\pi-3.14159|=0.10159$

Relative error= absolute error/3.14159 = 3.23E-2

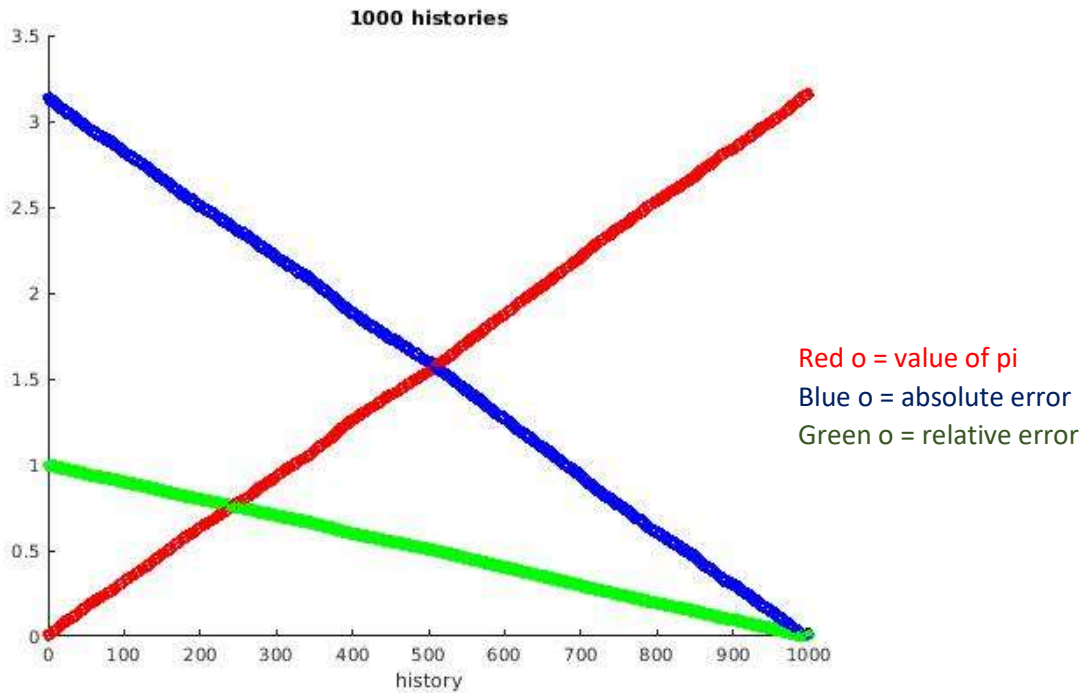


For 1000 histories:

Final value of $\pi=3.1600$

Absolute error= $|\pi-3.14159|=0.01841$

Relative error= absolute error/3.14159 = 5.86E-3

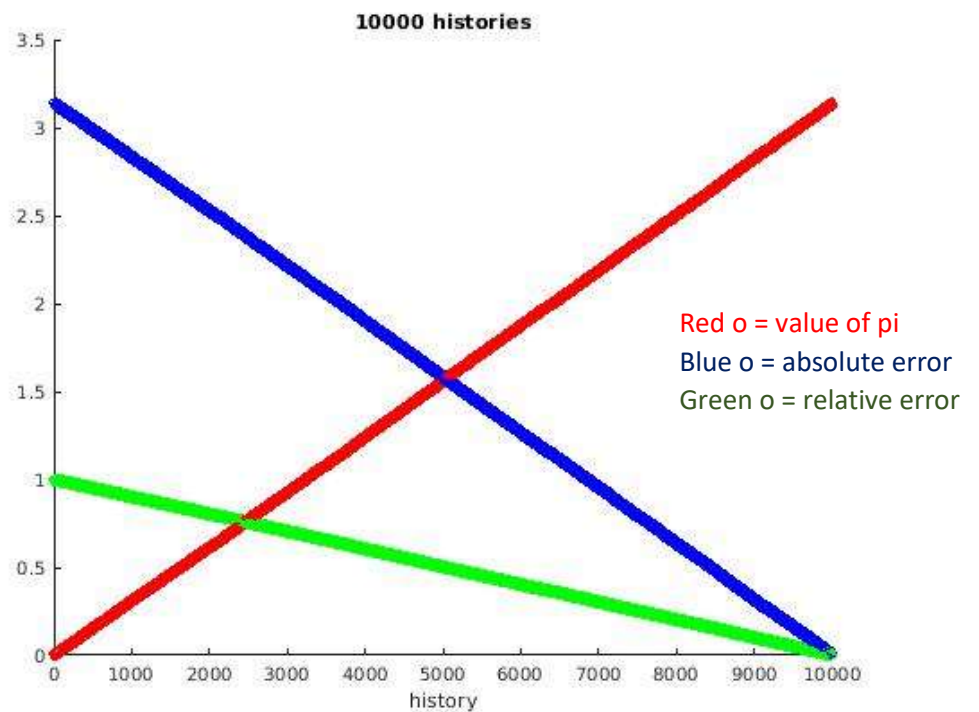


For 10000 histories:

Final value of $\pi = 3.1512$

Absolute error = $|\pi - 3.14159| = 0.00961$

Relative error = 3.059×10^{-3}



3.

a) Monte Carlo is inherently parallel because the particle histories are independent of each other. It treats each physical process as a probabilistic process and randomly sample each process using an independent stream of numbers. Hence the particle can be simulated in parallel and the results can be summed up at the end of the process. But the random number generator should be validated for ~~such~~ parallelization.

(b) ~~PE~~

$$\begin{aligned} \text{Probability of scattering } P(s) &= \frac{\Sigma_s}{\Sigma_t} = \frac{\Sigma_s}{\Sigma_t + \Sigma_a + \Sigma_f} \\ &= \frac{200}{200 + 150 + 530} \\ &= 0.227 \end{aligned}$$

$$\begin{aligned} \text{Probability of fission, } P(f) &= \frac{\Sigma_f}{\Sigma_t} = \frac{530}{200 + 150 + 530} \\ &= \underline{\underline{0.602}} \end{aligned}$$

$$\begin{aligned} P(\text{inelastic scattering}) &= \frac{20}{200} \cdot P(s) = 0.10 \times 0.227 \\ &= \underline{\underline{0.023}} \end{aligned}$$

$$\begin{aligned} P(\text{elastic scattering}) &= \frac{180}{200} \cdot P(s) = 0.90 \times 0.227 \\ &= \underline{\underline{0.204}} \end{aligned}$$