1

1. $\det(A-\lambda I)=0$

$$A-\lambda I = \begin{vmatrix} 0-\lambda & 3 \\ 4 & 4-\lambda \end{vmatrix}$$

$$(-\lambda)(4-\lambda)-12=0$$

$$-4\lambda+\lambda^2-12=0$$

$$\lambda = 6, -2$$

For $\lambda=6$,

$$(A-6I)x=0$$

$$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1+3x_2=0$$
$$4x_1-2x_2=0.$$

solving, we get, $x_1 = \frac{x_2}{2}$

$$x_1 = x_1\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For $\lambda=-2$,

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1+3x_2=0$$
$$4x_1+6x_2=0 \quad , \quad x=\frac{-3y}{2}$$

$$x = x_1\begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

~~24~~
~~+4~~
~~12~~
~~+5~~

~~i(g)~~

7. ~~3+6~~

(b) spectral radius = 6.

We care about spectral radius because it has an impact on the convergence of matrix power sequences and series.

In nuclear engineering, spectral radius = $\frac{\Sigma_s}{\Sigma_t}$

~~Hence~~ Since Σ_s cannot be greater than Σ_t, the max spectral radius will be 1.

spectral radii of the systems:

(i) $0.3/1.5 = 0.2$ (ii) $0.8/1.0 = 0.8$

(iii) $5.0/10.0 = 0.5$

(i) will converge most quickly and (ii) will converge most slowly. Smaller the spectral radius faster the convergence.

(c) In the k-eigenvalue problem, k is a multiplication factor. Multiplication factor is the ratio of neutrons produced in one generation to the neutrons produced in the previous generation. For a self sustaining ~~s~~critical system the multiplication factor must be greater than or equal to one.

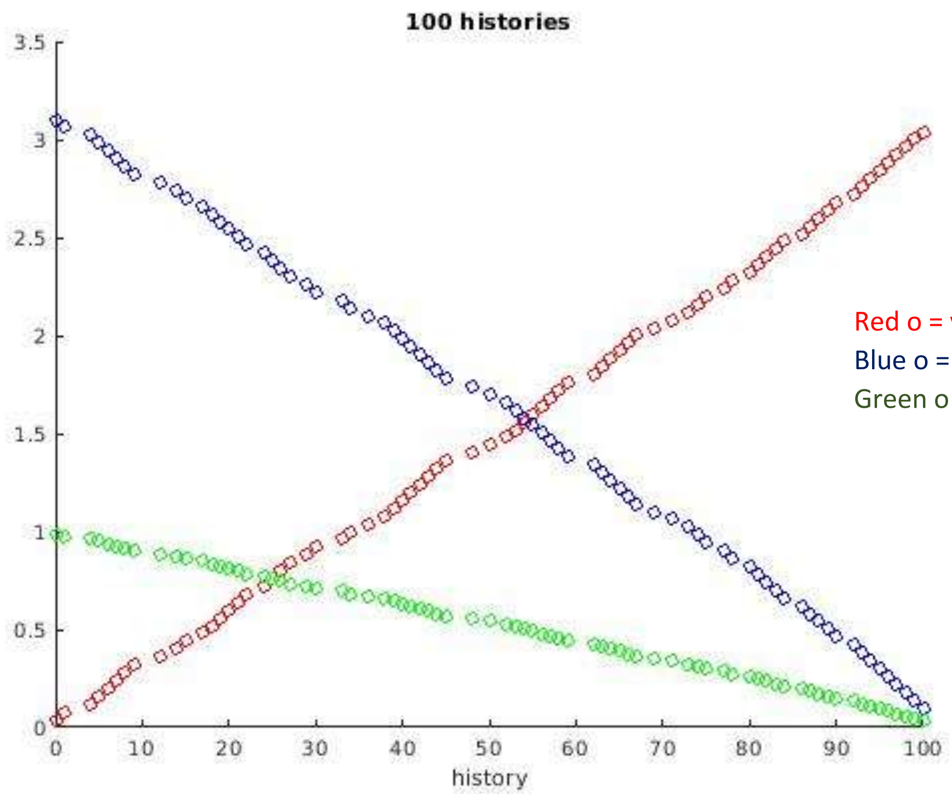
QUESTION 2

For 100 histories:

Final value of pi=3.04

Absolute error= |pi-3.14159|=0.10159

Relative error= absolute error/3.14159 = 3.23E-2



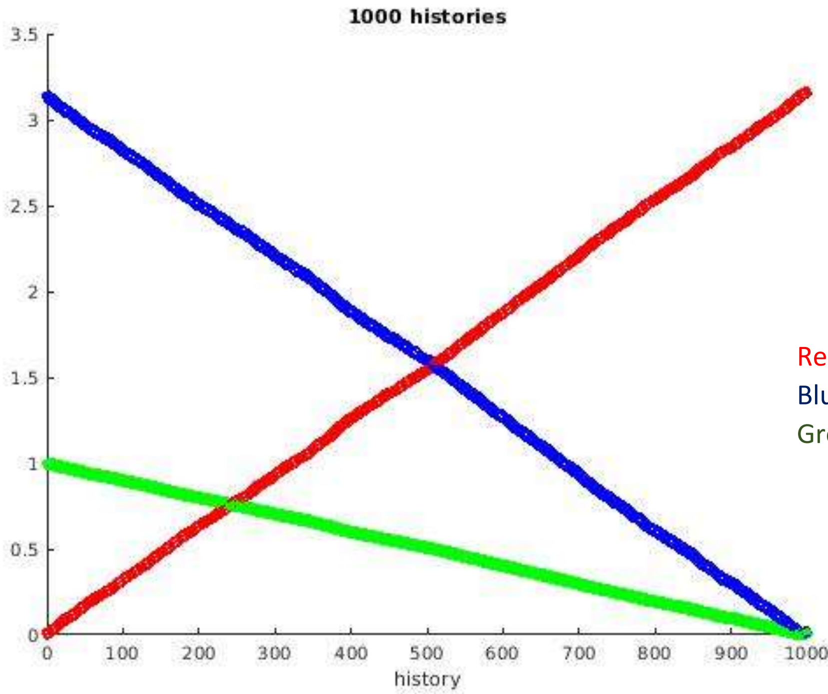
Red o = value of pi
Blue o = absolute error
Green o = relative error

For 1000 histories:

Final value of pi=3.1600

Absolute error= |pi-3.14159|=0.01841

Relative error= absolute error/3.14159 = 5.86E-3



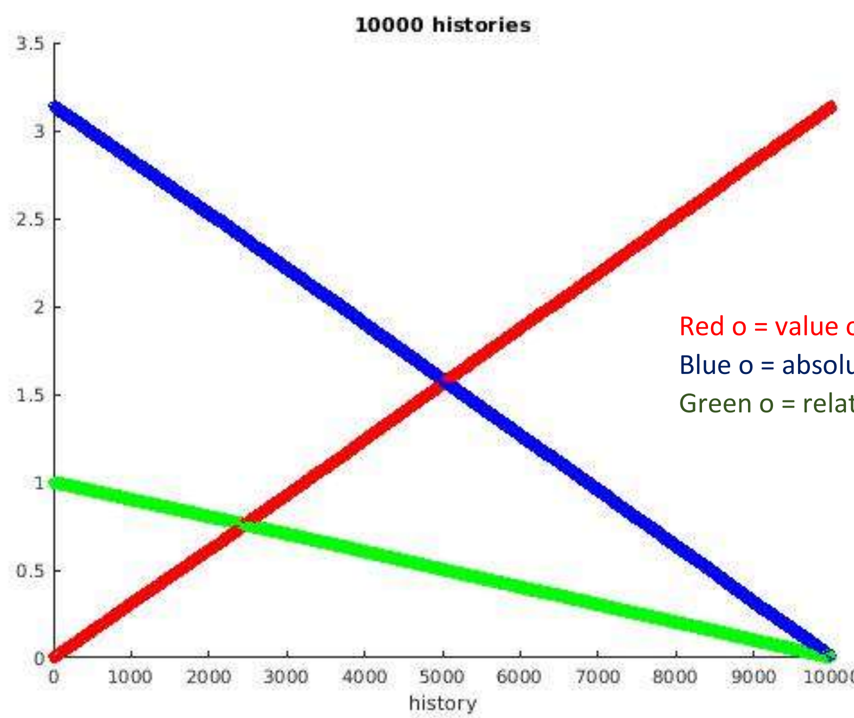
Red o = value of pi
Blue o = absolute error
Green o = relative error

For 10000 histories:

Final value of pi=3.1512

Absolute error= |pi-3.14159|=0.00961

Relative error= 3.059E-3



Red o = value of pi
Blue o = absolute error
Green o = relative error

MATlab code

```
clear
hold on;
counter = 0;       %count thr number points falling inside a unit circle
history = 10000;    %number of histories

for i=0:1:history
    rng('shuffle'); % calling for random number generation

    x = (1).*rand(); %generating random x values less than 1.0
    y = (1).*rand(); %generating random y values less than 1.0

      circle=((x*x)+(y*y))^0.5; %circle equation, calcuating the distance to the circumference

              if circle<1       %checking the coordinates falls within the unit circle
              counter=counter+1; %if falls within the circle, increases the points inside the circle by 1

               pi=(counter/history) * 4; %equation to calculate pi
              ab=abs(pi-3.14159); %calculating absolute error
               re=ab/3.14159;      %calculating relative error

        plot(i,pi, '-ro'); %plotting value of pi in each iteration
        plot(i,ab, 'bo');  %plotting value of absolute error in each iteration
        plot(i,re, 'go');   %plotting value of relative rror in each iteration
        end
    end

pi_final=(counter/history) * 4; %final value of pi
```

3.

a) Monte carlo is inherently parallel because the particle histories are independent of each other. It treats each physical process as a probabilistic process and randomly sample each process using an independent stream of numbers.
Hence the particle can be simulated in parallel and ~~&~~ the results can be summed up at the end of the process. but the random number generator should be validated for ~~such~~ parallelization.

(b) ~~P(s)~~
Probability of scattering $P(s) = \frac{\Sigma_s}{\Sigma_t} = \frac{\Sigma_s}{\Sigma_s + \Sigma_a + \Sigma_f}$

$= \frac{200}{200+150+530}$

$= 0.227$

Probability of fission, $P(f) = \frac{\Sigma_f}{\Sigma_t} = \frac{530}{200+150+530}$

$= 0.602$

$P(\text{inelastic scattering}) = \frac{20}{200} \cdot P(s) = 0.10 \times 0.227$
$= 0.023$

$P(\text{elastic scattering}) = \frac{180}{200} \cdot P(s) = 0.90 \times 0.227$
$= 0.204$