1. 0.858094

a) Integral of a PDF must be 1.

$$\int p(x)\,dx = \int_0^a 3x^2 + x\,dx = 1$$

$$\left[x^3 + \frac{x^2}{2}\right]_0^a = 1$$

$$a^3 + \frac{a^2}{2} - 0 = 1.$$

$$a^3 + \frac{a^2}{2} = 1.$$

$$2a^3 + a^2 = 2$$

~~$2a^2(a+1) = 2$~~

$$\therefore 2a^3 + a^2 - 2 = 0$$

$$\therefore a = 0.858 \qquad a = -0.679 \pm 0.839j$$

Hence max value of a is 0.858 (3 dp)

b) $$\int g(x)\,dx = \int_0^{0.858} 5\sqrt{x}\,dx = \left[\frac{5(x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{0.858}$$

$$= \left[\frac{10}{3}\sqrt{x^3}\right]_0^{0.858}$$

$$= 2.6535$$

$$\text{efficiency} = \frac{\int p(x)\,dx}{\int g(x)\,dx} = \frac{1}{2.6535} = 0.377 \text{ (3 dp)}$$

c) Used when the CDF cannot be inverted/difficult which is the case for many of them.

example:

PDF. $P(x) = \begin{cases} \frac{2}{\pi}(1-x^2)^{\frac{1}{2}} & -1<x<1 \\ 0, \text{ otherwise} \end{cases}$

CDF: $\int_0^x \frac{2}{\pi}(1-x^2)^{\frac{1}{2}}\,dx$

$= \frac{1}{\pi}\arcsin(x) + \frac{1}{2} + \frac{1}{2\pi}\sin($

$CDF = \frac{1}{\pi}\arcsin(x) + \frac{1}{2} + \frac{1}{2\pi}\sin(2\arcsin(x))$

You can see, it is difficult to find the inverse of the above. Hence, rejection sampling can be used here.

2. a) Accuracy is affected by systemic errors in simulation. Accuracy can be improved by using variance reduction method and having a confidence interval (say as 95%). This ensures on average 95% of the ~~variance~~ intervals will contain the true value of variance.

Precision can be improved by having more histories to run but increases tracking time. By a factor of 4. ~~Thirteen~~ Also, this can be also achieved by having variance reduction (saving time).

Precision $\propto \frac{1}{R^2 t}$, where t is tracking time and R is the relative error.

2-(b) Variance reduction is used to increase the precision of the estimates in Monte Carlo.

- When there is a negative correlation, i.e, convergence < 0, the variance is increased rather than decreased.

2 (bb) • Prior knowledge of the function being integrated is required for using variance reduction. This is because variance reduction is achieved by altering PDFs to favour events, changing weightage etc, and cannot be implemented without prior knowledge.

Question 3

Neutrons captured in region 2 = 3536*

Forward scattered neutrons from region 1 to region 2 = 1854*

* values were always in the order of 10^4 for different runs

```
% Monte Carlo particle tracking code
% Homework 5. Question 3

clear

sigt_1=45+4;         %sum of scattering and capture xs of region 1 respectively
lamda_1=1/sigt_1;    %mean free path of region 1

sigt_2=8+50;         %sum of scattering and capture xs of region 2 respectively
lamda_2=1/sigt_2;    %mean free path of region 2

%mean free path is used due to unknown atom and 1D geometry
%head on collissions and rebounded at 180 degrees

ncrossed=0;          %counter for number of neutrons crossed to region 2
ncaptured=0;         %counter for number of neutrons captured in region 2
nforward=0;          %counter for number of neutron forward scattered in region 1

for i=0:1:1000000            %ran for 100,000 histories

    pos=10.*rand()-5;        %random number generator between -5 and 5,
                             % negative sign indicates movement towards left
    db=5-abs(pos);  %distance to region 2 boundary

    % neutrons can only cross if the mean free path is greater than
    % distance to boundary

        if (db<lamda_1) && (pos>0) %condition for righward moving neutron
        ncrossed=ncrossed+1;
        ncaptured=ncaptured+(50/58);

        elseif (db<lamda_1) && (pos<0) %condition for leftward moving neutron
                                       %which could be scattered back into
                                       %region 2

       ncrossed=ncrossed+(45/49); %increased based on probability of capture/total
       nforward=nforward+(45/49);
       ncaptured=ncaptured+((45/49)*(50/58)); %increased based on probabilities of
                                               % scatter in region 1 and capture
                                               % in region 2

            end
end
```