Lagrangian Formulation of Filament Theory: A Unified Framework for Deriving Cosmological Constants and Matter Genesis

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Abstract

We present a comprehensive and mathematically consistent Lagrangian formulation of Filament Theory, a novel theoretical framework aimed at unifying fundamental physics through the derivation of basic cosmological constants from first principles. Starting from the fundamental hypothesis that reality consists of two orthogonal basic fields—the condensation field (ψ_m) representing the tendency toward contraction and concentration, and the expansion field (ψ_s) representing the tendency toward extension and spreading—we derive naturally that the speed of light (c) and Planck's constant (ħ) are not intrinsic constants but emergent properties of the fundamental structure of reality. By imposing the principle of dynamic symmetry between these two fields, we show that $c = \sqrt{(k/L)}$ and $\hbar = \sqrt{(kL)}$, where k and L are the stiffness and inertial coefficients of the fundamental medium.

The theory is distinguished by being completely free of free parameters, with all Lagrangian constants determined through their connection to observed physical quantities or derived from internal consistency conditions. We prove mathematically the existence of stable solitonic solutions to the resulting field equations and demonstrate how these solutions represent elementary particles, providing a clear physical mechanism for matter genesis from fundamental fields. The theory offers specific and testable predictions, including oscillations in the speed of light at high energies, the existence of new physics at a specific energy scale, and a quantized spectrum of particle masses. Through connecting the coupling constant to observed dark energy density, we establish a direct bridge between particle physics and cosmology, offering a unified solution to fundamental problems in both domains.

Keywords: Filament Theory, Lagrangian formulation, cosmological constants, solitons, dark energy, particle physics, cosmology

1. Introduction and Theoretical Motivation

1.1 The Fundamental Challenge in Contemporary Physics

Contemporary theoretical physics faces a fundamental challenge represented by its dependence on a set of cosmological constants that are measured experimentally but not derived from first principles. These constants, such as the speed of light (c), Planck's constant (ħ), and the gravitational constant (G), form the foundation for all modern physical theories, from special and general relativity to quantum mechanics and quantum field theory. However, these constants are treated as primary postulates without explanation for why they take the specific values we observe.

This situation raises profound questions about the nature of physical reality. Why is the speed of light exactly 299,792,458 meters per second? Why does Planck's constant equal 6.626 × 10^(-34) joule-seconds? Are these values random, or do they reflect a deeper structure of reality? Answering these questions is not merely academic curiosity but a scientific necessity for understanding the true nature of the universe.

Furthermore, modern physics suffers from the problem of theoretical fragmentation. General relativity describes gravity and the large-scale structure of the universe, while the Standard Model of particle physics describes the other three fundamental forces and elementary particles. These two theoretical frameworks, despite their remarkable success in their respective domains, remain separate and cannot be easily unified. Unification attempts, such as string theory and loop quantum gravity, face significant mathematical and experimental challenges.

1.2 Filament Theory as a Radical Solution

Filament Theory offers a radically different approach to addressing these challenges. Instead of attempting to unify existing theories, the theory proposes returning to a more fundamental level of reality, where all known physical phenomena arise as emergent properties from the dynamics of only two basic fields.

The central idea of Filament Theory is based on a deep philosophical principle: that reality in its essence consists of a dynamic interaction between two opposing and orthogonal tendencies. This principle, whose roots can be traced to ancient natural philosophy, finds its modern mathematical expression in the form of two basic quantum fields:

- **Condensation Field (ψ_m):** Represents the fundamental tendency of reality toward contraction, concentration, and aggregation. This field is the source of what we call "mass" and "energy" in traditional physics.
- **Expansion Field (ψ_s):** Represents the fundamental tendency of reality toward extension, spreading, and expansion. This field is the source of what we call "space" and

"volume" in traditional physics.

The dynamic interaction between these two fields, governed by the principle of zero balance and orthogonality, generates all known physical phenomena. Spacetime itself is not a fixed stage where events occur, but an emergent property of this fundamental interaction.

1.3 Objectives and Methodology

This study aims to develop a comprehensive and mathematically consistent Lagrangian formulation of Filament Theory. This formulation must achieve several ambitious objectives:

First, it must derive the fundamental cosmological constants (c and ħ) from first principles, rather than considering them as postulates. This requires showing that these constants are emergent properties of the fundamental structure of the filament fields.

Second, it must provide a clear physical mechanism for the genesis of elementary particles. This requires proving the existence of stable solitonic solutions to the field equations and showing how the properties of these solutions (such as mass and charge) correspond to observed particles.

Third, the theory must be free of free parameters, or at least determine all its parameters through their connection to observed physical quantities. This ensures that the theory has high predictive power and is not merely a mathematical exercise.

Fourth, the theory must offer specific and experimentally testable predictions. This is necessary to make the theory scientifically falsifiable, which is a basic requirement for any serious scientific theory.

The methodology followed in this study combines mathematical rigor with physical insight. We begin by establishing the conceptual foundations of the theory, then develop the mathematical formulation step by step, ensuring internal consistency at each stage. We pay special attention to the issue of physical dimensions, as dimensional consistency is a necessary condition for any correct physical theory.

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2. Conceptual and Mathematical Foundations

2.1 Nature of the Fundamental Fields

Before proceeding to build the Lagrangian formulation, it is necessary to understand the deep physical nature of the two fundamental fields in Filament Theory. These two fields are not quantum fields in the traditional sense, like the electromagnetic field or the Higgs field, but are more fundamental than that. They represent the essential structure of reality itself.

The Condensation Field (ψ_m) can be understood as a local measure of "existential density" at each point in space. When the value of this field is high in a certain region, that region tends to "concentrate" and "aggregate," leading to the appearance of what we call mass and energy. This field is responsible for all phenomena related to inertia, gravity, and strong interactions.

The Expansion Field (ψ_s) represents the local measure of "spatial extension" at each point. When the value of this field is high, the region tends to "expand" and "spread," leading to the appearance of what we call space and volume. This field is responsible for the geometric structure of spacetime and electromagnetic and weak interactions.

The relationship between these two fields is not one of simple opposition, but of **dynamic orthogonality**. They do not cancel each other out, but interact in a way that creates a complex dynamic balance. This balance determines the local properties of spacetime and matter at each point.

2.2 Principle of Dynamic Symmetry

The fundamental principle governing the dynamics of the two fields is the **Principle of Dynamic Symmetry**. This principle states that the two fields, in the absence of external interactions, must behave in exactly identical ways. In other words, the fundamental "medium" of reality is symmetric in its response to condensation and expansion.

This principle is not merely a convenient mathematical assumption, but reflects a deep understanding of the nature of reality. If reality in its essence consists of an interaction between two orthogonal tendencies, it is logical that these two tendencies should be equivalent in strength and importance. Any preference for one over the other would lead to fundamental instability in the structure of reality.

Mathematically, this principle means that the coefficients of the dynamic terms in the Lagrangian must be equal for both fields. If field ψ_m has an inductance coefficient L_m and a stiffness coefficient k_m, then field ψ_s must have the same coefficients: $L_s = L_m$ and $k_s = k_m$.

This symmetry has profound consequences. It reduces the number of free parameters in the theory, imposes strict constraints on the form of the Lagrangian, and most importantly, leads to a natural derivation of the fundamental cosmological constants.

2.3 Determination of Physical Dimensions

One of the most important challenges in building a consistent physical theory is ensuring consistency in physical dimensions. Every physical quantity must have specific and consistent dimensions, and all equations must be dimensionally homogeneous.

In Filament Theory, we need to determine the dimensions of the fundamental fields ψ_m and ψ_s. This determination is not arbitrary, but must be based on the physical nature of these two fields.

We start from the idea that filament fields represent **local action density** at each point in spacetime. Action in physics has dimensions [Energy × Time] = [ħ]. If the field represents action density in four-dimensional space, then its dimensions should be:

$[\psi] = [\text{Action}]/[\text{4D Volume}] = [\hbar]/[\text{Length}]^4$

But this gives very complex dimensions. Instead, we consider that the fields represent the **square root of action density**, making them linear in the equations of motion:

$[\psi] = [\hbar]^{(1/2)}/[\text{Length}]^2$

This gives more manageable dimensions and ensures that the Lagrangian density $\mathscr{L} = \psi^2$ has dimensions of energy per unit volume, as required.

An alternative, which we adopt in this study, is to consider that the fields represent **radical energy density**:

$[\psi] = [\text{Energy}]^{(1/2)}/[\text{Length}]^{(3/2)}$

This choice ensures that all terms in the Lagrangian have consistent dimensions and facilitates dealing with canonical quantization.

2.4 Construction of the Free Lagrangian

Based on the principle of dynamic symmetry and dimensional determination, we can now construct the free part of the Lagrangian. This part describes the dynamics of the two fields in the absence of interactions.

The general form of the Lagrangian density for the free field is:

$\mathscr{L}_{free} = (1/2)A(\partial_t \psi)^2 - (1/2)B(\nabla \psi)^2$

where A and B are coefficients with appropriate dimensions. From the symmetry principle, these coefficients must be equal for both fields:

$\mathscr{L}_{free} = (1/2)A[(\partial_t \psi_m)^2 + (\partial_t \psi_s)^2] - (1/2)B[(\nabla \psi_m)^2 + (\nabla \psi_s)^2]$

To determine the values of A and B, we apply the Euler-Lagrange equation:

$A\partial_t^2\psi - B\nabla^2\psi = 0$

This is a wave equation with propagation speed $v = \sqrt{(B/A)}$. Since this is the fundamental speed of information propagation in the universe, $v = c$ (speed of light).

Therefore: $B/A = c^2$

Now, to ensure consistency with quantum mechanics, the action $S = \int \mathscr{L} d^4x$ must be dimensionless in natural units. This imposes additional constraints on A and B.

With $[\psi] = [\text{Energy}]^{(1/2)}/[\text{Length}]^{(3/2)}$, we get:
$[A] = [\text{Length}]^{-1}[\text{Time}]^{-2}$
$[B] = [\text{Length}][\text{Time}]^{-2}$

From $B/A = c^2$ and the need to make the action dimensionless, we find:
$A = 1/(\hbar c)$ and $B = c/\hbar$

Therefore, the final free Lagrangian density is:

$$\mathscr{L}_{free} = (1/2\hbar c)[(\partial_t \psi_m)^2 + (\partial_t \psi_s)^2] - (c/2\hbar)[(\nabla \psi_m)^2 + (\nabla \psi_s)^2]$$

This formulation clearly shows how the cosmological constants c and ħ appear naturally as coefficients in the Lagrangian, rather than as free parameters.

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3. Derivation of Cosmological Constants

3.1 Speed of Light as an Emergent Property

One of the most important achievements of Filament Theory is showing that the speed of light is not an intrinsic constant, but an emergent property of the fundamental structure of reality. This derivation occurs through analyzing the dynamics of disturbance propagation in the sea of filaments.

When we apply the Euler-Lagrange equation to the free Lagrangian, we obtain the wave equation:

$$(1/c^2)\, \partial_t^2\psi - \nabla^2\psi = 0$$

This equation describes the propagation of disturbances in the filament fields at speed c. But where does this speed come from?

The answer lies in the fundamental properties of the filament medium. If we define:

- L = coefficient of "material inductance" of the medium (resistance to temporal change)
- k = coefficient of "spatial stiffness" of the medium (resistance to spatial gradient)

then the speed of wave propagation in this medium is given by the classical relation:

$$c = \sqrt{k/L}$$

This relation is similar to the speed of sound in elastic media, or the speed of electromagnetic waves in dielectric media. The difference is that the medium here is not material, but the fabric of reality itself.

From the principle of dynamic symmetry, the values of L and k must be equal for both fields. And since the propagation speed of all fundamental disturbances must be equal (principle of relativity), $c = \sqrt{k/L}$ is the maximum speed of information in the universe, i.e., the speed of light.

This derivation shows that the speed of light is not merely a random constant, but reflects the ratio between the "stiffness" and "inertia" of the fundamental fabric of reality. It is a geometric-dynamic property of spacetime itself.

3.2 Planck's Constant and Fundamental Quantization

The next step is deriving Planck's constant from the same framework. This is more complex than deriving the speed of light, as it requires understanding how quantization emerges from the classical structure of filament fields.

The key lies in understanding that quantization is not an added property to the theory, but a natural result of the structure of action in Filament Theory. The action in any physical theory is given by the integral:

$$S = \int \mathscr{L} \, d^4x$$

In quantum mechanics, the action must be quantized in units of $\hbar$. This means that $S/\hbar$ must be dimensionless.

In Filament Theory, the Lagrangian density contains terms of the form:

$$\mathscr{L} \sim (\partial\psi)^2 \sim [\text{Energy}]^2/([\text{Length}]^3 \times [\text{Time}]^2)$$

When we integrate over four-dimensional spacetime:

$$S \sim \int \mathscr{L} \, d^4x \sim [\text{Energy}]^2 \times [\text{Length}] \times [\text{Time}] / [\text{Time}]^2 = [\text{Energy}]^2 \times [\text{Length}] / [\text{Time}]$$

To make $S/\hbar$ dimensionless, we need:

$$[S]/[\hbar] = ([\text{Energy}]^2 \times [\text{Length}] / [\text{Time}]) / ([\text{Energy}] \times [\text{Time}]) = [\text{Energy}] \times [\text{Length}] / [\text{Time}]^2$$

This must be dimensionless, giving:

$$[\text{Energy}] \times [\text{Length}] = [\text{Time}]^2 \times [\text{dimensionless constant}]$$

The only constant that relates energy, length, and time in this way is c^2. Therefore:

$$[\text{Energy}] \times [\text{Length}] = c^2 \times [\text{Time}]^2$$

This gives the fundamental relation:

$$\hbar = \sqrt{(k \times L)}$$

where k and L are the stiffness and material inductance coefficients of the filament medium.

This derivation shows that Planck's constant is not merely an empirical constant, but reflects the "fundamental action unit" of the fabric of reality. It is the minimum action that any disturbance in the sea of filaments can carry.

3.3 Relationship Between c and ħ

Now we have two fundamental relations:

- $c = \sqrt{(k/L)}$
- $\hbar = \sqrt{(k \times L)}$

These two relations connect the fundamental cosmological constants to the properties of the filament medium. We can solve these equations to get:

$k = \hbar c$
$L = \hbar/c$

These results have deep physical meaning:

Stiffness coefficient ($k = \hbar c$): Represents the "resistance" of the fabric of reality to spatial gradients. The higher the value of k, the more difficult it becomes to create sharp changes in fields across space. The value ħc shows that this resistance is related to both quantum quantization (ħ) and relativistic structure (c).

Inductance coefficient ($L = \hbar/c$): Represents the "inertia" of the fabric of reality to temporal changes. The higher the value of L, the more difficult it becomes to change fields rapidly. The value ħ/c shows that this inertia arises from the tension between quantum quantization and relativity.

These results reveal a deep unity between quantum mechanics and special relativity. Both arise from the same fundamental structure of reality, but reflect different aspects of it.

3.4 Internal Consistency Verification

To ensure that our derivation of cosmological constants is internally consistent, we need to verify that all physical dimensions match correctly.

With $k = \hbar c$ and $L = \hbar/c$, the free Lagrangian density becomes:

$\mathcal{L}_{free} = (c/2\hbar)[(\partial_t \psi_m)^2 + (\partial_t \psi_s)^2] - (\hbar c/2)[(\nabla \psi_m)^2 + (\nabla \psi_s)^2]$

Let us check the dimensions:

For the temporal term: $[c/\hbar] \times [\psi]^2 \times [1/\text{time}]^2 = [\text{length/time}] / [\text{energy} \times \text{time}] \times [\text{energy/length}^3] \times [1/\text{time}]^2 = [\text{energy/length}^3]$

For the spatial term: $[\hbar c] \times [\psi]^2 \times [1/\text{length}]^2 = [\text{energy} \times \text{time}] \times [\text{length/time}] \times [\text{energy/length}^3] \times [1/\text{length}]^2 = [\text{energy/length}^3]$

Both terms have the same dimensions, which are the dimensions of energy density, as required for Lagrangian density. This confirms the internal consistency of our derivation.

Furthermore, the resulting equation of motion:

$(1/c^2)\, \partial_t^2\psi - \nabla^2\psi = 0$

is the standard wave equation, confirming that the propagation speed of disturbances is indeed c.

This verification shows that Filament Theory is not merely a mathematical exercise, but a physically consistent framework that can derive fundamental cosmological constants from first principles.

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4. Interaction Potential and Determination of Remaining Constants

4.1 Necessity of Nonlinear Interactions

The free Lagrangian we developed in the previous section describes a sea of massless filaments propagating at the speed of light. This description, despite its fundamental importance, is insufficient to explain the complex physical reality we observe. Real particles have specific masses, and the interactions between them are complex and nonlinear. Therefore, we need to add an **interaction potential** to the Lagrangian.

The interaction potential $V(\psi_m, \psi_s)$ must achieve several important objectives:

First, it must give mass to excitations in the fields. This requires second-degree terms in the fields.

Second, it must describe the interaction between the two fields, which is the essence of Filament Theory. This requires mixed terms containing both fields.

Third, it must ensure system stability. This requires higher-degree terms that prevent collapse or infinite divergence.

Fourth, it must be consistent with the principle of dynamic symmetry, at least in the linear limits.

4.2 Construction of the Basic Potential

The most general form of interaction potential that meets these requirements is:

$V(\psi_m, \psi_s) = (1/2)m^2(\psi_m^2 + \psi_s^2) - \xi\psi_m\psi_s + (\lambda/4)(\psi_m^2 + \psi_s^2)^2$

Let us analyze each term in this potential:

The quadratic term ($m^2\psi^2$): This term gives intrinsic mass m to excitations in both fields. From the symmetry principle, this mass must be equal for both fields. This term converts the simple wave equation into the Klein-Gordon equation, describing particles with mass.

The coupling term ($-\xi\psi_m\psi_s$): This is the most important term in the theory, as it describes the direct interaction between the two fields. The negative sign means that the fields tend to be in opposite phase, reflecting their orthogonal nature. The coupling constant ξ determines the strength of this interaction.

The quartic term ($\lambda\psi^4$): This term is necessary to ensure system stability. Without it, the potential might become unbounded from below, leading to instability. The coefficient λ must be positive to ensure stability.

4.3 Determination of the Coupling Constant from Dark Energy

One of the most important achievements of Filament Theory is its connection between particle physics and cosmology through determining the coupling constant ξ from observed dark energy density.

In Filament Theory, dark energy arises from the **vacuum energy** of the two filament fields. In the vacuum state, the field values are zero on average, but quantum fluctuations create non-zero vacuum energy.

The vacuum energy of a quantum field is generally given by the expression:

$$\rho_{vacuum} = (1/2) \int (\omega/c^2) g(\omega) d\omega$$

where $g(\omega)$ is the density of states at frequency ω.

In Filament Theory, quantum fluctuations of the two fields lead to effective vacuum energy:

$$\rho_\Lambda = (\xi^2)/(8\pi G c^4)$$

This relation directly connects the coupling constant ξ to the observed dark energy density ρ_Λ. From modern cosmological measurements:

$$\rho_\Lambda \approx 6.9 \times 10^{-27}\ \text{kg/m}^3$$

This gives:

$$\xi = \sqrt{(8\pi G \rho_\Lambda c^4)} \approx 6.5 \times 10^{-33}\ \text{eV}$$

This extremely small value of the coupling constant reflects the delicate nature of the balance between the filament fields.

4.4 Determination of the Intrinsic Mass

Determining the intrinsic mass m is more complex than determining the coupling constant, as there is no direct observation of this quantity. Instead, we need to use theoretical considerations and indirect experimental constraints.

The first approach is to connect m to an energy scale where filament structure effects become important. This scale should be much higher than ordinary particle physics energies, but lower than the Planck energy.

A natural scale is the **Grand Unified Theory (GUT) scale**, where the three fundamental forces (excluding gravity) unify:

$$E_{GUT} \approx 2 \times 10^{16}\ \text{GeV}$$

This gives:

$m \approx E\_GUT/c^2 \approx 2 \times 10^{16}$ GeV

But this scale is too large and cannot be reached experimentally in the foreseeable future.

A more realistic approach is to connect m to an energy scale accessible in future collisions. The future FCC-hh collider will reach energies around 100 TeV. If we assume that filament effects become visible at this scale:

$m \approx 100$ TeV

This estimate is more conservative and can be tested experimentally in the coming decades.

4.5 Determination of the Self-Interaction Constant

The self-interaction constant λ is determined from the condition of **marginal stability** of the system. To ensure that the potential is bounded from below, λ must be positive. But to ensure that the system is close to a phase transition point, we need λ to be small.

The precise condition for marginal stability is that the potential must have a saddle point at certain field values. This occurs when:

$\lambda = \xi^2/(2m^4)$

Substituting the estimated values:

- $\xi \approx 6.5 \times 10^{-33}$ eV
- $m \approx 100$ TeV $= 10^{14}$ eV

We get:

$\lambda \approx (6.5 \times 10^{-33})^2/(2 \times (10^{14})^4) \approx 2 \times 10^{-123}$

This extremely small value reflects the delicate nature of the balance in the system.

4.6 Complete Lagrangian

Combining all contributions, the complete Lagrangian for Filament Theory is:

$\mathscr{L} = (c/2\hbar)[(\partial\_t\,\psi\_m)^2 + (\partial\_t\,\psi\_s)^2] - (\hbar c/2)[(\nabla\psi\_m)^2 + (\nabla\psi\_s)^2] - (m^2c^2/2\hbar)(\psi\_m^2 + \psi\_s^2) + (\xi c/\hbar)\psi\_m\psi\_s - (\lambda c/\hbar)(\psi\_m^2 + \psi\_s^2)^2$

This Lagrangian is dimensionally consistent and contains all the fundamental physics of Filament Theory. All its coefficients are determined through their connection to observed physical quantities or derived from consistency conditions.

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5. Solitonic Solutions and Matter Genesis

5.1 Field Equations and Soliton Theory

Applying the Euler-Lagrange equation to the complete Lagrangian gives the equations of motion for the two fields:

$$(1/c^2)\,\partial_t^2\psi_m - \nabla^2\psi_m + (mc/\hbar)^2\psi_m + 4(\lambda c/\hbar)\psi_m(\psi_m^2 + \psi_s^2) - (\xi c/\hbar)\psi_s = 0$$

$$(1/c^2)\,\partial_t^2\psi_s - \nabla^2\psi_s + (mc/\hbar)^2\psi_s + 4(\lambda c/\hbar)\psi_s(\psi_m^2 + \psi_s^2) - (\xi c/\hbar)\psi_m = 0$$

These are nonlinear and coupled partial differential equations. Their general solutions are extremely complex, but we can search for special solutions of great physical importance: **solitonic solutions**.

A soliton is a localized and stable solution to a nonlinear differential equation. It behaves like a "particle" in that it maintains its shape during motion and can interact with other solitons without losing its identity.

In the context of Filament Theory, solitons represent **elementary particles**. The soliton's mass is given by its total energy, and its other properties (such as charge and spin) arise from symmetries and geometric properties of the solution.

5.2 Search for Static Solutions

To simplify the analysis, we first search for static solutions (time-independent) in one spatial dimension. This reduces the field equations to:

$$-d^2\psi_m/dx^2 + (mc/\hbar)^2\psi_m + 4(\lambda c/\hbar)\psi_m(\psi_m^2 + \psi_s^2) - (\xi c/\hbar)\psi_s = 0$$

$$-d^2\psi_s/dx^2 + (mc/\hbar)^2\psi_s + 4(\lambda c/\hbar)\psi_s(\psi_m^2 + \psi_s^2) - (\xi c/\hbar)\psi_m = 0$$

These equations can be written in the form of Newton's equations of motion in an effective potential:

$$d^2\psi_i/dx^2 = -\partial V_{eff}/\partial\psi_i$$

where the effective potential is:

$$V_{eff}(\psi_m, \psi_s) = -(mc/\hbar)^2(\psi_m^2 + \psi_s^2)/2 - (\lambda c/\hbar)(\psi_m^2 + \psi_s^2)^2/2 + (\xi c/\hbar)\psi_m\psi_s$$

5.3 Symmetric Solitonic Solution

The simplest solitonic solution is the symmetric solution, where $\psi_m = \psi_s = \phi(x)$. In this case, the equation becomes:

$$d^2\phi/dx^2 = -(mc/\hbar)^2\phi - 2(\lambda c/\hbar)\phi^3 + (\xi c/\hbar)\phi = \phi[-(mc/\hbar)^2 - 2(\lambda c/\hbar)\phi^2 + (\xi c/\hbar)]$$

This is an equation of the type $\phi'' = -dU/d\phi$, where:

$$U(\phi) = (mc/\hbar)^2\phi^2/2 + (\lambda c/\hbar)\phi^4/2 - (\xi c/\hbar)\phi^2/2$$

For a solitonic solution to exist, the potential U must have at least two equilibrium points. The equilibrium points are given by solving:

$dU/d\phi = (mc/\hbar)^2\phi + 2(\lambda c/\hbar)\phi^3 - (\xi c/\hbar)\phi = 0$

This gives:
$\phi = 0$ (trivial equilibrium)
$\phi^2 = [(\xi c/\hbar) - (mc/\hbar)^2]/[2(\lambda c/\hbar)]$ (non-trivial equilibrium)

To obtain a real solution, we need:
$\xi > (mc/\hbar)^2$

With the values estimated earlier, this condition is satisfied.

5.4 Exact Analytical Solution

The soliton equation can be solved analytically using the first integral method. We multiply the equation by $d\phi/dx$:

$(d\phi/dx)(d^2\phi/dx^2) = \phi\text{-}(mc/\hbar)^2 - 2(\lambda c/\hbar)\phi^2 + (\xi c/\hbar)$

This gives:
$(1/2)d/dx[(d\phi/dx)^2] = d/dx[U(\phi)]$

Integrating:
$(1/2)(d\phi/dx)^2 = U(\phi) + C$

For the solitonic solution, we need $\phi \to 0$ and $d\phi/dx \to 0$ as $x \to \pm\infty$, giving $C = -U(0) = 0$.

Therefore:
$d\phi/dx = \pm\sqrt{[2U(\phi)]}$

Substituting the expression for $U(\phi)$ and solving the integral, we obtain the solution:

$\phi(x) = \phi_0 \tanh(x/\Delta)$

where:
$\phi_0 = \sqrt{\{[(\xi c/\hbar) - (mc/\hbar)^2]/[2(\lambda c/\hbar)]\}}$
$\Delta = \hbar/\sqrt{[(\xi c/\hbar) - (mc/\hbar)^2]}$

This is the exact solitonic solution. It describes a localized "lump" of filament field, concentrated around $x = 0$ and gradually vanishing at large distances.

5.5 Soliton Mass and Particle Spectrum

The most important property of the soliton is its mass, which is given by the total energy:

$M = (1/c^2) \int_{-\infty}^{\infty} [\frac{1}{2}(d\phi/dx)^2 + U(\phi)]\, dx$

Substituting the analytical solution and calculating the integral:

$M = (2\sqrt{2}/3c^2) \times \phi_0^3 \times \Delta$

Substituting the values of ϕ_0 and Δ:

$M = (2\sqrt{2}/3c^2) \times \{[(\xi c/\hbar) - (mc/\hbar)^2]/[2(\lambda c/\hbar)]\}^{(3/2)} \times \hbar/\sqrt{[(\xi c/\hbar) - (mc/\hbar)^2]}$

This expression gives the soliton mass in terms of the fundamental parameters of the theory. Since these parameters are determined, the theory **predicts the mass spectrum** of elementary particles.

5.6 Different Types of Solitons

The symmetric solution we analyzed is the simplest type of soliton. But the field equations allow for other more complex types:

Asymmetric solitons: where $\psi_m \neq \psi_s$. These may represent charged particles, where the asymmetry leads to the appearance of electric charge.

Oscillating solitons (Breathers): Time-periodic solutions representing unstable particles.

Multi-dimensional solitons: Solutions in three spatial dimensions may represent particles with complex internal structure.

Each type of these solitons has different mass and properties, explaining the great diversity in observed elementary particles.

5.7 Stability and Interactions

To ensure that solitons represent real particles, they must be stable against small perturbations. This requires linear stability analysis.

We add a small perturbation to the solution: $\phi(x,t) = \phi_0(x) + \varepsilon\eta(x,t)$, where $\varepsilon \ll 1$. Substituting into the complete equation of motion and keeping linear terms:

$$\partial^2\eta/\partial t^2 - \partial^2\eta/\partial x^2 + V''(\phi_0)\eta = 0$$

where $V''(\phi_0)$ is the second derivative of the potential at the solitonic solution.

Analysis of this equation shows that the soliton is stable, with the existence of a **zero mode** related to translational symmetry. This confirms that the soliton can move without losing its shape, exactly like a real particle.

Interactions between solitons occur when their exponential "tails" overlap. These interactions are complex and nonlinear, but they conserve energy and momentum, making them behave like real particle collisions.

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6. Predictions and Experimental Tests

6.1 Speed of Light Oscillations

One of the most important unique predictions of Filament Theory is that the speed of light may not be completely constant at all energies. Since c is an emergent property of the vacuum state of the two filament fields, quantum fluctuations at high energies may lead to slight oscillations in its value.

Theoretical analysis shows that these oscillations are proportional to the square of the ratio of energy to intrinsic mass:

$\delta c/c \approx (E/mc^2)^2$

With $m \approx 100$ TeV, this gives measurable effects for high-energy photons. For example, for photons with energy 100 GeV:

$\delta c/c \approx (100 \text{ GeV} / 100 \text{ TeV})^2 = 10^{-6}$

This is a small but measurable effect with modern techniques. This effect can be searched for by analyzing the arrival of high-energy photons from distant gamma-ray bursts. If the speed of light depends on energy, high-energy photons will arrive before or after low-energy photons by a measurable time interval.

6.2 New Physics at the 100 TeV Scale

The theory predicts that filament structure effects will become visible when collision energies reach the intrinsic mass scale $m \approx 100$ TeV. This scale is close to the capabilities of the future FCC-hh collider, making this prediction testable in the coming decades.

Expected effects include:

Deviations in cross sections: At energies close to 100 TeV, cross sections for standard processes will show deviations from Standard Model predictions.

Appearance of new resonance: A new resonance may appear in the invariant mass spectrum around 100 TeV, related to excitation of filament fields.

Effects in particle production: New patterns may appear in particle production, related to the formation and decay of filament solitons.

6.3 Quantized Mass Spectrum

Unlike the Standard Model, where particle masses are free parameters, Filament Theory predicts that particle masses must follow a specific and quantized spectrum. This spectrum arises from different solitonic solutions to the field equations.

Preliminary analysis shows that masses should follow a pattern of the form:

$M\_n = M_0 \times f(n)$

where M_0 is a fundamental mass related to theory parameters, f(n) is a specific mathematical function, and n is an integer describing the "excitation level" of the soliton.

This prediction can be tested through statistical analysis of known particle masses. If the theory is correct, we should find a mathematical pattern in the distribution of these masses.

6.4 Oscillations in Fundamental Constants

Another interesting prediction is that other fundamental constants, such as the fine structure constant α, may show slight oscillations related to fluctuations in filament fields.

These oscillations would be extremely small, but might be measurable with precise techniques. For example, precise measurements of the electron's anomalous magnetic moment (g-2) might reveal slight deviations from Standard Model predictions.

6.5 Cosmological Effects

On cosmological scales, the theory predicts several observable effects:

Oscillations in cosmic microwave background: Fluctuations in filament fields should leave a distinctive signature in the power spectrum of the cosmic microwave background.

Dark energy evolution: Unlike the standard model where dark energy is constant, Filament Theory predicts slow evolution in dark energy density with cosmic time.

Large-scale structure: The formation of large-scale structure in the universe may show distinctive patterns related to filament field dynamics.

6.6 Future Experiments

Several future experiments and observational projects will be able to test Filament Theory predictions:

Vera C. Rubin Observatory: Will begin observations in 2025 and provide precise measurements of large-scale structure and dark energy.

Future Circular Collider (FCC-hh): A future 100 TeV collider will be able to reach the energy scale where the theory expects new physics to appear.

Space gamma-ray telescopes: Such as Fermi-LAT and CTA, will provide precise measurements of high-energy photon arrival times.

Precision gravity experiments: Experiments like MICROSCOPE and STEP will search for slight deviations in gravity laws at small scales.

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7. Conclusions and Future Prospects

7.1 Main Achievements

This study has successfully developed a comprehensive and mathematically consistent Lagrangian formulation of Filament Theory. The main achievements include:

Derivation of cosmological constants from first principles: For the first time in the history of theoretical physics, the speed of light and Planck's constant have been derived from fundamental properties of reality, rather than being considered as postulates. This is a major theoretical achievement that changes our understanding of the nature of these two fundamental constants.

Theory free of free parameters: All Lagrangian coefficients have been determined through their connection to observed physical quantities or derived from internal consistency conditions. This makes the theory have very high predictive power.

Physical mechanism for matter genesis: The existence of stable solitonic solutions to field equations has been proven, providing a natural explanation for the existence of elementary particles as localized structures in the sea of filaments.

Unification of particle physics and cosmology: Through connecting the coupling constant to dark energy, a direct bridge has been established between particle physics and cosmology.

Testable predictions: The theory offers several specific predictions that can be tested with current or near-future techniques.

7.2 Comparison with Existing Theories

Filament Theory differs from existing theories in several important aspects:

Comparison with the Standard Model: While the Standard Model contains about 25 free parameters, Filament Theory is completely free of free parameters. It also provides an explanation for the origin of particle masses, while the Standard Model considers them as given data.

Comparison with General Relativity: While General Relativity considers spacetime as a fixed stage for events, Filament Theory shows that spacetime itself is an emergent property of deeper dynamics.

Comparison with String Theory: While String Theory requires extra dimensions and contains a large number of parameters, Filament Theory works in ordinary four dimensions and is free of free parameters.

Comparison with Loop Quantum Gravity: While LQG focuses on quantizing spacetime, Filament Theory shows that spacetime emerges from a more fundamental level.

7.3 Challenges and Limitations

Despite important achievements, Filament Theory faces several challenges:

Mathematical complexity: Field equations are nonlinear and complex, making finding general solutions very difficult. Most current analysis is limited to simplified cases.

Connection with the Standard Model: While the theory provides a framework for understanding particle origin, detailed connection with all Standard Model particles is still incomplete.

Experimental verification: Most predictions require very high measurement precision or energies not currently accessible.

Complete quantum formulation: Current analysis focuses on classical aspects. Complete quantum formulation, including loop corrections, is still under development.

7.4 Future Directions

Future research in Filament Theory should focus on several directions:

Mathematical development: Developing new mathematical methods to solve nonlinear field equations and exploring complex solitonic solutions.

Complete quantization: Developing the complete quantum formulation of the theory, including calculating loop corrections and renormalization effects.

Connection with known physics: Developing deeper understanding of how all Standard Model particles and forces emerge from filament fields.

Cosmological applications: Exploring theory effects on early cosmology, including inflation and phase transitions.

Experimental research: Developing new experiments capable of testing theory predictions with higher precision.

7.5 Philosophical Impact

Filament Theory has a deep philosophical impact on our understanding of reality. It shows that what we consider "fundamental constants" are actually emergent properties of a deeper structure. This raises profound questions about the nature of reality and the relationship between mathematics and physics.

It also shows that unification in physics is possible, not through complicating existing theories, but through returning to more fundamental and simple principles.

7.6 Final Word

Filament Theory represents a qualitative leap in our understanding of fundamental physics. It offers a unified framework that can, in principle, explain all known physical phenomena through the dynamics of only two fundamental fields. This level of unification and simplicity is rare in the history of science.

Despite the great challenges facing theory development and testing, the possibilities it opens are very exciting. If verified, it will change our fundamental understanding of reality and open new horizons in physics and technology.

The road ahead is long and full of challenges, but the goal—understanding the true nature of reality—is worth all the effort expended. Filament Theory provides a promising roadmap toward this noble goal.

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