Lab-2: DC Motor Control – PD Position Control

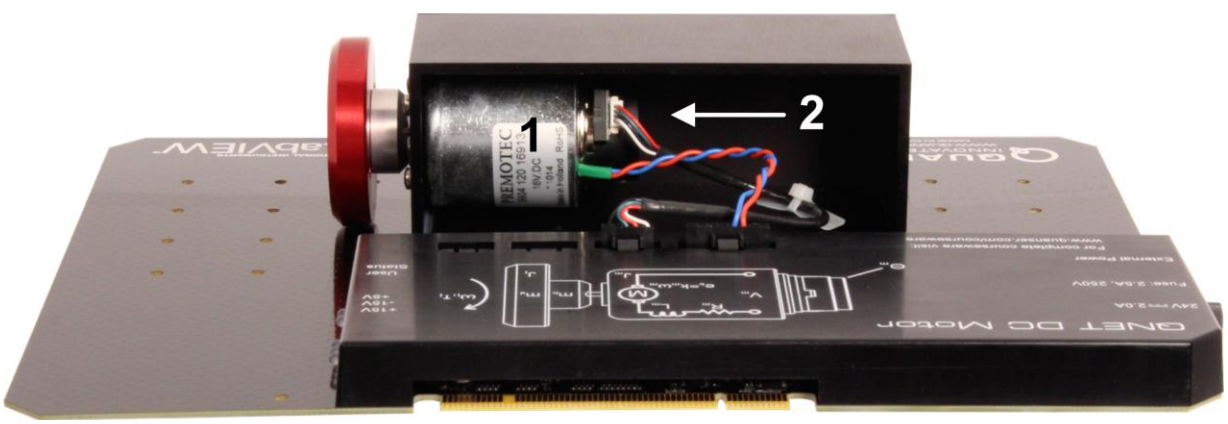
Topics Covered

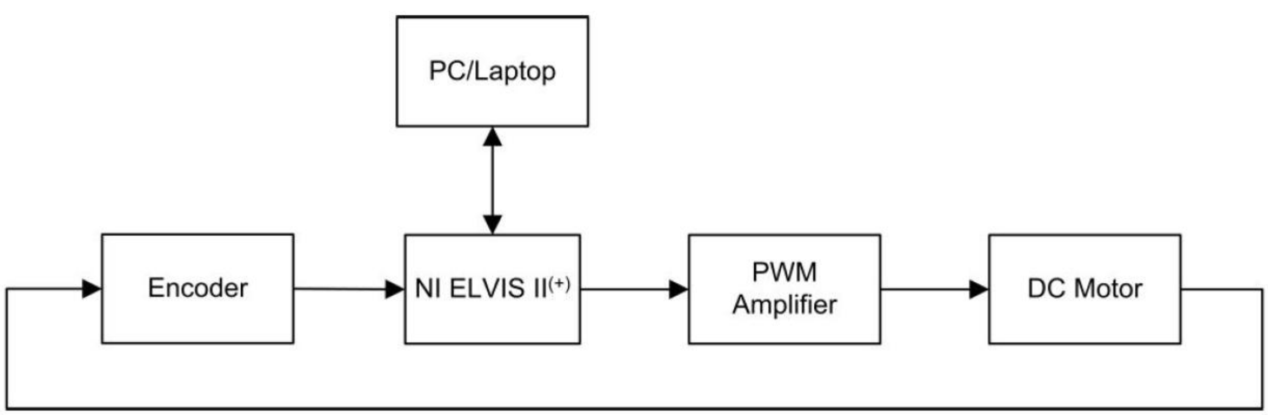
- QNET DC Motor position control.
- Proportional-derivative (PD) compensator design.
- Designing control according to specifications.

Prerequisites

- QNET DC Motor Qualitative PD Position Control laboratory experiment.
- Modeling and Qualitative PI Speed Control

Necessary Equipment:
- Labview software
- NI Elvis II+ board
- Quanser DC Motor
- 1 optical encoder
- Power module





1 Background

Control of motor position is a natural way to introduce the benefits of derivative action. In this experiment a proportional-integral-derivative controller is designed according to specifications. The closed-loop PID control block diagram is shown in Figure 1.1.

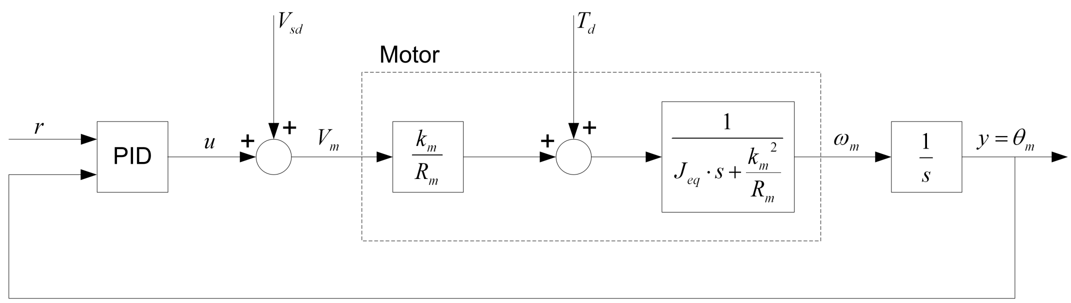


Figure 1.1: DC Motor PID closed-loop block diagram

The two-degree of freedom PID transfer function inside the PID block in Figure 1.1 is

$$u = k_p \left(b_{sp} r(t) - y(t) \right) + k_i \int_0^t \left(r(\tau) - y(\tau) \right) d\tau + k_d \left(b_{sd} \dot{r}(t) - \dot{y}(t) \right), \tag{1.1}$$

where k_p is the position proportional control gain, k_d is the derivative control gain, k_i is the integral control gain, b_{sp} is the set-point weight on the reference position $r(t)$, and b_{sd} is the set-point weight on the velocity reference of $r(t)$.

The dotted box labeled Motor in Figure 1.1 is the motor model in terms of the back-emf motor constant k_m, the electrical motor armature resistance R_m, and the equivalent moment of inertia of the motor pivot J_{eq}. The direct disturbance applied to the inertial wheel is represented by the disturbance torque variable T_d and the simulated disturbance voltage is denoted by the variable V_{sd}.

1.1 PD Control Design

The behavior of the controlling the motor position is first analyzed using a PD control. By setting $k_i = 0$ in the PID control equation Equation 1.1 and taking its Laplace transform, the PD transfer transfer function is

$$u = k_p(r - y) + k_d s(b_{sd} r - y). \tag{1.2}$$

Combining the position process model

$$\frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \tag{1.3}$$

with the PD control Equation 1.2 gives the closed-loop transfer function of the motor position system

$$G_{\theta,r}(s) = \frac{K \left(k_p + b_{sd} k_d s \right)}{\tau s^2 + (1 + K k_d) s + K k_p}. \tag{1.4}$$

Similarly to PI speed control, the standard characteristic equation $s^2 + 2s\zeta\omega_n + \omega_n^2$ can be achieved by setting the proportional gain to

$$k_p = \frac{\omega_0^2 \tau}{K}, \tag{1.5}$$

and the derivative gain to

$$k_d = \frac{-1 + 2\zeta\omega_0\tau}{K}. \tag{1.6}$$

1.2 Peak Time and Overshoot

The standard second-order transfer function has the form

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{1.7}$$

where ω_n is the natural undamped frequency and ζ is the damping ratio. The properties of its response depend on the values of the ω_n and ζ parameters. Consider when a second-order system, as shown in Equation 1.7, is subjected to a step input given by

$$R(s) = \frac{R_0}{s} \tag{1.8}$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 1.2, where the red trace is the response (output) $y(t)$ and the blue trace is the step input $r(t)$.

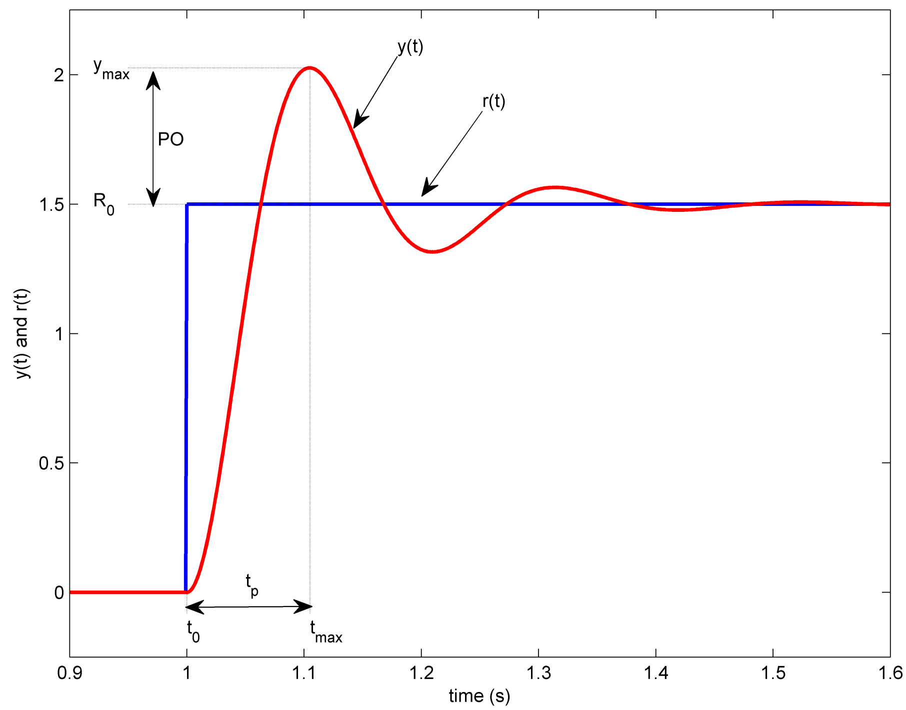


Figure 1.2: Standard second-order step response.

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 1.2, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}. \tag{1.9}$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100 e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}. \tag{1.10}$$

From the initial step time t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0. \tag{1.11}$$

This is called the peak time of the system and it depends on both the damping ratio and natural frequency of the system. It can be derived analytically as

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}. \tag{1.12}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

1.3 Position Control Virtual Instrument

Tracking a reference position square wave using PID control is first examined in this laboratory. Then, disturbance effects using PD and PID are studied through direct manual interaction or a simulated using a control switch in the VI. The LabVIEW™ virtual instrument for position control is shown in Figure 1.3.

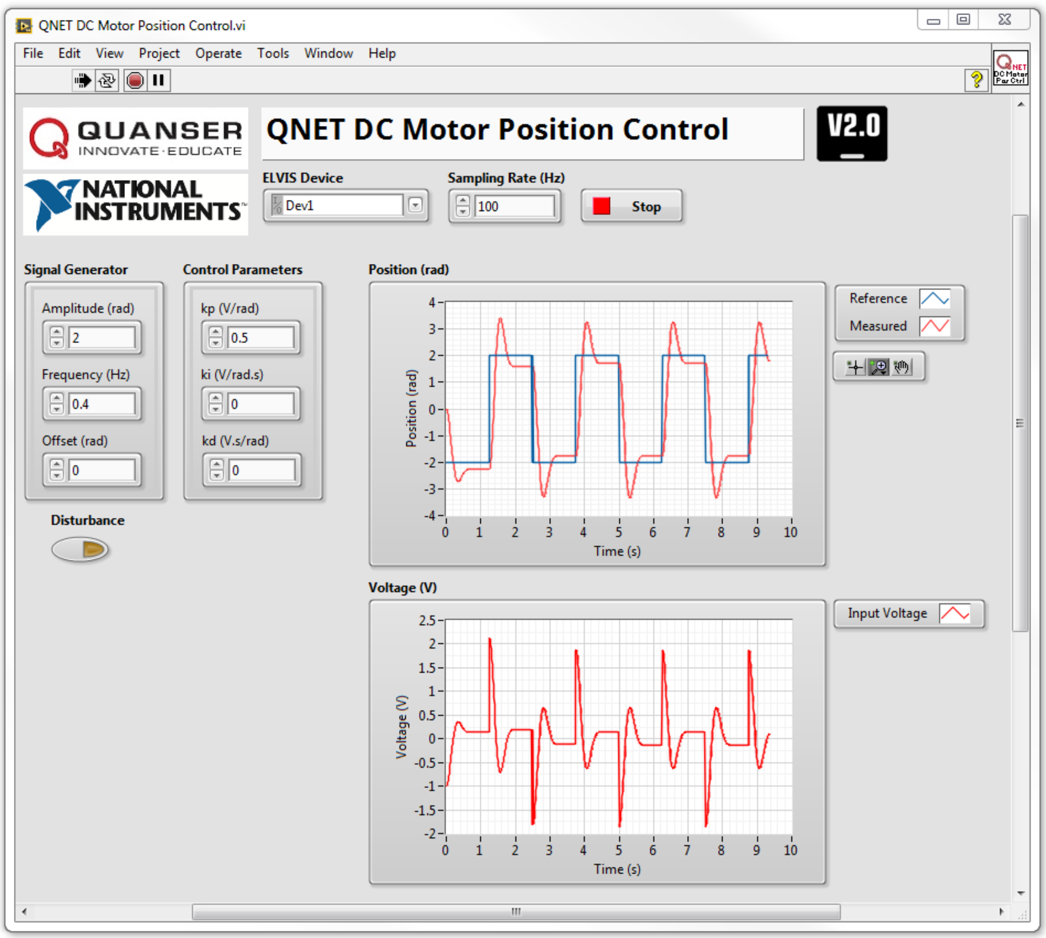


Figure 1.3: Virtual instrument for DC motor position control

See Wikipedia for more information on motion control, control theory and PID.

2 In-Lab Exercise

R 1. Calculate the expected peak time t_p and percentage overshoot PO given

- $\zeta = 0.60$,
- $\omega_0 = 25.0$ rad/s.

Optional: You can also design a VI that simulates the DC motor first-order model with a PD control and have it calculate the peak time and overshoot.

R 2. Assuming the model steady-state gain is $K = 26$ V/rad and time constant is $\tau = 0.145$ s, calculate the proportional and derivative control gains k_p and k_d, respectively, to meet the specifications above.

3. Open the `QNET DC Motor Position Control.vi`. **Make sure the correct Device is chosen.**

4. Run the VI. You should see the DC motor rotating back and forth.

5. In the *Signal Generator* section set:
   - Amplitude (rad) = 0.50
   - Frequency (Hz) = 0.40
   - Offset (rad) = 0.00

6. In the Control Parameters section, set the PD gains to the values found in Step 2. The PD controller is implemented with $b_{sd} = 0$.

R 7. Capture the position response found in the Position (rad) scope and control signal used in the Voltage (V) scope.

R 8. Measure the peak time and percentage overshoot of the measured position response. Are the specifications satisfied?

R 9. What effect does changing the specification zeta have on the measured position response and the generated control gains?

R 10. What effect does changing the specification ω_0 have on the measured position response and the generated control gains?

11. Stop the VI by clicking on the Stop button.

3 Lab Report

I. PROCEDURE
1. *PD Control According to Specifications*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experimental procedure in Step 7 in Section 2.
   - Effect of changing damping ratio and natural frequency specifications in Step 9-10 in Section 2.

II. RESULTS
Do not interpret or analyze the data in this section. Just provide the results.
1. Position control response plot from step 7 in Section 2.
2. PD disturbance response plot.
3. Provide applicable data collected in this laboratory from Table 1.

Table 1: DC motor Position Control results summary

Description	Symbol	Value	Unit
Section 2: PD Control Design			
Model gain used	K		rad/s
Model time constant used	τ		s
Proportional gain	k_p		V/(rad/s)
Integral gain	k_i		V/rad
Measured peak time	t_p		s
Measured percent overshoot	M_p		%

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:
1. Provide position control analysis with steady-state error.
2. Peak time and percent overshoot of speed control response in Step 8 in Section 2.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:
1. Whether the speed controller meets the specifications in Step 8 in Section 4.4.
2. Does the measured steady-state error using a PD control match what is expected.
3. Does the measured steady-state error using a PD control match what is expected when there is a disturbance