Lab-2: DC Motor Control – PI Speed Control

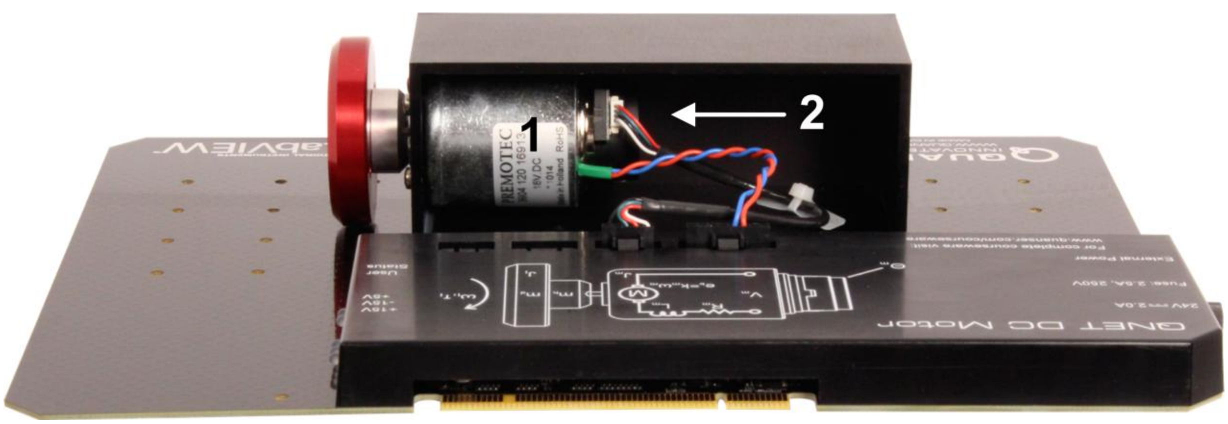
Topics Covered

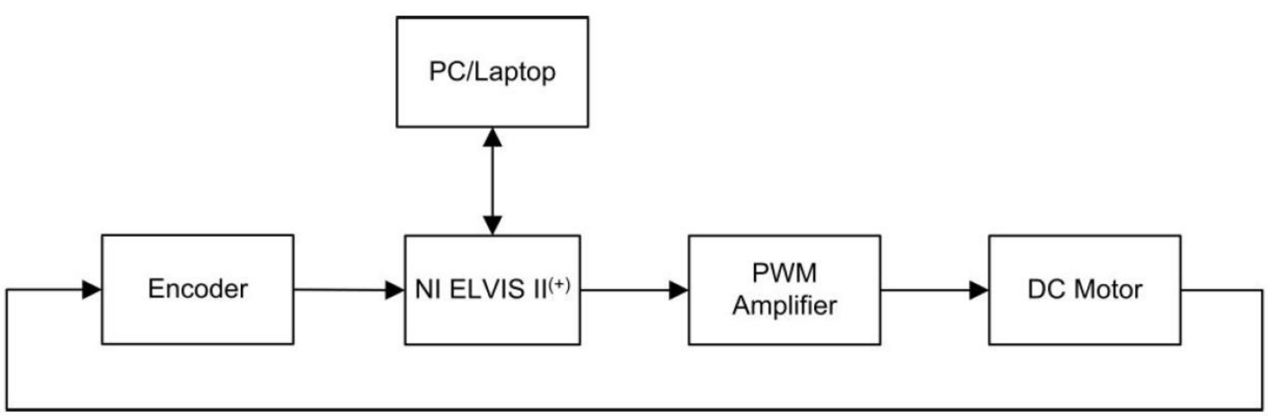
- QNET DC Motor position control.
- Proportional-integral (PI) compensator design.
- Designing control according to specifications.

Prerequisites

- QNET DC Motor Qualitative PI Speed Control laboratory experiment.

Necessary Equipment:
• Labview software
• NI Elvis II+ board
• Quanser DC Motor
• 1 optical encoder
• Power module





1 Background

The speed of the DC motor is controlled using a proportional-integral control system. The block diagram of the closed-loop system is shown in Figure 1.1.

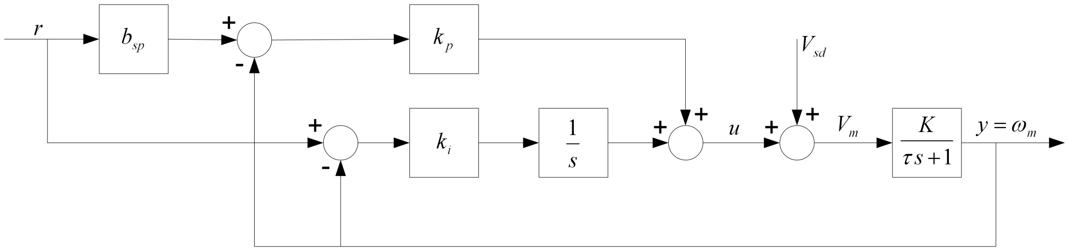


Figure 1.1: QNET DC Motor PI closed-loop block diagram

The transfer function representing the DC motor speed-voltage relation with steady-state gain K and time constant τ is

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \tag{1.1}$$

and will be used to design the PI controller. The input-output relation in the time-domain for a PI controller with set-point weighting is

$$u = k_p \left(b_{sp} r - y\right) + \frac{k_i \left(r - y\right)}{s}, \tag{1.2}$$

where k_p is the proportional gain, k_i is the integral gain, and b_{sp} is the set-point weight. The closed loop transfer function from the speed reference r to the angular motor speed output ω_m is

$$G_{\omega,r}(s) = \frac{K\left(k_p b_{sp} s + k_i\right)}{\tau s^2 + (K k_p + 1)s + K k_i}. \tag{1.3}$$

The standard desired closed loop characteristic polynomial is

$$s^2 + 2\zeta\omega_0 + \omega_0^2, \tag{1.4}$$

where ω_0 is the undamped closed loop frequency and ζ is the damping ratio. The denominator of the transfer function in Equation 1.3 is the characteristic equation of the system and matches the desired characteristic equation in Equation 1.4 with the following gains:

$$k_p = \frac{-1 + 2\zeta\omega_0\tau}{K} \tag{1.5}$$

and

$$k_i = \frac{\omega_0^2 \tau}{K}. \tag{1.6}$$

Large values of ω_0 give large values of controller gain. The damping ratio, ζ, and the set-point weight parameter, b_{sp}, can be used to adjust the speed and overshoot of the response to reference values.

There is no tachometer sensor present on the QNET DC Motor system that measures the speed. Instead the QNET DC Motor board has circuitry that computes the derivative of the encoder signal, i.e. a digital tachometer.

1.1 Peak Time and Overshoot

The standard second-order transfer function has the form

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{1.7}$$

where ω_n is the natural undamped frequency and ζ is the damping ratio. The properties of its response depend on the values of the ω_n and ζ parameters. Consider when a second-order system, as shown in Equation 1.7, is subjected to a step input given by

$$R(s) = \frac{R_0}{s} \tag{1.8}$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 1.2, where the red trace is the response (output) $y(t)$ and the blue trace is the step input $r(t)$.

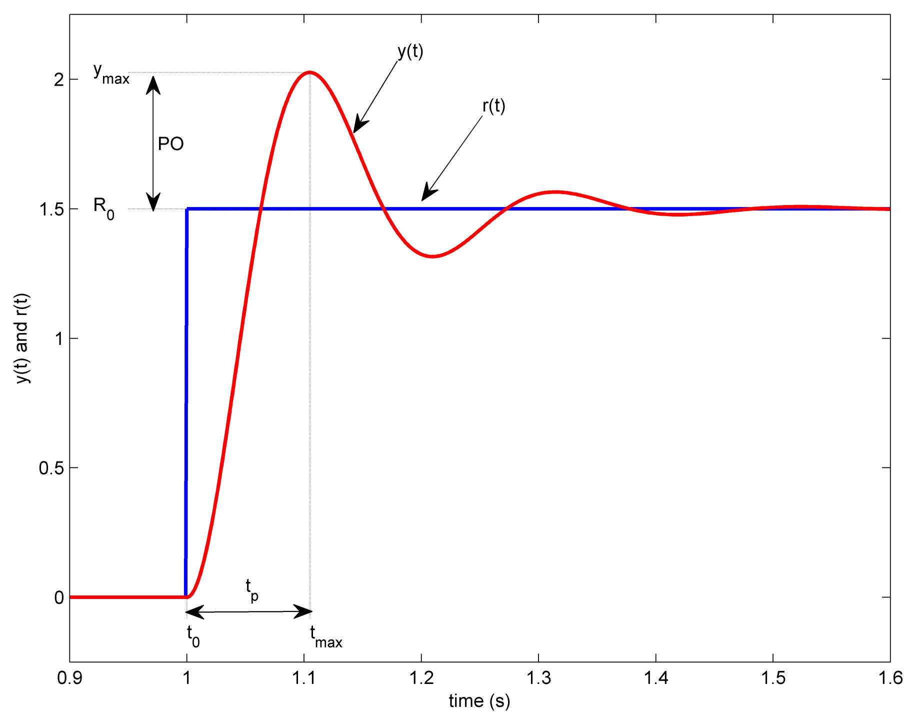


Figure 1.2: Standard second-order step response.

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 1.2, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}. \tag{1.9}$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100 e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}. \tag{1.10}$$

From the initial step time t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0. \tag{1.11}$$

This is called the peak time of the system and it depends on both the damping ratio and natural frequency of the system. It can be derived analytically as

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}. \tag{1.12}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

1.2 Speed Control Virtual Instrument

In the following laboratory, you will tracking a square wave with various PI gains as well as investigate the effects of set-point weights. The virtual instrument for speed control is shown in Figure 1.3.

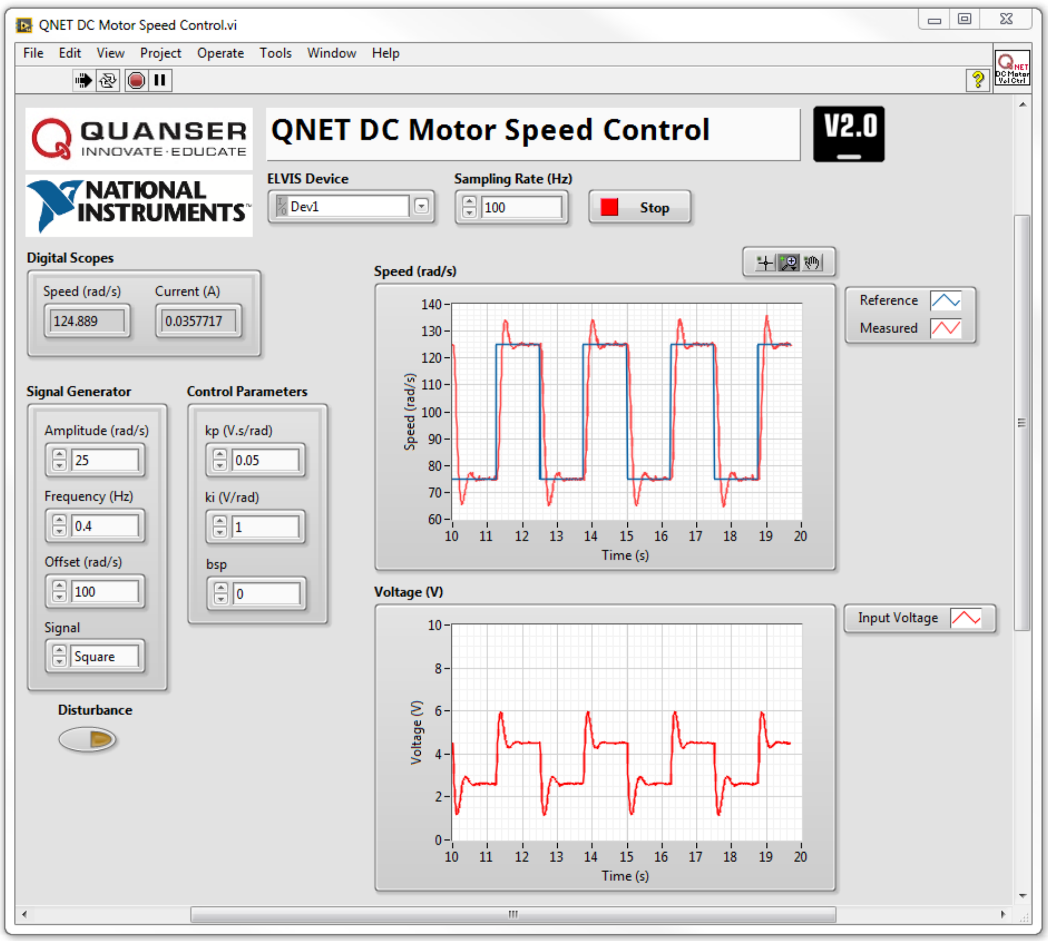


Figure 1.3: Virtual instrument for QNET DC Motor speed control

2 In-Lab Exercise

2.1 PI Control According to Specifications

R 1. Calculate the expected peak time t_p and percent overshoot PO given the following design specifications:

- $\zeta = 0.75$,
- $\omega_0 = 16.0$ rad/s.

Optional: You can also design a VI that simulates the QNET DC Motor first-order model with a PI control and have it calculate the peak time and overshoot.

R 2. Calculate the proportional and integral control gains k_p and k_i, respectively, according to the design specifications for the model parameters $K = 26.0$ rad/(V.s) and $\tau = 0.145$ s.

3. Ensure the `QNET DC Motor Speed Control.vi` is open. **Make sure the correct Device is chosen.**
4. Run the VI. The DC motor should begin spinning and the scopes should look similar to Figure 1.3.
5. In Signal Generator set:
   - Amplitude (rad/s) $= 25.0$ rad/s
   - Frequency (Hz) $= 0.40$ Hz
   - Offset (rad/s) $= 100.0$
   - Signal ='Square'
6. In the Control Parameters section, enter the PI control gains found in Step 2 and make sure bsp = 0.
7. Stop the VI when you collected two sample cycles by clicking on the Stop button.

R 8. Capture the measured speed response. Make sure you include both the Speed (rad/s) and the control signal Voltage (V) scopes.

R 9. Measure the peak time and percentage overshoot of the observed response. Are the specifications satisfied? If they are not, adjust the proportional gain k_p and integral gain k_i to meet the specifications and capture your system response plots. What gains did you use?

R 10. What effect does increasing the specification ζ have on the measured speed response? How about on the control gains?

Hint: Start by examining Equation 1.10.

R 11. What effect does increasing the specification w0 have on the measured speed response and the generated control gains?

Hint: Start by examining Equation 1.12.

12. Stop the VI by clicking on the Stop button.

2.2 Set-Point Weight

1. Ensure the `QNET DC Motor Speed Control.vi` is open. **Make sure the correct Device is chosen.**
2. Run the VI. The DC motor should begin rotating.
3. In the Signal Generator section set:
   - Amplitude (rad/s) $= 25.0$

- Frequency (Hz) $= 0.40$
- Offset (rad/s) $= 100.0$
- Signal = 'Square'

4. In the Control Parameters section set:

    - kp (V.s/rad) $= 0.10$
    - ki (V/rad) $= 1.50$
    - bsp $= 0.00$

5. Increment the set-point weight parameter bsp in steps of 0.05. Vary the parameter between 0 and 1.

R 6. Examine the effect that raising b_{sp} has on the shape of the measured speed signal in the Speed (rad/s) scope. Explain what the set-point weight parameter is doing.

7. Stop the VI by clicking on the Stop button.

3 Lab Report

I. PROCEDURE

1. *PI Control According to Specifications*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experimental procedure in Step 8 in Section 2.1.
   - Effect of changing damping ratio specification in Step 10 in Section 2.1.
   - Effect of changing natural frequency specification in Step 11 in Section 2.1.
2. *Set-Point Weight*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in Step 6 in Section 2.2.

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.
1. Speed control response plot from step 8 in Section 2.1.
2. Provide applicable data collected in this laboratory from Table 1.

Table 1: DC motor Speed Control results summary

Description	Symbol	Value	Unit
Section 2.1: PI Control Design			
Model gain used	K		rad/s
Model time constant used	τ		s
Proportional gain	k_p		V/(rad/s)
Integral gain	k_i		V/rad
Measured peak time	t_p		s
Measured percent overshoot	M_p		%

III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:
1. Explain effect of changing proportional and integral gains..
2. Peak time and percent overshoot of speed control response in Step 9 in Section 2.1.
3. Effect of changing set-point weight in Step 6 in Section 2.2.

IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions for the following:
1. Whether the speed controller meets the specifications in Step 7 in Section 2.1.
2. Explain why there is steady-state error in the system in Step 5 of Section 3.6.