Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$
  $\equiv \sim p \vee (r \wedge q)$
  $\equiv p \rightarrow (r \wedge q)$

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

  There is no real number x such that x = x + 1. So ∃x Q(x) is false for R.

Quantifiers

- $P(x) : x^2 + 1 < 10$, $D = \{1, 2, 3\}$

  What is the truth value of $\forall x\, P(x)$ if the domain is D?

  If the domain consists of n elements,
  then $\forall x\, P(x) \equiv P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

  $P(1) : 2 < 10$, true
  $P(2) : 5 < 10$, true
  $P(3) : 10 < 10$, false

  Since $1 \wedge 1 \wedge 0 \equiv 0$, then $\forall x\, P(x)$ is false for D.

Quantifiers

- $Q(x) : x^2 < 3$, $D = \{1, 2, 3\}$

  What is the truth value of $\exists x\, Q(x)$ if the domain is D?

If the domain consists of n elements,
then $\exists x\, P(x) \equiv P(x_1) \lor P(x_2) \lor \ldots \lor P(x_n)$

$P(1) : 1 < 3$, true
$P(2) : 4 < 3$, false
$P(3) : 9 < 3$, false

Since $1 \lor 0 \lor 0 \equiv 1$, then $\exists x\, P(x)$ is true for D.

Quantifiers

- Every student in this class has entered the entrance exam

  $\forall x\ P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

$$\begin{aligned} \sim(\forall x\ P(x)) &\equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)) \\ &\equiv \sim P(x_1) \vee \sim P(x_2) \vee \ldots \vee \sim P(x_n) \\ &\equiv \exists x \sim P(x) \end{aligned}$$

Quantifiers

- There is a student in this class who has taken the entrance exam.

  $\exists x\, P(x)$, ' x has taken the entrance exam'

Negation

- It's not the case that There is a student in this class who has taken the entrance exam

  None of the students in this class has taken the entrance exam.

$$\begin{aligned}
\sim(\exists x\, P(x)) &\equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)) \\
&\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \ldots \wedge \sim P(x_n) \\
&\equiv \forall x \sim P(x)
\end{aligned}$$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists x \sim(x^2 > x)$
  $\equiv \exists x\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv \forall x \sim(x^2 = 7)$
  $\equiv \forall x\ x^2 \neq 7$

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

- For two integers x and y, if $x > 0$ and $y > 0$, then $\mathrm{x} + y > 0$

$$(x > 0) \wedge (y > 0) \rightarrow (x + y > 0)$$

$$\forall x\, \forall y\, ((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x \, \exists y \, (x + y = 6)$$

or

$$\exists y \, \exists x \, (x + y = 6)$$

- $\forall x \, \exists y \, (x + y = 6)$

  For every integer x, there exists an integer y such that $x + y = 6$ (It's true)

- $\exists y \, \forall x \, (x + y = 6)$

  There exists an integer y so that for all integers x, $x + y = 6$ (It's false)