Solution for Midterm 2

1. Use mathematical induction to prove that $\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\mathrm{i}(\mathrm{i}+1)} = \frac{\mathrm{n}}{\mathrm{n}+1}$ for every positive integer n.

Solution *P(n) :* $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

Basic Step, P(1) : $\frac{1}{1.2} = \frac{1}{1+1} = \frac{1}{2}$
Inductive Step, $P(k) \rightarrow P(k+1)$

assume $P(k)$ *is true, i.e* $\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$

$$\left[\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}\right] \rightarrow \left[\sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}\right]$$
$$\rightarrow \left[\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)}\right]$$
$$\rightarrow \left[\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{k+1}{k+2}\right], \textit{thus } P(k+1) \textit{is also true}$$

2. What value is returned by the following algorithm? What is its basic operation? How many times is the basic operation executed? Give the worst-case running time of the algorithm using Big Oh notation.

Maradona (n)
input *: a positive integer n*
r ← 0
for i = 1 to n
    for j = i + 1 to n
        for k = i + j − 1 to n
            r ← r + 1
return r

Solution

basic operation : r ← r + 1 (incrementing r at each step)

the basic operation executed

$$T(n) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i+j-1}^{n} 1 = \frac{n^3-n}{3} = O(n^3) \textit{ times}$$

the algorithm returns the value

$$\frac{n^3-n}{3}$$