COM234 ELECTRONICS

Kirchhoff’s Law

Kirchhoff's Laws

For circuit analysis Ohm's law may not be enough to provide a complete solution

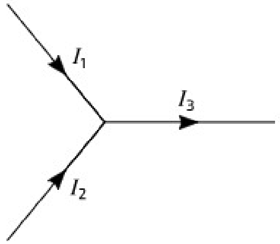
Kirchhoff's laws are fundamental to circuit theory. They quantify how current flows through a circuit and how voltage varies around a loop in a circuit.

1. *Kirchhoff's current law (1st Law) states that current flowing into a node (or a junction) must be equal to current flowing out of it. This is a consequence of charge conservation.*
2. *Kirchhoff's voltage law (2nd Law) states that the sum of all voltages around any closed loop in a circuit must equal zero. This is a consequence of charge conservation and also conservation of energy.*

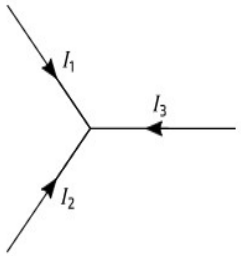
Kirchhoff's Current Law (KCL)

(Conservation of Charge)

Kirchhoff's current law states that for the diagram below, the currents in the three wires must be related by



The standard way of displaying Kirchhoff's current law is by having all currents either flowing towards or away from the node, as shown below:



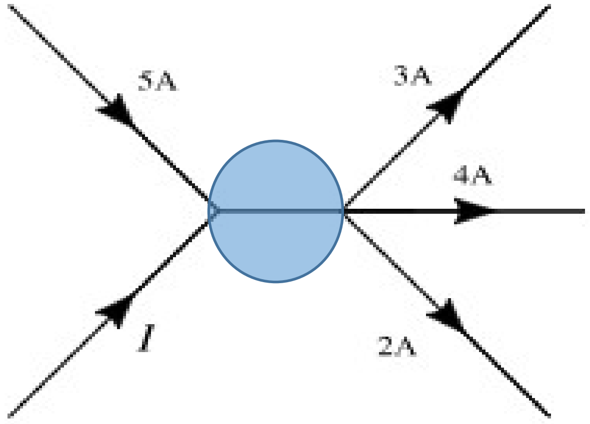
Here, at least one of the currents will have to be negative (i.e, away from the node and in the opposite direction to the arrows on this diagram)and Kirchhoff's current law can be written as:

This can be generalized to the case with n wires all connected at a node by writing:

Kirchhoff's Current Law (KCL)

What is the value of I in the circuit segment shown below ?

What goes in must come out,
or
the total coming in is zero

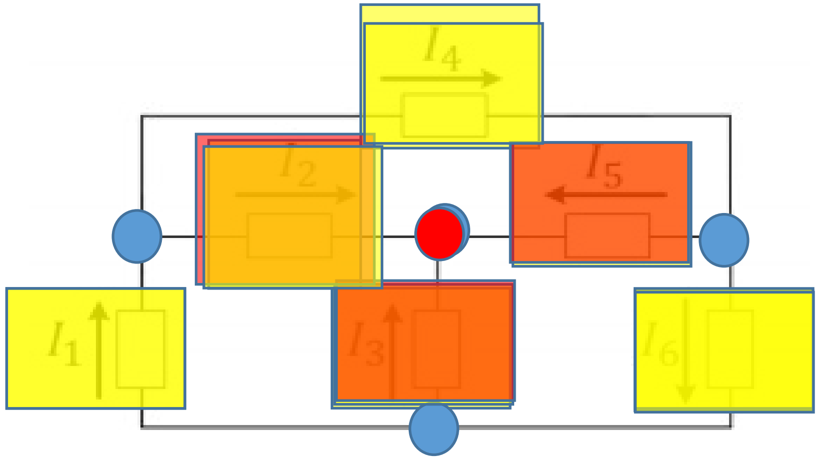


Current in = Current out
Conservation of Charge

Kirchhoff's Current Law (KCL)

The algebraic sum of all the currents at any node in a circuit equals zero. !!!

KCL equations are often used at nodes, but can also be used for a sub-circuit



A. $I_1 = I_2 + I_4$

B. $I_4 = I_5 + I_6$

C. $I_1 + I_3 = I_6$

D. $I_3 + I_5 = I_2$

E. $I_6 - I_4 = I_3 + I_2$

Q1: Which of the equations is NOT a correct application of KCL?

Kirchhoff's Voltage Law (KVL)

(Conservation of Energy)

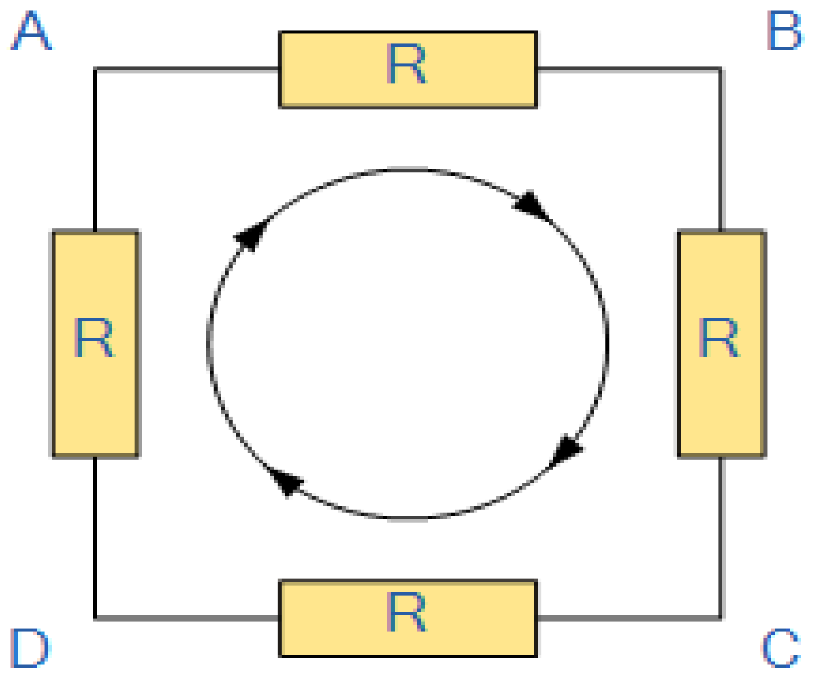
The algebraic sum of ALL the potential differences around the loop must be equal to zero as:

$$\Sigma V = 0$$

Note here that the term "algebraic sum" means to take into account the polarities and signs of the sources and voltage drops around the loop.

Kirchhoff's Voltage Law (KVL)

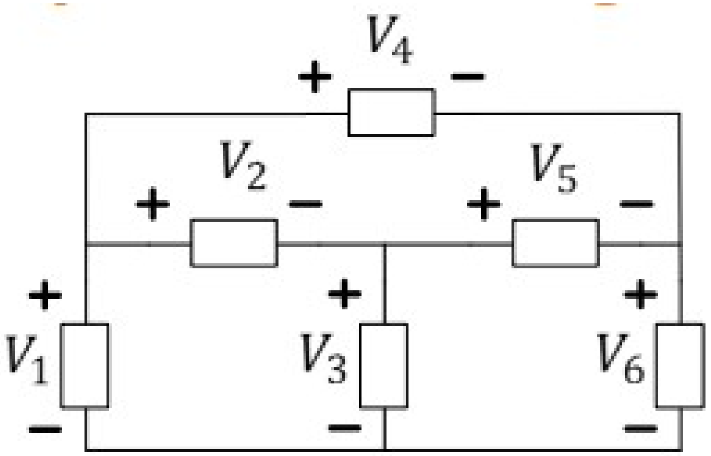
The sum of all the Voltage
Drops around the loop
is equal to Zero



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Kirchhoff's Voltage Law (KVL)

Keeping track of voltage drop ***polarity*** is important in writing correct KVL equations.

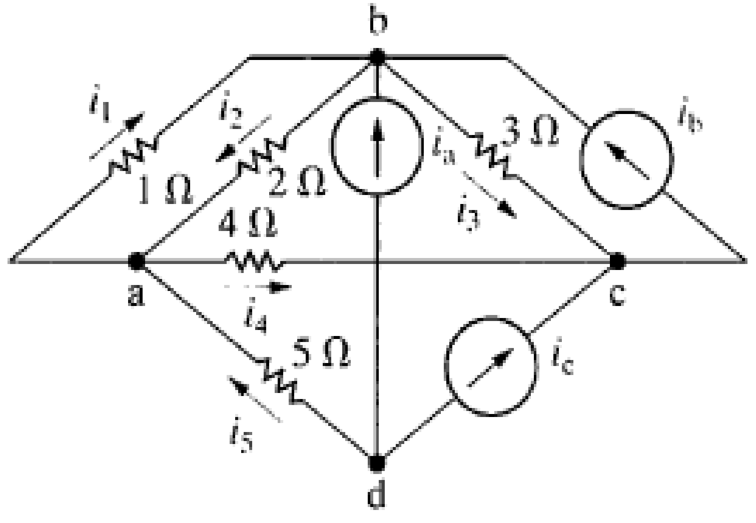


A. $V_1 - V_2 - V_3 = 0$
B. $V_1 = V_2 + V_5 + V_6$
C. $V_1 - V_4 = V_6$
D. $V_3 + V_2 = V_1$
E. $V_3 + V_5 = V_6$

Q2: Which of the equations is NOT a correct application of KVL?

Q3. What are the values of the voltages $V1$, $V2$ and $V6$ if $V3$ = 2 V, $V4$ = 6 V, $V5$ = 1 V?

EXAMPLE (Using KCL)



Question:

Sum the currents at each node in the circuit shown in Fig. Note that there is no connection dot (•) in the center of the diagram, where the 4 ohm branch crosses the branch containing the ideal Current source Ia.

Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

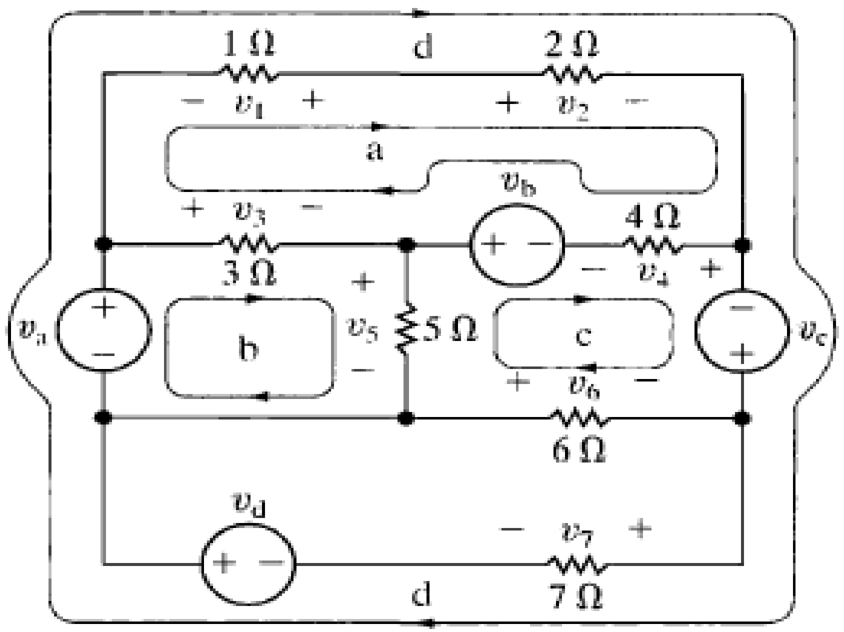
node a $\quad i_1 + i_4 - i_2 - i_5 = 0,$

node b $\quad i_2 + i_3 - i_1 - i_b - i_a = 0,$

node c $\quad i_b - i_3 - i_4 - i_c = 0,$

node d $\quad i_5 + i_a + i_c = 0.$

EXAMPLE (Using KVL



Question:

Sum the voltages around each designated path in the circuit shown in Fig.

Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are

path a $\quad -v_1 + v_2 + v_4 - v_b - v_3 = 0,$

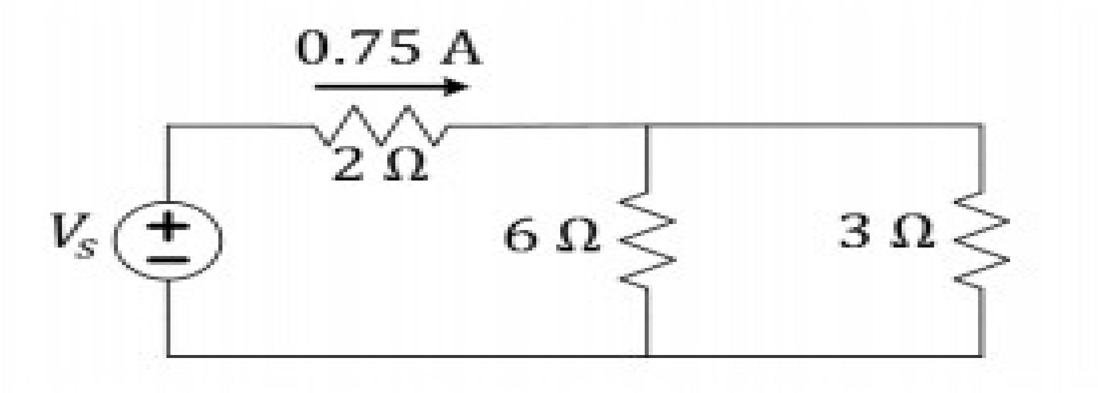
path b $\quad -v_a + v_3 + v_5 = 0,$

path c $\quad v_b - v_4 - v_c - v_6 - v_5 = 0,$

path d $\quad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$

EXAMPLES

- Use KCL, KVL and Ohm's Law to solve below questions



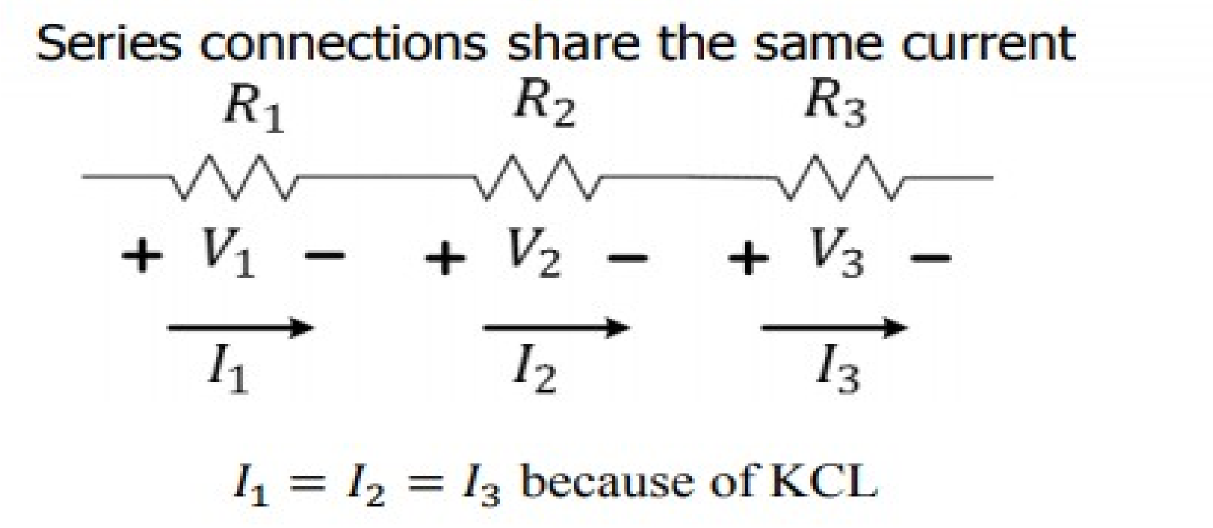
Q4: What is the value of the source voltage?
Q5: How much power is the source supplying?
Q6: How much power is each resistance consuming?

Current and Voltage Dividers

- Series Connections, Equivalent Resistance, Voltage Divider
- Parallel Connections, Equivalent Resistance, Current Divider
- Power Dissipation in Series and Parallel Resistive Loads

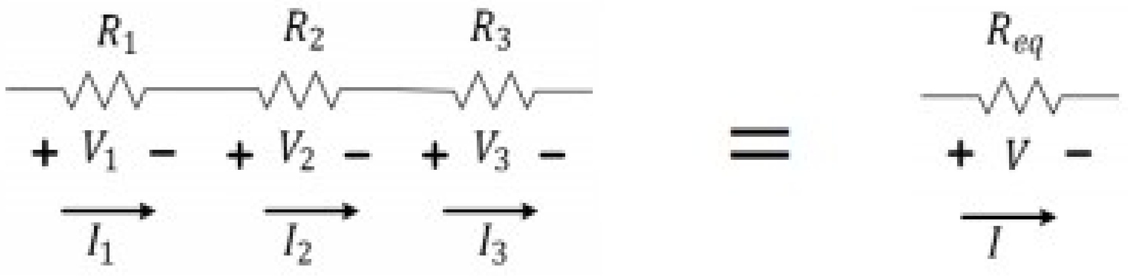
Series Connection

Series connections share the same current



Equivalent Resistance of Series Resistors

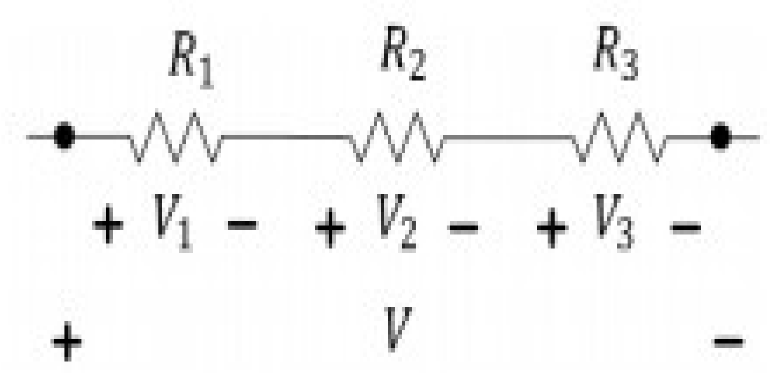
Resistances in series add up

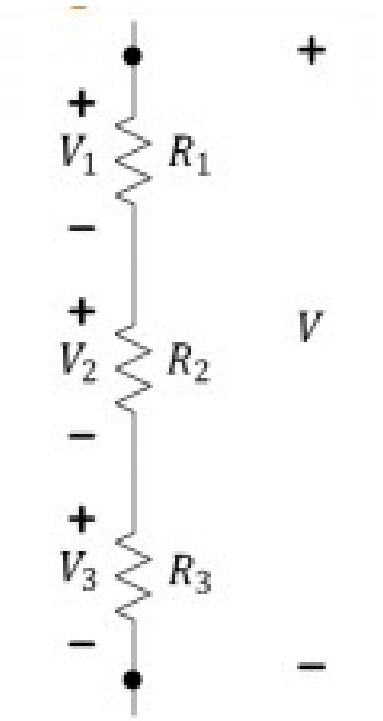


$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Voltage Divider Rule (VDR)

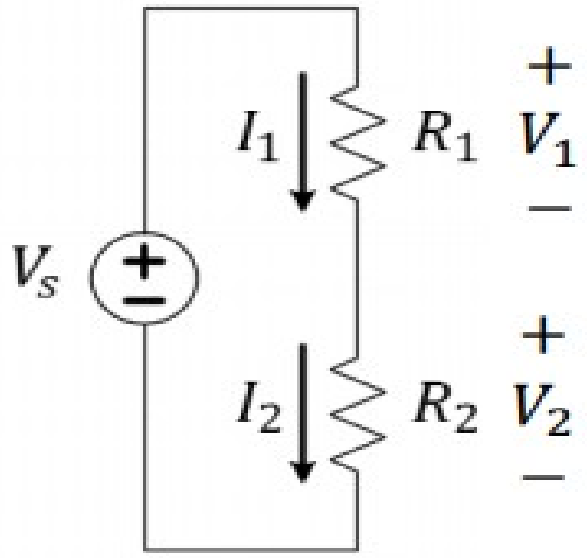
When a voltage divides across resistors in series, more voltage drop appears across the largest resistor





Example

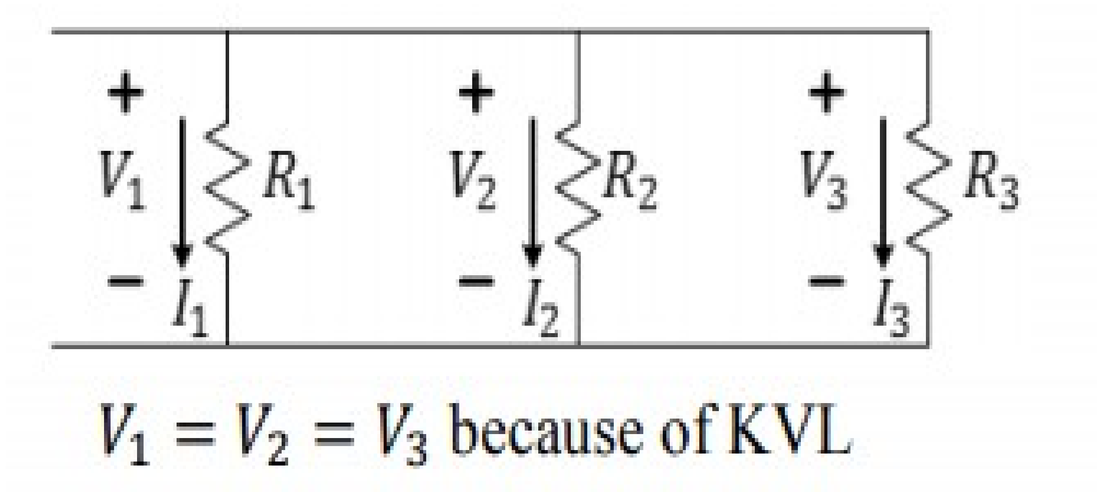
If $R_1 < R_2$, which of the following is true?



A. $V_1 < V_2$ and $I_1 < I_2$
B. $V_1 < V_2$ and $I_1 = I_2$
C. $V_1 = V_2$ and $I_1 = I_2$
D. $V_1 > V_2$ and $I_1 = I_2$
E. $V_1 > V_2$ and $I_1 > I_2$

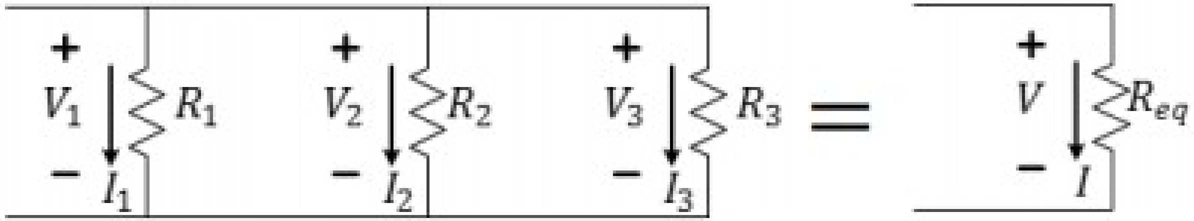
Parallel Connection

Parallel connections share the same voltage potentials at two end nodes (shared by the elements)



Equivalent Resistance of Parallel Resistors

Adding resistance in parallel always brings resistance down!



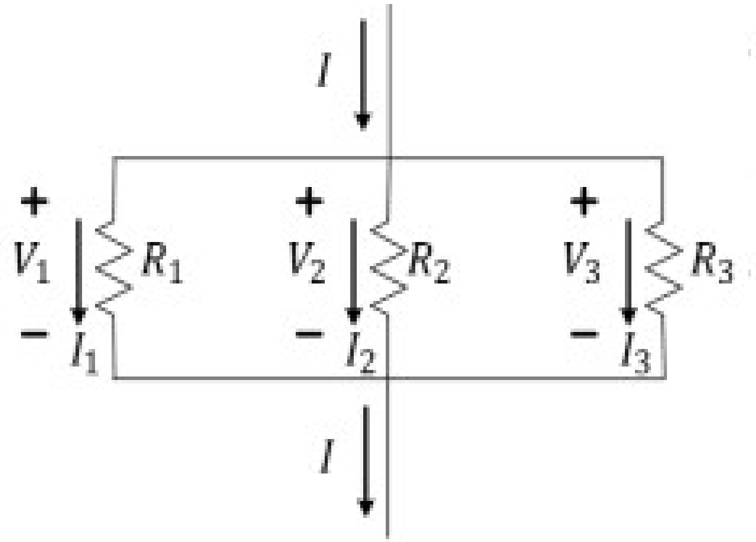
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

If $N = 2$, $\qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Current Divider Rule (CDR)

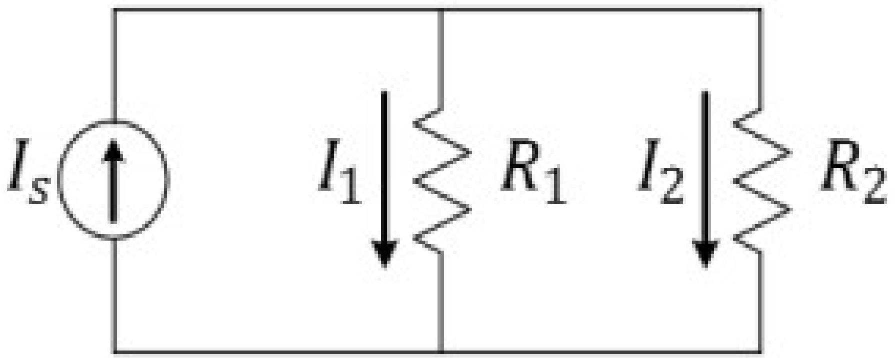
When a current divides into two or more paths, more current will go down the path of lowest resistance.

$$I_k = \frac{R_{eq}}{R_k} \cdot I$$



EXAMPLE

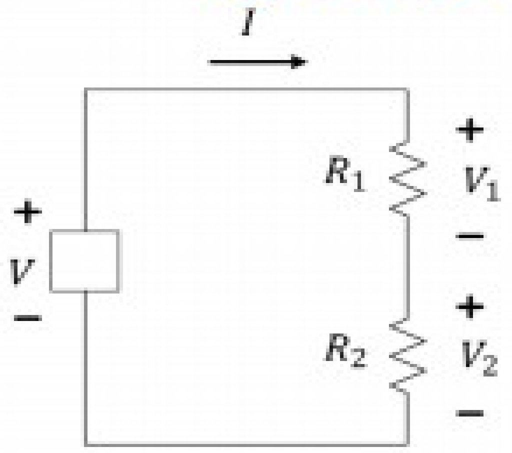
If $R_1 < R_2$, which of the following is true?



A. $I_1 < I_2 < I_s$
B. $I_1 < I_s < I_2$
C. $I_2 < I_1 < I_s$
D. $I_2 < I_s < I_1$
E. $I_s < I_2 < I_1$

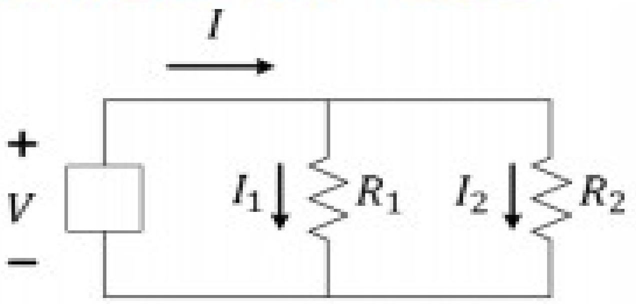
EXAMPLE

VDR and CDR for Two Resistances



$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$



$$I_1 = \frac{R_2}{R_1 + R_2} I \qquad I_2 = \frac{R_1}{R_1 + R_2} I$$

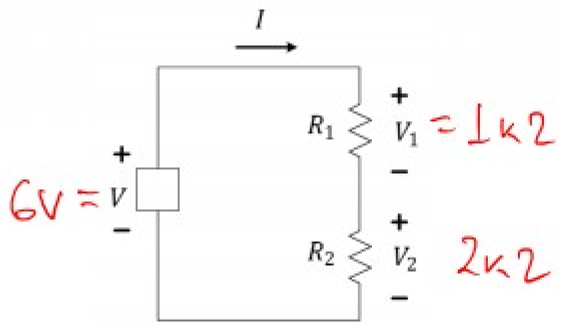
If 6V falls across a series combination of 1kΩ and 2kΩ, what is V across 2kΩ?

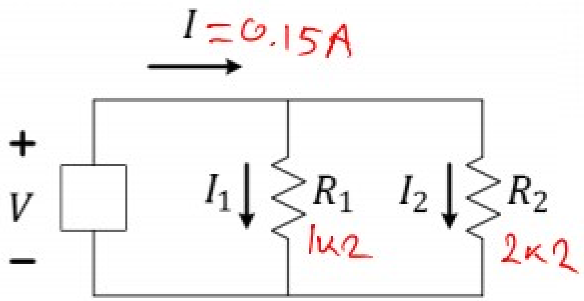
If 0.15A flows through a parallel combo of 1kΩ and 2kΩ, what is I through 2kΩ?

If a source supplies 60W to a series combination of 10Ω and 30Ω, what is the power absorbed by the 10Ω resistor? What is absorbed by the 30Ω resistor?

If a source supplies 300mW to a parallel combination of 3kΩ and 2kΩ, what is the power absorbed by the 3kΩ resistor? What is absorbed by the 2kΩ resistor?

EXAMPLE





If 6V falls across a series combination of 1kΩ and 2kΩ, what is V across 2kΩ?

$V_2 = ?$

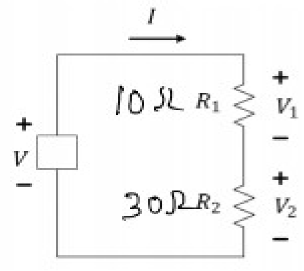
$$V_{2k} = V\frac{R_2}{R_1+R_2} = 6V\frac{2k\Omega}{1k\Omega+2k\Omega} = \underline{\underline{4V}}$$

If 0.15A flows through a parallel combo of 1kΩ and 2kΩ, what is I through 2kΩ?

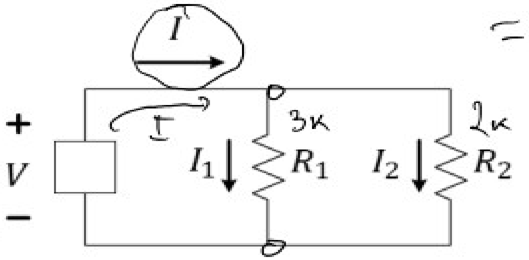
$I_2 = ?$

$$I_{2k} = I\frac{R_1}{R_1+R_2} = (0.15A)\cdot\frac{1k}{1k+2k} = \underline{\underline{0.05A}}$$

EXAMPLE



$P = 300\,mW$



$P_{R2} = I_2 \cdot V =$
$= \frac{3}{5} I V = \frac{3}{5}\, 300\,mW$
$\boxed{= 180\,mW}$

$P_{R1} = I_1 \cdot V =$
$= \frac{2}{5} I \cdot V =$
$= \frac{2}{5} \cdot 300\,mW = \boxed{120\,mW}$

If a source supplies 60W to a series combination of 10Ω and 30Ω, what is the power absorbed by the 10Ω resistor? What is absorbed by the 30Ω resistor?

$P = V \cdot I = 300\,mW$

$I_1 = I \frac{R_2}{R_1 + R_2} = I \frac{2k}{5k}$ $\quad I_2 = I \frac{R_1}{R_1 + R_2} = I \frac{3k}{5k}$

If a source supplies 300mW to a parallel combination of 3kΩ and 2kΩ, what is the power absorbed by the 3kΩ resistor? What is absorbed by the 2kΩ resistor?

Q9 $\quad P = V \cdot I = 60\,W$

$P_{R1} + P_{R2} = 60\,W$

$V_1 = V \frac{R_1}{R_1 + R_2} = V \frac{10}{40}$

$V_2 = V \frac{R_2}{R_1 + R_2} = V \frac{30}{40}$

$P = 60\,W$

$P_{30} = V_2 \cdot I = \frac{30}{40} \cdot V \cdot I = \frac{30}{40} \cdot 60\,W = 45\,W$

$P_{10} = V_1 \cdot I = \frac{10}{40} \cdot V \cdot I = \frac{1}{4}\, 60\,W = 15\,W$

battery → $\boxed{+ 60\,W}$