

Final

The exam should be done individually. You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten. **Solutions must be submitted electronically before 4 pm on August 25.** No credit will be given to solutions obtained verbatim from the Internet or other sources.

1.(20p) Let R be the relation defined on $A = Z \times Z$ in the following way :

$$((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1 \cdot y_1 = x_2 \cdot y_2$$

Determine whether the relation R is an equivalence relation on A or not.

2. (20p) Use mathematical induction to prove $\sum_{i=1}^n i^2 \cdot 2^i = n^2 \cdot 2^{n+1} - n \cdot 2^{n+2} + 3 \cdot 2^{n+1} - 6$ for every positive integer n.

3. (20p) Consider the graphs G and H given with the following adjacency lists:

<u>the graph G</u>	<u>the graph H</u>
1 : 3, 5, 6	A : D, E, F
2 : 4, 5, 6	B : D, E, F
3 : 1, 4, 5	C : D, E, F
4 : 2, 3, 6	D : A, B, C
5 : 1, 2, 3	E : A, B, C
6 : 1, 2, 4	F : A, B, C

Determine whether two given graphs are isomorphic or not. Justify your answer.

4. For the question, first construct an undirected graph G using your student id as follows (*'14670345' will be used here as an example to show you how the graph is constructed. Note that if your id contains letters, first remove the letters, then apply the following steps. For instance, if your id is '18YZ0345', consider it as '180345'*):

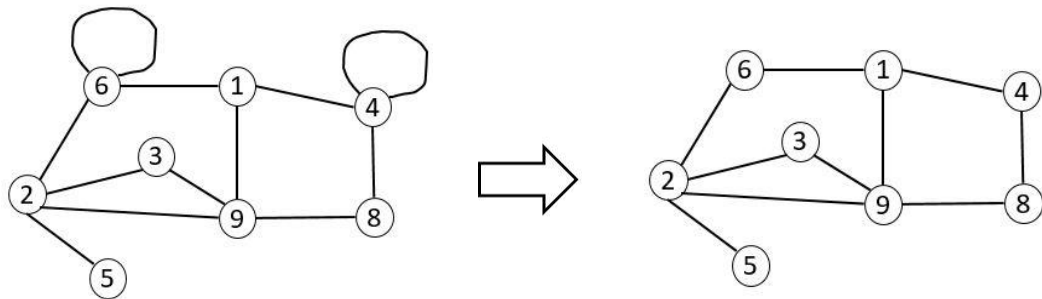
- multiply your id with '98765'

$$14670345 * 98765 = 1448916623925$$

- consider each consecutive two number as an edge in the graph

$$1 \rightarrow 4 \rightarrow 4 \rightarrow 8 \rightarrow 9 \rightarrow 1 \rightarrow 6 \rightarrow 6 \rightarrow 2 \rightarrow 3 \rightarrow 9 \rightarrow 2 \rightarrow 5$$

- remove the reflexive edges (having same starting and ending node) from the corresponding graph



- (10p)** Give the adjacency matrix of the graph G
- (10p)** Does the graph have an Euler path? If so, give such a path. If not, determine the minimum number edges that must be deleted to form a graph which has an Euler path.
- (10p)** What is the Chromatic number $\chi(G)$ of G ?
- (5p)** What is $\kappa(G)$, the minimum number of vertices in a vertex cut of G ?
- (5p)** What is $\lambda(G)$, the minimum number of edges in an edge cut of G ?