Solution 4

1. Let X be a set and (Y, R_1) be a poset. Consider the set $Fun(X, Y)$ that consists of all the functions defined from the set X to the set Y. Let R_2 be a relation on $Fun(X, Y)$ defined as $\forall f, g \in Fun(X, Y)$, $(f, g) \in R_2$ if $\big(f(a), g(a)\big) \in R_1$, $\forall a \in X$. Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation R_2 satisfies. Justify your answer.

Solution

Since (Y, R_1) is a poset, R_1 is a partial order relation defined on Y.

$\forall f \in Fun(X, Y)$, $\big(f(a), f(a)\big) \in R_1$ *since* R_1 *is reflexive. Then* $(f, f) \in R_2$, R_2 *is reflexive.*

$\forall (f, g) \in R_2$ *such that* $f \neq g$, $\big(f(a), g(a)\big) \in R_1$ *(from the definition). If* $\big(f(a), g(a)\big) \in R_1$, *then* $\big(g(a), f(a)\big) \notin R_1$ *since* R_1 *is anti-symmetric. Then* $(g, f) \notin R_2$, R_2 *is anti-symmetric.*

Since R_2 *is anti-symmetric, it is not symmetric.*

$\forall (f, g), (g, h) \in R_2$, $\big(f(a), g(a)\big), \big(g(a), h(a)\big) \in R_1$ *(from the definition). If* $\big(f(a), g(a)\big), \big(g(a), h(a)\big) \in R_1$, *then* $\big(f(a), h(a)\big) \in R_1$ *since* R_1 *is transitive. Then* $(f, h) \in R_2$, R_2 *is transitive.*

2. What is the maximum possible number of vertices for a connected undirected graph with 19 edges such that each vertex has degree at least 4 ? Draw a graph to demonstrate one possible case.

Solution

Employ the formula $\sum deg(v) = 2|E|$ *to solve the question.* $\sum deg(v) = 38$.
Assume there are 10 vertices, then $|E|$ *must be at least 20 since each vertex has degree at least 4. So, the number of vertices should be less than 10. Thus, the maximum possible number of vertices will be 9. One example satisfying the given values will be the following one with 7 vertices of degree 4 and 2 vertices of degree 5 :*