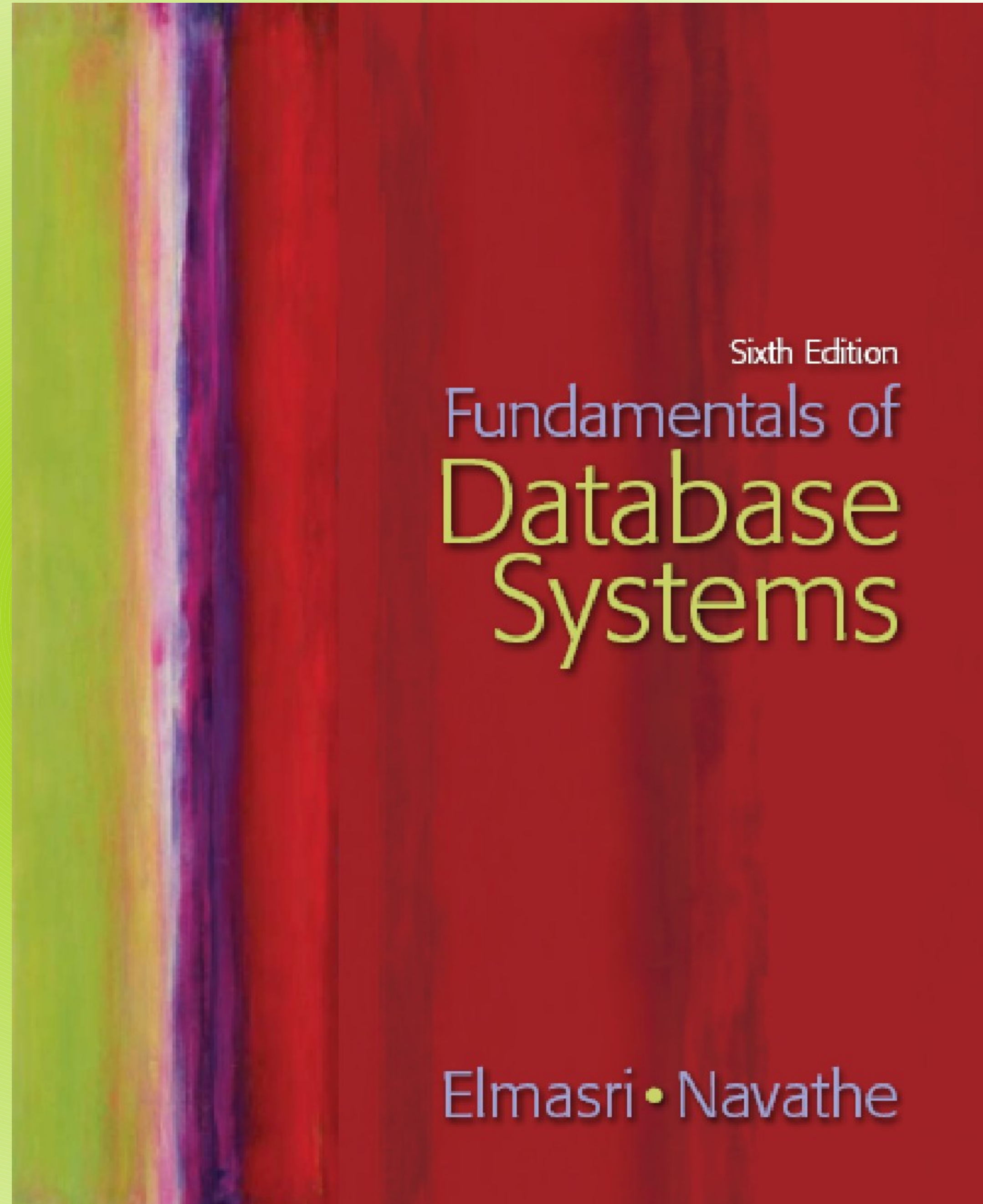
Chapter 6

The Relational Algebra and Relational Calculus



Addison-Wesley
is an imprint of
PEARSON

Chapter 6 Outline

- Unary Relational Operations: SELECT and PROJECT
- Relational Algebra Operations from Set Theory
- Binary Relational Operations: JOIN and DIVISION
- Additional Relational Operations

Chapter 6 Outline (cont'd.)

- Examples of Queries in Relational Algebra
- The Tuple Relational Calculus
- The Domain Relational Calculus

Chapter 6 Outline (cont'd.)

- Examples of Queries in Relational Algebra
- The Tuple Relational Calculus
- The Domain Relational Calculus

The Relational Algebra and Relational Calculus

Relational algebra

Basic set of operations for the relational model

Relational algebra expression

Sequence of relational algebra operations

Relational calculus

Higher-level declarative language for specifying relational queries

Unary Relational Operations: SELECT and PROPERTY

The SELECT Operation

Subset of the tuples from a relation that satisfies a selection condition:

$$\sigma_{<\text{selection condition}>}(R)$$

- Boolean expression contains clauses of the form <attribute name> <comparison op> <constant value>

  or

- <attribute name> <comparison op> <attribute name>

Unary Relational Operations: SELECT and PROJECT (cont'd.)

Example:

$$\sigma_{(\text{Dno}=4\ \textbf{AND}\ \text{Salary}>25000)\ \textbf{OR}\ (\text{Dno}=5\ \textbf{AND}\ \text{Salary}>30000)}(\text{EMPLOYEE})$$

<selection condition> applied independently to each individual tuple *t* in *R*

- If condition evaluates to TRUE, tuple selected

Boolean conditions **AND**, **OR**, and **NOT**

Unary

- Applied to a single relation

Unary Relational Operations: SELECT and PROJECT (cont'd.)

- **Selectivity**
  - Fraction of tuples selected by a selection condition
- SELECT operation commutative

$$\sigma_{<\text{cond1}>}(\sigma_{<\text{cond2}>}(R)) = \sigma_{<\text{cond2}>}(\sigma_{<\text{cond1}>}(R))$$

- **Cascade** SELECT operations into a single operation with **AND** condition

$$\sigma_{<\text{cond1}>}(\sigma_{<\text{cond2}>}(\ldots(\sigma_{<\text{cond}n>}(R))\ \ldots)) = \sigma_{<\text{cond1}>\ \text{AND}<\text{cond2}>\ \text{AND}\ldots\text{AND}\ <\text{cond}n>}(R)$$

Addison-Wesley is an imprint of PEARSON

Unary Relational Operations: SELECT and PROJECT (cont'd.)

For example, the following operation:

$$\sigma_{Dno=4\ \textbf{AND}\ Salary>25000}\ (\text{EMPLOYEE})$$

would correspond to the following SQL query:

```
SELECT   *
FROM     EMPLOYEE
WHERE    Dno=4 AND Salary>25000;
```

The PROJECT Operation

Selects columns from table and discards the other columns:

$$\pi_{\text{<attribute list>}}(R)$$

$$\pi_{\text{Lname, Fname, Salary}}(\text{EMPLOYEE})$$

Degree

- Number of attributes in <attribute list>

Duplicate elimination

- Result of PROJECT operation is a set of distinct tuples

The PROJECT Operation

In SQL, the PROJECT attribute list is specified in the SELECT clause of a query. For example, the following operation:

$$\pi_{\text{Sex, Salary}}(\text{EMPLOYEE})$$

would correspond to the following SQL query:

```
SELECT    DISTINCT Sex, Salary
FROM      EMPLOYEE
```

Figure 6.1
Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4 \textbf{ AND } Salary>25000) \text{ OR } (Dno=5 \textbf{ AND } Salary>30000)}$ (EMPLOYEE). (b) $\pi_{Lname,\ Fname,\ Salary}$(EMPLOYEE). (c) $\pi_{Sex,\ Salary}$(EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Sequences of Operations and the RENAME Operation

In-line expression:

$$\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$$

Sequence of operations:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Salary}}(\text{DEP5\_EMPS})$$

Rename attributes in intermediate results

RENAME operation

$$\text{TEMP} \leftarrow \sigma_{\text{Dno}=5}(\text{EMPLOYEE})$$
$$R(\text{First\_name, Last\_name, Salary}) \leftarrow \pi_{\text{Fname, Lname, Salary}}(\text{TEMP})$$

$$\rho_{S(B1,\,B2,\,\ldots,\,Bn)}(R) \quad \text{or} \quad \rho_S(R) \quad \text{or} \quad \rho_{(B1,\,B2,\,\ldots,\,Bn)}(R)$$

Sequences of Operations and the RENAME Operation

Figure 6.2
Results of a sequence of operations. (a) $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$. (b) Using intermediate relations and renaming of attributes.

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)
TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Relational Algebra Operations from Set Theory

UNION, **INTERSECTION**, and **MINUS**

- Merge the elements of two sets in various ways
- Binary operations
- Relations must have the same type of tuples

UNION

- $R \cup S$
- Includes all tuples that are either in *R* or in *S* or in both *R* and *S*
- Duplicate tuples eliminated

Relational Algebra Operations from Set Theory

$DEP5\_EMPS \leftarrow \sigma_{Dno=5}(EMPLOYEE)$
$RESULT1 \leftarrow \pi_{Ssn}(DEP5\_EMPS)$
$RESULT2(Ssn) \leftarrow \pi_{Super\_ssn}(DEP5\_EMPS)$
$RESULT \leftarrow RESULT1 \cup RESULT2$

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Figure 6.3
Result of the UNION operation $RESULT \leftarrow RESULT1 \cup RESULT2$.

Relational Algebra Operations from Set Theory (cont'd.)

- INTERSECTION
  - $R \cap S$
  - Includes all tuples that are in both R and S
- SET DIFFERENCE (or MINUS)
  - $R - S$
  - Includes all tuples that are in R but not in S

Relational Algebra Operations from Set Theory (cont'd.)

Figure 6.4
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

The CARTESIAN PRODUCT (CROSS PRODUCT) Operation

CARTESIAN PRODUCT

- **CROSS PRODUCT** or **CROSS JOIN**
- Denoted by ×
- Binary set operation
- Relations do not have to be union compatible
- Useful when followed by a selection that matches values of attributes

$$R(A_1, A_2, \ldots, A_n) \times S(B_1, B_2, \ldots, B_m) \longrightarrow Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m),$$

$$\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{Sex}=\text{'F'}}(\text{EMPLOYEE})$$
$$\text{EMPNAMES} \leftarrow \pi_{\text{Fname, Lname, Ssn}}(\text{FEMALE\_EMPS})$$
$$\text{EMP\_DEPENDENTS} \leftarrow \text{EMPNAMES} \times \text{DEPENDENT}$$
$$\text{ACTUAL\_DEPENDENTS} \leftarrow \sigma_{\text{Ssn}=\text{Essn}}(\text{EMP\_DEPENDENTS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Dependent\_name}}(\text{ACTUAL\_DEPENDENTS})$$

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	. . .
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	. . .
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	. . .
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	. . .
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	. . .
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	. . .
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	. . .
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	. . .
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	. . .
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	. . .
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	. . .
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	. . .
Joyce	English	453453453	333445555	Alice	F	1986-04-05	. . .
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	. . .
Joyce	English	453453453	333445555	Joy	F	1958-05-03	. . .
Joyce	English	453453453	987654321	Abner	M	1942-02-28	. . .
Joyce	English	453453453	123456789	Michael	M	1988-01-04	. . .
Joyce	English	453453453	123456789	Alice	F	1988-12-30	. . .
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	. . .

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .

Figure 6.5
The Cartesian Product (Cross Product) operation.

Binary Relational Operations: JOIN and DIVISION

The **JOIN** Operation $R \bowtie_{\text{<join condition>}} S$

- Denoted by $\bowtie$
- Combine related tuples from two relations into single “longer” tuples
- General join condition of the form <condition> **AND** <condition> **AND**...**AND** <condition>
- Example:

$$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$$
$$\text{RESULT} \leftarrow \pi_{\text{Dname, Lname, Fname}}(\text{DEPT\_MGR})$$

Binary Relational Operations: JOIN and DIVISION

$$\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$$

$$\text{RESULT} \leftarrow \pi_{\text{Dname, Lname, Fname}}(\text{DEPT\_MGR})$$

DEPT_MGR

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Figure 6.6
Result of the JOIN operation $\text{DEPT\_MGR} \leftarrow \text{DEPARTMENT} \bowtie_{\text{Mgr\_ssn=Ssn}} \text{EMPLOYEE}$.

Binary Relational Operations: JOIN and DIVISION (cont'd.)

THETA JOIN

- Each <condition> of the form $A_i \theta B_j$
- A_i is an attribute of R
- B_j is an attribute of S
- A_i and B_j have the same domain
- θ (theta) is one of the comparison operators:
  - $\{=, <, \leq, >, \geq, \neq\}$

Variations of JOIN: The EQUIJOIN and NATURAL JOIN

EQUIJOIN

- Only = comparison operator used
- Always have one or more pairs of attributes that have identical values in every tuple

NATURAL JOIN

- Denoted by *
- Removes second (superfluous) attribute in an EQUIJOIN condition

Variations of JOIN: The EQUIJOIN and NATURAL JOIN

$$\text{PROJ\_DEPT} \leftarrow \text{PROJECT} * \rho_{(\text{Dname, Dnum, Mgr\_ssn, Mgr\_start\_date})}(\text{DEPARTMENT})$$

=

$$\text{DEPT} \leftarrow \rho_{(\text{Dname, Dnum, Mgr\_ssn, Mgr\_start\_date})}(\text{DEPARTMENT})$$
$$\text{PROJ\_DEPT} \leftarrow \text{PROJECT} * \text{DEPT}$$

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

Variations of JOIN: The EQUIJOIN and NATURAL JOIN

DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Variations of JOIN: The EQUIJOIN and NATURAL JOIN (cont'd.)

Join selectivity

- Expected size of join result divided by the maximum size $n_R * n_S$

Inner joins

- Type of match and combine operation
- Defined formally as a combination of CARTESIAN PRODUCT and SELECTION

A Complete Set of Relational Algebra Operations

Set of relational algebra operations {σ, π, ∪, ρ, –, ×} is a **complete set**

Any relational algebra operation can be expressed as a sequence of operations from this set

$$R \cap S \equiv (R \cup S) - ((R - S) \cup (S - R))$$

$$R \bowtie_{<\text{condition}>} S \equiv \sigma_{<\text{condition}>}(R \times S)$$

The DIVISION Operation

- Denoted by ÷
- Example: retrieve the names of employees who work on all the projects that 'John Smith' works on
- Apply to relations $R(Z) \div S(X)$
  - Attributes of S are a subset of the attributes of R

$$\text{SMITH} \leftarrow \sigma_{\text{Fname='John' AND Lname='Smith'}}(\text{EMPLOYEE})$$
$$\text{SMITH\_PNOS} \leftarrow \pi_{\text{Pno}}(\text{WORKS\_ON} \bowtie_{\text{Essn=Ssn}} \text{SMITH})$$

$$\text{SSN\_PNOS} \leftarrow \pi_{\text{Essn, Pno}}(\text{WORKS\_ON})$$

$$\text{SSNS(Ssn)} \leftarrow \text{SSN\_PNOS} \div \text{SMITH\_PNOS}$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname}}(\text{SSNS} * \text{EMPLOYEE})$$

Figure 6.8
The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

Operations of Relational Algebra

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{(\text{<join attributes 1>}),(\text{<join attributes 2>})} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{(\text{<join attributes 1>}),(\text{<join attributes 2>})} R_2$ OR $R_1 * R_2$

Operations of Relational Algebra (cont'd.)

Table 6.1 Operations of Relational Algebra

UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Notation for Query Trees

Query tree

- Represents the input relations of query as leaf nodes of the tree
- Represents the relational algebra operations as internal nodes

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$STAFFORD\_PROJS \leftarrow \sigma_{Plocation='Stafford'}(PROJECT)$

$CONTR\_DEPTS \leftarrow (STAFFORD\_PROJS \bowtie_{Dnum=Dnumber} DEPARTMENT)$

$PROJ\_DEPT\_MGRS \leftarrow (CONTR\_DEPTS \bowtie_{Mgr\_ssn=Ssn} EMPLOYEE)$

$RESULT \leftarrow \pi_{Pnumber,\ Dnum,\ Lname,\ Address,\ Bdate}(PROJ\_DEPT\_MGRS)$

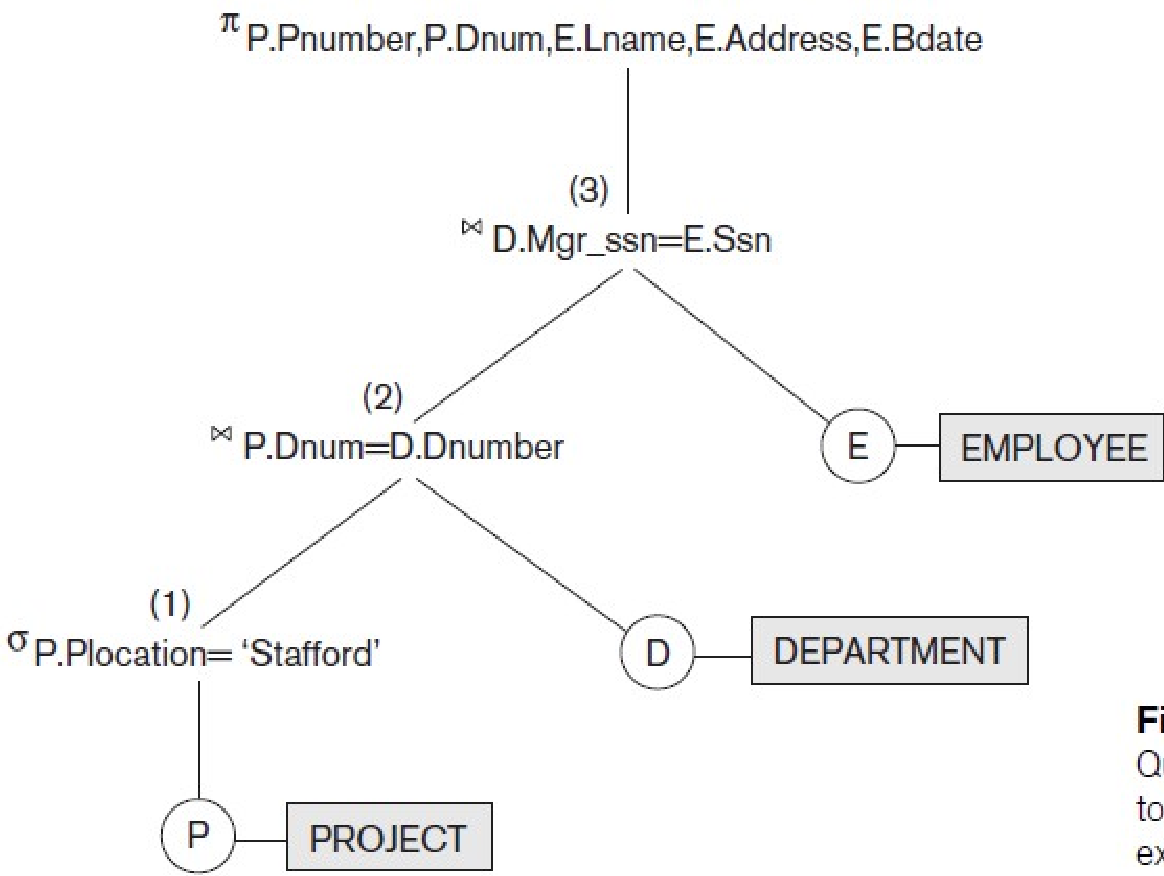


Figure 6.9
Query tree corresponding to the relational algebra expression for Q2.

Additional Relational Operations

Generalized projection

Allows functions of attributes to be included in the projection list

$$\pi_{F1,\, F2,\, ...,\, Fn}\,(R)$$

Example:

EMPLOYEE (Ssn, Salary, Deduction, Years_service)

$$\text{REPORT} \leftarrow \rho_{(\text{Ssn, Net\_salary, Bonus, Tax})}(\pi_{\text{Ssn, Salary} - \text{Deduction}, 2000 * \text{Years\_service}, 0.25 * \text{Salary}}(\text{EMPLOYEE})).$$

Aggregate functions and **grouping**

Common functions applied to collections of numeric values

Include SUM, AVERAGE, MAXIMUM, and MINIMUM, COUNT

Additional Relational Operations (cont'd.)

Group tuples by the value of some of their attributes

Apply aggregate function independently to each group

$$_{<\text{grouping attributes}>}\mathfrak{I}_{<\text{function list}>}(R)$$

Figure 6.10
The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\,\Im\,_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.

b. $_{\text{Dno}}\,\Im\,_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

c. $\Im\,_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

[8]Note that this is an arbitrary notation we are suggesting. There is no standard notation.

Recursive Closure Operations

Operation applied to a **recursive relationship** between tuples of same type

to specify the Ssn s of all employees e' directly supervised—at level one—by the employee e whose name is ‘James Borg

$$\text{BORG\_SSN} \leftarrow \pi_{\text{Ssn}}(\sigma_{\text{Fname}=\text{'James'} \text{ AND } \text{Lname}=\text{'Borg'}}(\text{EMPLOYEE}))$$
$$\text{SUPERVISION}(\text{Ssn1}, \text{Ssn2}) \leftarrow \pi_{\text{Ssn},\text{Super\_ssn}}(\text{EMPLOYEE})$$
$$\text{RESULT1}(\text{Ssn}) \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2}=\text{Ssn}} \text{BORG\_SSN})$$

to retrieve all employees supervised by Borg at level 2—that is, all employees e'' supervised by some employee e' who is directly supervised by Borg

$$\text{RESULT2}(\text{Ssn}) \leftarrow \pi_{\text{Ssn1}}(\text{SUPERVISION} \bowtie_{\text{Ssn2}=\text{Ssn}} \text{RESULT1})$$

SUPERVISION

(Borg's Ssn is 888665555)

Ssn1 (Ssn)	Ssn2 (Super_ssn)
123456789	333445555
333445555	888665555
999887777	987654321
987654321	888665555
666884444	333445555
453453453	333445555
987987987	987654321
888665555	null

RESULT1

Ssn
333445555
987654321

(Supervised by Borg)

RESULT2

Ssn
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT

Ssn
123456789
999887777
666884444
453453453
987987987
333445555
987654321

(RESULT1 ∪ RESULT2)

Figure 6.11
A two-level recursive query.

OUTER JOIN Operations

Outer joins

Keep all tuples in *R*, or all those in *S*, or all those in both relations regardless of whether or not they have matching tuples in the other relation

Types

- **LEFT OUTER JOIN, RIGHT OUTER JOIN, FULL OUTER JOIN**

Example

$$\text{TEMP} \leftarrow (\text{EMPLOYEE} \mathbin{\unicode{x27D5}}_{\text{Ssn=Mgr\_ssn}} \text{DEPARTMENT})$$

$$\text{RESULT} \leftarrow \pi_{\text{Fname, Minit, Lname, Dname}}(\text{TEMP})$$

OUTER JOIN Operations

$$\text{TEMP} \leftarrow (\text{EMPLOYEE} ⟕_{\text{Ssn=Mgr\_ssn}} \text{DEPARTMENT})$$

$$\text{RESULT} \leftarrow \pi_{\text{Fname, Minit, Lname, Dname}}(\text{TEMP})$$

Figure 6.12
The result of a LEFT OUTER JOIN operation.

RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

The OUTER UNION Operation

Take union of tuples from two relations that have some common attributes

- Not union (type) compatible

Partially compatible

- All tuples from both relations included in the result
- Tuples with the same value combination will appear only once

STUDENT(Name, Ssn, Department, Advisor)

INSTRUCTOR(Name, Ssn, Department, Rank)

STUDENT_OR_INSTRUCTOR(Name, Ssn, Department, Advisor, Rank)

Addison-Wesley is an imprint of PEARSON

Examples of Queries in Relational Algebra

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

$$RESEARCH\_DEPT \leftarrow \sigma_{Dname=\text{'Research'}}(DEPARTMENT)$$
$$RESEARCH\_EMPS \leftarrow (RESEARCH\_DEPT \bowtie_{Dnumber=Dno} EMPLOYEE)$$
$$RESULT \leftarrow \pi_{Fname,\ Lname,\ Address}(RESEARCH\_EMPS)$$

As a single in-line expression, this query becomes:

$$\pi_{Fname,\ Lname,\ Address}\ (\sigma_{Dname=\text{'Research'}}(DEPARTMENT \bowtie_{Dnumber=Dno}(EMPLOYEE))$$

Examples of Queries in Relational Algebra (cont'd.)

Query 2. For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$\text{STAFFORD\_PROJS} \leftarrow \sigma_{\text{Plocation}=\text{'Stafford'}}(\text{PROJECT})$
$\text{CONTR\_DEPTS} \leftarrow (\text{STAFFORD\_PROJS} \bowtie_{\text{Dnum}=\text{Dnumber}} \text{DEPARTMENT})$
$\text{PROJ\_DEPT\_MGRS} \leftarrow (\text{CONTR\_DEPTS} \bowtie_{\text{Mgr\_ssn}=\text{Ssn}} \text{EMPLOYEE})$
$\text{RESULT} \leftarrow \pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(\text{PROJ\_DEPT\_MGRS})$

Query 3. Find the names of employees who work on *all* the projects controlled by department number 5.

$\text{DEPT5\_PROJS} \leftarrow \rho_{(\text{Pno})}(\pi_{\text{Pnumber}}(\sigma_{\text{Dnum}=5}(\text{PROJECT})))$
$\text{EMP\_PROJ} \leftarrow \rho_{(\text{Ssn, Pno})}(\pi_{\text{Essn, Pno}}(\text{WORKS\_ON}))$
$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$
$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$

Examples of Queries in Relational Algebra (cont'd.)

Query 6. Retrieve the names of employees who have no dependents.

This is an example of the type of query that uses the MINUS (SET DIFFERENCE) operation.

$\text{ALL\_EMPS} \leftarrow \pi_{\text{Ssn}}(\text{EMPLOYEE})$
$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$
$\text{EMPS\_WITHOUT\_DEPS} \leftarrow (\text{ALL\_EMPS} - \text{EMPS\_WITH\_DEPS})$
$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{EMPS\_WITHOUT\_DEPS} * \text{EMPLOYEE})$

Query 7. List the names of managers who have at least one dependent.

$\text{MGRS(Ssn)} \leftarrow \pi_{\text{Mgr\_ssn}}(\text{DEPARTMENT})$
$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$
$\text{MGRS\_WITH\_DEPS} \leftarrow (\text{MGRS} \cap \text{EMPS\_WITH\_DEPS})$
$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{MGRS\_WITH\_DEPS} * \text{EMPLOYEE})$

Summary

Formal languages for relational model of data:

- Relational algebra: operations, unary and binary operators
- Some queries cannot be stated with basic relational algebra operations
  - But are important for practical use