Midterm

The exam should be done individually. You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten. **Solutions must be submitted electronically before 4 pm on August 11**. No credit will be given to solutions obtained verbatim from the Internet or other sources.

1. (20p) Prove that if $n^2 + 2n$ is odd integer, then $n + 1$ is even integer.

2. (20p) Let R be the relation on the set of integers defined as $\forall a, b \in \mathbb{Z}, (a, b) \in R$ if $a.b < 0$. Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation satisfies. Justify your answer.

Employ your id to calculate a specific number that will be used in the further questions as follows *('18YZ0345' will be used here as an example to show you how the number is calculated):*

- remove the letters from your number (if it does not contain any letter, just keep it as it is)

  $18YZ0345 \longrightarrow 180345$

- multiply the result with '12345'

  $180345 * 12345 \longrightarrow 2226359025$

- remove all the zeros from the resulting number

  $2226359025 \longrightarrow 222635925$

- cut out the last 4 digits and assign them to the letters A, B, C, D, respectively.

  $5 \longrightarrow A, \quad 9 \longrightarrow B, \quad 2 \longrightarrow C, \quad 5 \longrightarrow D$

- put the numbers in place of the corresponding letters to solve the following questions.

3. (15p) Solve the recurrence relation $a_n = Aa_{n-1} + Ba_{n-2}$ where $a_0 = C$ and $a_1 = D$.

4. (15p) How many integer solutions are there for the equation $x_1 + x_2 + x_3 + x_4 = 30$ if $x_1 \geq A$, $x_2 \geq B$, and $x_2, x_3 \geq 0$?

5. (15p) How many bit strings (that consist of the symbols '0' and '1') of length $(A + B)$ have more zeroes than ones? *(-the bit string '0101100' of length 7 has more zeros than ones-)*

6. (15p) Suppose A% of the people in a community has a particular disease and there is a fairly accurate diagnostic test for it. B% of the time this test gives a positive result for the people having this disease, and C% of the time this test gives a negative result for the people not having this disease. What is the probability that a person, who had a positive result from the test, has the disease?