

Solution 3

Employ your id to calculate a specific number that will be used in the question as follows ('14990013' will be used here as an example to show you how the number is calculated):

- multiply your id with '54321'
 $14950103 * 54321 \rightarrow 812104545063$
- remove all the zeros and ones from the resulting number
 $812104545063 \rightarrow 82454563$
- cut out the last 2 digits and assign them to the letters A, B, respectively.
 $6 \rightarrow A, \quad 3 \rightarrow B$

1. How many integer solutions are there for the equation $x_1 + x_2 + x_3 = A + B$
 $x_1, x_2, x_3 \geq 0$?

Solution

$$x_1 + x_2 + x_3 = 9 \text{ where } x_1, x_2, x_3 \geq 0$$

apply combination with repetition formula

$$n = 3 \text{ and } r = 9, \binom{n+r-1}{r} = \binom{11}{9} = 55$$

2. Suppose half of the people in a community has a particular disease and there is a fairly accurate diagnostic test for it. A% of the time this test gives a negative result for the people having this disease, and B% of the time this test gives a positive result for the people not having this disease. What is the probability that a person, who had a positive result from the test, has the disease?

Solution

E : person has the disease (\bar{E} : person does not have the disease)

F : test gives a positive result (\bar{F} : test gives a negative result)

$$p(E|F) = \frac{p(F|E)p(E)}{p(F|E)p(E) + p(F|\bar{E})p(\bar{E})} = ?$$

$$p(E) = 0.5, p(\bar{E}) = 0.5, \text{ thus } p(E|F) = \frac{p(F|E)p(E)}{p(F|E)p(E) + p(F|\bar{E})p(\bar{E})} = \frac{p(F|E)}{p(F|E) + p(F|\bar{E})}$$

$$p(F|\bar{E}) = 0.03 \text{ and } p(\bar{F}|E) = 0.06, \text{ then } p(F|E) = 0.94$$

$$\text{Thus, } p(E|F) = \frac{0.94}{0.94 + 0.03} = 0.969$$