Solution for Midterm 1

1. Prove that if $n^2 + 2n$ is odd integer, then $n + 1$ is even integer.

Solution

(Proof by Contraposition) $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$

if n + 1 is not even integer, then $n^2 + 2n$ *is not odd integer*

assume $\neg q$ *is true, i.e n + 1 is not even integer*

$$n + 1 = 2k + 1, \exists k \in Z$$
$$n = 2k$$
$$n^2 + 2n = 4k^2 + 4k$$
$$n^2 + 2n = 2(2k^2 + 2k)$$
$$n^2 + 2n = 2m, \exists m \in Z, \text{ thus } \neg p \text{ is true.}$$

2. Employ your id to calculate a specific number that will be used in the question as follows *('14990013' will be used here as an example to show you how the number is calculated):*

- multiply your id with '12345'

  $14990113 * 12345 \longrightarrow 185051710485$

- remove all the zeros from the resulting number

  $185051710485 \longrightarrow 1855171485$

- cut out the last 4 digits and assign them to the letters A, B, C, D, respectively.

  $1 \longrightarrow A, \quad 4 \longrightarrow B, \quad 8 \longrightarrow C, \quad 5 \longrightarrow D$

- exchange the numbers with the corresponding letters in the following recurrence relation and solve it

  $a_n = Aa_{n-1} + Ba_{n-2}$ where $a_0 = C$ and $a_1 = D$

Solution

$a_n = a_{n-1} + 4a_{n-2}$ *where* $a_0 = 8$ *and* $a_1 = 5$

the characteristic equation : $r^2 - r - 4 = 0$

the roots of the equation will be : $r_1 = \frac{1-\sqrt{17}}{2}, \; r_2 = \frac{1+\sqrt{17}}{2}$

if we apply the initial values :

$$c_1 + c_2 = 8$$
$$c_1\left(\frac{1-\sqrt{17}}{2}\right) + c_2\left(\frac{1+\sqrt{17}}{2}\right) = 5,$$

c_1 *and* c_2 *found as :* $c_1 = 4 - \frac{\sqrt{17}}{17}, \; c_2 = 4 + \frac{\sqrt{17}}{17}$

then $a_n = \left(4 - \frac{\sqrt{17}}{17}\right)\left(\frac{1-\sqrt{17}}{2}\right)^n + \left(4 + \frac{\sqrt{17}}{17}\right)\left(\frac{1+\sqrt{17}}{2}\right)^n$