Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

p → q

assume p is true

$$
\begin{aligned}
n &= 2k + 1, \exists k \in Z \\
n^2 &= (2k + 1)^2 \\
n^2 &= 4k^2 + 2k + 1 \\
n^2 &= 2(2k^2 + k) + 1 \\
n^2 &= 2m + 1, \exists m \in Z
\end{aligned}
$$

q is also true

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a+b \text{ is even integer.}}_{q}$

p → q

assume p is true

$$a = 2x + 1 \text{ and } b = 2y + 1 \ \exists x, y \in Z$$
$$a + b = 2x + 1 + 2y + 1$$
$$a + b = 2x + 2y + 2$$
$$a + b = 2(x + y + 1)$$
$$a + b = 2m, \exists m \in Z$$

q is also true

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

p → q   assume p is true

$$m = x^2 \text{ and } n = y^2,\ \exists x, y \in Z$$
$$m.n = x^2y^2$$
$$m.n = (x.y)^2$$
$$m.n = k^2,\ \exists k \in Z$$

q is also true

Proofs

Proof by Contraposition

$\text{If } \underbrace{3n+2 \text{ is an odd integer}}_{p}, \text{ then } \underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$

assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$
$$3n = 2k - 1$$
$$n = \frac{2k-1}{3}$$

Proofs

Proof by Contraposition $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n+2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n+2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n+2 = 6k+8$$
$$3n+2 = 2(3k+4)$$
$$3n+2 = 2m, \exists m \in Z$$

~p is also true

Proofs

Proof by Contraposition $p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $\quad x < 50$ and $y < 50$

$x + y < 100$

~p is also true

Proofs

Proof by Contradiction

- Prove that if $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

Assuming 'p ∧ ~q is not true' leads us a contradiction.

$3n + 2$ is an odd integer and n is even integer. (p ∧ ~q)

$n = 2k, \exists\, k \in Z$. So $3n + 2 = 6k + 2 = 2(3k + 1) = 2m, \exists\, m \in Z$

$3n + 2$ is an even integer. (Contradiction!)

Proofs

Proof of Equivalence (to prove two statements p and q are equal, the statement of the form p↔q should be proved)

$(p \leftrightarrow q) \leftrightarrow (p \to q) \wedge (q \to p)$

$\underbrace{\text{n is odd integer}}_{p}$ if and only if $\underbrace{5n + 4 \text{ is odd integer}}_{q}$

$\boxed{\sim p \to \sim q}$

$p \to q$ (direct proof)	$q \to p$ (proof by contraposition)
assume p is true	assume ~p is true
$n = 2k + 1, \exists k \in Z$	$n = 2k, \exists k \in Z$
$5n + 4 = 10k + 9$	$5n + 4 = 10k + 4$
$5n + 4 = 2(5k + 4) + 1$	$5n + 4 = 2(5k + 2)$
$5n + 4 = 2m + 1, \exists m \in Z$	$5n + 4 = 2m, \exists m \in Z$
q is true	~q is true