COM234 ELECTRONICS

Superposition

Thevenin’s and Norton’s Theorems

SUPERPOSITION

The idea of superposition rests on the linearity property.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables. With these in mind, we apply the superposition principle in three steps:

SUPERPOSITION

Steps to Apply Super position Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. Keep in mind that superposition is based on linearity.

Example 1: Use the superposition theorem to find v in the circuit.

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6V voltage source and the 3A current source, respectively. To obtain v_1, we set the current source to zero, as shown in **Fig (a)**. Applying **KVL** to the loop in **Fig.(a)** gives

$$12i_1 - 6 = 0 \Rightarrow i_1 = 0.5\ A$$

Thus:

$$v_1 = 4i_1 = 2\ V$$

To get v_2, we set the voltage source to zero, as in **Fig. (b)**. Using current division,

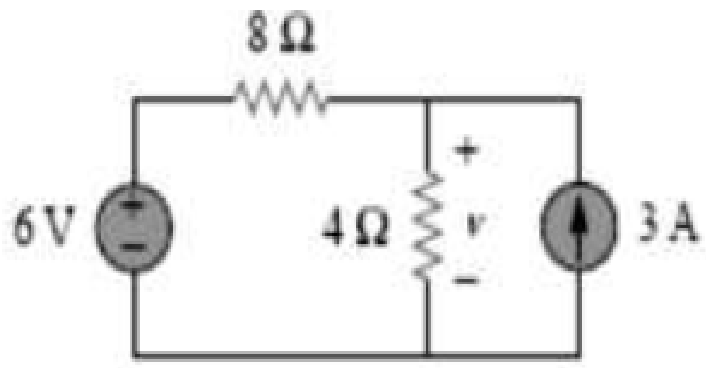
$$i_3 = \frac{8}{4+8} 3A = 2A$$

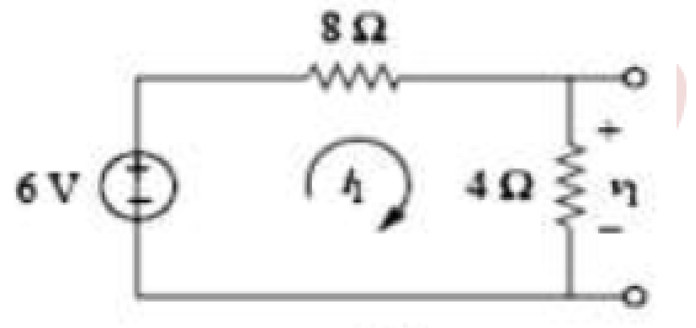
Hence,

$$v_2 = 4i_3 = 8\ V$$

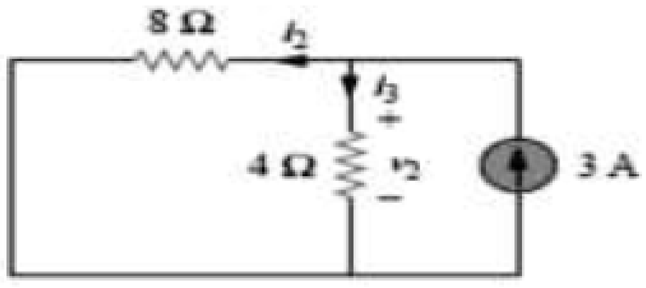
And we find

$$v = v_1 + v_2 = 2 + 8 = 10\ V$$





(a)

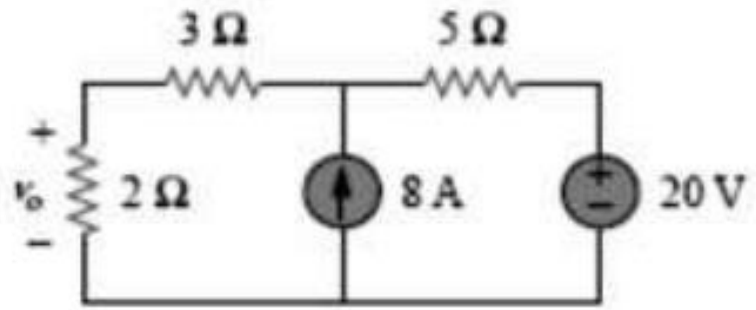


(b)

Practice 1:

Using the superposition theorem, find $\mathbf{v_o}$ in the circuit in Figure below.

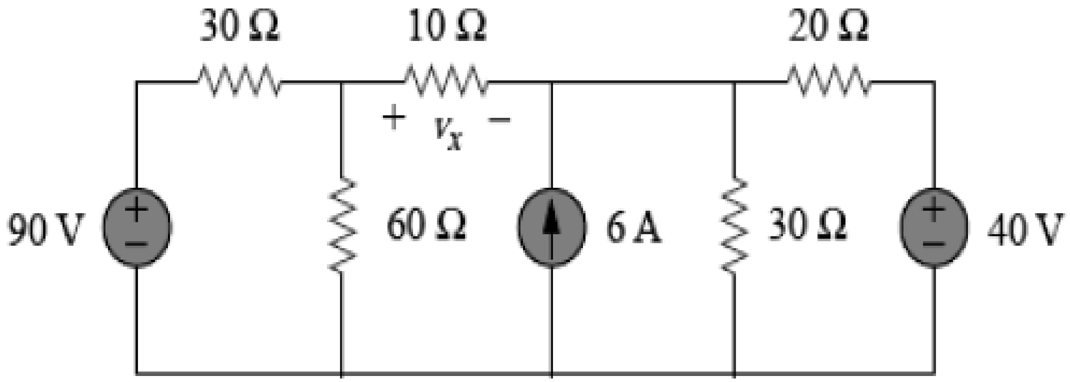
Answer:12 V



Practice 2:

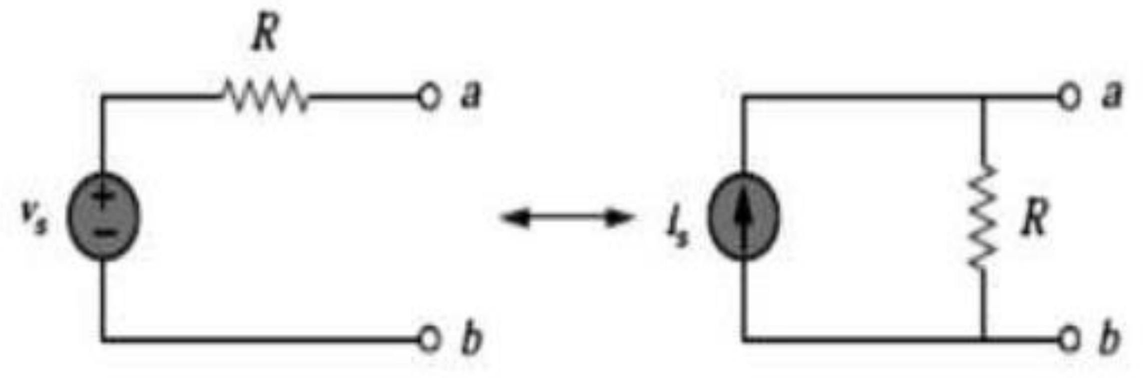
Use superposition to obtain v_x in the circuit of Figure below..

Answer: 0.75 A



SOURCE TRANSFORMATION

We have noticed that series-parallel combination. Source transformation is another tool for simplifying circuits. We can substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in **Fig.** below. Either substitution is known as a ***source transformation***.



Key Point: *A source transformation is the process of replacing a voltage source vs in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.*

SOURCE TRANSFORMATION

We need to find relationship between v_s and i_s that guarantees the two configurations in Fig below are equivalent with respect to nodes a, b.

Suppose R_L, is connected between nodes a, b in Fig. Using Ohm Law, the Current in R_L is.

$$i_L = \frac{v_s}{(R+R_L)}$$

R and R_L in series

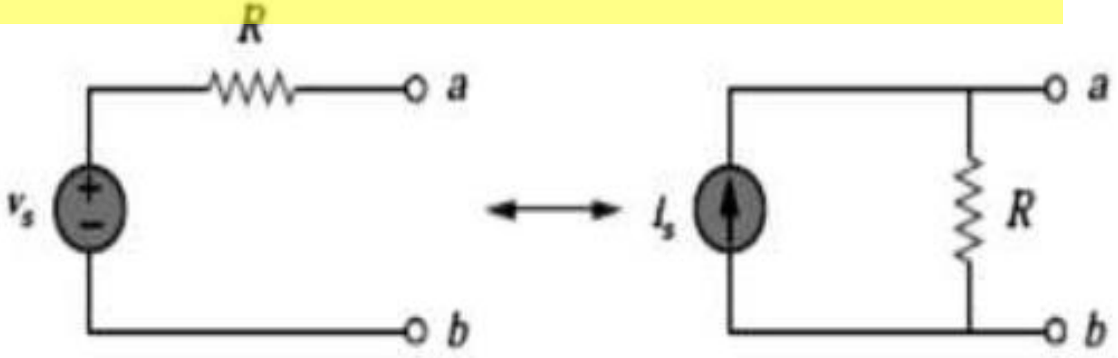
If it is to be replaced by a current source then load current must be $\frac{V}{(R+R_L)}$

Now suppose the same resistor R_L, is connected between nodes **a**, **b** in **Fig. (b)**. Using current division, the current in R_L, is

$$i_L = i_s \frac{R}{(R+R_L)}$$

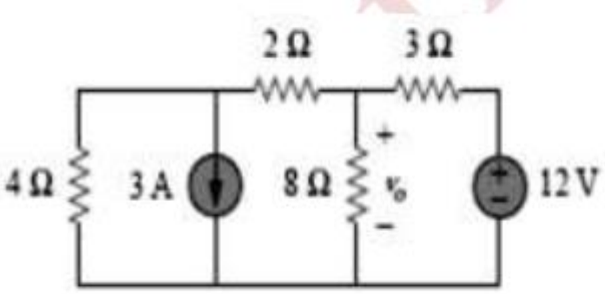
If the two circuits in Fig are equivalent, these resistor and currents must be the same. Equating the right hand sides of equations and simplifying

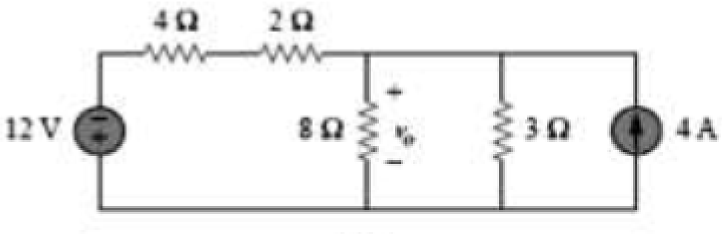
$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R$$



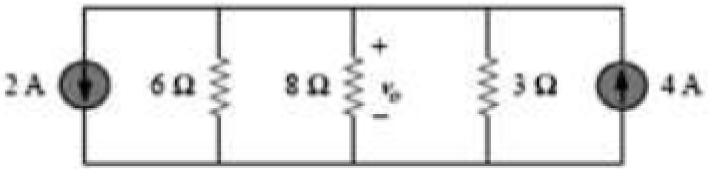
EXAMPLE: Use source transformation to find v_o in the circuit in Fig

Solution: We first transform the current and voltage sources to obtain the circuit in **Fig. (a)**. Combining the 4Ω and 2Ω resistors in series and transforming the 12V voltage source gives us **Fig. (b)**. We now combine the 3Ω and 6Ω resistors in parallel to get 2Ω. We also combine the 2A and 4A current sources to get a 2A source. Thus, by repeatedly applying source transformations, we obtain the circuit in **Fig. (c)**.





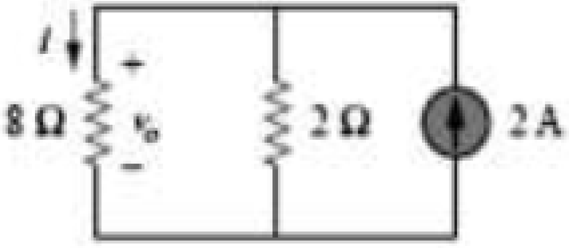
(a)



(b)

Alternatively, since the 8Ω and 2Ω resistors in **Fig. (c)** are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8||2)(2\text{ A}) = ((8\times 2)/10)*2 = 3.2\text{ V}$$

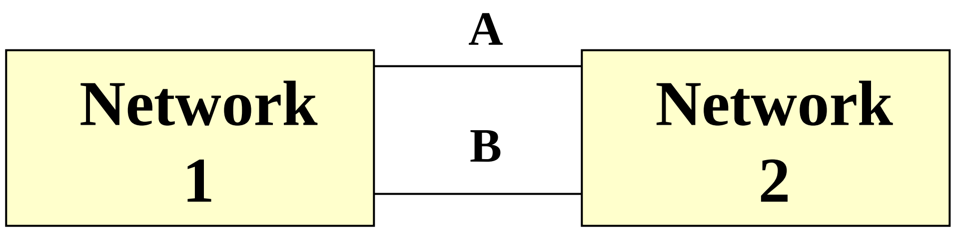


(c)

THEVENIN AND NORTON

THEVENIN'S THEOREM

Consider the following:

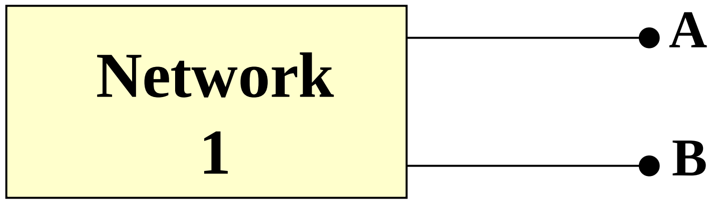


For purposes of discussion, at this point, we consider that both networks are composed of resistors and independent voltage and current sources

THEVENIN & NORTON

THEVENIN'S THEOREM:

Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.



Network 1 can be as complicated in structure as one can imagine. Maybe 45 meshes, 387 resistors, 91 voltage sources and 39 current sources.

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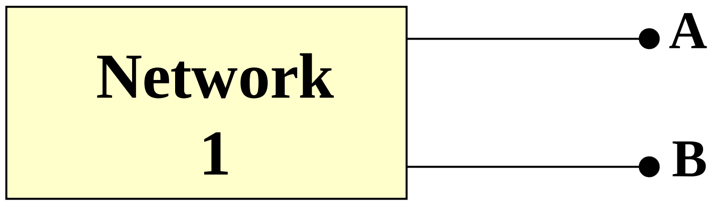
THEVENIN'S THEOREM:



Now place a voltmeter across terminals A-B and read the voltage. We call this the open-circuit voltage.

No matter how complicated Network 1 is, we read one voltage. It is either positive at A, (with respect to B) or negative at A.

We call this voltage V_{os} and we also call it $V_{THEVENIN} = V_{TH}$

THEVENIN & NORTON

THEVENIN'S THEOREM:

- **We now deactivate all sources of Network 1.**
- **To deactivate a voltage source, we remove the source and replace it with a short circuit.**
- **To deactivate a current source, we remove the source.**

THEVENIN & NORTON

THEVENIN'S THEOREM:

Consider the following circuit.

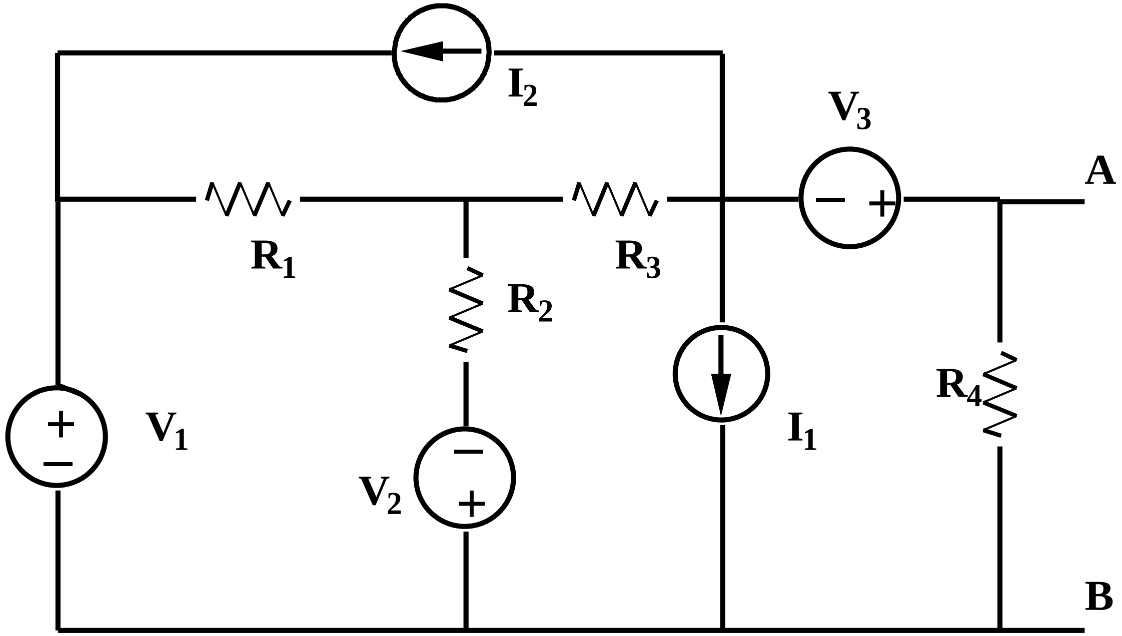


Figure : A typical circuit with independent sources

How do we deactivate the sources of this circuit?

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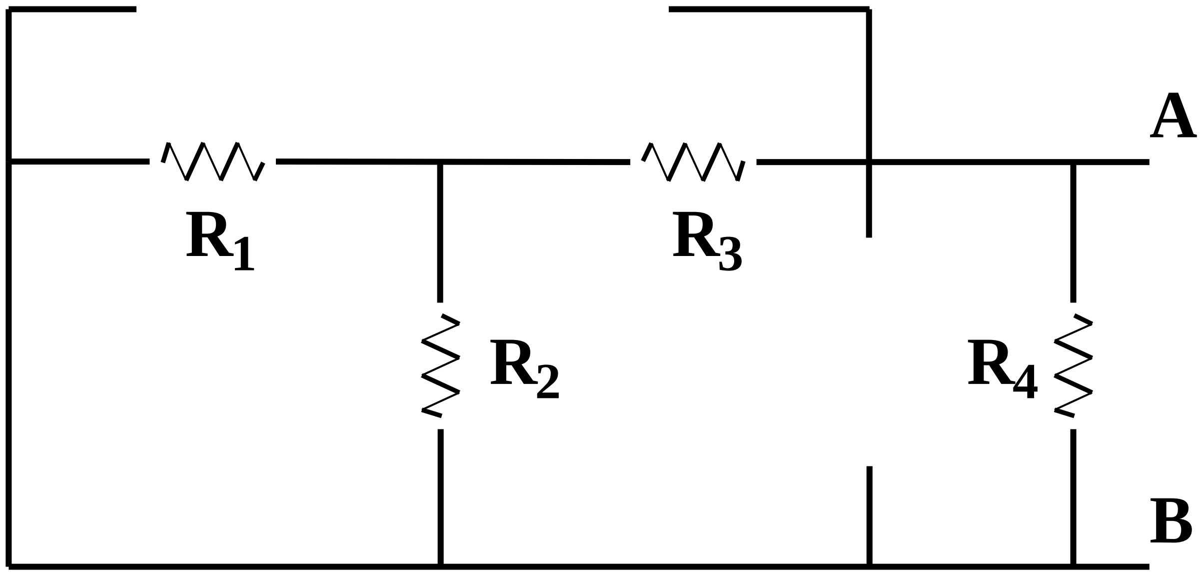
THEVENIN'S THEOREM:

When the sources are deactivated the circuit appears as in Figure below.



Now place an ohmmeter across A-B and read the resistance. If $R_1 = R_2 = R_4 = 20\ \Omega$ and $R_3 = 10\ \Omega$ then the meter reads $10\ \Omega$.

THEVENIN & NORTON

THEVENIN'S THEOREM:

We call the ohmmeter reading, under these conditions, $R_{THEVENIN}$ and shorten this to R_{TH}. Therefore, the important results are that we can replace Network 1 with the following network.

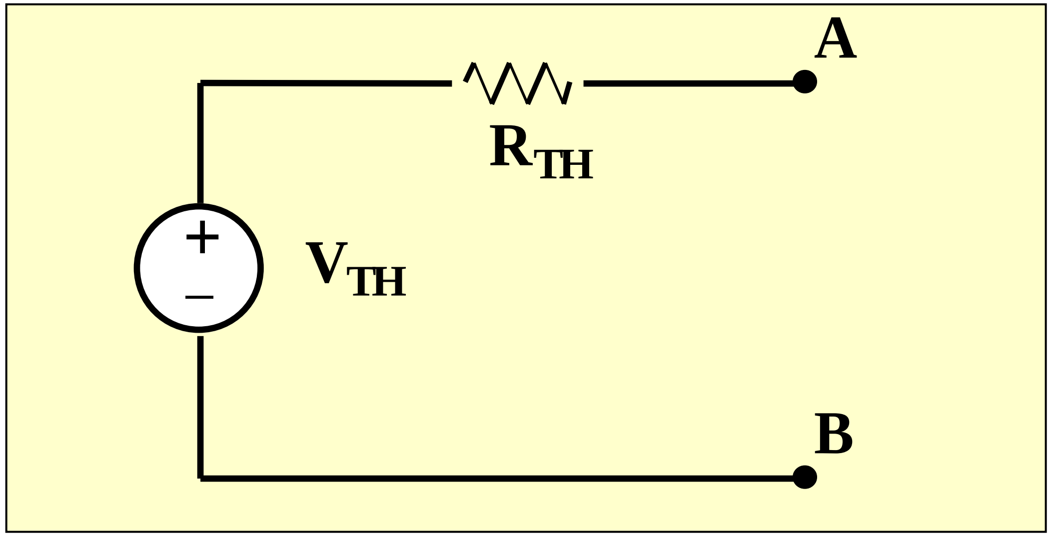


Figure : The Thevenin equivalent structure.

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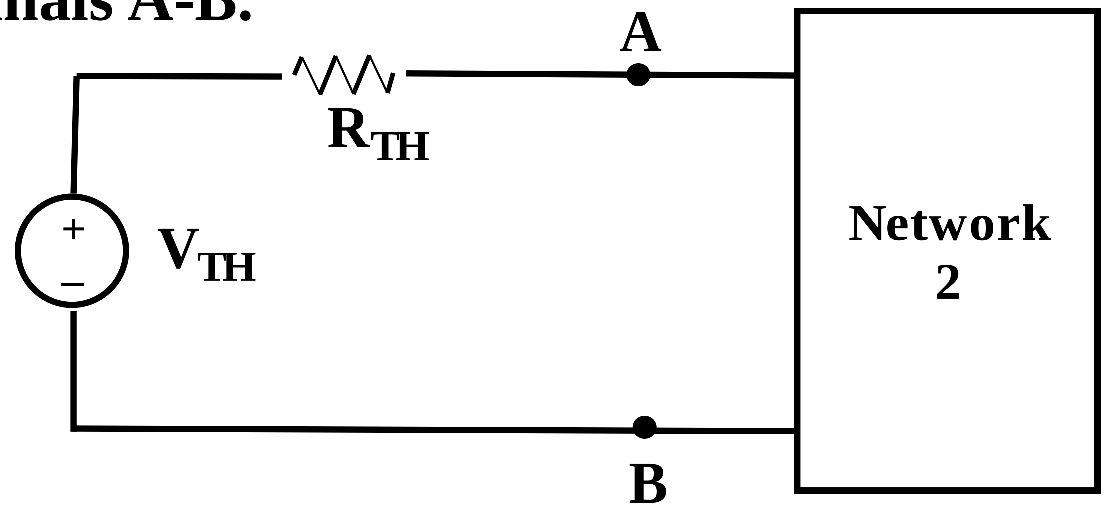
THEVENIN'S THEOREM:

We can now tie (reconnect) Network 2 back to terminals A-B.



We can now make any calculations we desire within Network 2 and they will give the same results as if we still had Network 1 connected.

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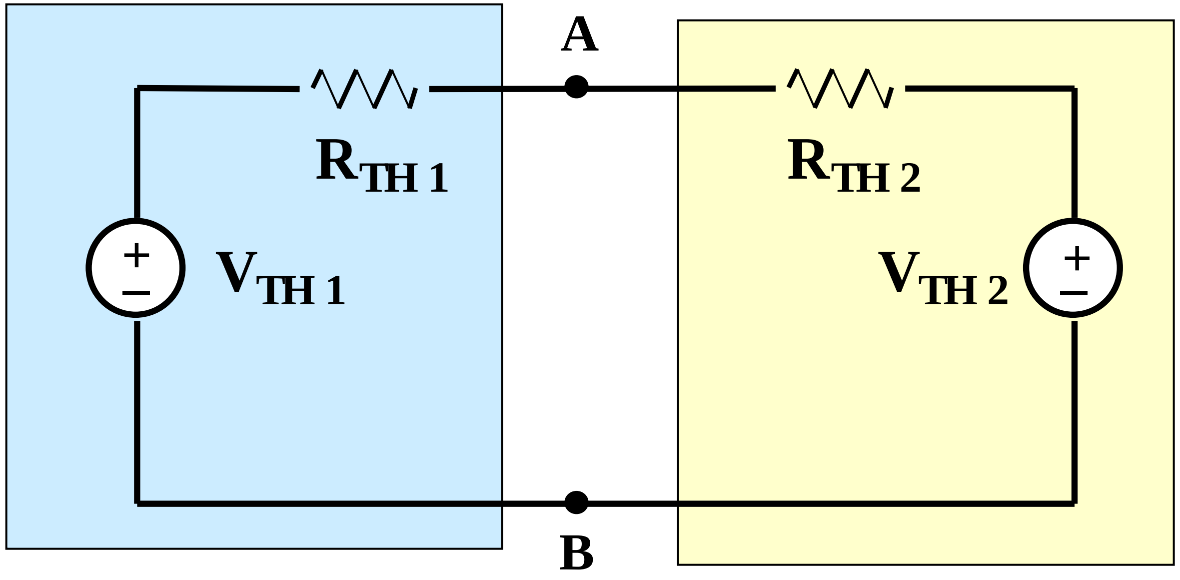
THEVENIN'S THEOREM:

It follows that we could also replace Network 2 with a Thevenin voltage and Thevenin resistance. The results would be as shown in Figure below.



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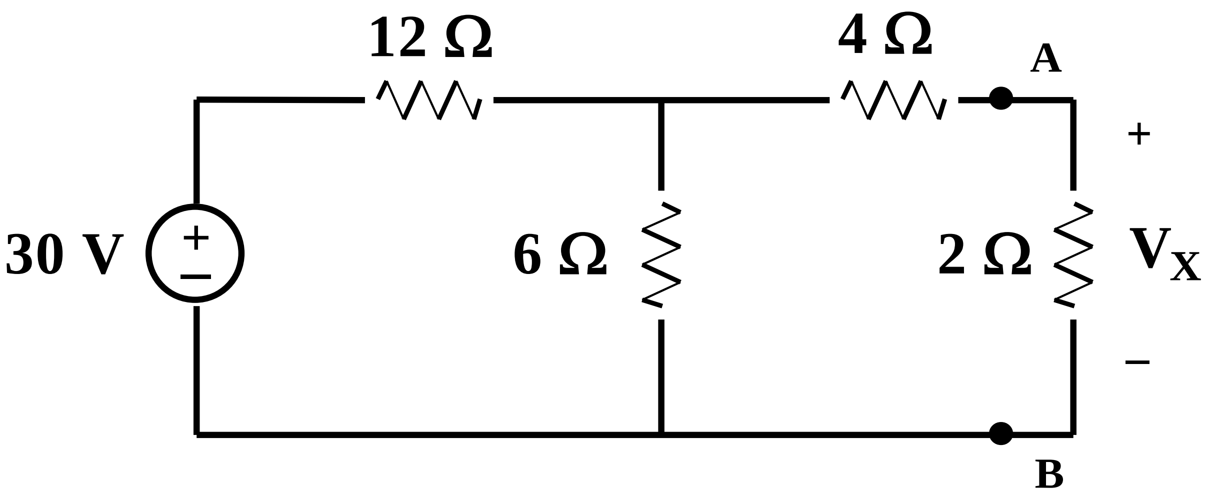
THEVENIN'S THEOREM: Example.

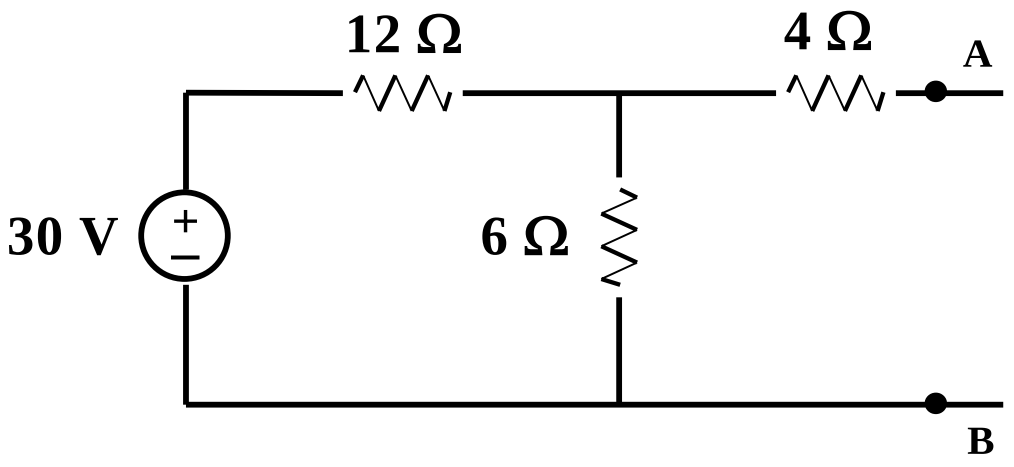
Find V_X by first finding V_{TH} and R_{TH} to the left of A-B.



First remove everything to the right of A-B.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued



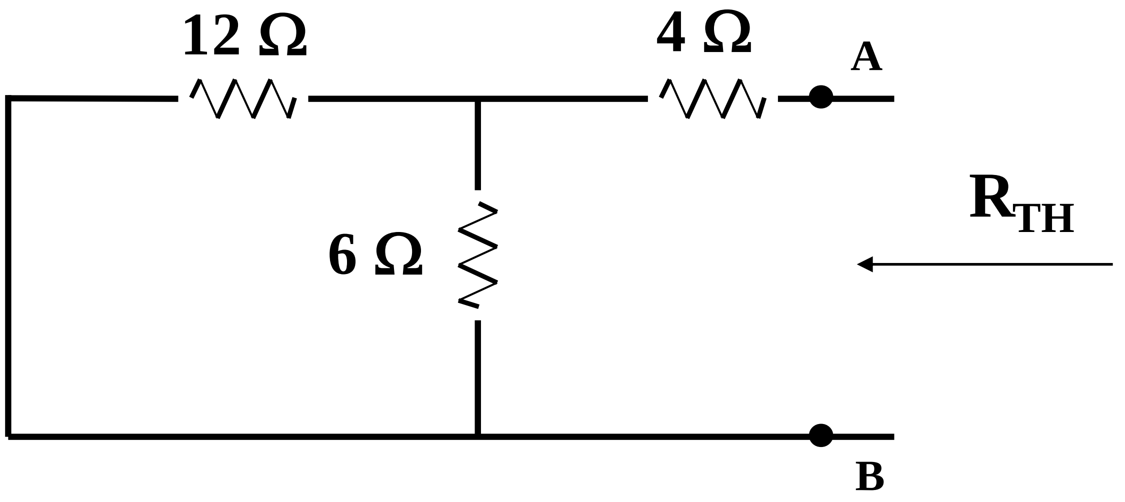
$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus there can be no voltage across the resistor.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.



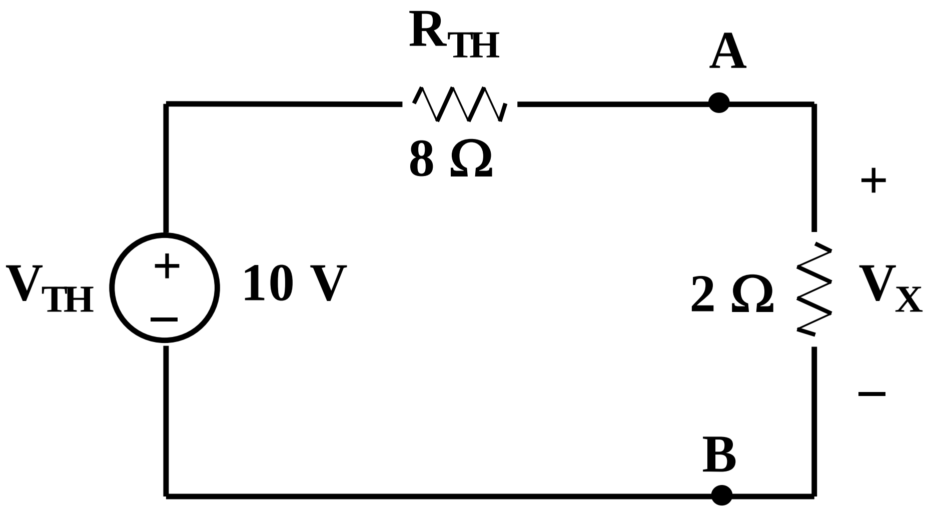
We see,

$$R_{TH} = 12||6 + 4 = 8\ \Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

After having found the Thevenin circuit, we connect this to the load in order to find V_X.



$$V_X = \frac{(10)(2)}{2+8} = 2V$$

THEVENIN & NORTON

THEVENIN'S THEOREM:

In some cases it may become tedious to find R_{TH} by reducing the resistive network with the sources deactivated. Consider the following:

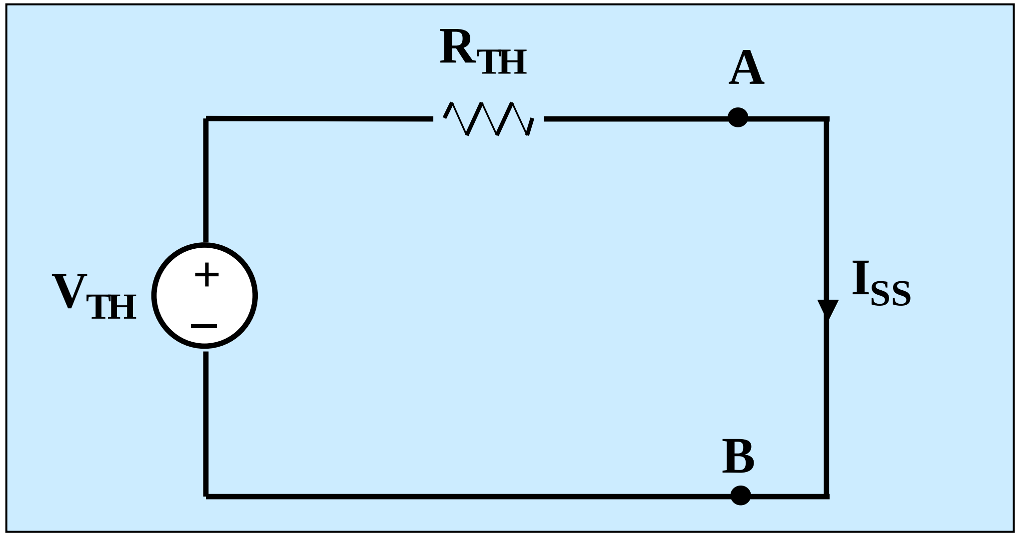


Figure : A Thevenin circuit with the output shorted.

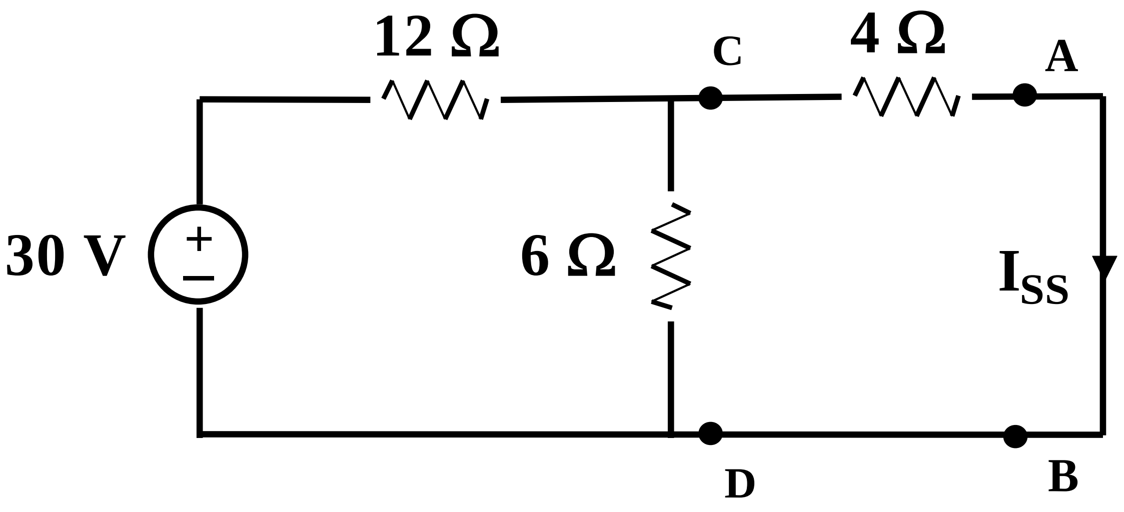
We see;

$$R_{TH} = \frac{V_{TH}}{I_{SS}}$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.2.

For the circuit in Figure, find R_{TH}.

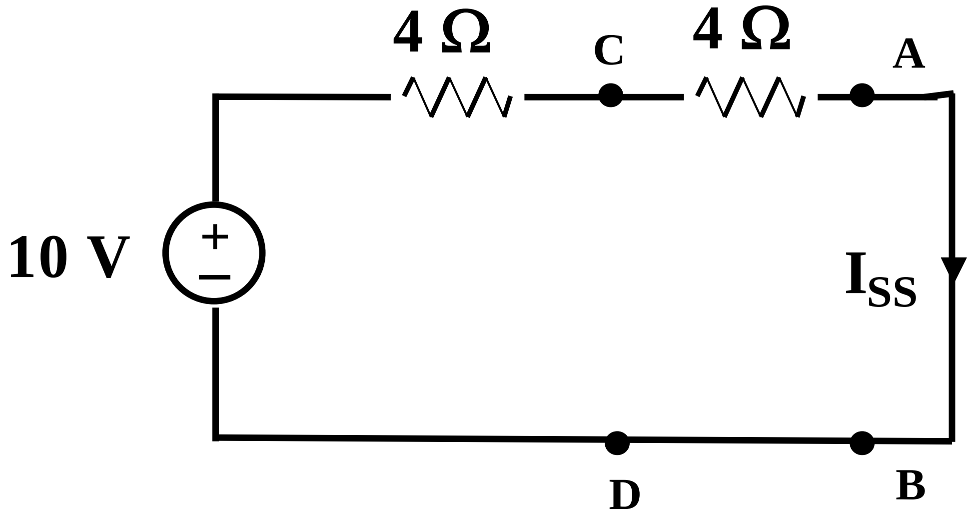


The task now is to find I_{SS}. One way to do this is to replace the circuit to the left of C-D with a Thevenin voltage and Thevenin resistance.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

Applying Thevenin's theorem to the left of terminals C-D and reconnecting to the load gives,



$$R_{TH} = \frac{V_{TH}}{I_{SS}} = \frac{10}{10/8} = 8\Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example

For the circuit below, find V_{AB} by first finding the Thevenin circuit to the left of terminals A-B.

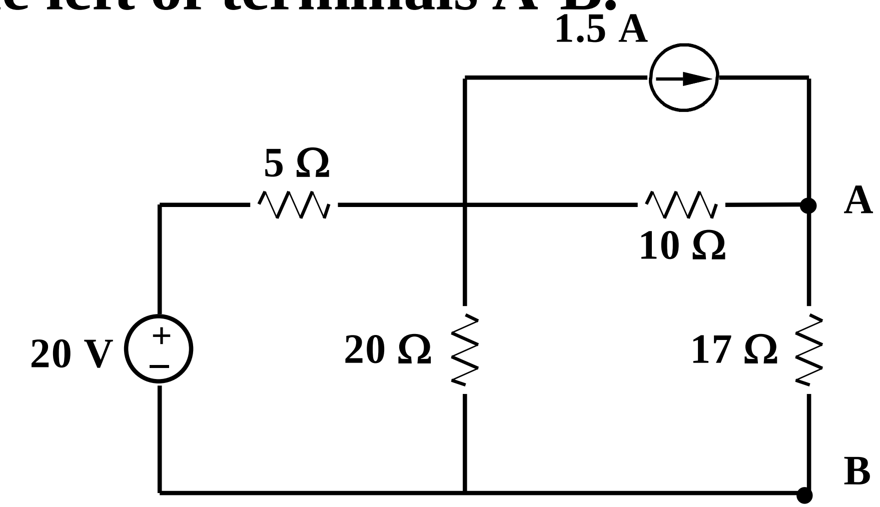


Figure : Circuit for Example .

We first find V_{TH} with the 17 Ω resistor removed. Next we find R_{TH} by looking into terminals A-B with the sources deactivated.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

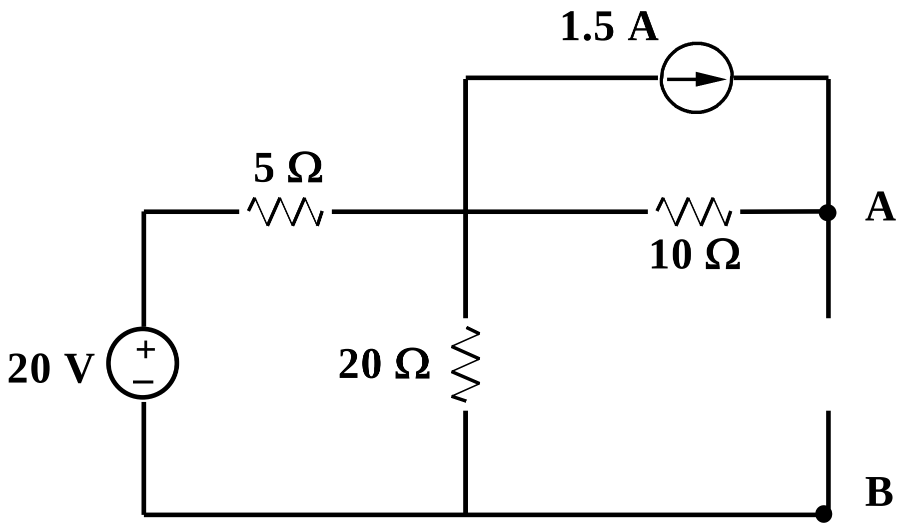


Figure : Circuit for finding V_{OS} for Example

$$V_{OS} = V_{AB} = V_{TH} = (1.5)(10) + \frac{20(20)}{(20+5)}$$

$$\therefore \quad V_{TH} = 31V$$

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THEVENIN'S THEOREM: Example 10.3 continued

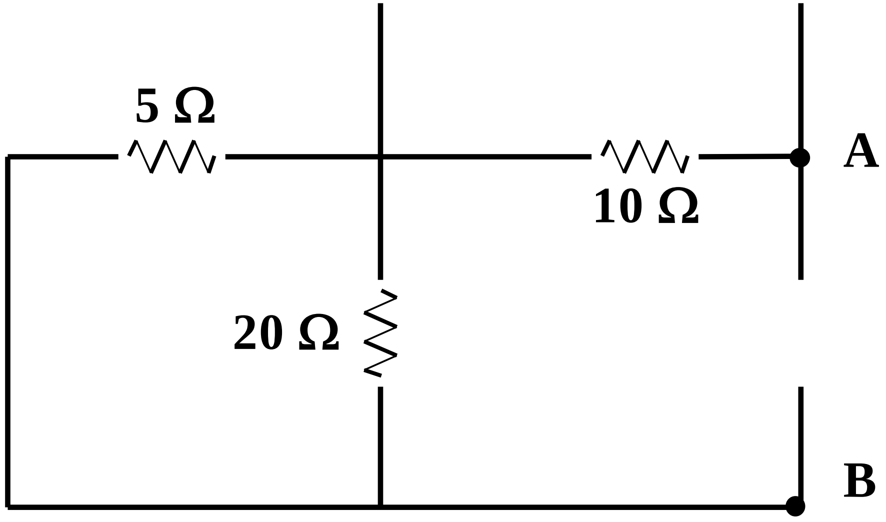

Figure : Circuit for find R_{TH} for Example.

$$R_{TH} = 10 + \frac{5(20)}{(5+20)} = 14\Omega$$

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THEVENIN'S THEOREM: Example continued

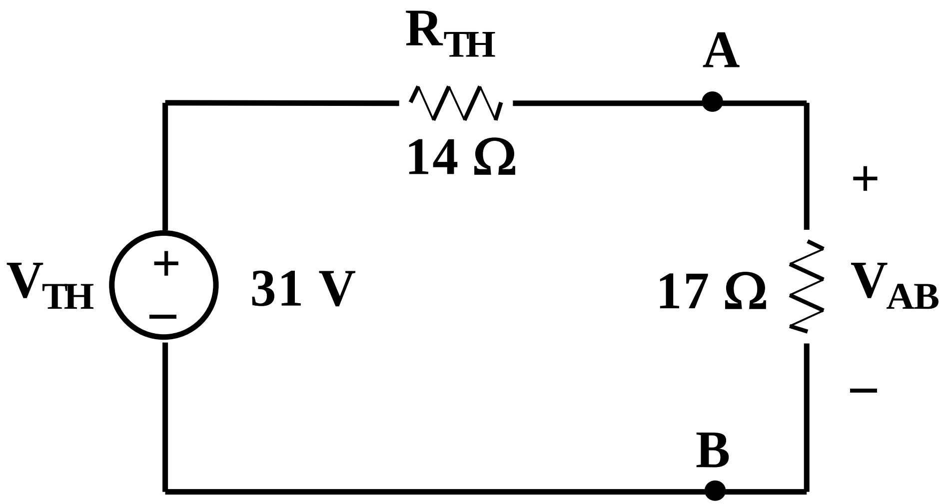

Figure : Thevenin reduced circuit for Example

We can easily find that,

$$V_{AB} = 17V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example : Working with a mix of independent and dependent sources.

Find the voltage across the 100 Ω load resistor by first finding the Thevenin circuit to the left of terminals A-B.

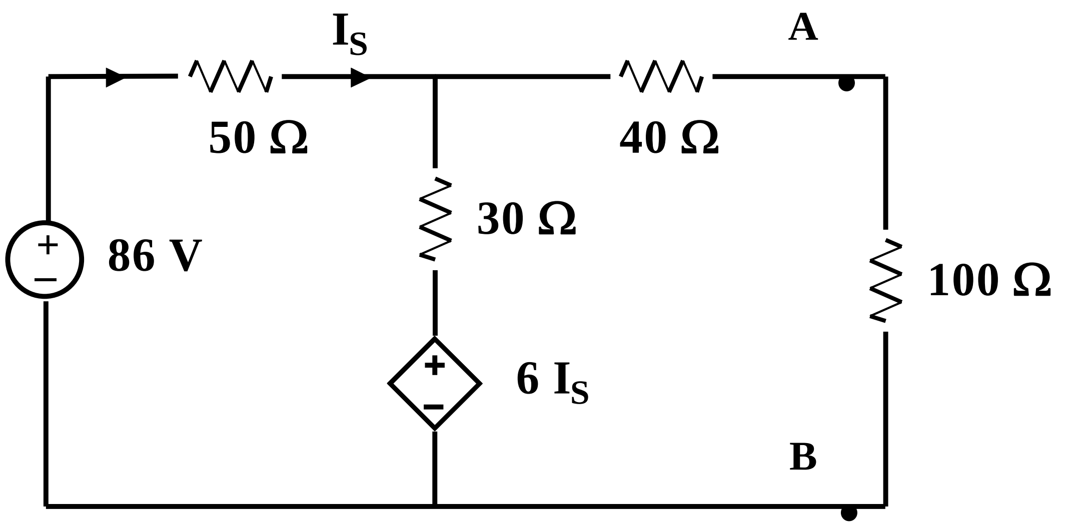

Figure : Circuit for Example

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.4: continued

First remove the 100 Ω load resistor and find $V_{AB} = V_{TH}$ to the left of terminals A-B.

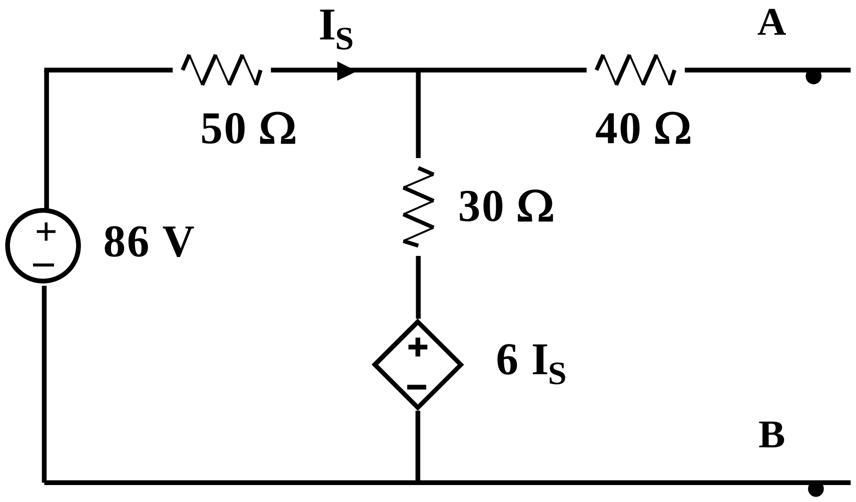

Figure 10.20: Circuit for find V_{TH}, Example 10.4.

$$-86 + 80I_S + 6I_S = 0 \;\rightarrow\; I_S = 1\,A$$

$$V_{AB} = 6I_S + 30I_S = \;\rightarrow\; 36V$$

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THEVENIN'S THEOREM: Example : continued

To find R_{TH} we deactivate all independent sources but retain all dependent sources as shown in Figure .

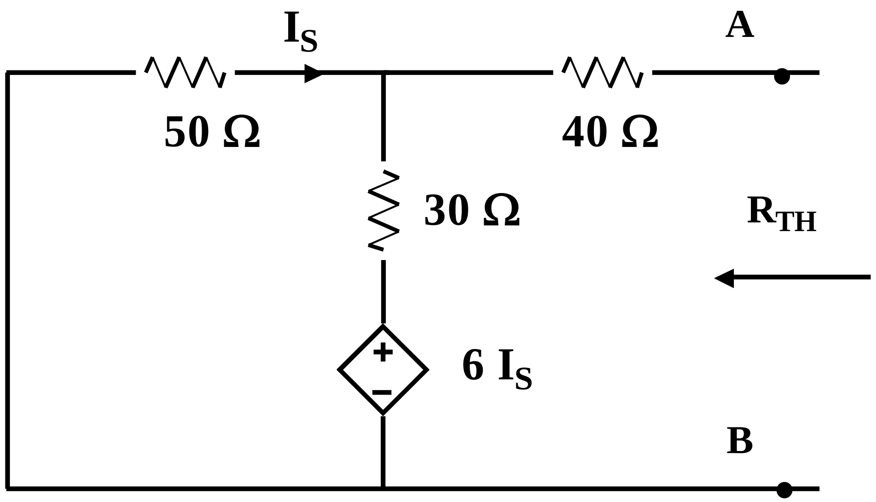

Figure : Example , independent sources deactivated.

We cannot find R_{TH} of the above circuit, as it stands. We must apply either a voltage or current source at the load and calculate the ratio of this voltage to current to find R_{TH}.

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THEVENIN'S THEOREM: Example 10.4: continued

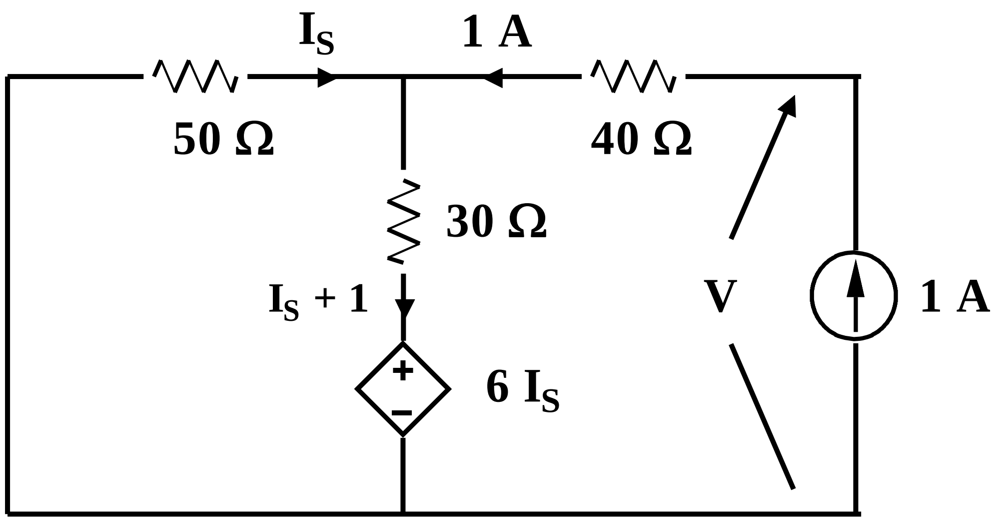

Figure 10.22: Circuit for find R_{TH}, Example 10.4.

Around the loop at the left we write the following equation:

$$50I_S + 30(I_S + 1) + 6I_S = 0$$

From which

$$I_S = \frac{-15}{43}\ A$$

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THEVENIN'S THEOREM: Example 10.4: continued

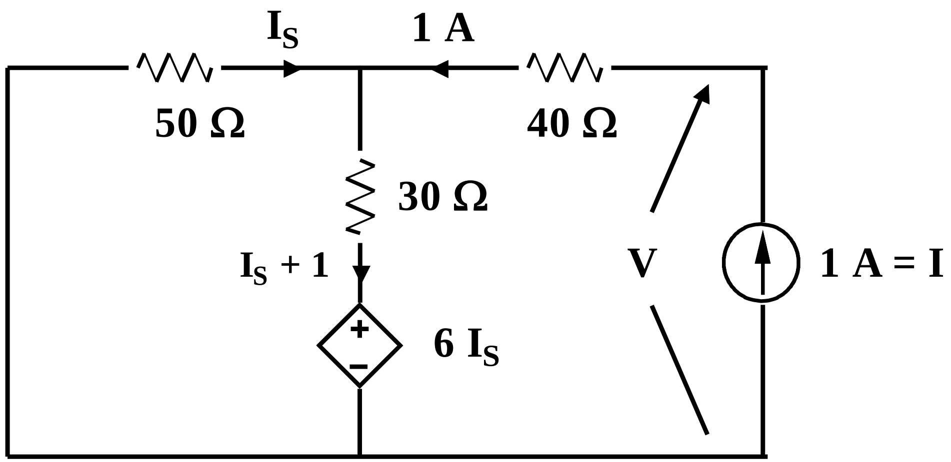

Figure 10.23: Circuit for find R_{TH}, Example 10.4.

Using the outer loop, going in the cw direction, using drops;

$$50\left(\frac{-15}{43}\right) - 1(40) + V = 0 \qquad \text{or} \qquad V = 57.4 \; volts$$

$$R_{TH} = \frac{V}{I} = \frac{V}{1} = 57.4 \; \Omega$$

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THEVENIN'S THEOREM: Example 10.4: continued

The Thevenin equivalent circuit tied to the 100 Ω load resistor is shown below.

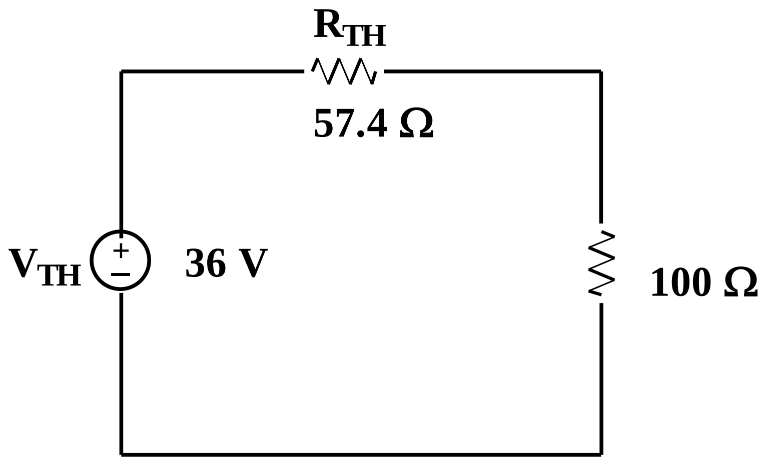

Figure 10.24: Thevenin circuit tied to load, Example 10.4.

$$V_{100} = \frac{36x100}{57.4 + 100} = 22.9\ V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: **Example 10.5: Finding the Thevenin circuit when only resistors and dependent sources are present. Consider the circuit below. Find V_{xy} by first finding the Thevenin circuit to the left of x-y.**

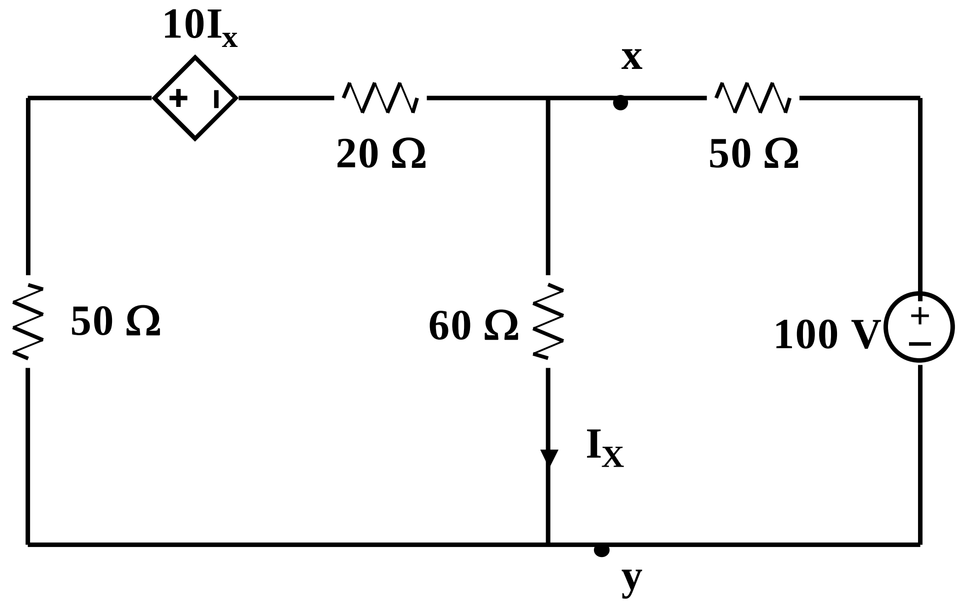

Figure 10.25: Circuit for Example 10.5.

For this circuit, it would probably be easier to use mesh or nodal analysis to find V_{xy}. However, the purpose is to illustrate Thevenin's theorem.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

We first reconcile that the Thevenin voltage for this circuit must be zero. There is no "juice" in the circuit so there cannot be any open circuit voltage except zero. This is always true when the circuit is made up of only dependent sources and resistors.

To find R_{TH} we apply a 1 A source and determine V for the circuit below.

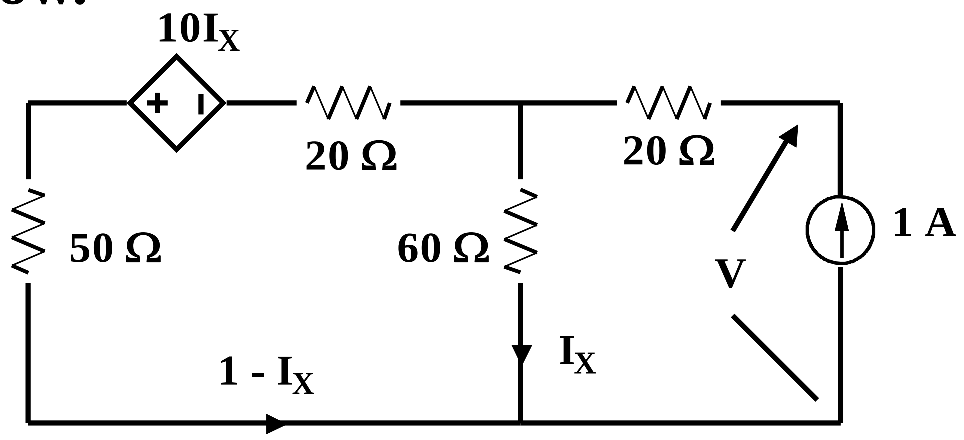

Figure 10.26: Circuit for find R_{TH}, Example 10.5.

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THEVENIN'S THEOREM: Example 10.5: continued

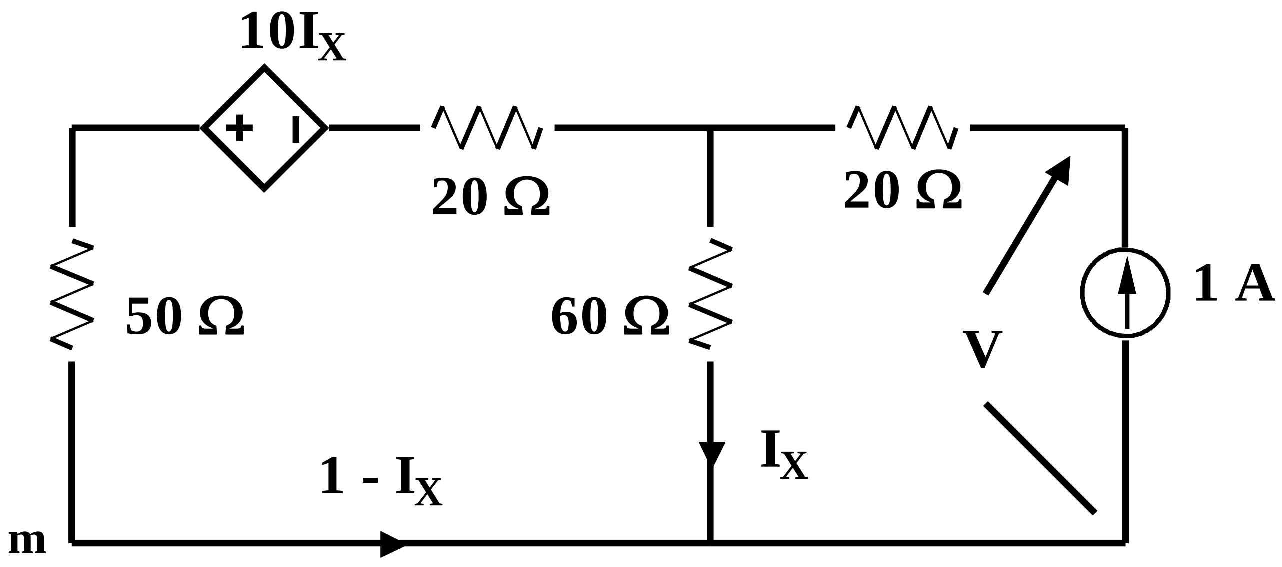


Figure 10.27: Circuit for find R_{TH}, Example 10.5.

Write KVL around the loop at the left, starting at "m", going cw, using drops:

$$-50(1-I_X)+10I_X-20(1-I_X)+60I_X = 0$$

$$I_X = 0.5\ A$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

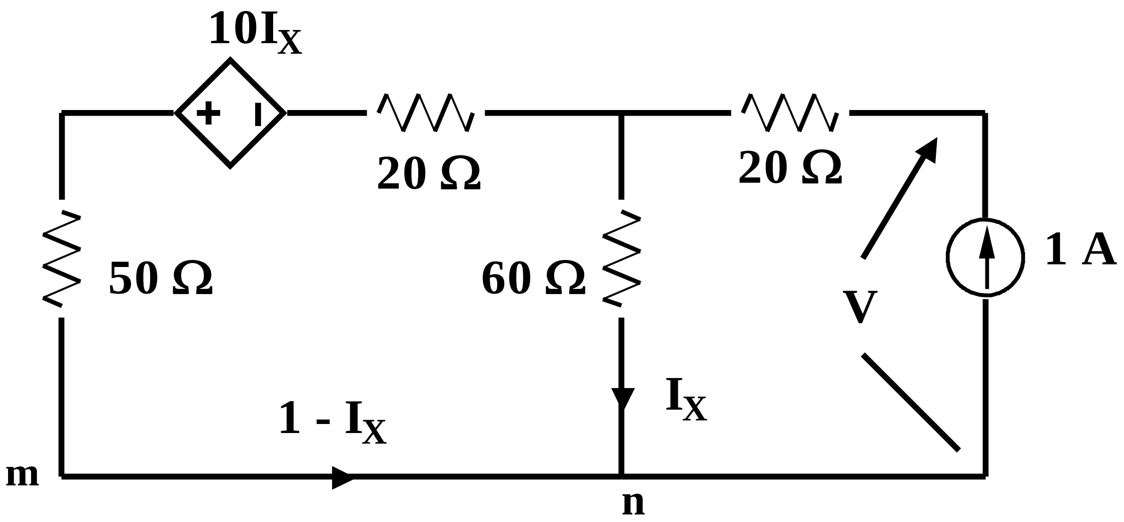


Figure 10.28: Determining R_{TH} for Example 10.5.

We write KVL for the loop to the right, starting at n, using drops and find;

$$-60(0.5) - 1x20 + V = 0$$

or

$$V = 50\ volts$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

We know that, $R_{TH} = \frac{V}{I}$, **where V = 50 and I = 1.**

Thus, $R_{TH} = 50\ \Omega$. **The Thevenin circuit tied to the load is given below.**

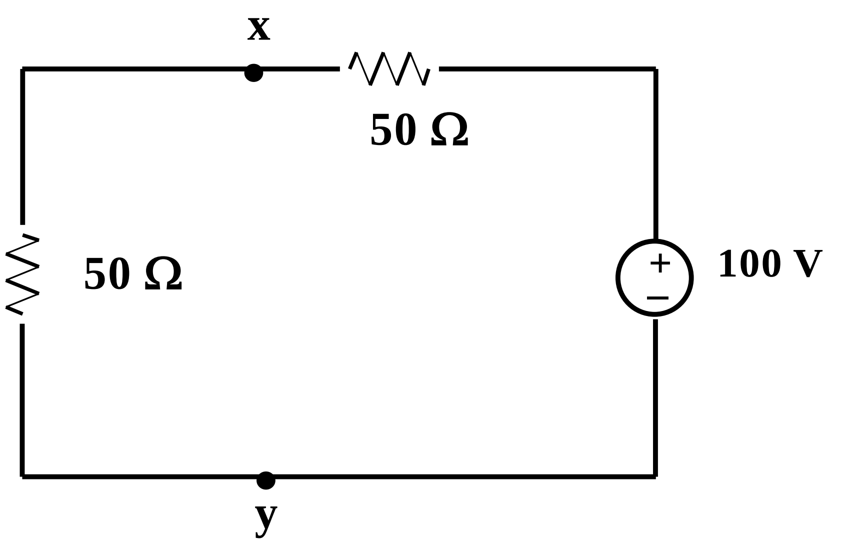


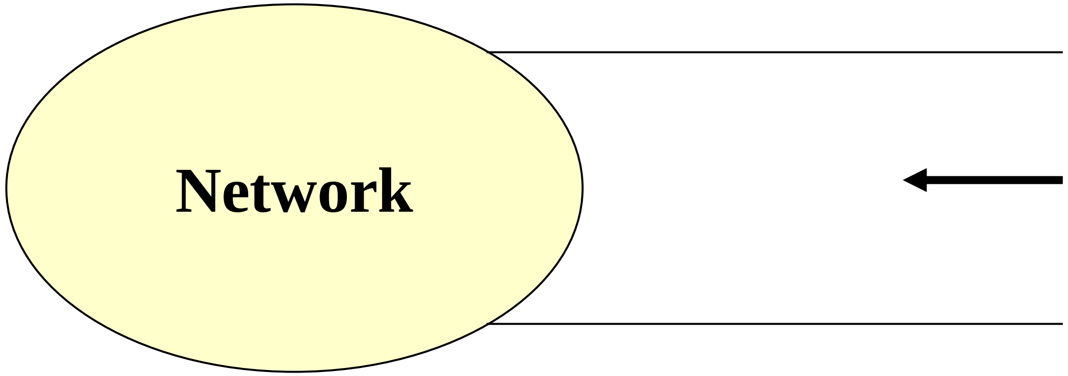
Figure 10.29: Thevenin circuit tied to the load, Example 10.5.

Obviously, $V_{XY} = 50$ **V**

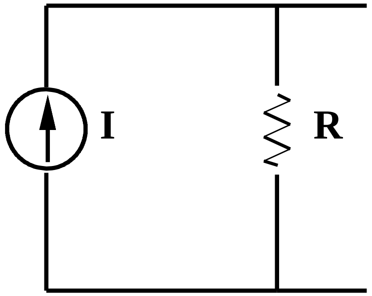
THEVENIN & NORTON

NORTON'S THEOREM:

Assume that the network enclosed below is composed of independent sources and resistors.



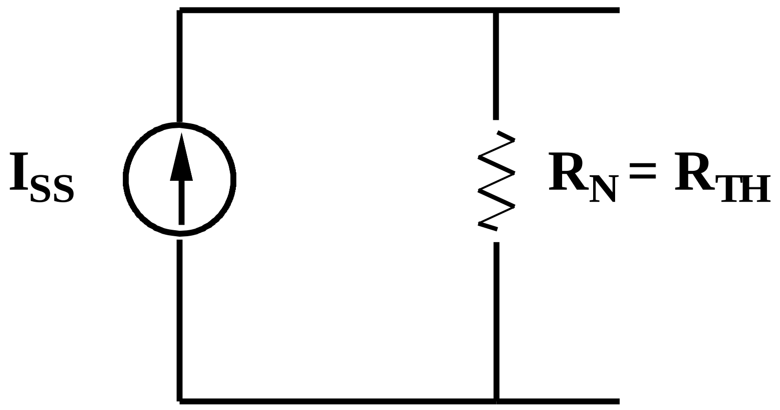
Norton's Theorem states that this network can be replaced by a current source shunted by a resistance R.



THEVENIN & NORTON

NORTON'S THEOREM:

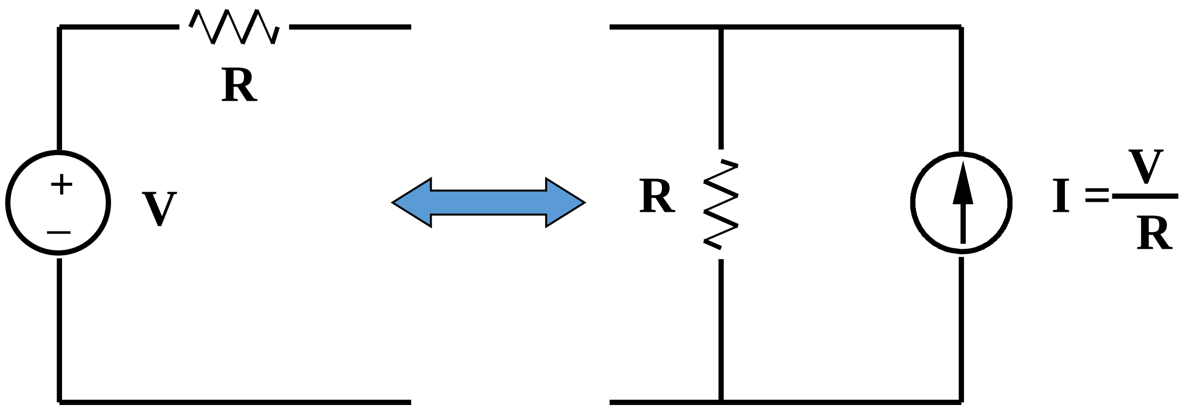
In the Norton circuit, the current source is the short circuit current of the network, that is, the current obtained by shorting the output of the network. The resistance is the resistance seen looking into the network with all sources deactivated. This is the same as R_{TH}.



THEVENIN & NORTON

NORTON'S THEOREM:

We recall the following from source transformations.



In view of the above, if we have the Thevenin equivalent circuit of a network, we can obtain the Norton equivalent by using source transformation.

However, this is not how we normally go about finding the Norton equivalent circuit.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6.

Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 50 Ω resistor.

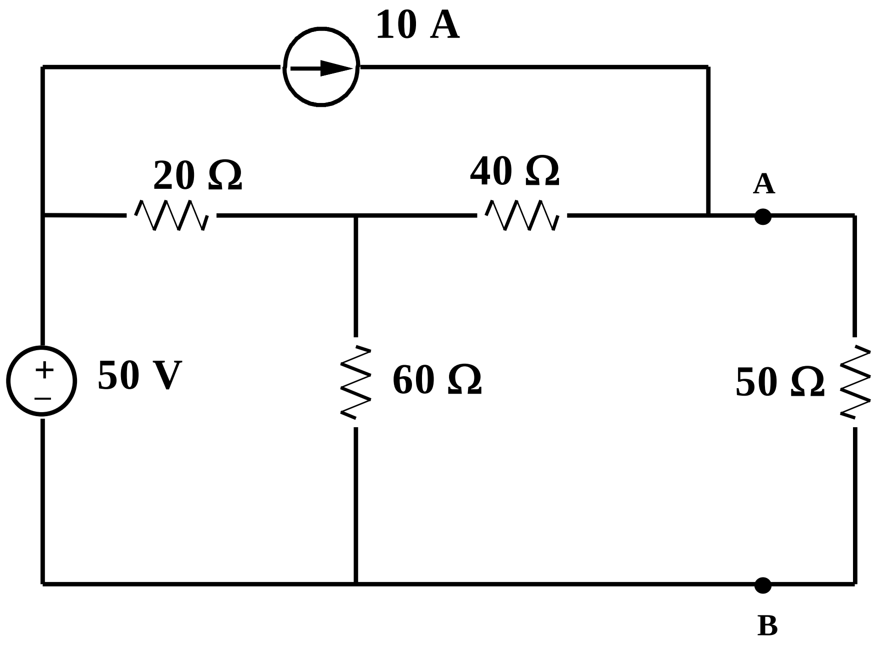


Figure 10.30: Circuit for Example 10.6.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6. continued

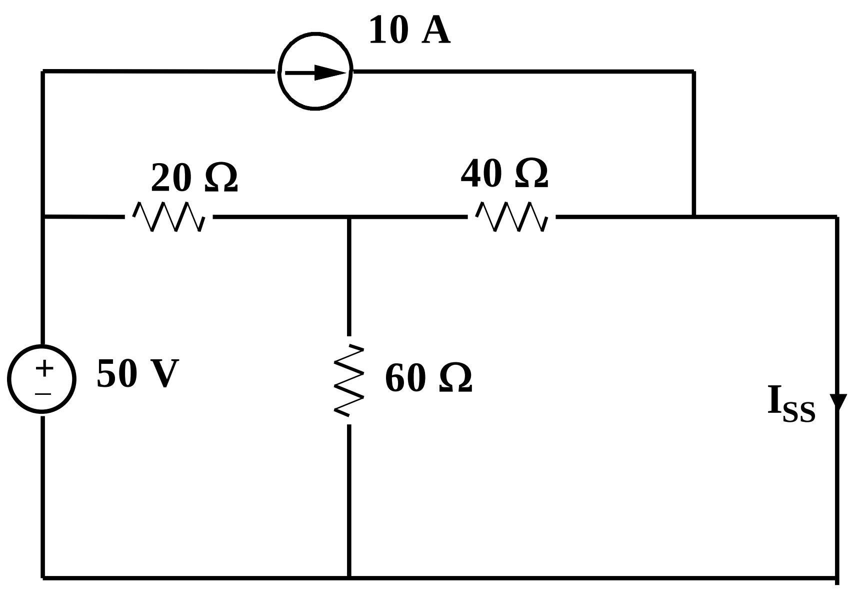


Figure 10.31: Circuit for find I_{NORTON}.

It can be shown by standard circuit analysis that

$$I_{SS} = 10.7\ A$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6. continued

It can also be shown that by deactivating the sources,
We find the resistance looking into terminals A-B is

$$R_N = 55\ \Omega$$

R_N and R_{TH} will always be the same value for a given circuit.
The Norton equivalent circuit tied to the load is shown below.

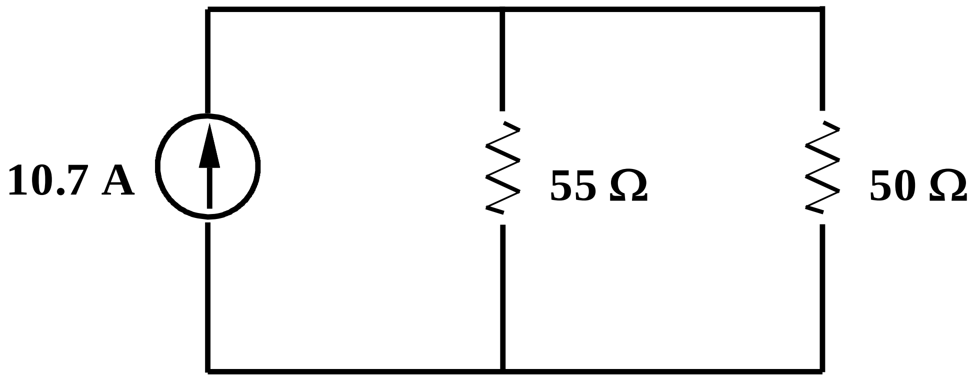


Figure 10.32: Final circuit for Example 10.6.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. **This example illustrates how one might use Norton's Theorem in electronics. the following circuit comes close to representing the model of a transistor.**

For the circuit shown below, find the Norton equivalent circuit to the left of terminals A-B.

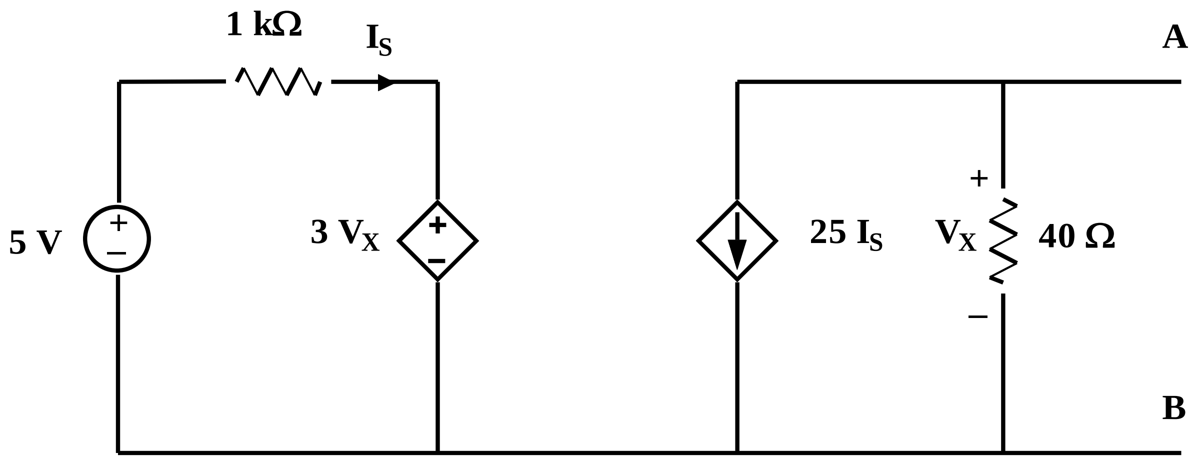
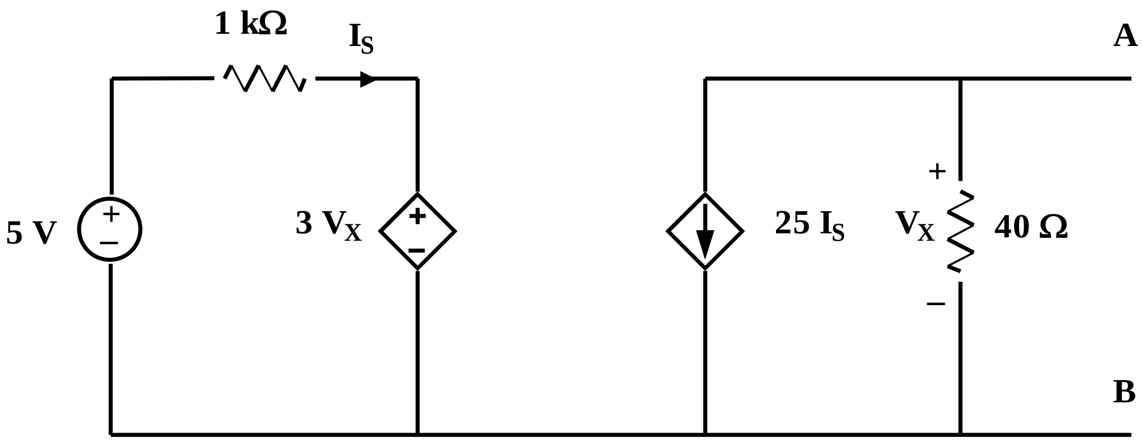


Figure 10.33: Circuit for Example 10.7.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued



We first find;

$$R_N = \frac{V_{OS}}{I_{SS}}$$

We first find V_{OS}:

$$V_{OS} = V_X = (-25 I_S)(40) = -1000 I_S$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued

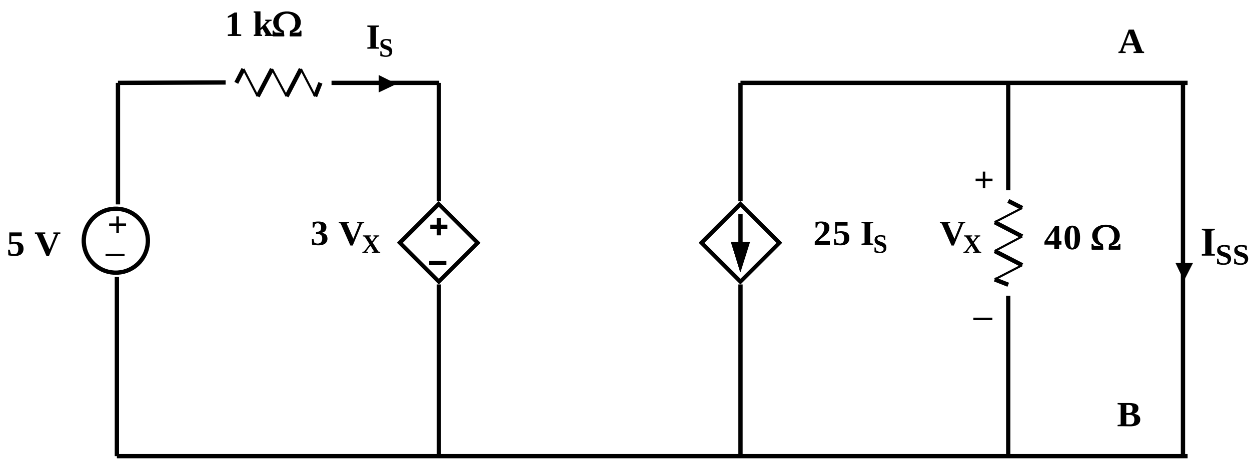


Figure 10.34: Circuit for find I_{SS}, Example 10.7.

We note that $I_{SS} = - 25I_S$. Thus,

$$R_N = \frac{V_{OS}}{I_{SS}} = \frac{-1000 I_S}{-25 I_S} = 40\ \Omega$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued

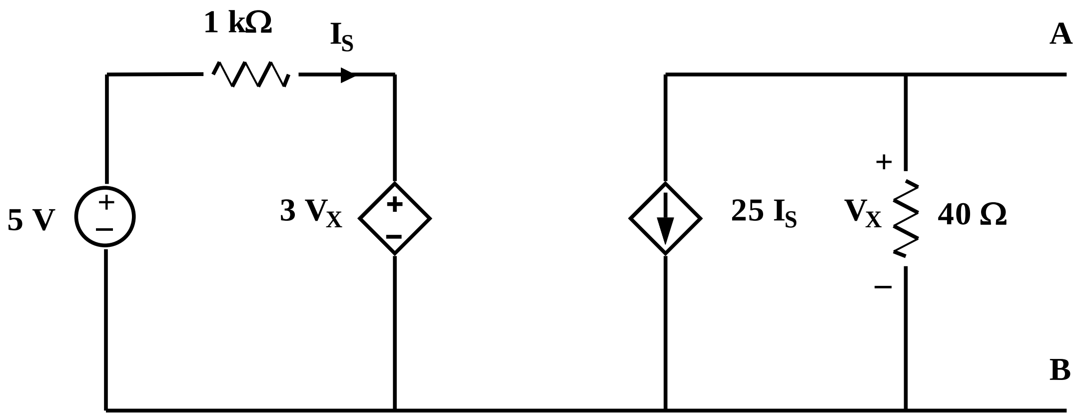


Figure 10.35: Circuit for find V_{OS}, Example 10.7.

From the mesh on the left we have;

$$-5 + 1000I_S + 3(-1000I_S) = 0$$

From which,

$$I_S = -2.5\ mA$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued

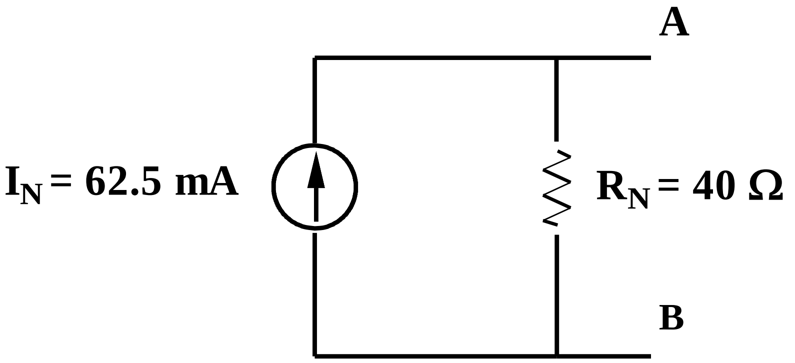
We saw earlier that,

$$I_{SS} = -25 I_S$$

Therefore;

$$I_{SS} = 62.5\ mA$$

The Norton equivalent circuit is shown below.



Norton Circuit for Example 10.7

THEVENIN & NORTON

Extension of Example 10.7:

Using source transformations we know that the Thevenin equivalent circuit is as follows:

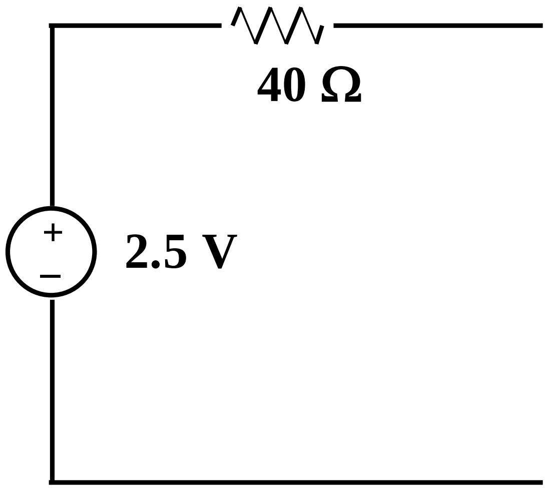


Figure 10.36: Thevenin equivalent for Example 10.7.