Definitions

- A set A is a subset of a set B if and only if every element of A is also an element of B.

$$A \subseteq B \leftrightarrow \forall x[x \in A \rightarrow x \in B]$$

$$\begin{aligned} A \not\subseteq B &\leftrightarrow \sim\forall x[x \in A \rightarrow x \in B] \\ &\leftrightarrow \exists x \sim[x \in A \rightarrow x \in B] \\ &\leftrightarrow \exists x \sim[\sim x \in A \vee x \in B] \qquad (p \rightarrow q \equiv \sim p \vee q) \\ &\leftrightarrow \exists x[x \in A \wedge \sim x \in B] \\ &\leftrightarrow \exists x[x \in A \wedge x \notin B] \end{aligned}$$

Definitions

- of A is

$A = \{x | x = 4k + 1 \ for\ some\ k \in Z\}$,
$B = \{x | x = 4k - 3 \ for\ some\ k \in Z\}$
Show that whether the sets A and B are equal or not.

(A⊆B) For any $x \in A$, $x = 4k + 1 \ for\ some\ k \in Z$

$$x = 4k + 1 + 3 - 3$$
$$x = 4(k + 1) - 3$$
$$x = 4m - 3 \ for\ some\ m \in Z, \text{ so } x \in B$$

(B⊆A) For any $x \in B$, $x = 4k - 3 \ for\ some\ k \in Z$

$$x = 4k - 3 + 1 - 1$$
$$x = 4(k - 1) + 1$$
$$x = 4m + 1 \ for\ some\ m \in Z, \text{ so } x \in A$$

Thus, A=B.

Definitions

- A set A is a subset of a set B if and only if every element of A is also an element of B.

  $A \subseteq B \leftrightarrow \forall x[x \in A \rightarrow x \in B]$

- $\emptyset \subseteq A$ and $A \subseteq A$.

- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

- A set A is a proper subset of a set B if and only if $A \subseteq B$ and $A \neq B$

  $B = \{x \in Z^+ | x < 10\}$ and $A = \{1, 2, 3, 4, 5\}$, $A \subseteq B$

Definitions

- The cardinality of a set A is defined as the size of A. It's denoted by $|A|$. (only for finite set)

  For the set $A = \{x \in Z^+ | x < 10\}$, $|A| = 9$

- The power set of a given set is the set of all possible subsets.

$S=\{1\}$	$P(S)=\{\emptyset, \{1\}\}$	$\lvert P(S)\rvert=2$
$S=\{a, b\}$	$P(S)=\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$	$\lvert P(S)\rvert=4$

- If $|S|=n$, then $|P(S)|=2^n$

Set Operations

- $A \cup \emptyset = A$
  $A \cap U = A$

  $p \vee 0 \equiv p$
  $p \wedge 1 \equiv p$

- $A \cup U = U$
  $A \cap \emptyset = \emptyset$

  $p \vee 1 \equiv 1$
  $p \wedge 0 \equiv 0$

- $A \cup A = A$
  $A \cap A = A$

  $p \vee p \equiv p$
  $p \wedge p \equiv p$

- $A \cup B = B \cup A$
  $A \cap B = B \cap A$

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- $A \cap \overline{A} = \emptyset$
  $A \cup \overline{A} = U$

  $p \wedge \sim p \equiv 0$
  $p \vee \sim p \equiv 1$

- $\overline{(\overline{A})} = A$

  $\sim(\sim p) \equiv p$

- $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan)
  $\overline{A \cap B} = \overline{A} \cup \overline{B}$

  $\sim(p \vee q) \equiv \sim p \wedge \sim q$
  $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Set Operations

- $A \cup \emptyset = A$
  $A \cap U = A$
- $A \cup U = U$
  $A \cap \emptyset = \emptyset$
- $A \cup A = A$
  $A \cap A = A$
- $A \cup B = B \cup A$
  $A \cap B = B \cap A$

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cap \overline{A} = \emptyset$
  $A \cup \overline{A} = U$
- $\overline{(\overline{A})} = A$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan)
  $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

Definitions

Show that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

($\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$) assume $x \in \overline{A \cup B}$, then $(x \notin A \cup B)$

$$\begin{aligned}(x \notin A \cup B) &\leftrightarrow \sim((x \in A) \lor (x \in B)) \\ &\leftrightarrow (x \notin A) \land (x \notin B) \\ &\leftrightarrow (x \in \overline{A}) \land (x \in \overline{B}) \\ &\leftrightarrow x \in \overline{A} \cap \overline{B}\end{aligned}$$

($\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$) assume $x \in \overline{A} \cap \overline{B}$, then $(x \in \overline{A}) \land (x \in \overline{B})$

$$\begin{aligned}(x \in \overline{A}) \land (x \in \overline{B}) &\leftrightarrow (x \notin A) \land (x \notin B) \\ &\leftrightarrow \sim(x \in A) \land \sim(x \in B) \\ &\leftrightarrow \sim((x \in A) \lor (x \in B)) \\ &\leftrightarrow (x \notin A \cup B) \leftrightarrow (x \in \overline{A \cup B})\end{aligned}$$

Cartesian Products

- The cartesian product of A and B, denoted by AxB, is the set of all pairs (x,y) where $x \in A$ and $y \in B$

$$A \, x \, B = \{(x, y) | x \in A \;\; \wedge \;\; y \in B\}$$

- A={a,b}, B={1, 2, 3}

AxB = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

$$\boxed{|A\text{x}B| = |A| \,.\, |B|}$$

- *The Cartesian products of the sets* $A_1, A_2, \ldots, A_n$ *is the set of ordered n-tuples* $(a_1, a_2, \ldots, a_n)$ *where* $a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n$.

$$A_1 x \ldots x A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i, i = 1..n\}$$