BLM231/COM231 Discrete Structures

Sample Questions for Midterm

1. Let A be a nonempty set and B be a fixed subset of A. Define a relation R on $P(A)$ such that for any $X, Y \in P(A)$, $(X, Y) \in R$ if $B \cap X = B \cap Y$. Show that R is an equivalence relation.

2. Let R be the relation defined on $A = Z \times Z$ in the following way :

$$\big((x_1, y_1), (x_2, y_2)\big) \in R \Leftrightarrow x_1 . y_2 = x_2 . y_1$$

Determine whether the relation R is an equivalence relation on A or not.

3. Consider the poset $\big(\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq\big)$.

a) Find the maximal elements of the poset.

b) Find the minimal elements of the poset.

c) Find the all upper bounds of $\{\{2\}, \{4\}\}$.

d) Find the all lower bounds of $\{\{1,3,4\}, \{2,3,4\}\}$.

4. Solve the recurrence relation $a_n = 3a_{n-1} + 10a_{n-2}$ where $a_0 = 1$ and $a_1 = 4$.

5. How many integer solutions are there for the eq. $x_1 + x_2 + x_3 = 15$ if $x_1 \geq 5$, $x_2 \geq 3$ and $x_3 \geq 0$?

6. Let $n \in \mathbb{Z}^+$ and $n \leq 500$. How many such n are there which are not divisible by 3, 5, or 8?

7. In how many ways can 12 different books be distributed among 4 people so that each gets exactly 3 books?

8. Let S be a subset of Z^+ and $|S| \geq 3$. Show that there exist distinct $x, y \in S$ such that $x + y$ is even.

9. How many committees of 12 people contain even number of women if it's selected from 10 men and 10 women?

10. Suppose $a, b \in Z$. Prove that if $a^2(b^2 - 2b)$ is odd, then a and b are both odd.

11. Use mathematical induction to prove that 43 divides $6^{n+1} + 7^{2n-1}$ for every positive integer n.

12. Prove that if $n^2 + 3n + 1$ is odd integer, then n is odd integer.