COM234 ELECTRONICS

The Node Voltage Method

The Node Voltage Method

In this part, we will cover the following topics:

- Some basic definitions
- The steps for writing the Node-Voltage Equations
- Tips on picking the best reference node
- How to handle dependent sources

This material is covered in your textbook in the following sections:

- **Electric Circuits 10th Ed. by Nilsson and Riedel: Sections 4.1 through 4.3**

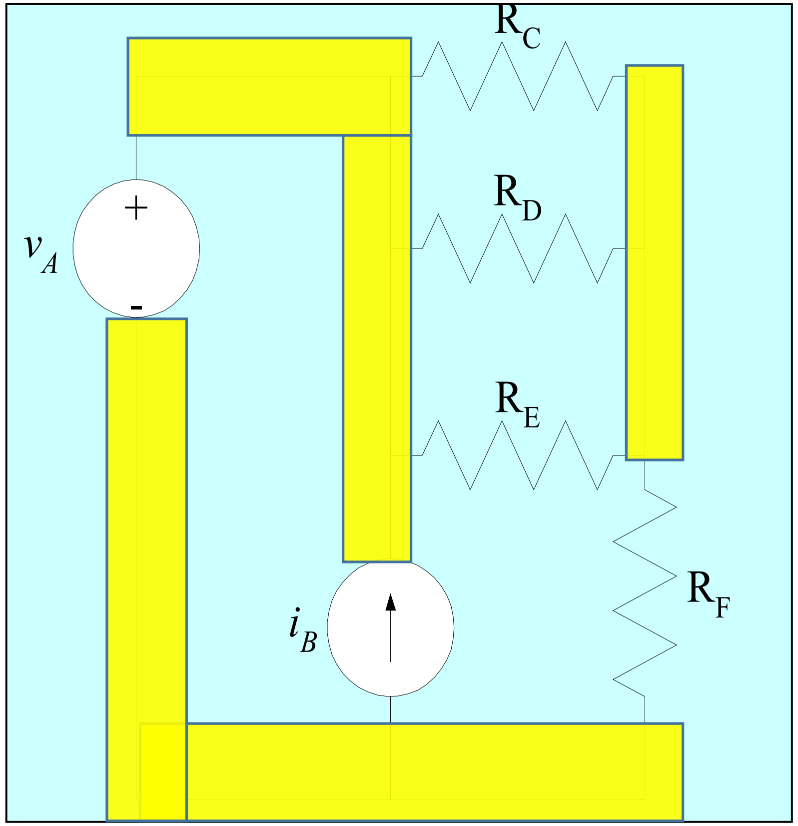
Some Basic Definitions

- **Node** – a place where two or more components meet
- **Essential Node** – a place where three or more components meet
- **Reference Node** – a special essential node that we choose as a reference point for voltages

You may be familiar with the word node from its use as a location in computer networks. It has a similar meaning there, a place where computers are connected.

Some Basic Definitions

- A node is defined as a place where two or more components are connected.
- In the example circuit schematic given here, there are three nodes. These nodes are shown in yellow here.
- The key thing to remember is that we connect components with wires. It doesn't matter how many wires are being used; it only matters how many components are connected together.
- Some students count more than three nodes in a circuit like this. When they do, it is usually because they have considered two points connected by a wire to be two nodes.
- How many nodes are there in this circuit here?
- There are also three essential nodes. Each of these three nodes has at least 3 components connected to it.



The Node-Voltage Method (NVM)

The Node-Voltage Method (NVM) is a systematic way to write all the equations needed to solve a circuit, and to write just the number of equations needed. The idea is that any other current or voltage can be found from these node voltages.

This method is not that important in very simple circuits, but in complicated circuits it gives us an approach that will get us all the equations that we need, and no extras.

It is also good practice for the writing of KCL and KVL equations. Many students believe that they know how to do this, but make errors in more complicated situations. Our work on the NVM will help correct some of those errors.

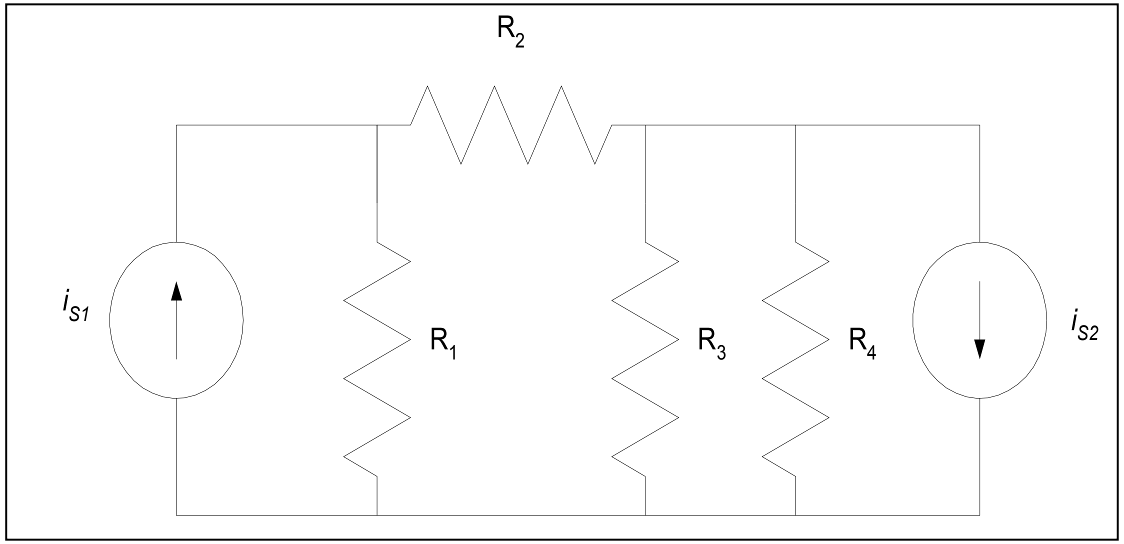
The Node-Voltage Method (NVM)

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

NVM – 1st Example

For most students, it seems to be best to introduce the NVM by doing examples. We will start with simple examples, and work our way up to complicated examples. Our first example circuit is given here.



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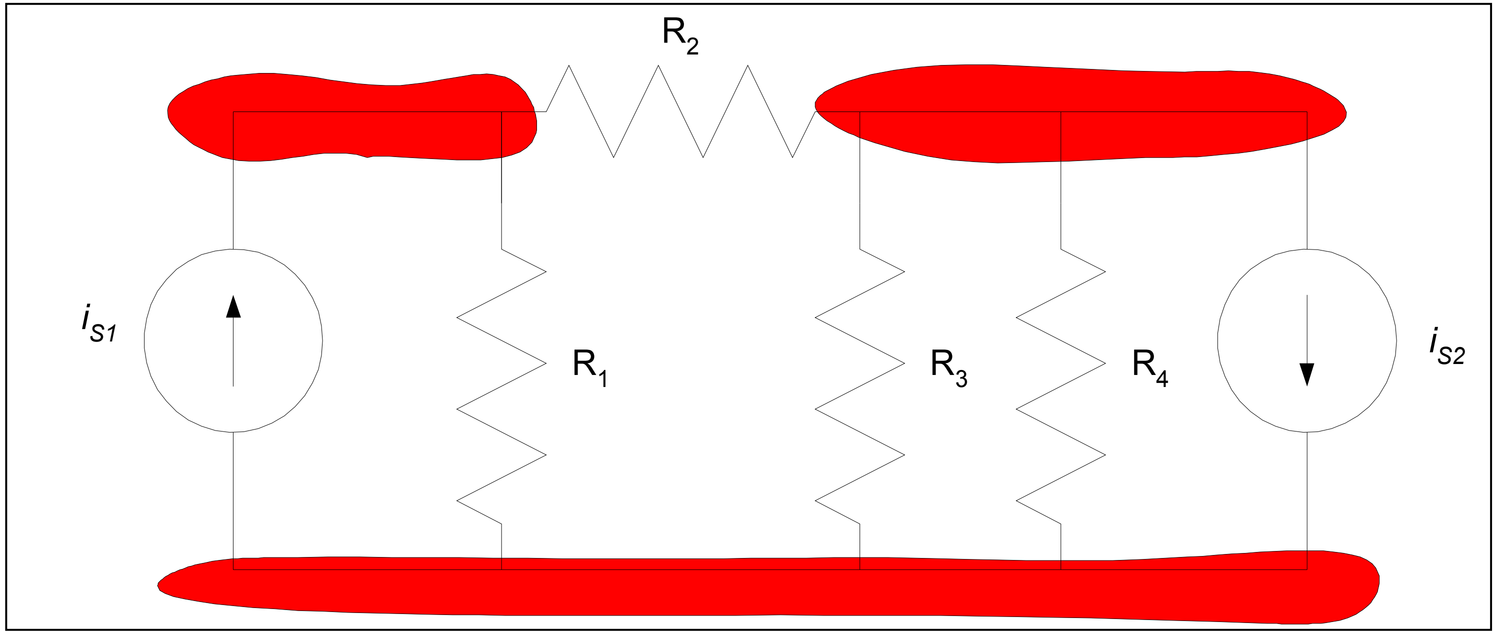

There are three essential nodes, each of which is shown in red.

NVM – 1st Example

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- **<u>Define one essential node as the reference node.</u>**
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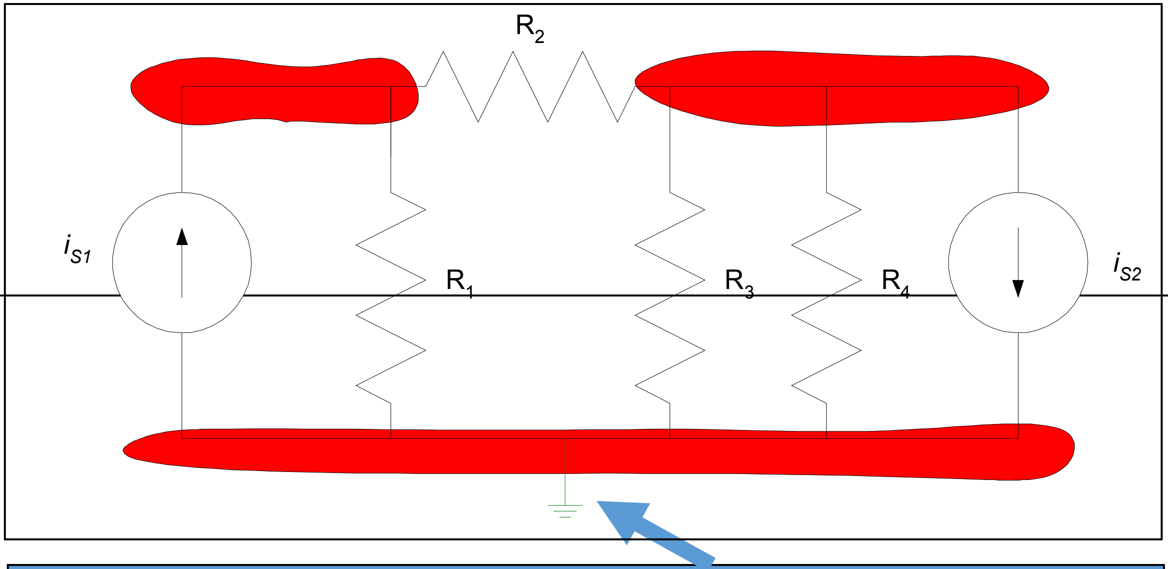

This symbol is used to designate the reference node. There are different symbols used for this designation. This choice of symbols is not important. Making a designation **<u>is</u>** important.

We could choose any of the three essential nodes as the reference node. However, there are better choices. Remember that we need to write a KCL equation for each essential node, except for the reference node. The best idea, then, is to pick the node with the most connections, to eliminate the most difficult equation. Here this is the bottom node. It is labeled to show that it is the reference node.

NVM – 1st Example

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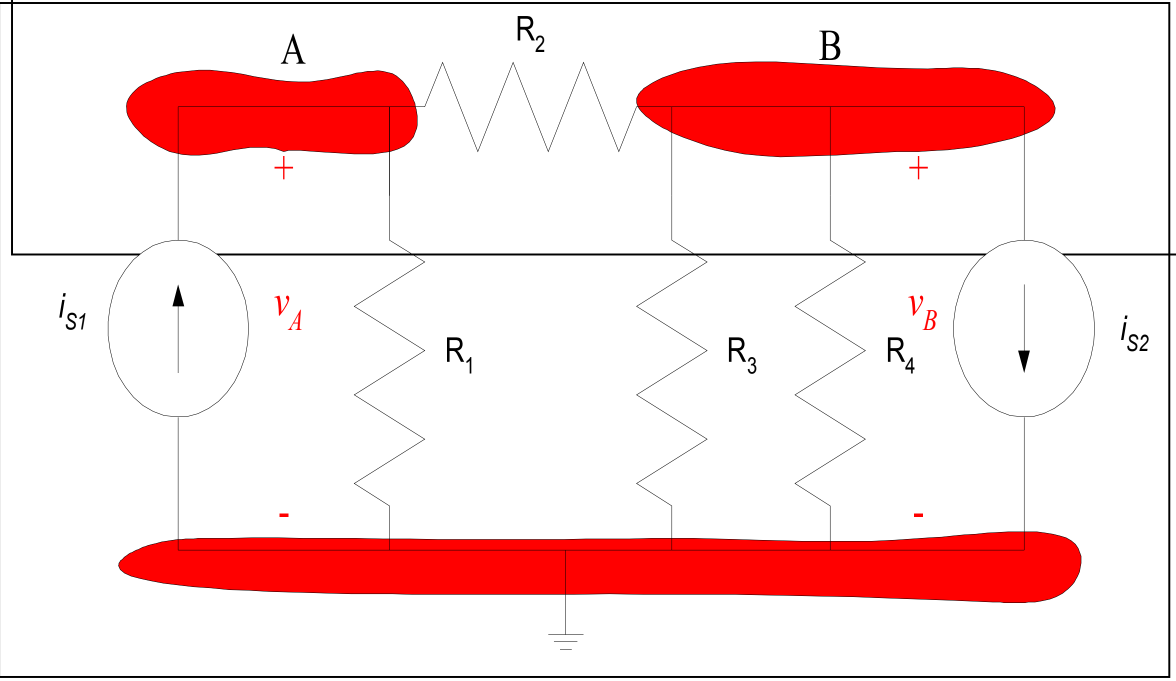
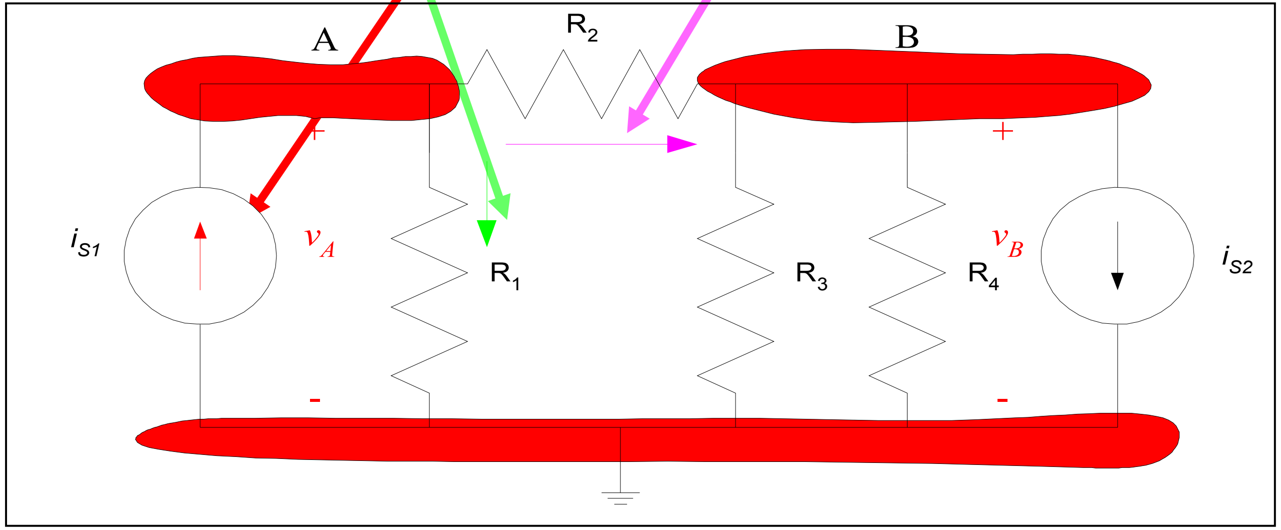


We have labeled the node voltages, v_A and v_B. They are shown in red. For clarity, we have also named the nodes themselves, A and B.

Note: As with any voltage, the polarity must be defined. We have defined the voltages by showing the voltages with a "+" and "-" sign for each. Strictly speaking, this should not be necessary. The words in step 3 make the polarity clear. Some texts do not label the voltages on the schematic. For clarity, we will label the voltages in these notes.

NVM – 1st Example

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



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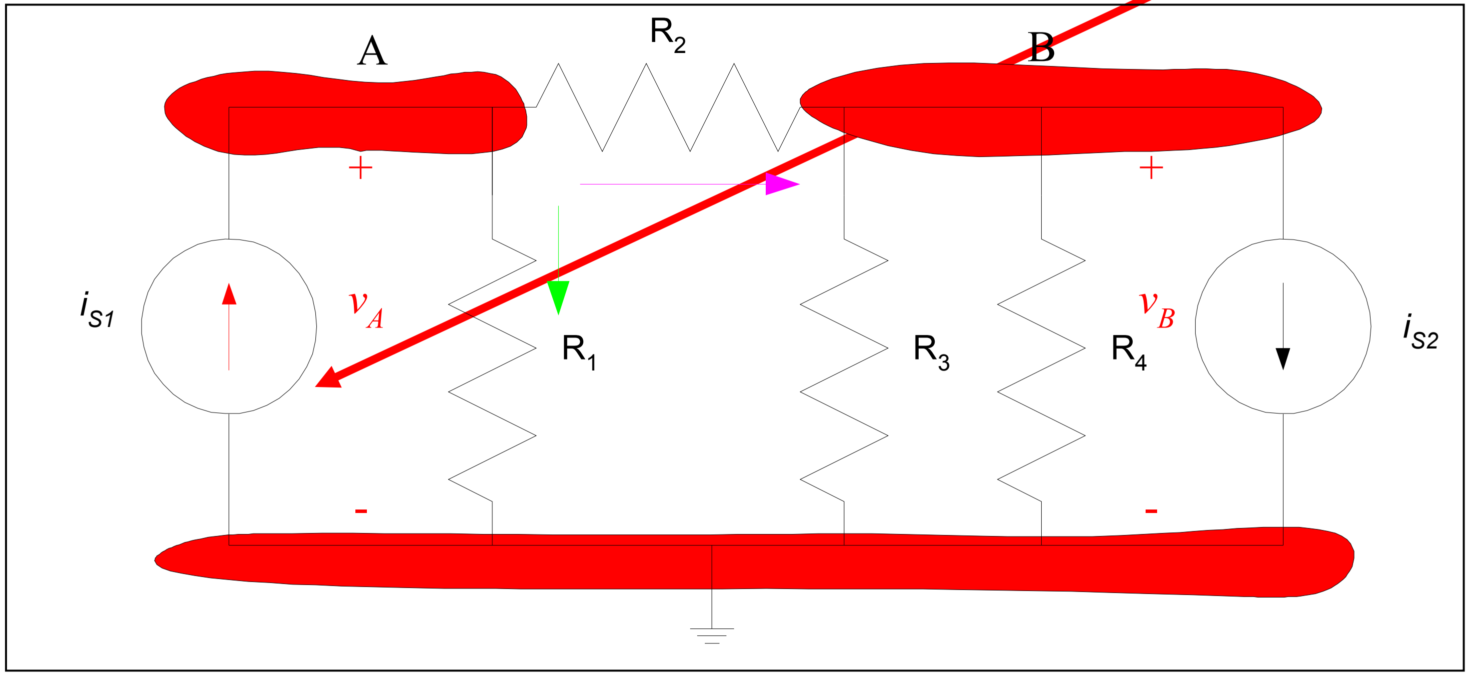
NVM – 1st Example

The first term comes from Ohm's Law. The voltage v_A is the voltage across R_1. Thus, the current shown in green is v_A/R_1, out of node A, and thus has a + sign in this equation.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

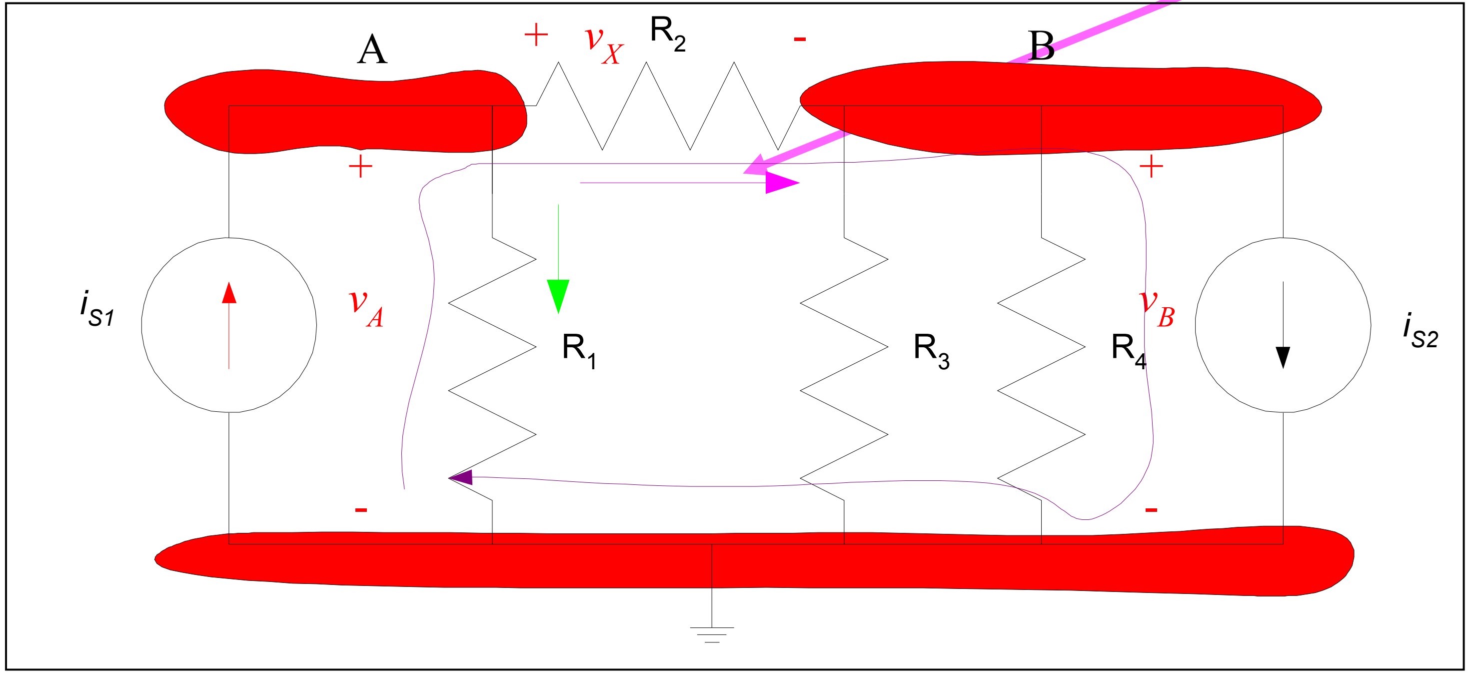


The current through the current source is, by definition, given by the value of that current source. Since the reference polarity of the current is entering node A, it has a "-" sign.

NVM – 1st Example

This current expression also comes from Ohm's Law. The voltage v_X is the voltage across the resistor R_2, and results in a current in the polarity shown.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



To prove to yourself that $v_X = v_A - v_B$, take KVL around the loop shown. The voltage at A with respect to B, is $v_A - v_B$, where v_A and v_B are both node voltages.

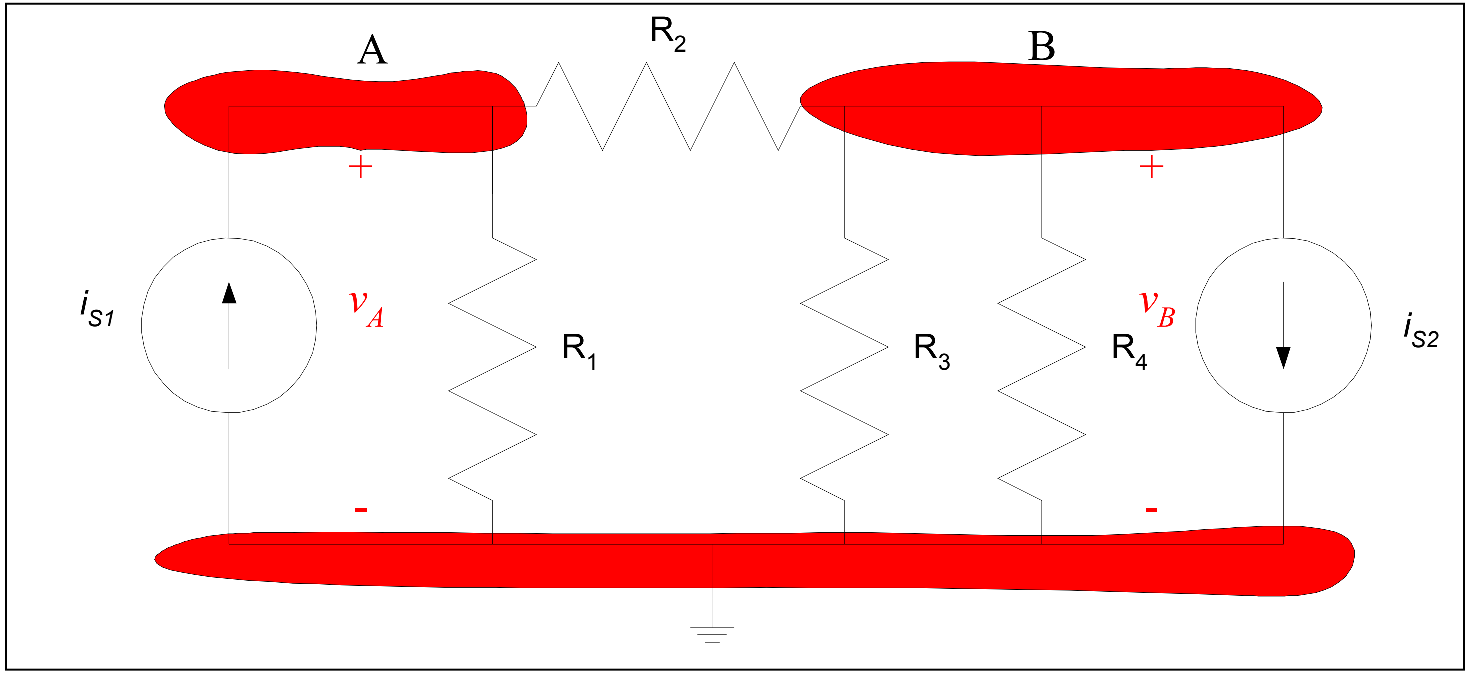
NVM – 1st Example

The KCL equation for the A node was:

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

The KCL equation for the B node is:

$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$



Be very careful that you understand the signs of all these terms. One of the big keys in these problems is to get the signs correct. If you have questions, review this material.

NVM – 1st Example

The Node-Voltage Method steps are:

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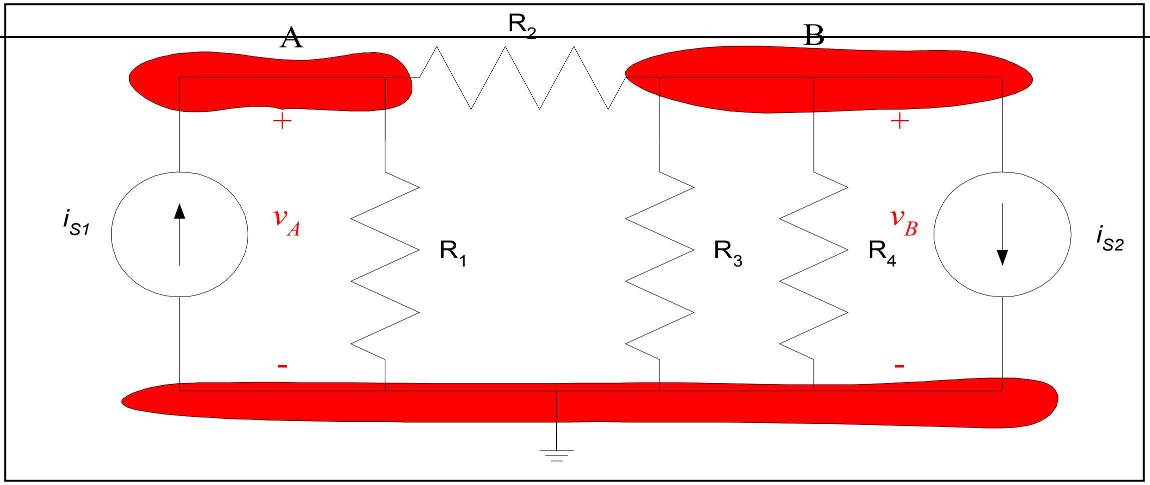
- **Write an equation for each current or voltage upon which dependent sources depend, as needed.**

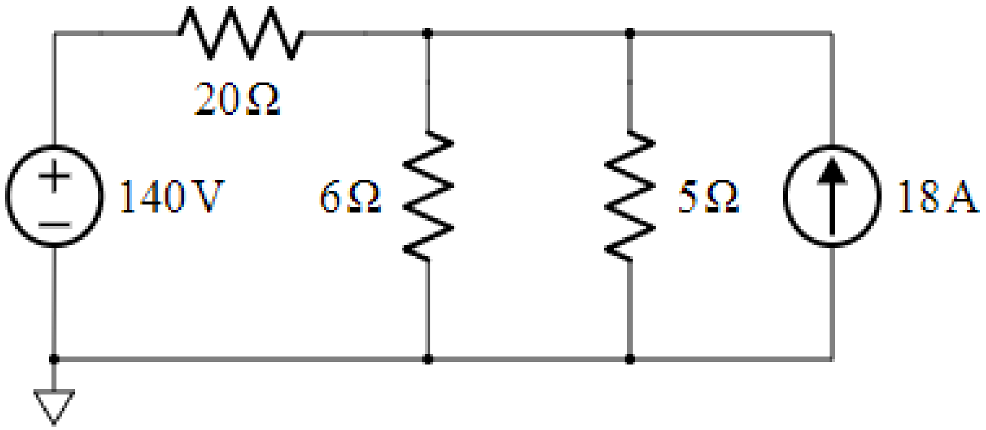
$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

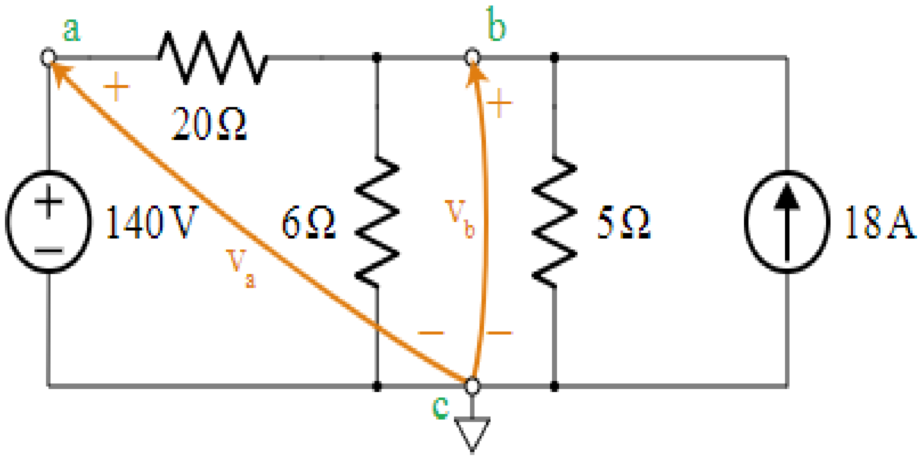
$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$



Note that we have assumed that all the values of the resistors and sources have been given. If not, we will need to get more information before we can solve.

NVM 2nd Example



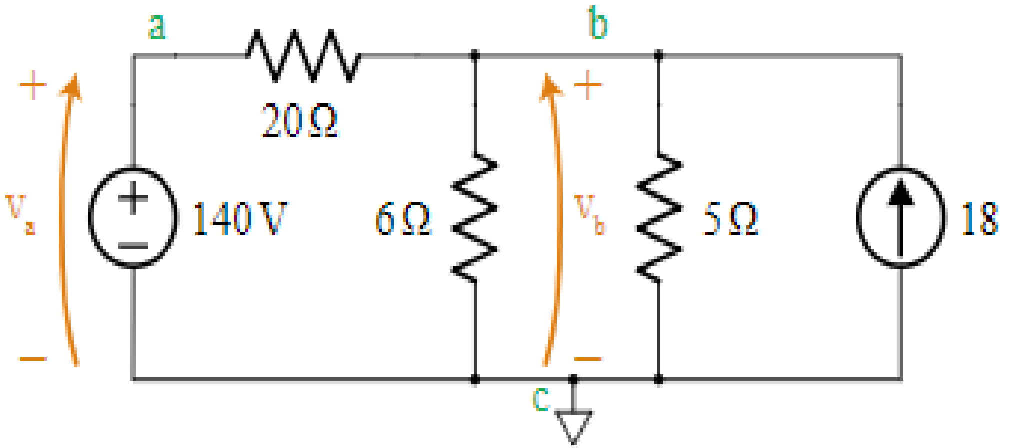


Our example circuit has three nodes, a, b and c, so N=3. Node c has 4 connections and it connects directly to both sources. This make it a good candidate to play the role of reference node. Node c has been marked with the ground symbol to let everyone know our choice for reference node.

We also call out N-1 =2 node voltages on the schematic, labeled in orange as Va and Vb

There is an obvious opprtunity here to simplify the two parallel resistors, 6 ohm with 5 ohm. We will not do that, because we want to study the Node Voltage Method procedure

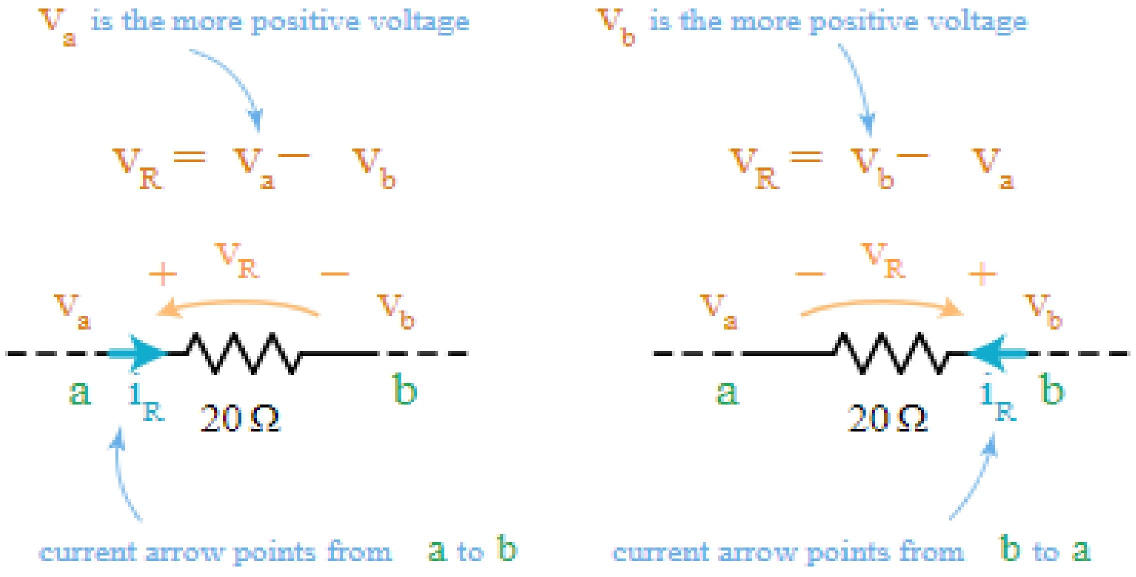
NVM 2nd Example



Node voltages control the current arrow

Notice something missing from the schematic. There is no orange label on the voltage across the 20 Ω resistor. When we need to know that voltage, we express it in terms of the node voltages.

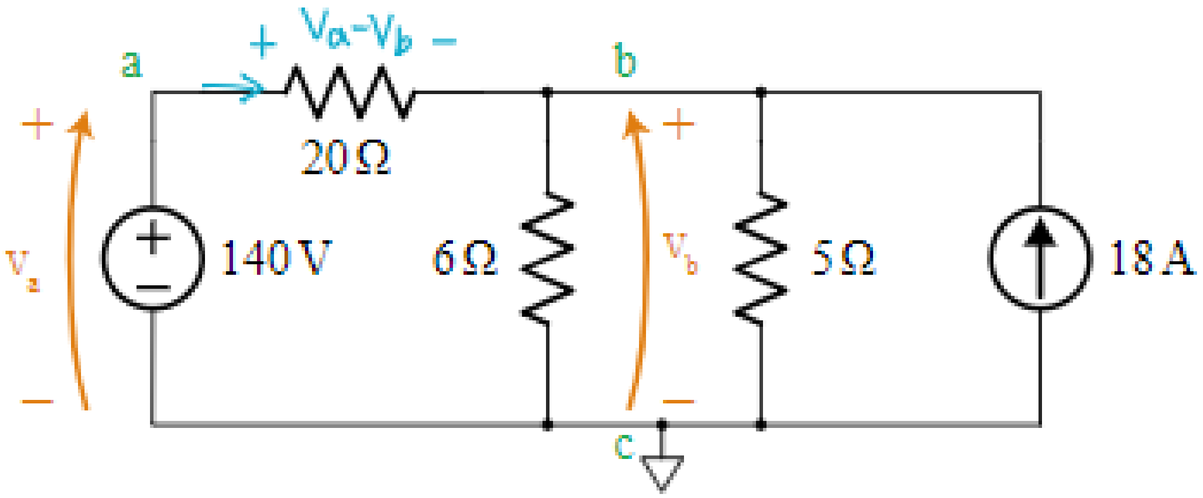
$v_R = v_a - v_b$ or $v_R = v_b - v_a$



NVM 2nd Example

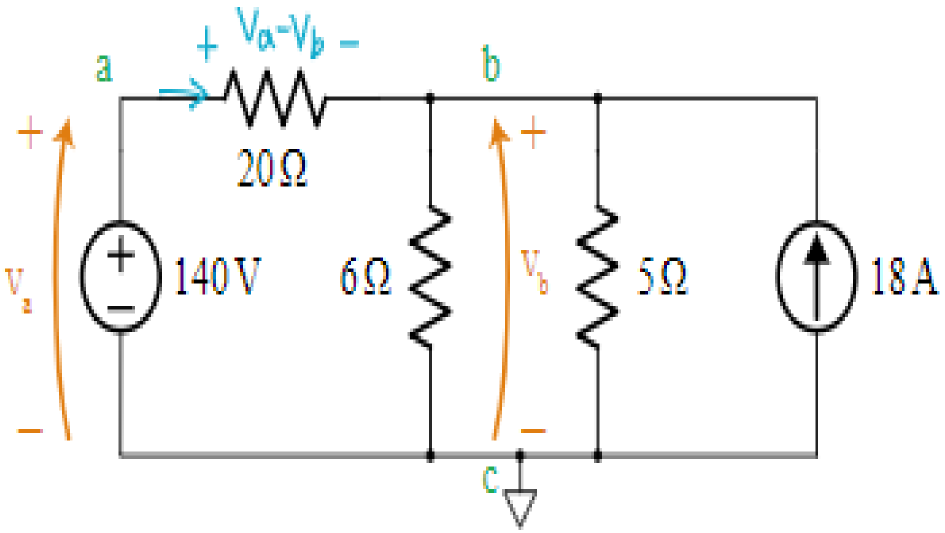
Solve the easy nodes

The voltage v_a is easy to figure out. Node a connects to a voltage source that connects to reference node c. That makes it an easy node. The voltage at node a is $v_a = 140\,\text{V}$.



NVM 2nd Example

As you write each term in the KCL equation, do Ohm's Law in your head and immediately write the current in terms of node voltages divided by resistance.



We now write a KCL equation for the remaining unsolved node, b. Node voltage v_b is the independent variable.

The current (blue arrow) flowing *into* node b from the $20\,\Omega$ resistor can be written as $+\dfrac{(140 - v_b)}{20}$.

The current in the $6\,\Omega$ and $5\,\Omega$ resistors instantly goes into the equation as $-\dfrac{v_b}{6}$ and $-\dfrac{v_b}{5}$.

We have just one node to deal with, node b. KCL says the sum of the currents flowing *into* node $b = 0$.

$$+\frac{(140 - v_b)}{20} - \frac{v_b}{6} - \frac{v_b}{5} + 18 = 0$$

NVM 2nd Example

Find the node voltages

Our system of equations happens to be just one equation. Let's solve it to find the node voltage.

$$+\frac{140}{20}-\frac{v_b}{20}-\frac{v_b}{6}-\frac{v_b}{5}=-18$$

$$-\frac{v_b}{20}-\frac{v_b}{6}-\frac{v_b}{5}=-18-7$$

$$\left(-\frac{3}{60}-\frac{10}{60}-\frac{12}{60}\right)\cdot v_b=-25$$

$$v_b=-25\cdot\left(-\frac{60}{25}\right)$$

$$v_b=60\,\mathrm{V}$$

NVM 2nd Example

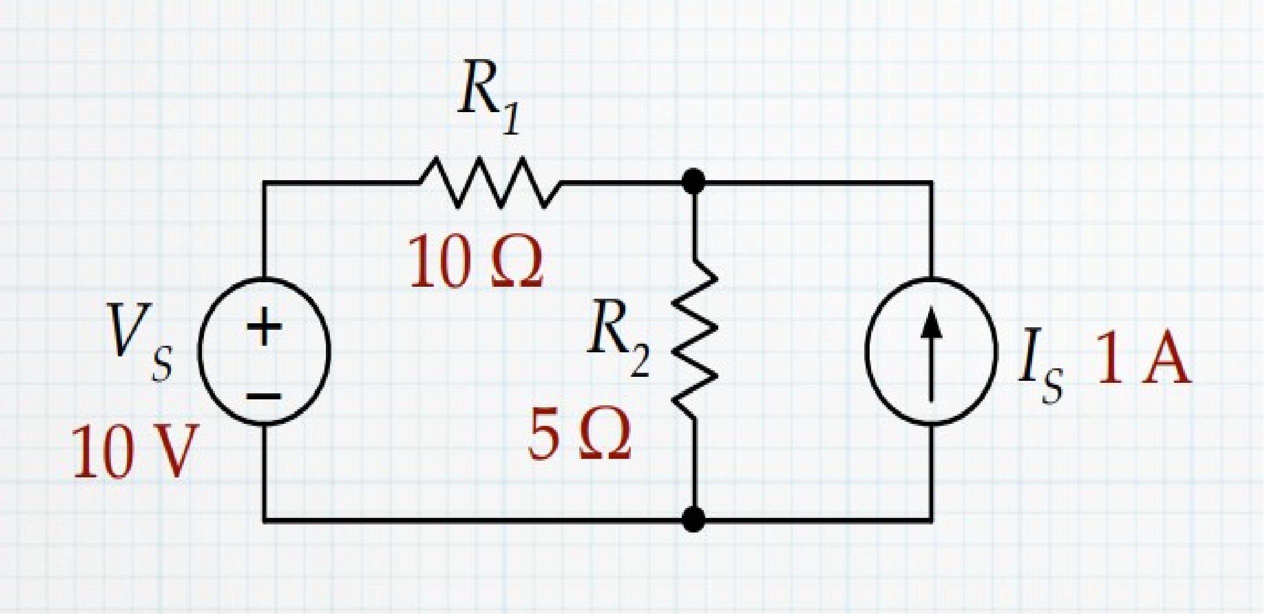
Solve for unknown currents using Ohm's Law

Now we have both node voltages, and we can solve for all the unknown currents using Ohm's Law.

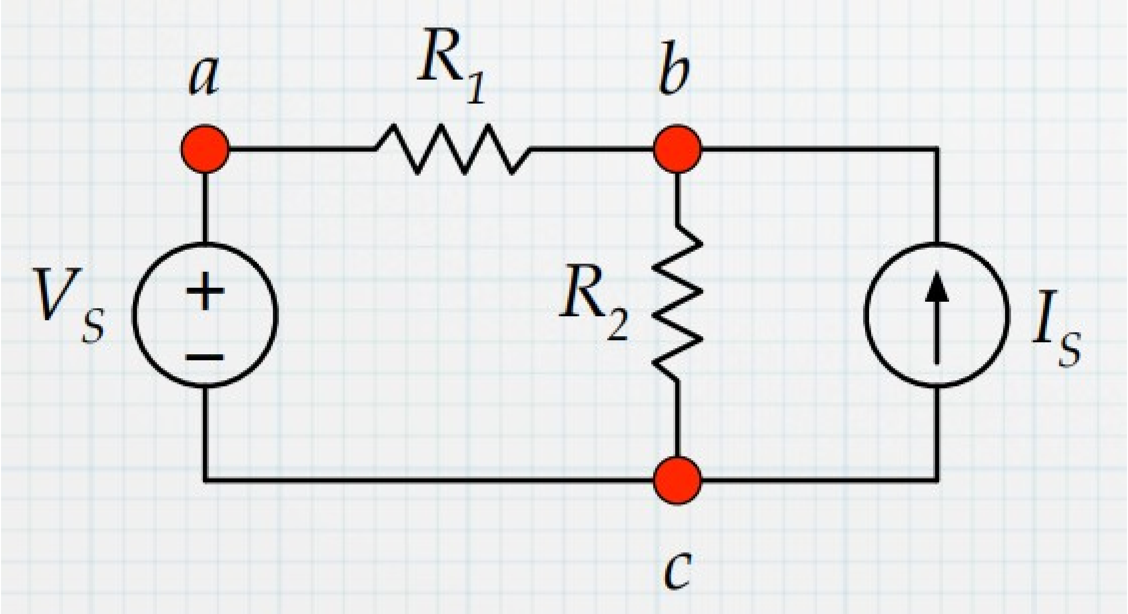
$$i_{20\,\Omega} = \frac{(v_a - v_b)}{20} = \frac{(140 - 60)}{20} = 4\ \mathrm{A}$$

$$i_{6\,\Omega} = \frac{v_b}{6} = \frac{60}{6} = 10\ \mathrm{A}$$

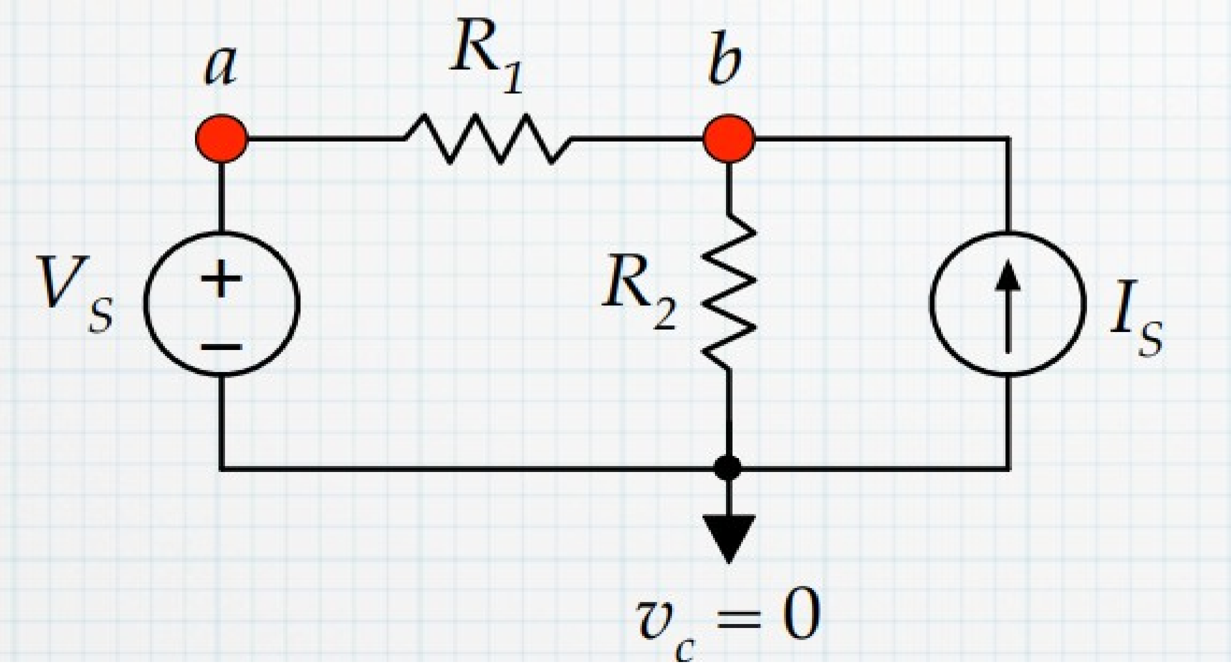
$$i_{5\,\Omega} = \frac{v_b}{5} = \frac{60}{5} = 12\ \mathrm{A}$$



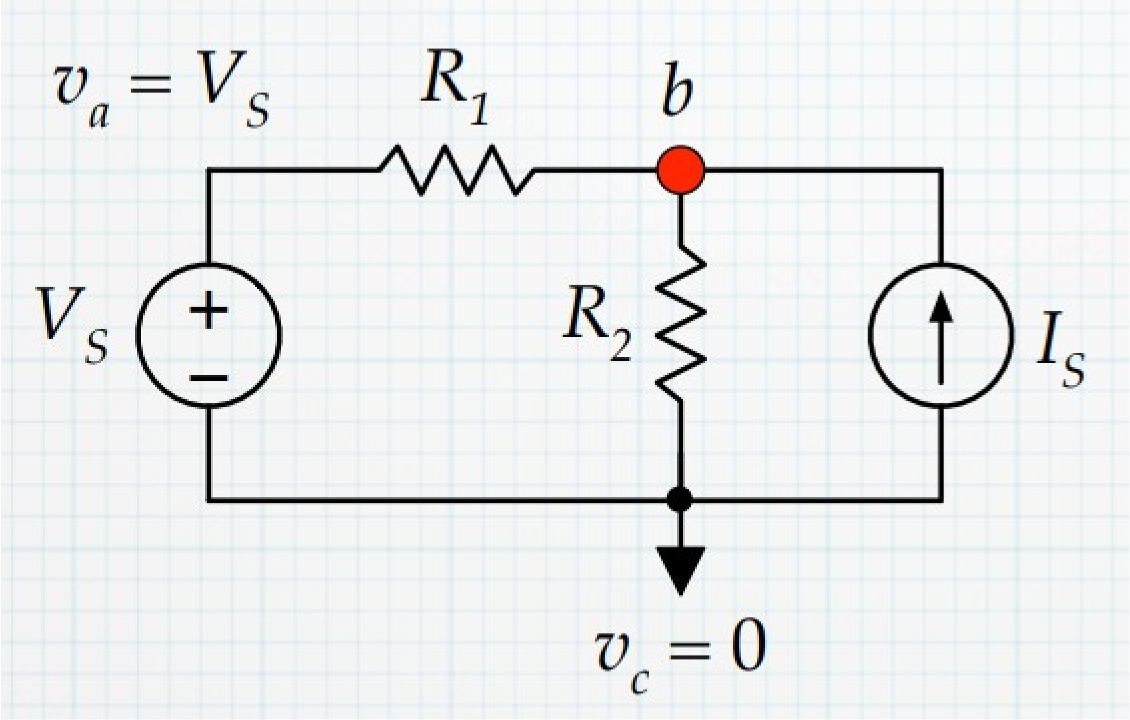
R_1
10 Ω
V_S
10 V
R_2
5 Ω
I_S 1 A



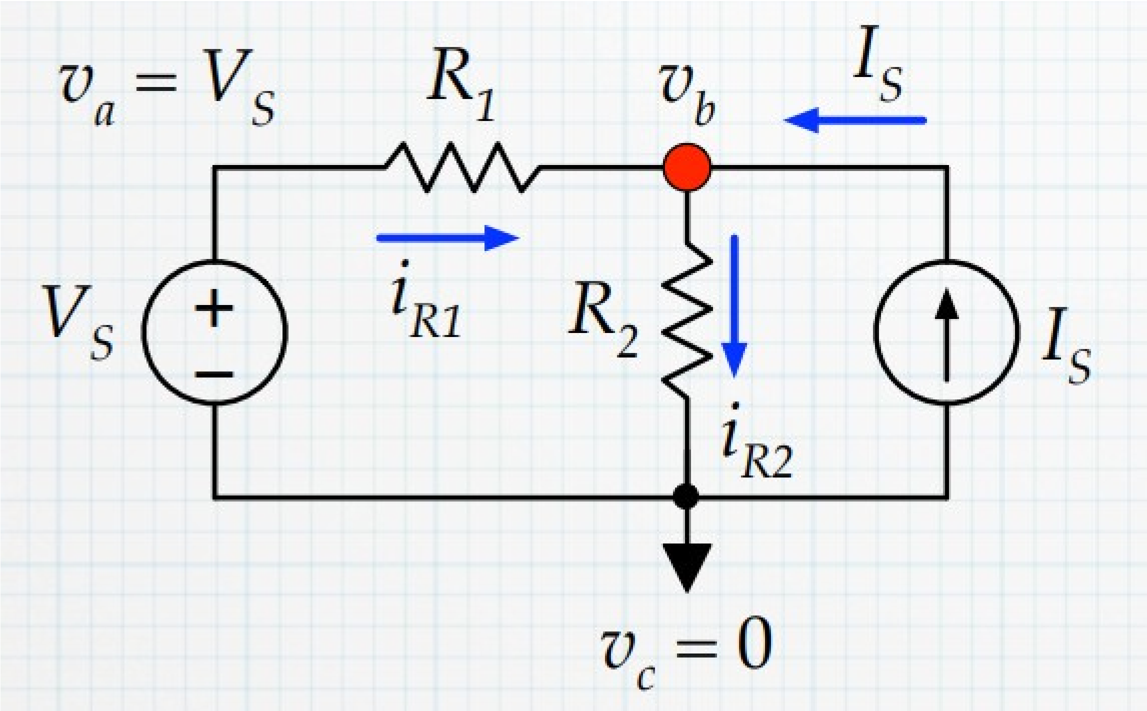
a
R_1
b
V_S
R_2
I_S
c



a
R_1
b
V_S
R_2
I_S
$v_c = 0$



$v_a = V_S$
R_1
b
V_S
R_2
I_S
$v_c = 0$



$$i_{R1} + I_S = i_{R2}$$

$$i_{R1} = \frac{V_S - v_b}{R_1}$$

$$i_{R2} = \frac{v_b - 0}{R_2}$$

$$\frac{V_S - v_b}{R_1} + I_S = \frac{v_b}{R_2}$$

$$V_S - v_b + R_1 I_S = \frac{R_1}{R_2} v_b$$

$$v_b = \frac{V_S + R_1 I_S}{1 + \frac{R_1}{R_2}}$$

$$V_S + R_1 I_S = \left(1 + \frac{R_1}{R_2}\right) v_b$$

$$v_b = \frac{10\text{V} + (10\Omega)\,(1\text{A})}{\left(1 + \frac{10\Omega}{5\Omega}\right)} = \boxed{6.67\text{V}}$$

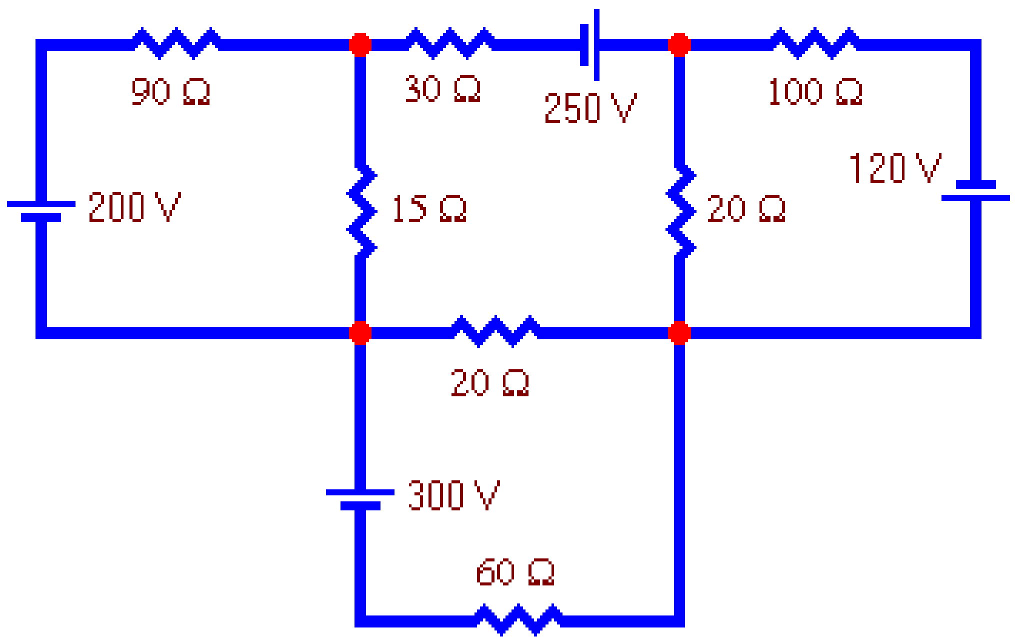
$$v_{R1} = V_S - v_b = 10\text{ V} - 6.67\text{ V} = 3.33\text{ V}$$

$$v_{R2} = v_b - 0 = 6.67\text{ V}$$

$$i_{R1} = \frac{v_{R1}}{R_1} = \frac{3.33\text{ V}}{10\Omega} = 0.333\text{ A}$$

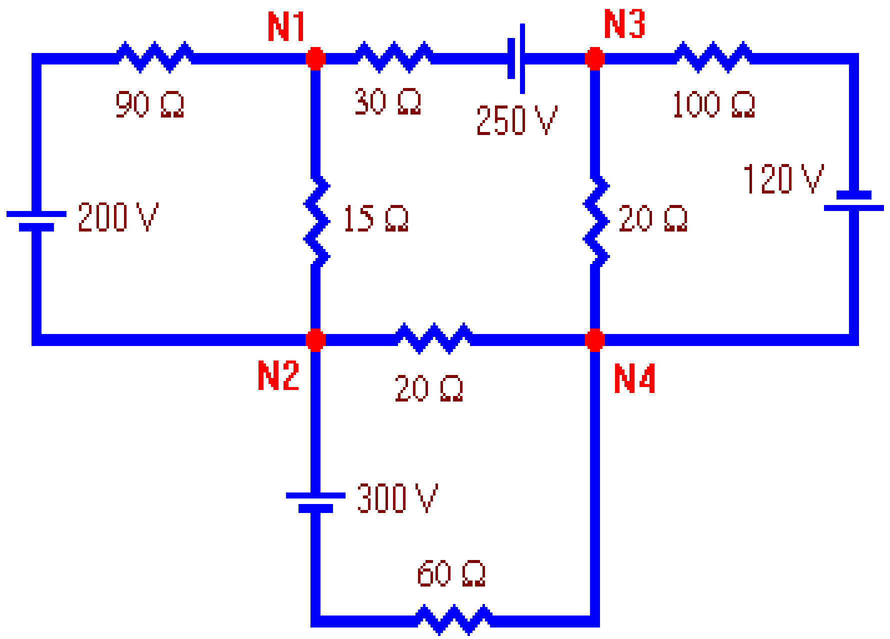
$$i_{R2} = \frac{v_{R2}}{R_2} = \frac{6.67\text{ V}}{5\Omega} = 1.33\text{ A}$$

Example Use nodal analysis to find the voltage at each node of this circuit.



Solution:

- The number of nodes is 4.
- We will number the nodes as shown to the right.



- We will choose node 2 as the reference node and assign it a voltage of zero.
- Write down Kirchoff's Current Law for each node. Call V_1 the voltage at node 1, V_3 the voltage at node 3, V_4 the voltage at node 4, and remember that $V_2 = 0$. The result is the following system of equations:

$$\frac{V1}{15} + \frac{V1-200}{90} + \frac{V1-V3+250}{30} = 0$$

$$\frac{V3-V1-250}{30} + \frac{V3-V4}{20} + \frac{V3-V4+120}{100} = 0$$

$$\frac{V4-V3}{20} + \frac{V4}{20} + \frac{V4-V3-120}{100} + \frac{V4+300}{60} = 0$$

The first equation results from KCL applied at node 1, the second equation results from KCL applied at node 3 and the third equation results from KCL applied at node 4. Collecting terms this becomes:

$$\left(\frac{1}{15}+\frac{1}{90}+\frac{1}{30}\right)V1-\left(\frac{1}{30}\right)V3=\frac{200}{90}-\frac{250}{30}$$

$$-\left(\frac{1}{30}\right)V1+\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{100}\right)V3-\left(\frac{1}{20}+\frac{1}{100}\right)V4=\frac{250}{30}-\frac{120}{100}$$

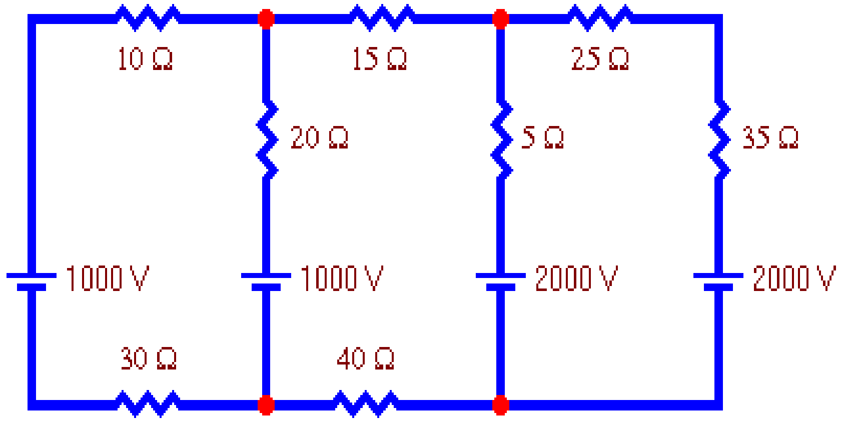
$$-\left(\frac{1}{20}+\frac{1}{100}\right)V3+\left(\frac{1}{20}+\frac{1}{20}+\frac{1}{100}+\frac{1}{60}\right)V4=\frac{120}{100}-\frac{300}{60}$$

This form for the system of equations could have been gotten immediately by using the inspection method.

- Solving the system of equations using Gaussian elimination or some other method gives the following voltages:

V_1 = -35.88 volts, V_3 = 63.74 volts and V_4 = 0.19 volts

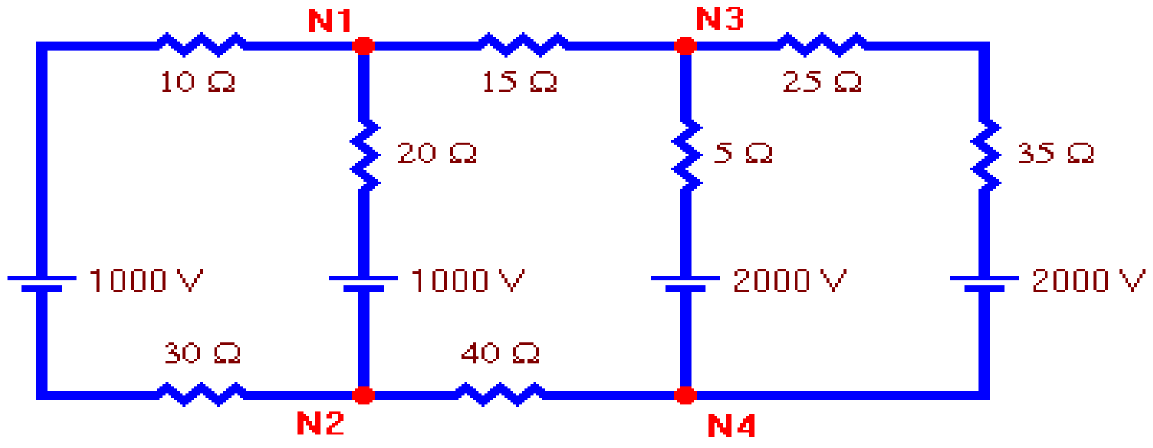
Example Use nodal analysis to find the voltage at each node of this circuit.



Solution:

- The number of nodes is 4.

- We will number the nodes as shown to the right.



- We will choose node 4 as the reference node and assign it a voltage of zero.

- Write down Kirchoff's Current Law for each node. Call V_1 the voltage at node 1, V_2 the voltage at node 2, V_3 the voltage at node 3, and remember that $V_4 = 0$. The result is the following system of equations:

$$\frac{V1-V2-1000}{20} + \frac{V1-V2-1000}{40} + \frac{V1-V3}{15} = 0$$

$$\frac{V2-V1+1000}{20} + \frac{V2-V1+1000}{40} + \frac{V2}{40} = 0$$

$$\frac{V3-V1}{15} + \frac{V3-2000}{5} + \frac{V3-2000}{60} = 0$$

The first equation results from KCL applied at node 1, the second equation results from KCL applied at node 2 and the third equation results from KCL applied at node 3. Collecting terms this becomes:

$$\left(\frac{1}{20}+\frac{1}{40}+\frac{1}{15}\right)V1-\left(\frac{1}{20}+\frac{1}{40}\right)V2-\left(\frac{1}{15}\right)V3=\frac{1000}{20}+\frac{1000}{40}$$

$$-\left(\frac{1}{20}+\frac{1}{40}\right)V1+\left(\frac{1}{20}+\frac{1}{40}+\frac{1}{40}\right)V2=-\frac{1000}{20}-\frac{1000}{40}$$

$$-\left(\frac{1}{15}\right)V1+\left(\frac{1}{15}+\frac{1}{5}+\frac{1}{60}\right)V3=\frac{2000}{5}+\frac{2000}{60}$$

This form for the system of equations could have been gotten immediately by using the inspection method.

- Solving the system of equations using Gaussian elimination or some other method gives the following voltages:

V_1 = 1731 volts, V_2 = 548 volts and V_3 = 1937 volts