BLM2534 ELEKTRONİK

Syllabus

Week	Subjects	Week	Subjects
1	Basic Circuit Elements, Resistors, Ohm's Law	8	Diodes
2	Kirchhoff's Law (Voltage and Current)	9	Bipolar Junction Transistor (BJT)
3	Node Voltage Method	10	Bipolar Junction Transistor (BJT)
4	Mesh Current Method	11	BJT biasing
5	Thevenin and Norton Equivalents	12	BJT amplifiers
6	Thevenin and Norton Equivalents	13	Operational amplifiers
7	Superposition	14	Operational amplifiers

GRADING

Midterm Exam	**15 %**
Lab Performance + Quiz + Homeworks + etc	**15 %**
Final Exam	**80 %**

1) ANY ABSENCIES FROM A LAB SESSION WITHOUT ANY OFFICIAL EXCUSE WILL BRING 0 GRADE FROM that lab. This rule will strictly be applied throughout the course.

2) Pre lab /Pre tutorial preparation is a must .

Book: ELECTRIC CIRCUITS (James W. Nilsson & Susan A. Riedel, **Prentice Hall**)

Lecture Notes (Slides)

The International System of Units(SI)

Quantity	Basic Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Thermodynamic temperature	Degree kelvin	K
Amount of substance	Mole	Mol
Luminous intensity	Candela	cd

The SI units are based on these 7 defined quantities

Derived Units in SI

Quantity	Name	Symbol	Expression
Frequency	Hertz	Hz	$1/s$
Force	Newton	N	$kg \cdot m/s^2$
Pressure, stress	Pascal	Pa	$N/m^2 = kg/m \cdot s^2$
Energy, work	Joule	J	$N \cdot m = kg \cdot m^2/s^2$
Power, radiant flux	Watt	W	$J/s = kg \cdot m^2/s^3$
Electric charge	Coulomb	C	$A \cdot s$
Voltage, electric potential	Volt	V	$W/A = kg \cdot m^2/A \cdot s^3$
Capacitance	Farad	F	$C/V = s^4A^2/m^2kg$
Electric resistance	Ohm	Ω	$V/A = m^2kg/s^3A^2$
Conductance	Siemens or mho	S or Ω	$1/\Omega = s^3A^2/m^2kg$
Magnetic field	Tesla	T	$N/A \cdot m = kg/s^2A$
Magnetic flux	Weber	Wb	$T \cdot m^2 = m^2kg/s^2A$
Inductance	Henry	H	$V \cdot s/A = m^2kg/s^2A^2$

Standardized Prefixes to Signify Powers of 10

atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

SI units Example

Example 1.1 Using SI Units and Prefixes for Powers of 10

If a signal can travel in a cable at 80% of the speed of light, what length of cable, in cm represents 1 ns?

Solution

First, note that 1 ns=10^{-9} s. Also recall that the speed of light c= 3 x 10^8 m/s. Then, 80% of the speed of light is 0.8c = (0.8)(3 x 10^8) = 2.4 x 10^8 m/s. Using a product of ratio, we can convert 80% of the speed of light from meters-per-second to cm-per-nanosecond. The result is the distance in cm traveled in 1 ns:

$$\frac{2.4 \times 10^8\, meters}{1\, second} \cdot \frac{1\, second}{10^9\, nanoseconds} \cdot \frac{100\, centimeters}{1\, meters}$$

$$\frac{(2.4 \times 10^8)(100)}{10^9} = 24\, cm/nanoseconds$$

Therefore, a signal traveling at 80% of the speed of light will cover 24 cm of cable in 1 nanosecond.

Circuit Analysis

- **Ideal Circuit:** The elements that comprise the circuit model are called **ideal circuit components.** An ideal circuit component is a mathematical model of an actual electrical component, like a battery or a light bulb.

- **Actual Circuit (physical prototype):** The **physical prototype** is an actual electrical system, constructed from actual electrical components. Measurement techniques are used to determine the actual, quantitative behavior of the physical system.

Current Voltage and Power

Electric Charge:

- The concept of electric charge is the physical basis for describing electrical phenomena.
- Charge is represented by the symbol q. It is measured in coulombs (C).
- Charge is either positive or negative.
- The charge of an electron is $q_e = 1.602 \times 10^{-19}$ C.

Charge is not easy to measure directly. In engineering the related signal variable **current** is used instead.

Current

- The symbol used for Current is i or i(t)
- Current is a measure of the **flow of electric charge over** time. It is defined as **$i=dq/dt$**
- The units of current are amperes (A).
- 1 A = 1C/s (1 Ampere = 1 coulomb/second).

  Since $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$, 1A corresponds to the flow of 6.24×10^{18} electrons per second.

Current

- Since charge can be positive or negative, Current can be ***positive or negative***.
- By convention, the ***direction of current*** is the direction of the net flow of positive charge. This is called conventional current.
- The flow of negative charge (electrons) in the opposite direction is called ***electronic current***.

Energy

- Moving charge from a point A to a point B in a circuit requires energy.
- Energy is represented by the symbol w. It is measured in *joules (J)*.

Voltage

- Measuring energy is not convenient.
- In engineering the related signal variable ***voltage***, denoted by v, is used.
- The voltage between two points A and B is defined as ***v=dw/dq***, i.e., it is the change in energy per unit charge as charge passes through a circuit from point A to point B.
- The units of voltage are ***volts (V)***. 1V=1J/C

Power

Power p, measured in watts (W), is the time rate of change of energy: p=dw/dt,

$$p = \frac{dw}{dt}$$

where

p = power in Watts,

w = the energy in joules,

t = the time in seconds

Thus 1 W = 1 J/s

The power associated with the flow of charge follows directly from the definition of voltage and Current

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right)$$

p=v.i

Where

p = power in Watts,

v = the voltage in volts,

i = the Current in amperes

CIRCUIT ELEMENTS

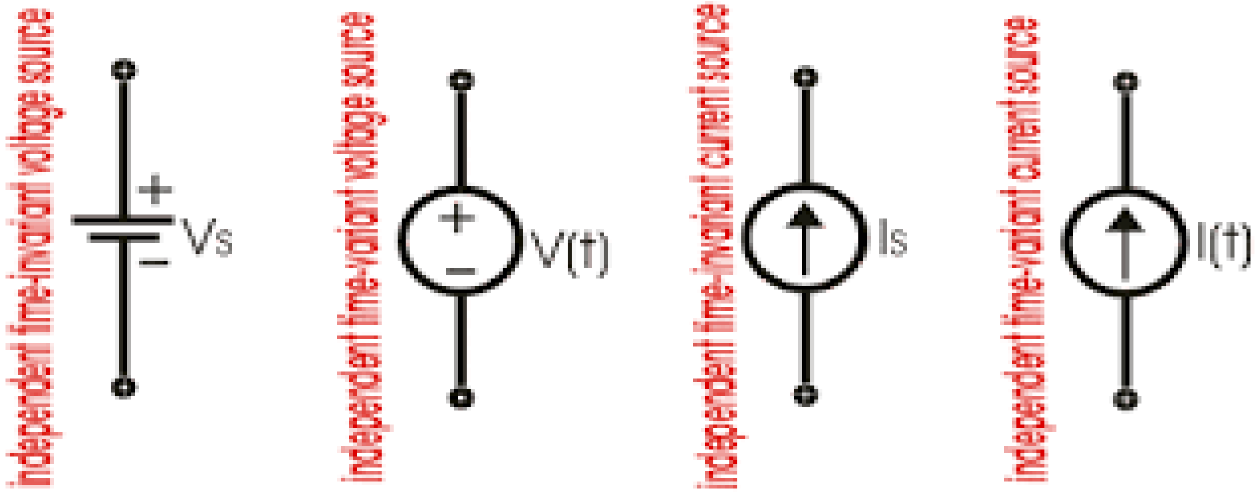
- There are five ideal basic circuit elements:
  - Voltage Sources
  - Current Sources,
  - Resistors
  - Inductors,
  - Capacitors

For now we will start with Voltage Sources, Current Sources and Resistors.

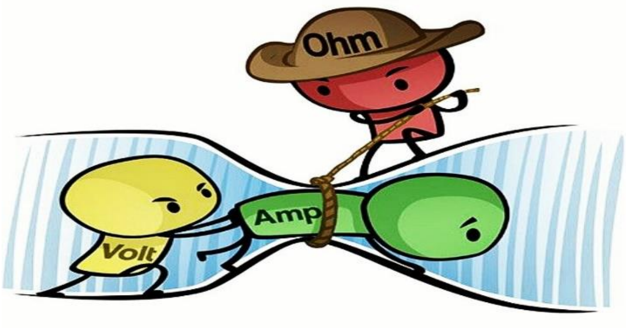
Voltage and Current Sources

- An **electrical source** is a device that is capable of converting nonelectric energy to electric energy and vice versa.

- An **ideal voltage source** is a circuit element that maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals.
- An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

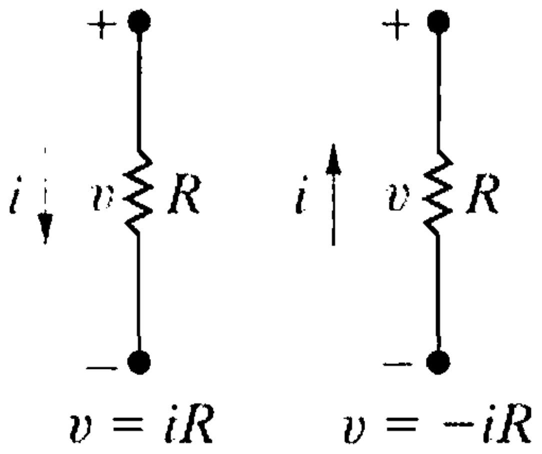


Electrical Resistance (Ohm's Law)



Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element used to model this behavior is the resistor.

For purposes of circuit analysis, we must reference the current in the resistor to the terminal voltage. We can do so in two ways: either in the direction of the voltage drop across the resistor or in the direction of the voltage rise across the resistor, as shown in Fig. If we choose the former, the relationship between the voltage and current is



$$v = iR$$ ← Ohm's Law

If we choose the second method, we must write

$$v = -iR$$

where

v = the voltage in volts,

i = the current in amperes,

R = the resistance in ohms.

Electrical Resistance (Ohm's Law)

- Ohm's law is the mathematical relationship between voltage and current for a resistor. In SI units, resistance is measured in ohms. The Greek letter omega (Ω) is the standard symbol for an ohm.

- Ohm's law expresses the voltage as a function of the current. However, expressing the current as a function of the voltage also is convenient.

$$i = \frac{v}{R}$$

- The reciprocal of the resistance is referred to as conductance, is symbolized by the letter G, and is measured in Siemens (S).Thus,

$$G = \frac{1}{R}\ \mathrm{S}$$

- As an example 8 Ω resistor has a conductance value of 0.125 S. In much of the professional literature, the unit used for conductance is the mho (ohm spelled backward), which is symbolized by an inverted omega (℧). Therefore we may also describe an 8 Ω resistor as having a conductance of 0.125 mho, (℧).

Electrical Resistance (Ohm's Law)

We may calculate the power at the terminals of a resistor in several ways. The first approach is to use the defining equation and simply calculate the product of the terminal voltage and current.

$$p = vi$$
$$v = iR$$

$$p = (iR)i \rightarrow \quad p = i^2R$$

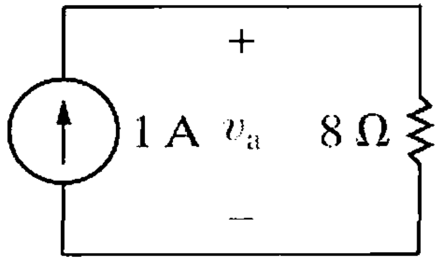
A third method of expressing the power at the terminals of a resistor is in terms of the voltage and resistance. The expression is independent of the polarity references, so

$$p = vi$$

$$i = \frac{v}{R}$$

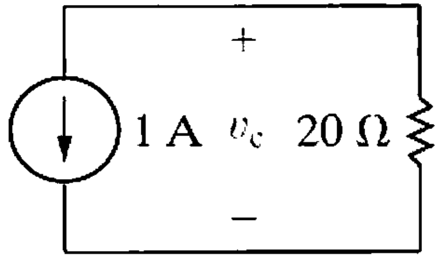
$$p = v\frac{v}{R} \quad \rightarrow p = \frac{v^2}{R}$$

Electrical Resistance (Ohm's Law)



Example: Calculating Voltage, Current, and Power for a Simple Resistive Circuit

a) Calculate the values of *v* and *i*.
b) Determine the power dissipated in each resistor.



a)
The voltage *va* in first Fig is a drop in the direction of the current in the resistor. Therefore,

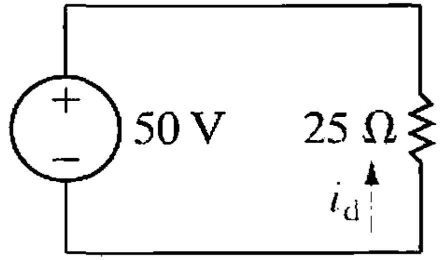
$$v_a = 1A.8\Omega = 8V$$

The voltage *vc* in second Fig. is a rise in the direction of the current in the resistor. Hence,

$$v_c = -1A.20\Omega = -20V$$

The current *id* in the 25 ohm resistor in third Fig. is in the direction of the voltage rise across the resistor. Therefore

$$i_d = \frac{-50V}{25\Omega} = -2A$$



b) The power dissipated in each of the three resistors is

$$p_{8\Omega} = 1^2.8 = 8W$$

$$p_{20\Omega} = 1^2.20 = 20W$$

$$p_{25\Omega} = (-2^2).25 = 100W$$

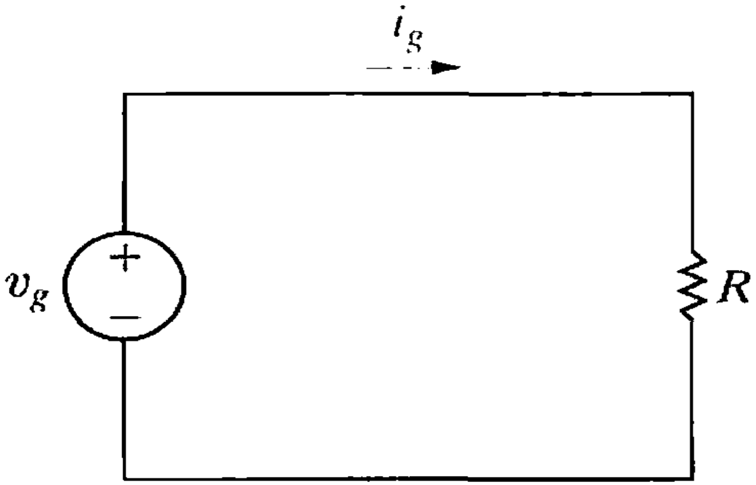
EXAMPLE

For the circuit shown

a) If v_g = 1 kV and i_g = 5 mA, find the value of *R* and the power absorbed by the resistor.

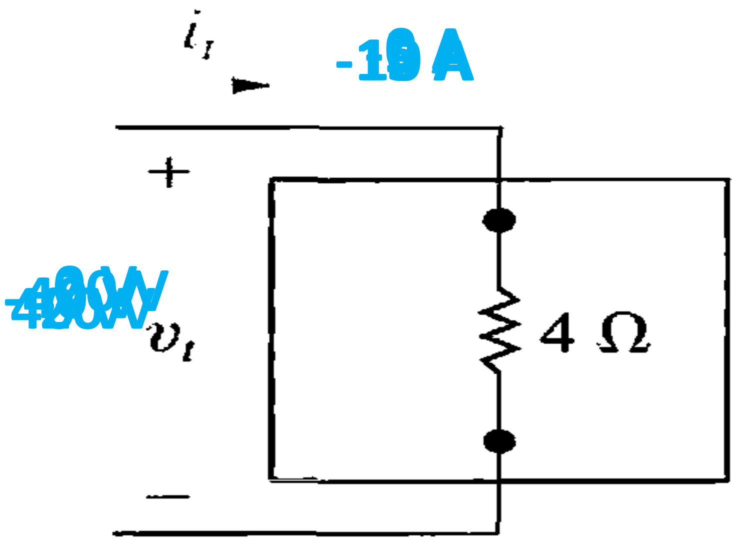
b) If i_g = 75 mA and the power delivered by the voltage source is 3 W, find v_g, *R*, and the power absorbed by the resistor.

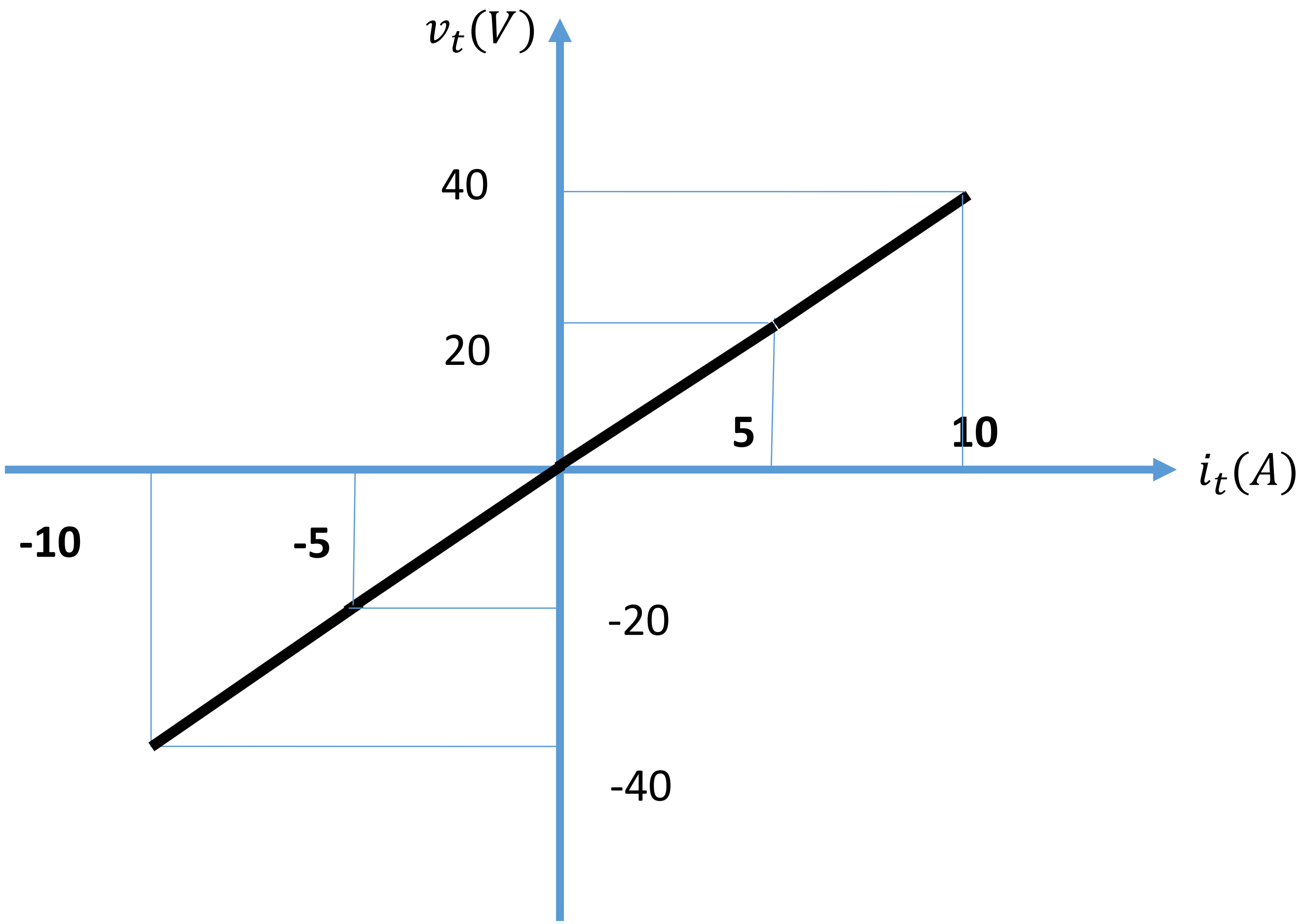
c) If *R*= 300 ohm and the power absorbed by *R* is 480 mW, find *L* and v_g.



EXAMPLE (The Voltage-Current Relationship)

$v_t(V)$	i_t(A)
-40	-10
-20	-5
0	0
20	5
40	10

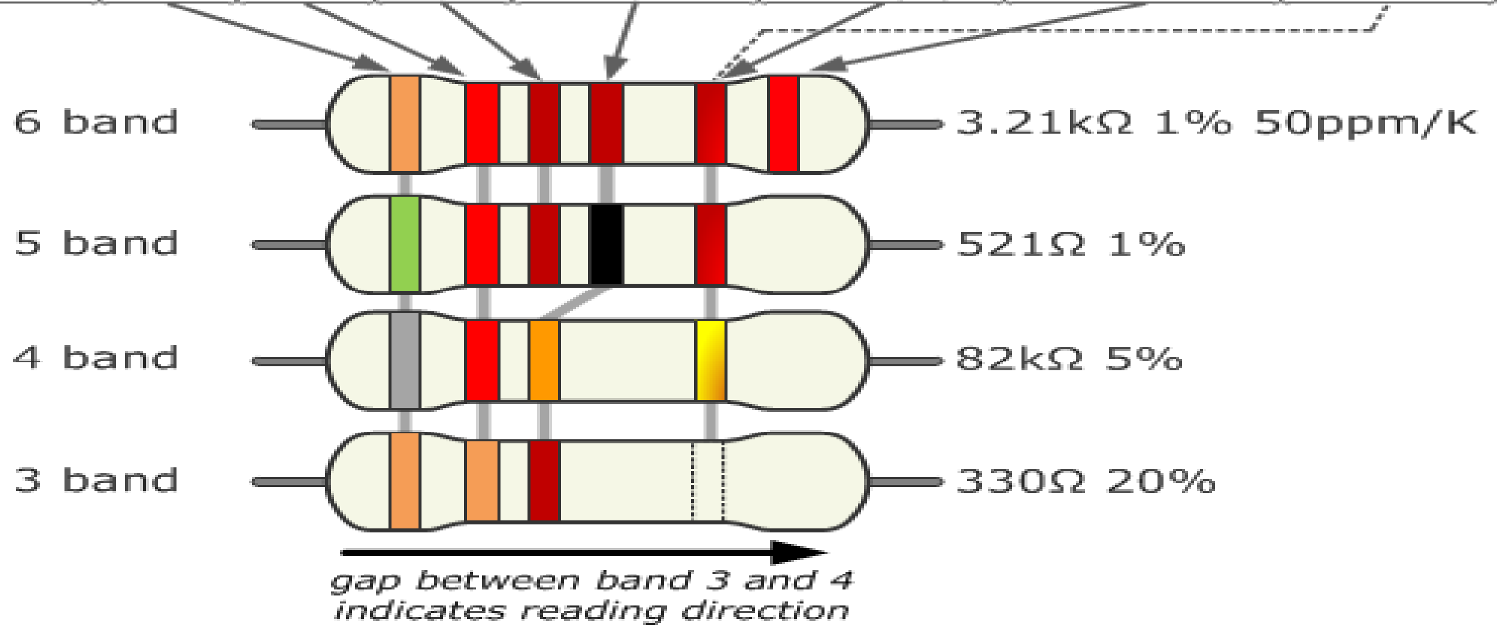




The **relationship** between **voltage**, **current**, and **resistance** is described by Ohm's law.

Resistor Color Codes

Color	Signficant figures			Multiply	Tolerance (%)	Temp. Coeff. (ppm/K)	Fail Rate (%)
black	0	0	0	x 1		250 (U)	
brown	1	1	1	x 10	1 (F)	100 (S)	1
red	2	2	2	x 100	2 (G)	50 (R)	0.1
orange	3	3	3	x 1K		15 (P)	0.01
yellow	4	4	4	x 10K		25 (Q)	0.001
green	5	5	5	x 100K	0.5 (D)	20 (Z)	
blue	6	6	6	x 1M	0.25 (C)	10 (Z)	
violet	7	7	7	x 10M	0.1 (B)	5 (M)	
grey	8	8	8	x 100M	0.05 (A)	1(K)	
white	9	9	9	x 1G			
gold			3th digit only for 5 and 6 bands	x 0.1	5 (J)		
silver				x 0.01	10 (K)		
none					20 (M)		



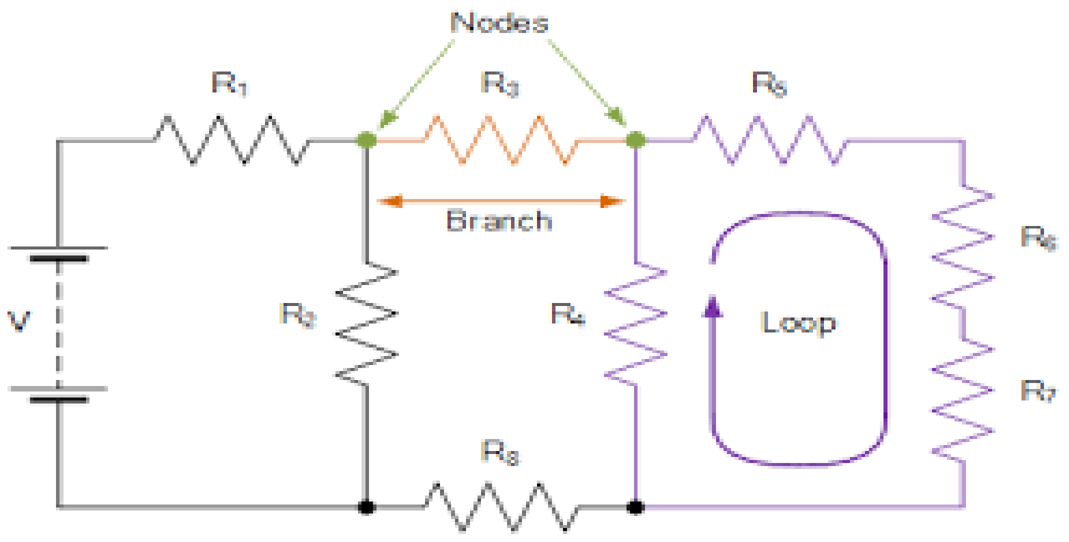
Terms in Circuit Diagrams

Node : Point of connection between elements.

Branch : Connection between two nodes

Loop : Closed loops in circuit diagram

Number of Branches = Number of Loops + Number of Nodes -1



COM234 ELECTRONICS

Kirchhoff's Law

Kirchhoff's Laws

For circuit analysis Ohm's law may not be enough to provide a complete solution

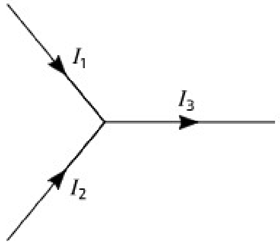
Kirchhoff's laws are fundamental to circuit theory. They quantify how current flows through a circuit and how voltage varies around a loop in a circuit.

1. *Kirchhoff's current law (1st Law) states that current flowing into a node (or a junction) must be equal to current flowing out of it. This is a consequence of charge conservation.*
2. *Kirchhoff's voltage law (2nd Law) states that the sum of all voltages around any closed loop in a circuit must equal zero. This is a consequence of charge conservation and also conservation of energy.*

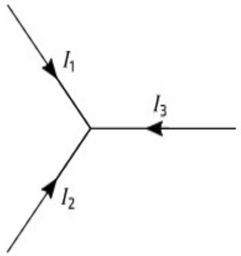
Kirchhoff's Current Law (KCL)

(Conservation of Charge)

Kirchhoff's current law states that for the diagram below, the currents in the three wires must be related by



The standard way of displaying Kirchhoff's current law is by having all currents either flowing towards or away from the node, as shown below:



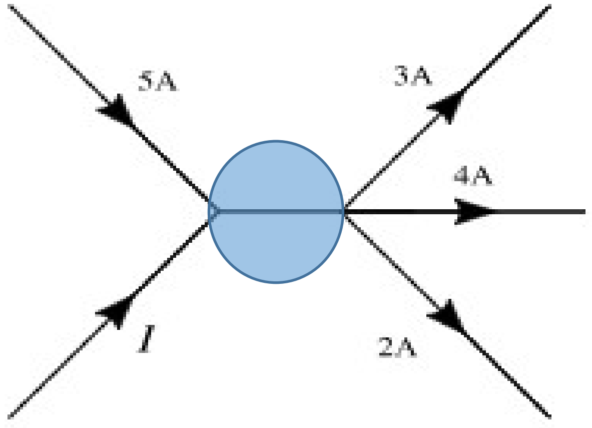
Here, at least one of the currents will have to be negative (i.e, away from the node and in the opposite direction to the arrows on this diagram)and Kirchhoff's current law can be written as:

This can be generalized to the case with n wires all connected at a node by writing:

Kirchhoff's Current Law (KCL)

What is the value of I in the circuit segment shown below ?

What goes in must come out,
or
the total coming in is zero

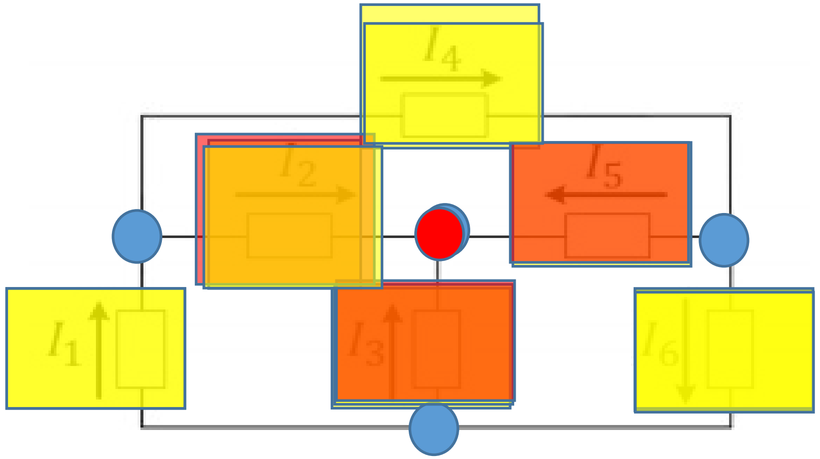


Current in = Current out
Conservation of Charge

Kirchhoff's Current Law (KCL)

The algebraic sum of all the currents at any node in a circuit equals zero. !!!

KCL equations are often used at nodes, but can also be used for a sub-circuit



A. $I_1 = I_2 + I_4$

B. $I_4 = I_5 + I_6$

C. $I_1 + I_3 = I_6$

D. $I_3 + I_5 = I_2$

E. $I_6 - I_4 = I_3 + I_2$

Q1: Which of the equations is NOT a correct application of KCL?

Kirchhoff's Voltage Law (KVL)

(Conservation of Energy)

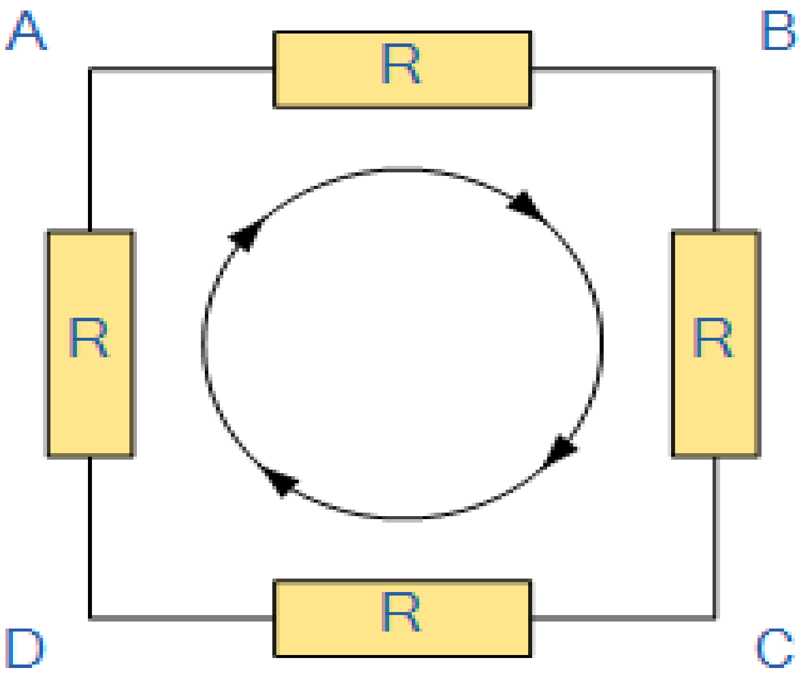
The algebraic sum of ALL the potential differences around the loop must be equal to zero as:

$$\Sigma V = 0$$

Note here that the term "algebraic sum" means to take into account the polarities and signs of the sources and voltage drops around the loop.

Kirchhoff's Voltage Law (KVL)

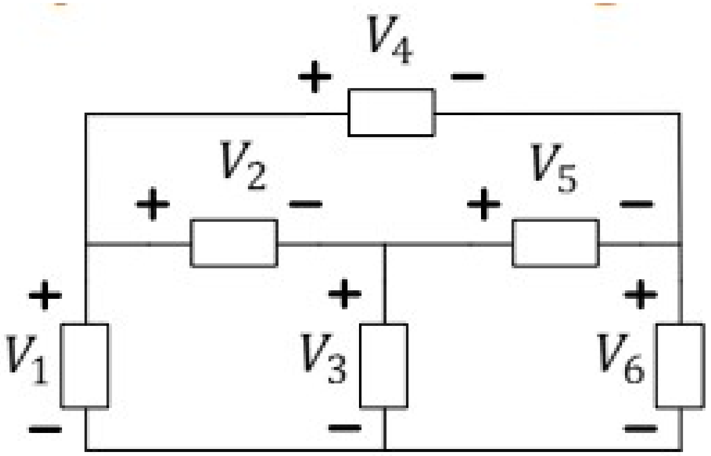
The sum of all the Voltage
Drops around the loop
is equal to Zero



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Kirchhoff's Voltage Law (KVL)

Keeping track of voltage drop ***polarity*** is important in writing correct KVL equations.



A. $V_1 - V_2 - V_3 = 0$
B. $V_1 = V_2 + V_5 + V_6$
C. $V_1 - V_4 = V_6$
D. $V_3 + V_2 = V_1$
E. $V_3 + V_5 = V_6$

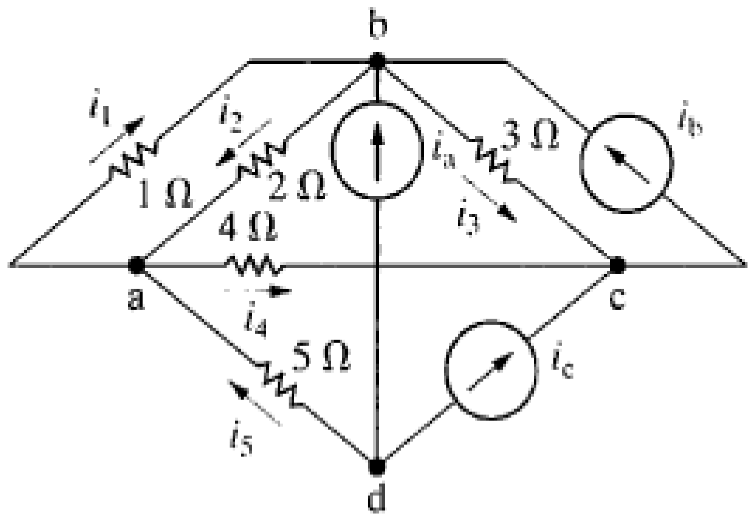
Q2: Which of the equations is NOT a correct application of KVL?

Q3. What are the values of the voltages $V1$, $V2$ and $V6$ if $V3$ = 2 V, $V4$ = 6 V, $V5$ = 1 V?

EXAMPLE (Using KCL)

Question:

Sum the currents at each node in the circuit shown in Fig. Note that there is no connection dot (•) in the center of the diagram, where the 4 ohm branch crosses the branch containing the ideal Current source Ia.



Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

node a $\quad i_1 + i_4 - i_2 - i_5 = 0,$

node b $\quad i_2 + i_3 - i_1 - i_b - i_a = 0,$

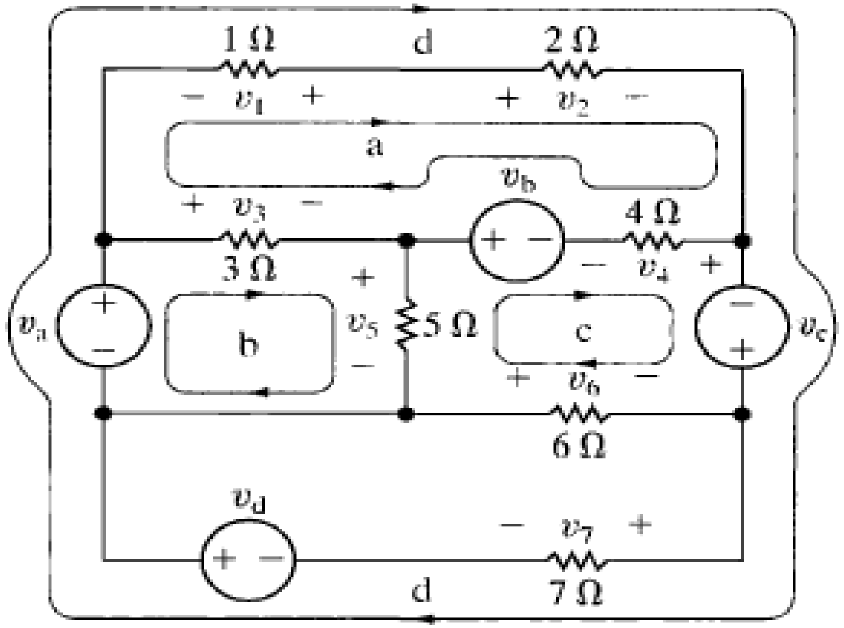
node c $\quad i_b - i_3 - i_4 - i_c = 0,$

node d $\quad i_5 + i_a + i_c = 0.$

EXAMPLE (Using KV

Question:

Sum the voltages around each designated path in the circuit shown in Fig.



Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are

path a $$-v_1 + v_2 + v_4 - v_b - v_3 = 0,$$

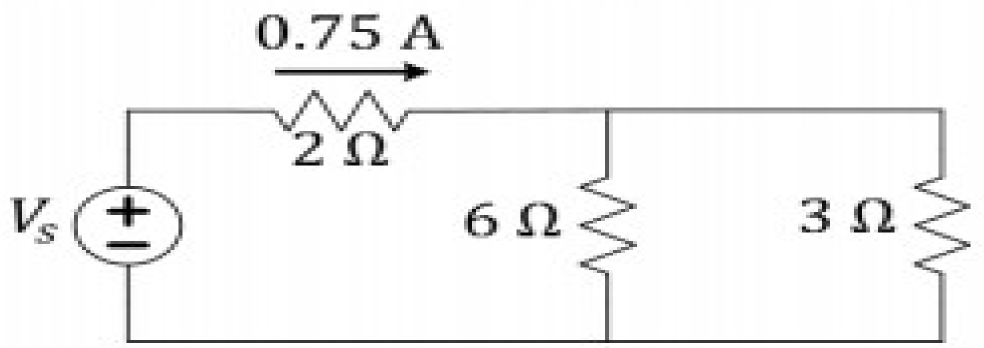
path b $$-v_a + v_3 + v_5 = 0,$$

path c $$v_b - v_4 - v_c - v_6 - v_5 = 0,$$

path d $$-v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$$

EXAMPLES

- Use KCL, KVL and Ohm's Law to solve below questions



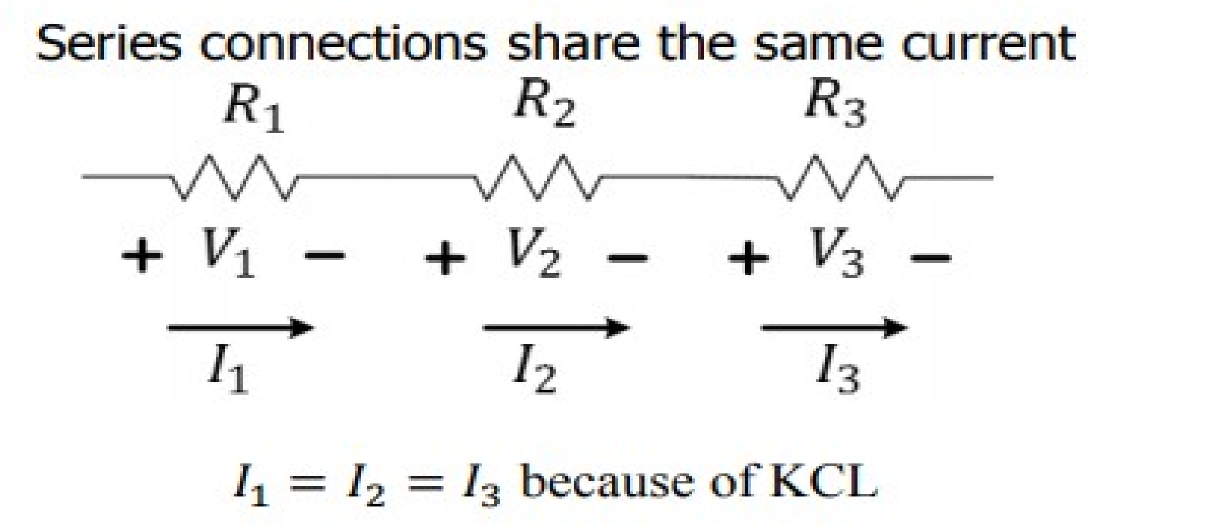
Q4: What is the value of the source voltage?
Q5: How much power is the source supplying?
Q6: How much power is each resistance consuming?

Current and Voltage Dividers

- Series Connections, Equivalent Resistance, Voltage Divider
- Parallel Connections, Equivalent Resistance, Current Divider
- Power Dissipation in Series and Parallel Resistive Loads

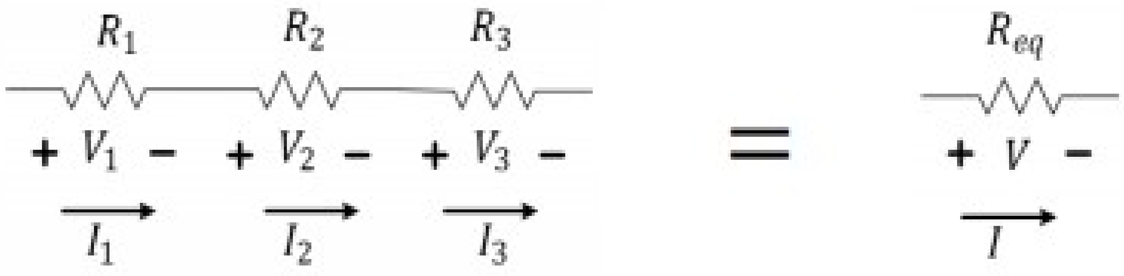
Series Connection

Series connections share the same current



Equivalent Resistance of Series Resistors

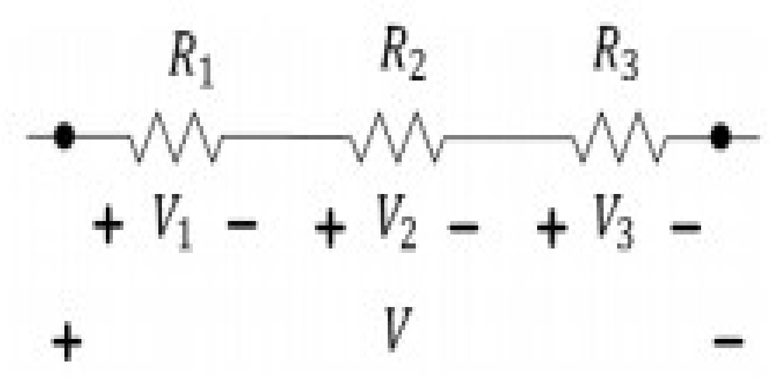
Resistances in series add up

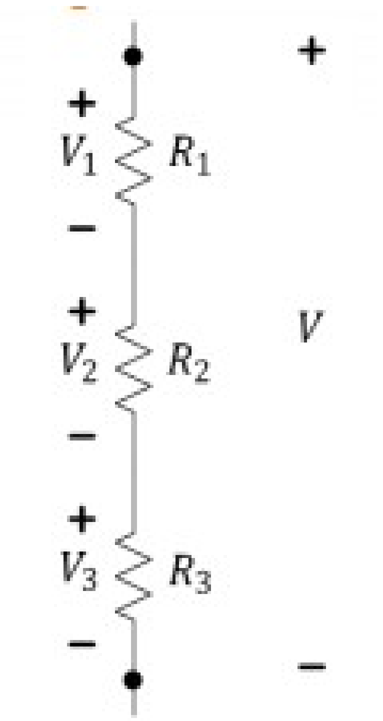


$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Voltage Divider Rule (VDR)

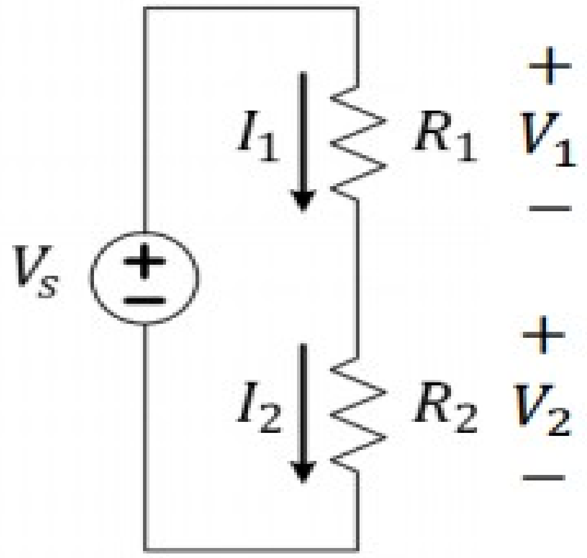
When a voltage divides across resistors in series, more voltage drop appears across the largest resistor





Example

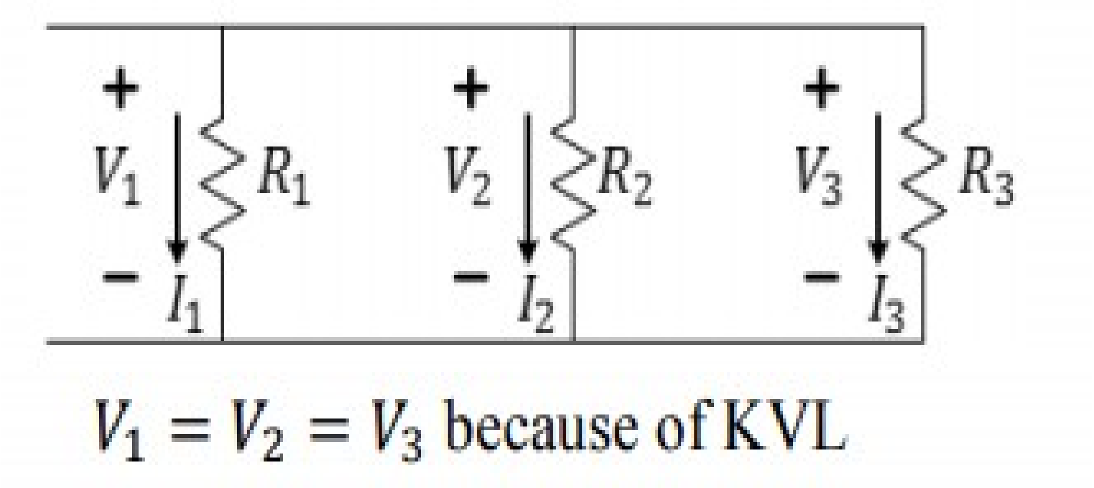
If $R_1 < R_2$, which of the following is true?



A. $V_1 < V_2$ and $I_1 < I_2$
B. $V_1 < V_2$ and $I_1 = I_2$
C. $V_1 = V_2$ and $I_1 = I_2$
D. $V_1 > V_2$ and $I_1 = I_2$
E. $V_1 > V_2$ and $I_1 > I_2$

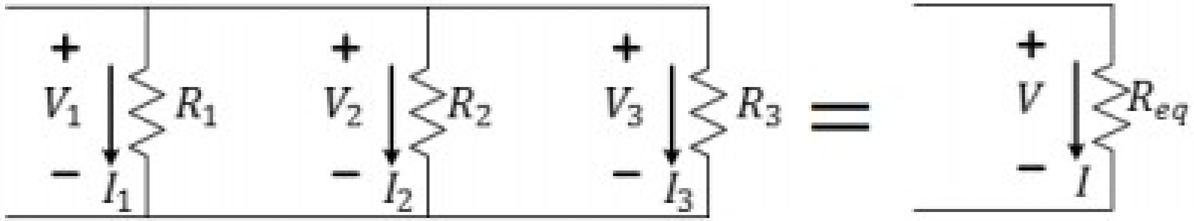
Parallel Connection

Parallel connections share the same voltage potentials at two end nodes (shared by the elements)



Equivalent Resistance of Parallel Resistors

Adding resistance in parallel always brings resistance down!



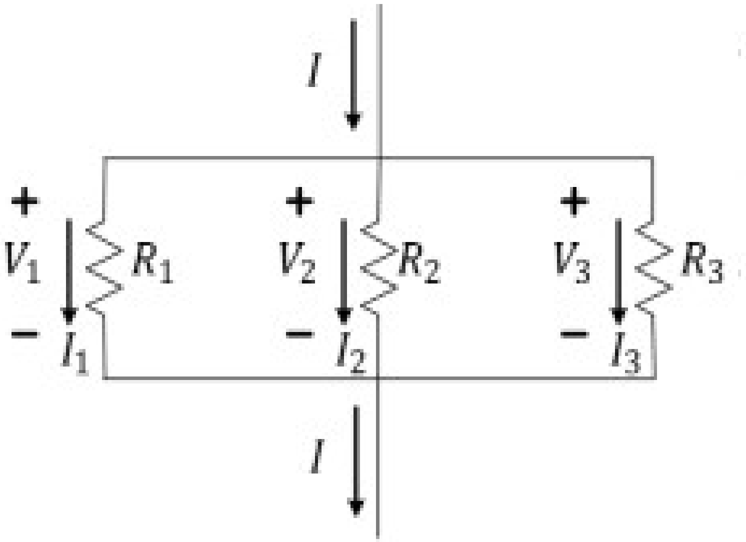
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

If $N = 2$, $\quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Current Divider Rule (CDR)

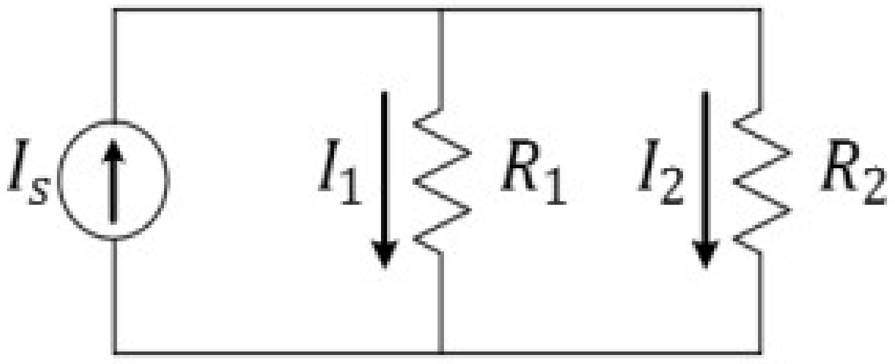
When a current divides into two or more paths, more current will go down the path of lowest resistance.

$$I_k = \frac{R_{eq}}{R_k} \cdot I$$



EXAMPLE

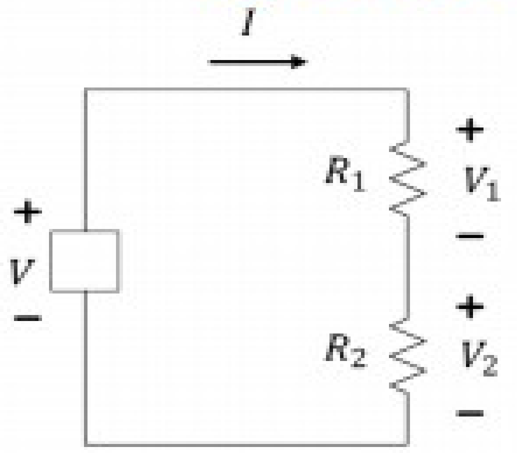
If $R_1 < R_2$, which of the following is true?



A. $I_1 < I_2 < I_s$
B. $I_1 < I_s < I_2$
C. $I_2 < I_1 < I_s$
D. $I_2 < I_s < I_1$
E. $I_s < I_2 < I_1$

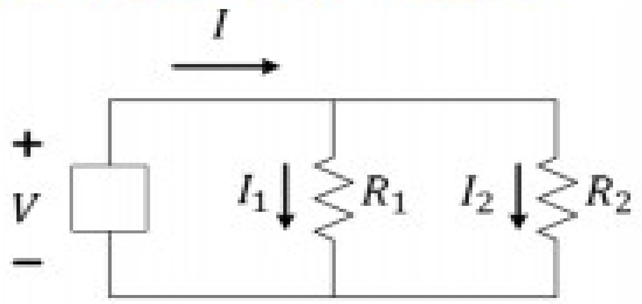
EXAMPLE

VDR and CDR for Two Resistances



$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$



$$I_1 = \frac{R_2}{R_1 + R_2} I \qquad I_2 = \frac{R_1}{R_1 + R_2} I$$

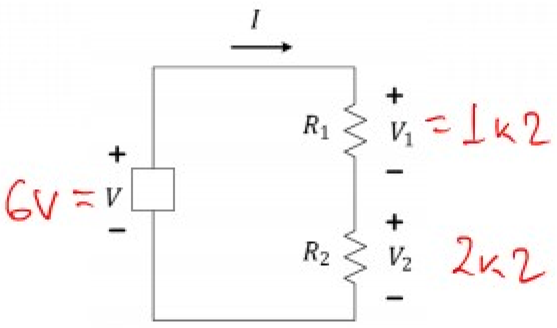
If 6V falls across a series combination of 1kΩ and 2kΩ, what is V across 2kΩ?

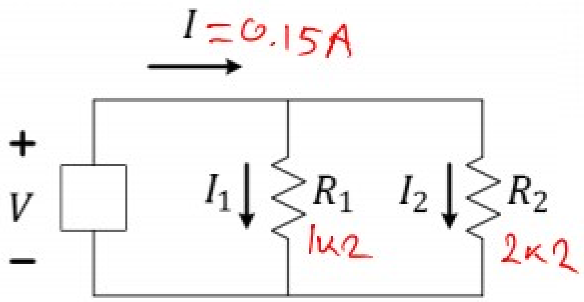
If 0.15A flows through a parallel combo of 1kΩ and 2kΩ, what is I through 2kΩ?

If a source supplies 60W to a series combination of 10Ω and 30Ω, what is the power absorbed by the 10Ω resistor? What is absorbed by the 30Ω resistor?

If a source supplies 300mW to a parallel combination of 3kΩ and 2kΩ, what is the power absorbed by the 3kΩ resistor? What is absorbed by the 2kΩ resistor?

EXAMPLE





If 6V falls across a series combination of 1kΩ and 2kΩ, what is V across 2kΩ?

$V_2 = ?$

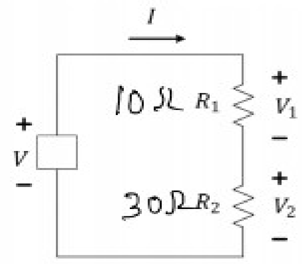
$$V_{2k} = V \frac{R_2}{R_1 + R_2} = 6V \frac{2k\Omega}{1k\Omega + 2k\Omega} = \underline{\underline{4V}}$$

If 0.15A flows through a parallel combo of 1kΩ and 2kΩ, what is I through 2kΩ?

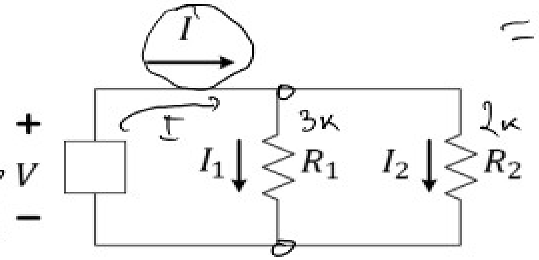
$I_2 = ?$

$$I_{2k} = I \frac{R_1}{R_1 + R_2} = (0.15A) \cdot \frac{1k}{1k + 2k} = \underline{\underline{0.05A}}$$

EXAMPLE



$\boxed{P=300mW}$



$P_{R2} = I_2 \cdot V =$
$= \frac{3}{5} I V = \frac{3}{5}\, 300mW$
$\boxed{= 180mW}$

$P_{R1} = I_1 \cdot V =$
$= \frac{2}{5} I \cdot V =$
$= \frac{2}{5} \cdot 300mW = \boxed{120mW}$

If a source supplies 60W to a series combination of 10Ω and 30Ω, what is the power absorbed by the 10Ω resistor? What is absorbed by the 30Ω resistor?

$P = V \cdot I = 300mW$

$I_1 = I \frac{R_2}{R_1+R_2} = I \frac{2k}{5k}$ $\quad I_2 = I \frac{R_1}{R_1+R_2} = I \frac{3k}{5k}$

If a source supplies 300mW to a parallel combination of 3kΩ and 2kΩ, what is the power absorbed by the 3kΩ resistor? What is absorbed by the 2kΩ resistor?

Q9

$P = V \cdot I = 60W$

$P_{R1} + P_{R2} = 60W$

$V_1 = V \frac{R_1}{R_1+R_2} = V \frac{10}{40}$

$V_2 = V \frac{R_2}{R_1+R_2} = V \frac{30}{40}$

$P = 60W$

$P_{30} = V_2 \cdot I = \frac{30}{40} \cdot V \cdot I = \frac{30}{40} \cdot 60W = 45W$

$P_{10} = V_1 \cdot I = \frac{10}{40} \cdot V \cdot I = \frac{1}{4}\, 60W = 15W$

battery → $\boxed{+\ 60W}$

COM234 ELECTRONICS

The Node Voltage Method

The Node Voltage Method

In this part, we will cover the following topics:

- Some basic definitions
- The steps for writing the Node-Voltage Equations
- Tips on picking the best reference node
- How to handle dependent sources

This material is covered in your textbook in the following sections:

- **Electric Circuits 10th Ed. by Nilsson and Riedel: Sections 4.1 through 4.3**

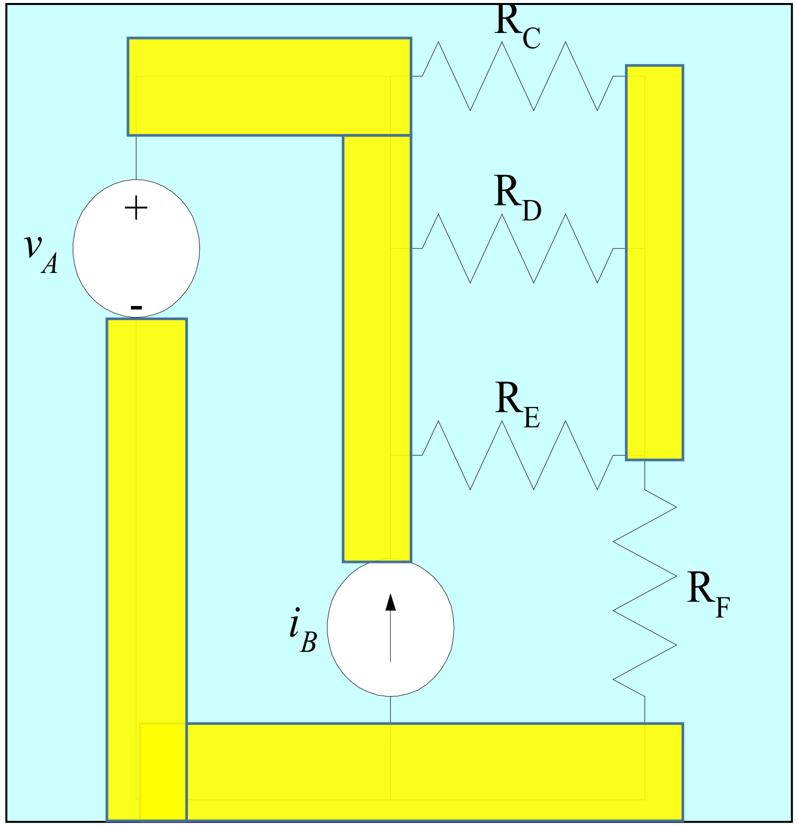
Some Basic Definitions

- **Node** – a place where two or more components meet
- **Essential Node** – a place where three or more components meet
- **Reference Node** – a special essential node that we choose as a reference point for voltages

You may be familiar with the word node from its use as a location in computer networks. It has a similar meaning there, a place where computers are connected.

Some Basic Definitions

- A node is defined as a place where two or more components are connected.
- In the example circuit schematic given here, there are three nodes. These nodes are shown in yellow here.
- The key thing to remember is that we connect components with wires. It doesn't matter how many wires are being used; it only matters how many components are connected together.
- Some students mistakenly count more than three nodes in a circuit like this. When they do, it is usually because they have considered two points connected by a wire to be two nodes.
- How many nodes are there in this circuit here?
- There are also three essential nodes. Each of these three nodes has at least 3 components connected to it.



The Node-Voltage Method (NVM)

The Node-Voltage Method (NVM) is a systematic way to write all the equations needed to solve a circuit, and to write just the number of equations needed. The idea is that any other current or voltage can be found from these node voltages.

This method is not that important in very simple circuits, but in complicated circuits it gives us an approach that will get us all the equations that we need, and no extras.

It is also good practice for the writing of KCL and KVL equations. Many students believe that they know how to do this, but make errors in more complicated situations. Our work on the NVM will help correct some of those errors.

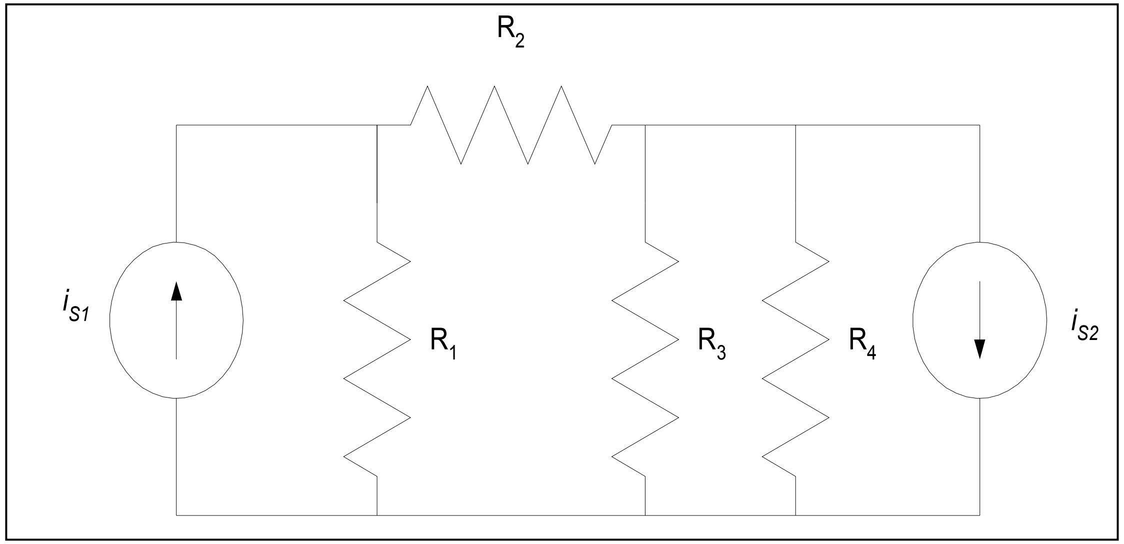
The Node-Voltage Method (NVM)

The Node-Voltage Method steps are:

1. Find the essential nodes.
2. Define one essential node as the reference node.
3. Define the node voltages, the essential nodes with respect to the reference node. Label them.
4. Apply KCL for each non-reference essential node.
5. Write an equation for each current or voltage upon which dependent sources depend, as needed.

NVM – 1st Example

For most students, it seems to be best to introduce the NVM by doing examples. We will start with simple examples, and work our way up to complicated examples. Our first example circuit is given here.



The Node-Voltage Method steps are:

- **Find the essential nodes.**
- **Define one essential node as the reference node.**
- **Define the node voltages, the essential nodes with respect to the reference node. Label them.**
- **Apply KCL for each non-reference essential node.**
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NVM – 1st Example

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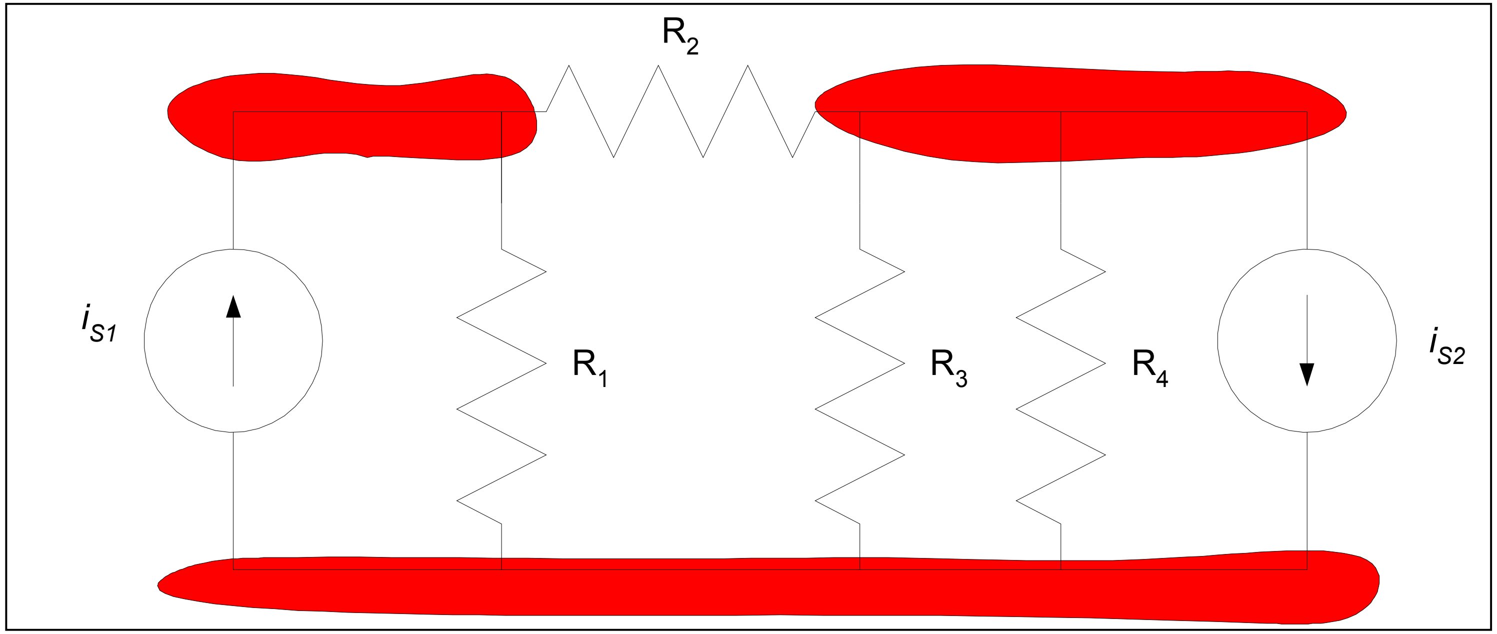
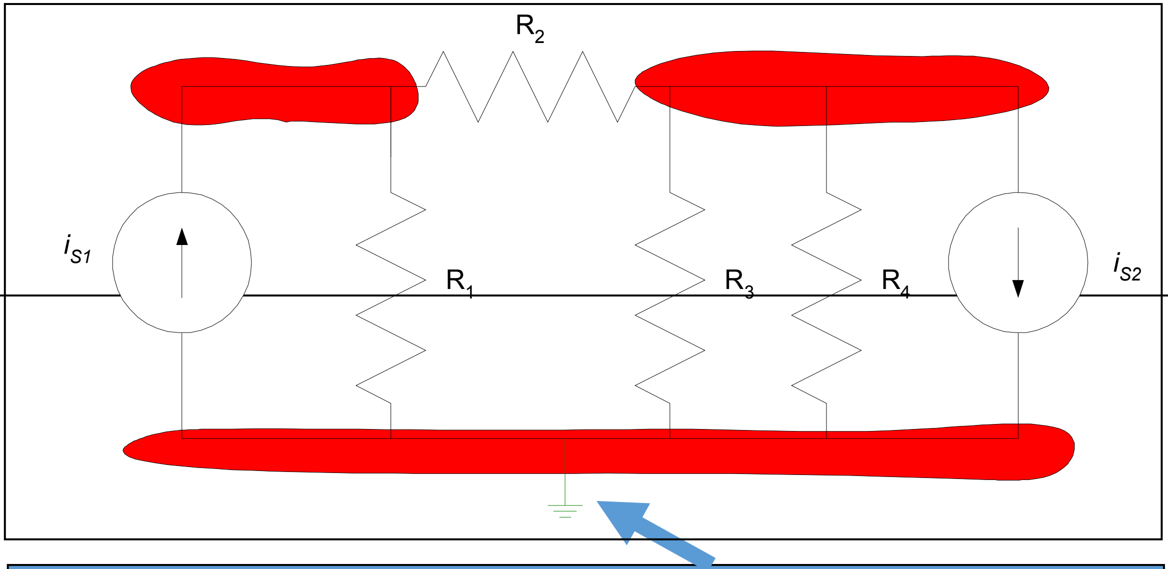


There are three essential nodes, each of which is shown in red.

NVM – 1st Example

The Node-Voltage Method steps are:

- Find the essential nodes.
- **<u>Define one essential node as the reference node.</u>**
- Define the node voltages, the essential nodes with respect to the reference node. Label them.
- Apply KCL for each non-reference essential node.
- Write an equation for each current or voltage upon which dependent sources depend, as needed.



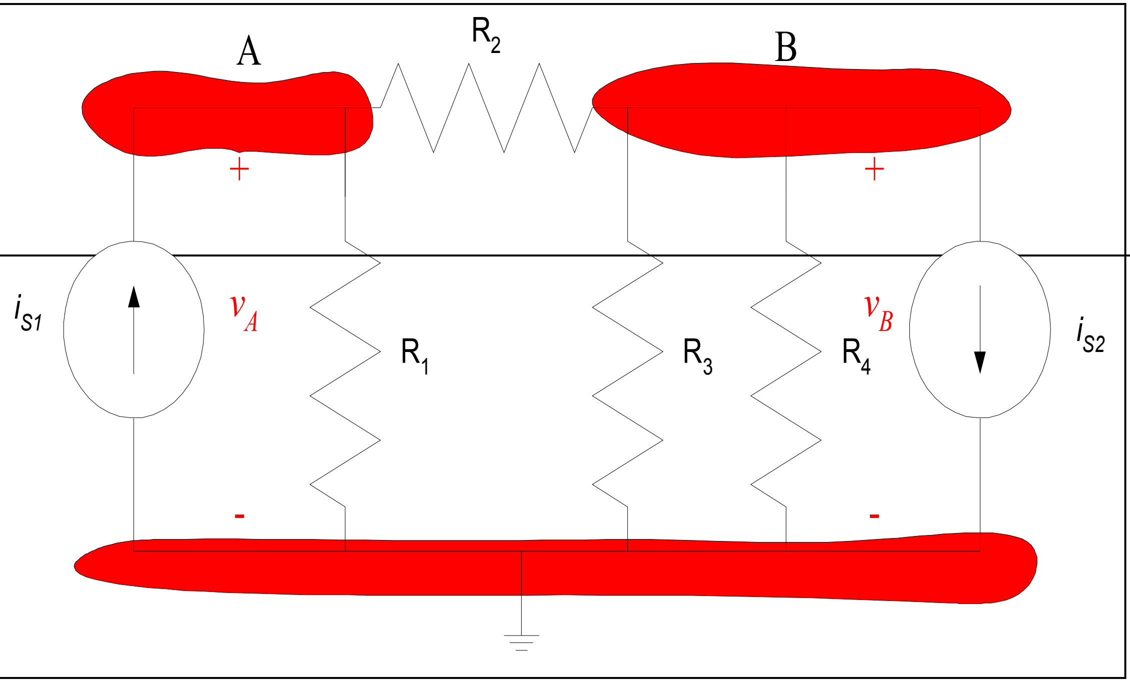
This symbol is used to designate the reference node. There are different symbols used for this designation. This choice of symbols is not important. Making a designation **<u>is</u>** important.

We could choose any of the three essential nodes as the reference node. However, there are better choices. Remember that we need to write a KCL equation for each essential node, except for the reference node. The best idea, then, is to pick the node with the most connections, to eliminate the most difficult equation. Here this is the bottom node. It is labeled to show that it is the reference node.

NVM – 1st Example

The Node-Voltage Method steps are:

- Find the essential nodes.
- Define one essential node as the reference node.
- **<u>Define the node voltages, the essential nodes with respect to the reference node. Label them.</u>**
- Apply KCL for each non-reference essential node.
- Write an equation for each current or voltage upon which dependent sources depend, as needed.

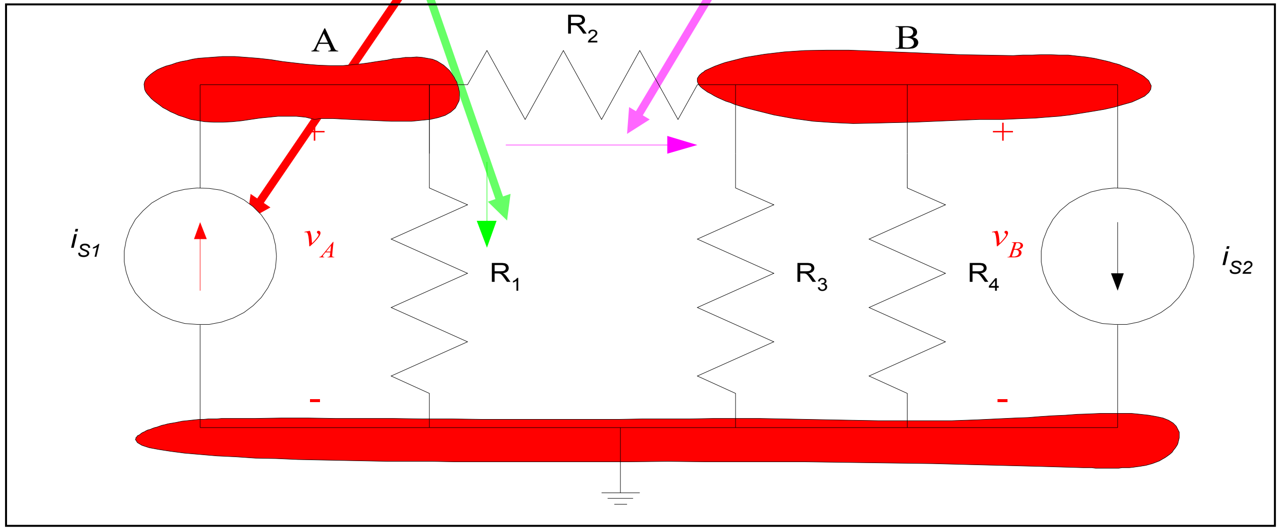


We have labeled the node voltages, v_A and v_B. They are shown in red. For clarity, we have also named the nodes themselves, A and B.

Note: **As with any voltage, the polarity must be defined. We have defined the voltages by showing the voltages with a "+" and "-" sign for each. Strictly speaking, this should not be necessary. The words in step 3 make the polarity clear. Some texts do not label the voltages on the schematic. For clarity, we will label the voltages in these notes.**

NVM – 1st Example

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



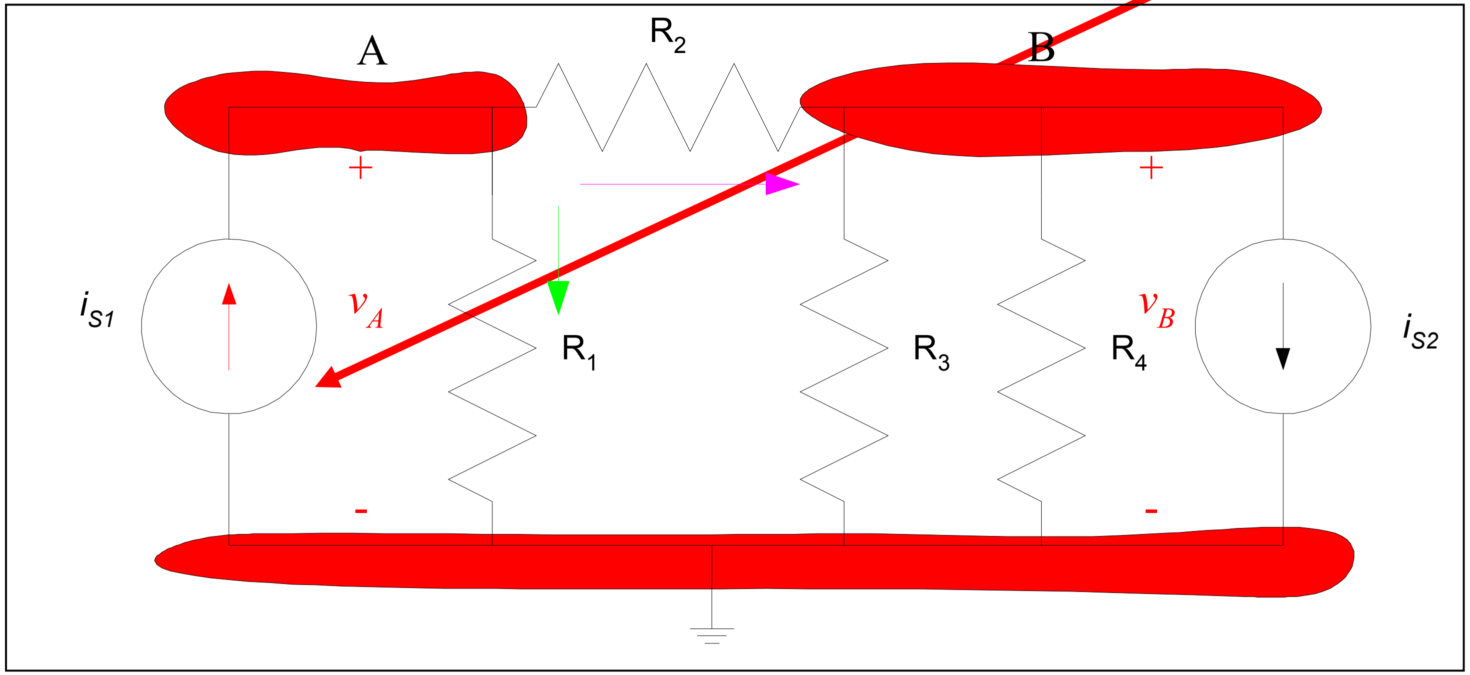
The Node-Voltage Method steps are:

- Find the essential nodes.
- Define one essential node as the reference node.
- Define the node voltages, the essential nodes with respect to the reference node. Label them.
- **Apply KCL for each non-reference essential node.**
- Write an equation for each current or voltage upon which dependent sources depend, as needed.

NVM – 1st Example

The first term comes from Ohm's Law. The voltage v_A is the voltage across R_1. Thus, the current shown in green is v_A/R_1, out of node A, and thus has a + sign in this equation.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

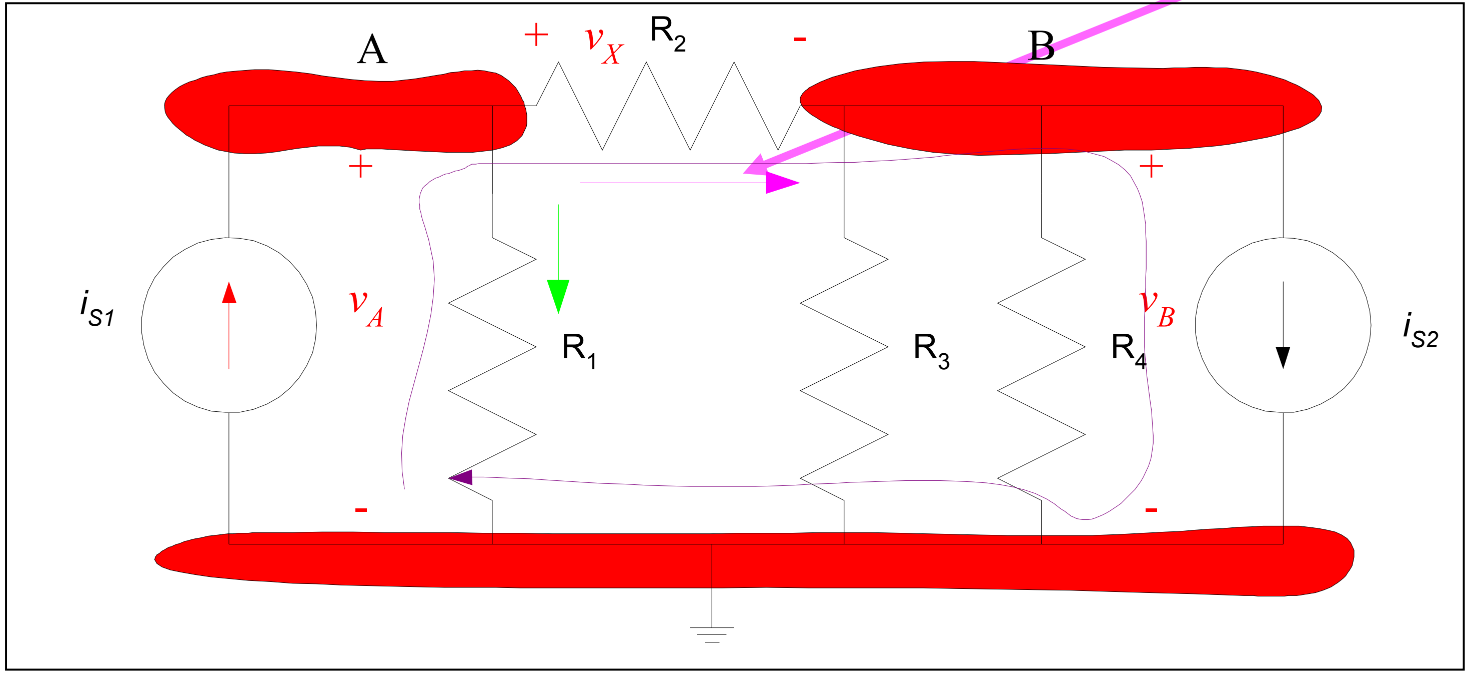


The current through the current source is, by definition, given by the value of that current source. Since the reference polarity of the current is entering node A, it has a "-" sign.

NVM – 1st Example

This current expression also comes from Ohm's Law. The voltage v_X is the voltage across the resistor R_2, and results in a current in the polarity shown.

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$



To prove to yourself that $v_X = v_A - v_B$, take KVL around the loop shown. The voltage at A with respect to B, is $v_A - v_B$, where v_A and v_B are both node voltages.

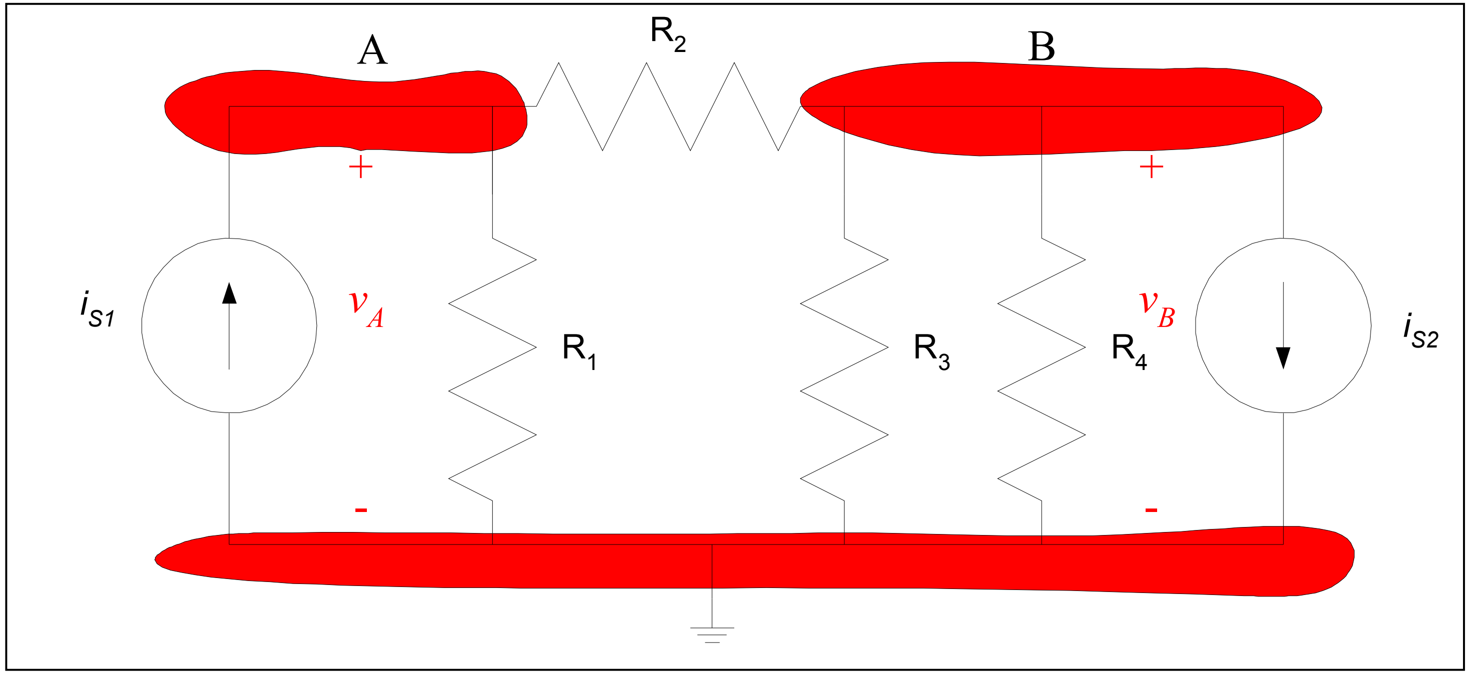
NVM – 1st Example

The KCL equation for the A node was:

$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

The KCL equation for the B node is:

$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$



Be very careful that you understand the signs of all these terms. One of the big keys in these problems is to get the signs correct. If you have questions, review this material.

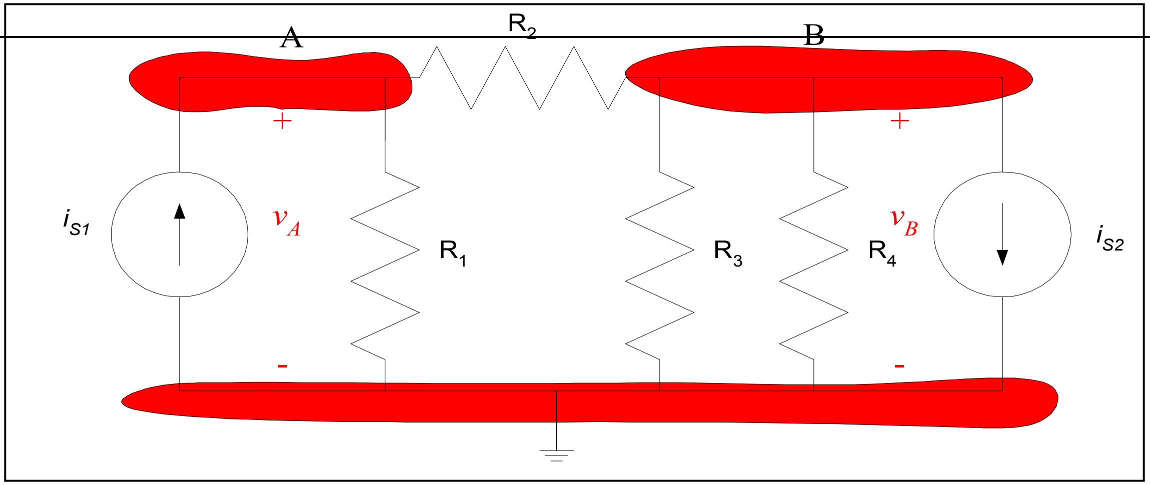
NVM – 1st Example

The Node-Voltage Method steps are:

- Find the essential nodes.
- Define one essential node as the reference node.
- Define the node voltages, the essential nodes with respect to the reference node. Label them.
- Apply KCL for each non-reference essential node.
- **Write an equation for each current or voltage upon which dependent sources depend, as needed.**

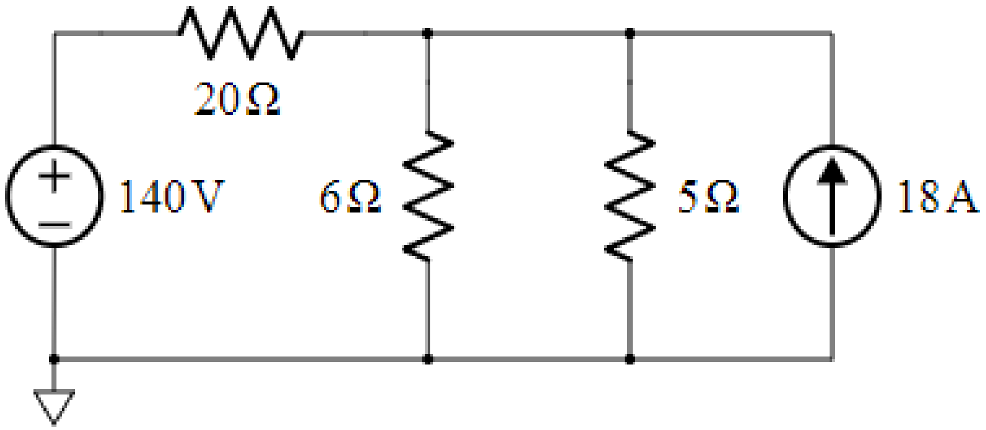
$$\frac{v_A}{R_1} - i_{S1} + \frac{v_A - v_B}{R_2} = 0$$

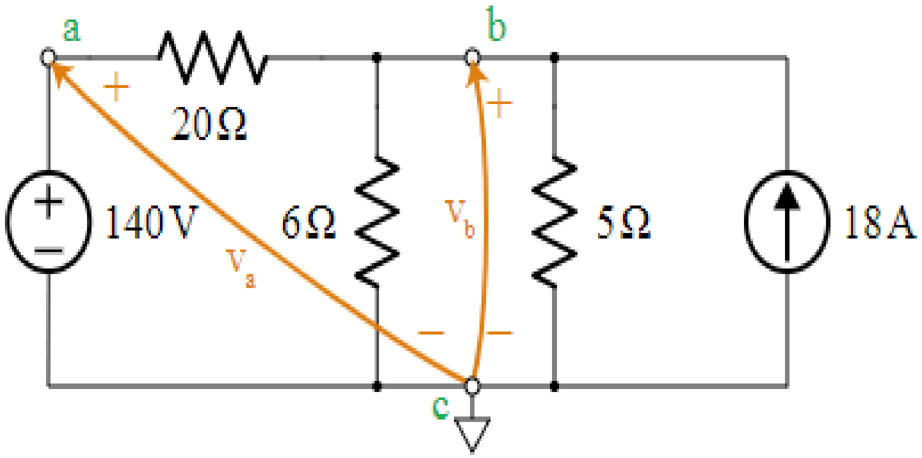
$$i_{S2} + \frac{v_B}{R_4} + \frac{v_B}{R_3} + \frac{v_B - v_A}{R_2} = 0$$



Note that we have assumed that all the values of the resistors and sources have been given. If not, we will need to get more information before we can solve.

NVM 2^{nd} Example



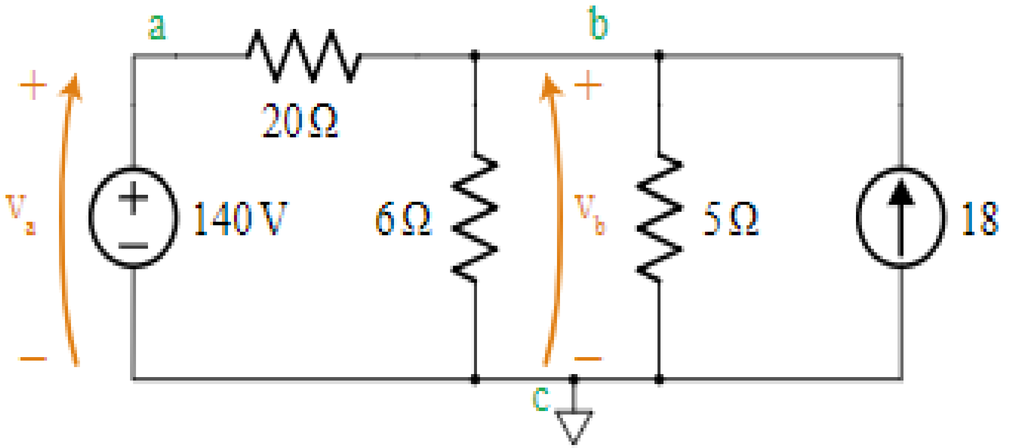


Our example circuit has three nodes, a, b and c, so N=3. Node c has 4 connections and it connects directly to both sources. This make it a good candidate to play the role of reference node. Node c has been marked with the ground symbol to let everyone know our choice for reference node.

We also call out N-1 =2 node voltages on the schematic, labeled in orange as Va and Vb

There is an obvious opprtunity here to simplify the two parallel resistors, 6 ohm with 5 ohm. We will not do that, because we want to study the Node Voltage Method procedure

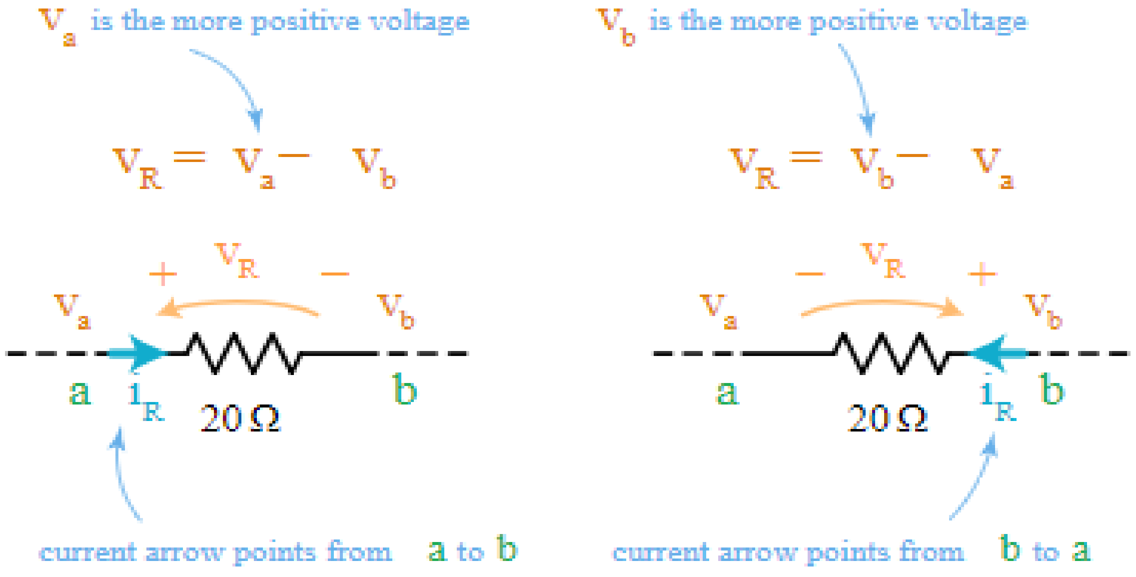
NVM 2nd Example



Node voltages control the current arrow

Notice something missing from the schematic. There is no orange label on the voltage across the 20 Ω resistor. When we need to know that voltage, we express it in terms of the node voltages.

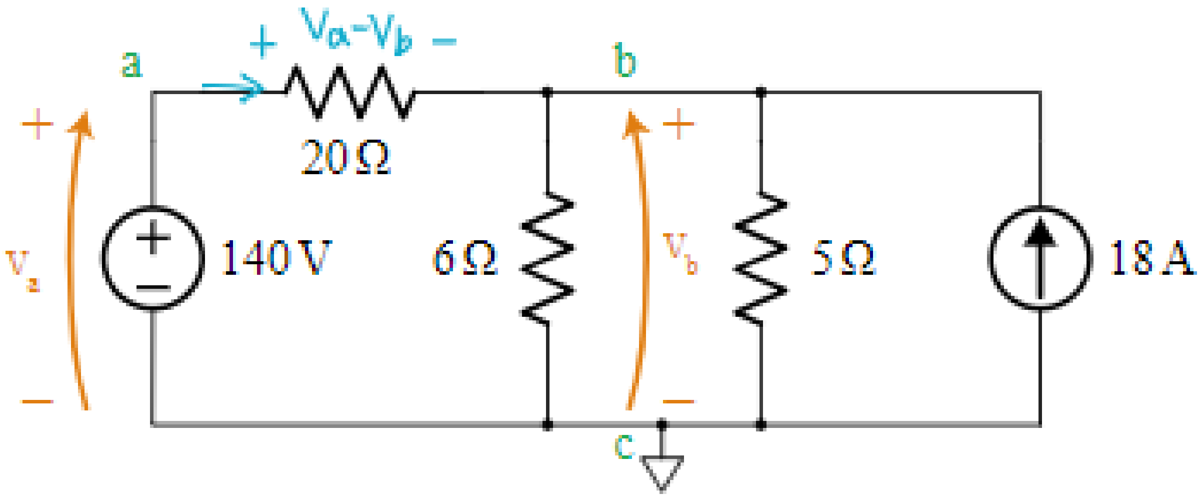
$v_R = v_a - v_b$ or $v_R = v_b - v_a$



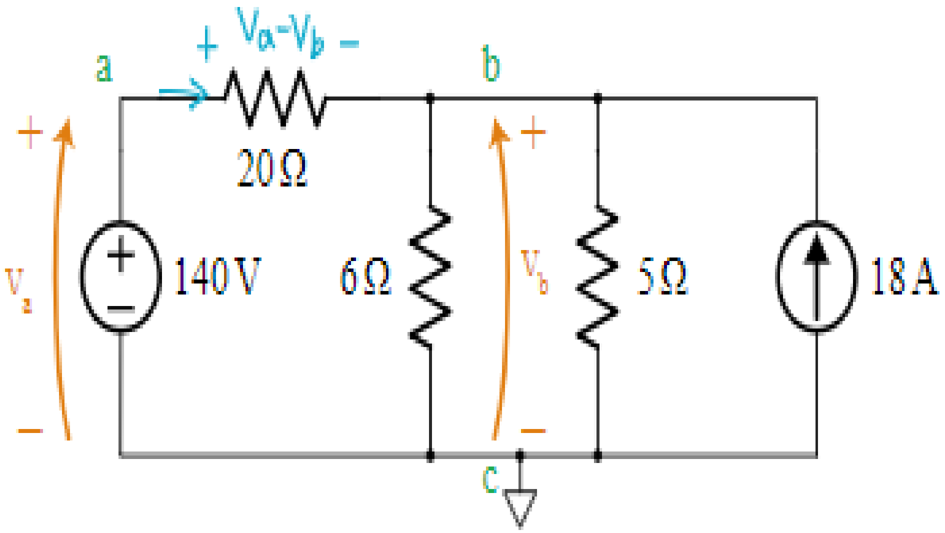
NVM 2nd Example

Solve the easy nodes

The voltage v_a is easy to figure out. Node a connects to a voltage source that connects to reference node c. That makes it an easy node. The voltage at node a is $v_a = 140\,\text{V}$.



NVM 2nd Example



As you write each term in the KCL equation, do Ohm's Law in your head and immediately write the current in terms of node voltages divided by resistance.

We now write a KCL equation for the remaining unsolved node, b. Node voltage v_b is the independent variable.

The current (blue arrow) flowing *into* node b from the $20\,\Omega$ resistor can be written as $+\dfrac{(140 - v_b)}{20}$.

The current in the $6\,\Omega$ and $5\,\Omega$ resistors instantly goes into the equation as $-\dfrac{v_b}{6}$ and $-\dfrac{v_b}{5}$.

We have just one node to deal with, node b. KCL says the sum of the currents flowing *into* node $b = 0$.

$$+\frac{(140 - v_b)}{20} - \frac{v_b}{6} - \frac{v_b}{5} + 18 = 0$$

NVM 2nd Example

Find the node voltages

Our system of equations happens to be just one equation. Let's solve it to find the node voltage.

$$+\frac{140}{20}-\frac{v_b}{20}-\frac{v_b}{6}-\frac{v_b}{5}=-18$$

$$-\frac{v_b}{20}-\frac{v_b}{6}-\frac{v_b}{5}=-18-7$$

$$\left(-\frac{3}{60}-\frac{10}{60}-\frac{12}{60}\right)\cdot v_b=-25$$

$$v_b=-25\cdot\left(-\frac{60}{25}\right)$$

$$v_b=60\,\mathrm{V}$$

NVM 2nd Example

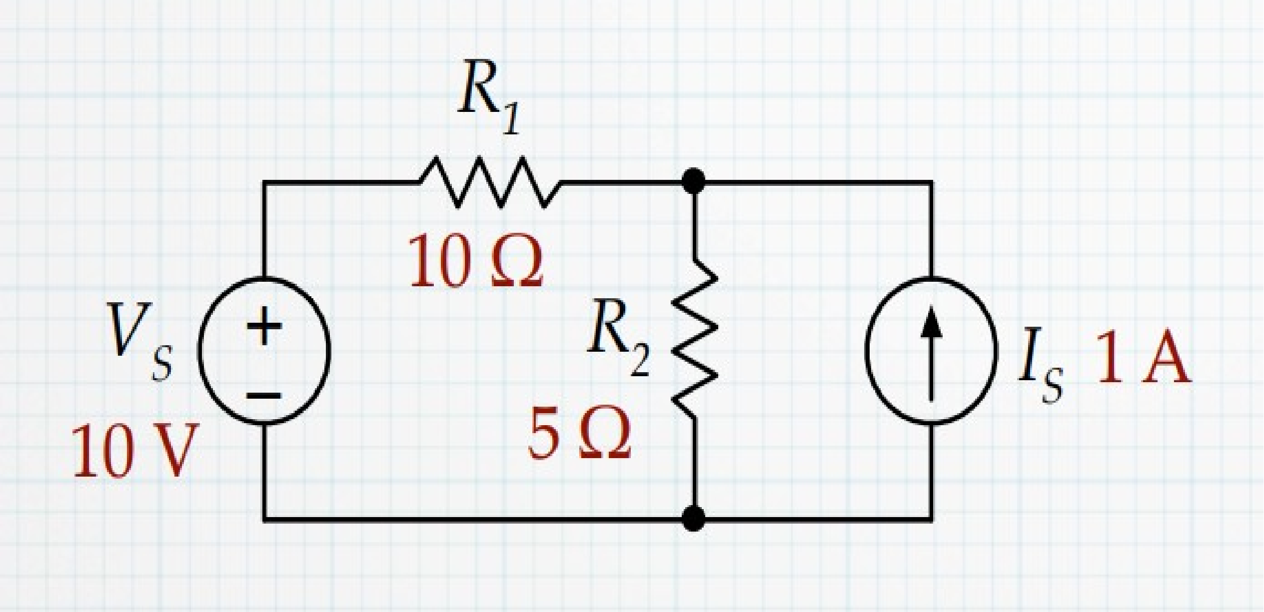
Solve for unknown currents using Ohm's Law

Now we have both node voltages, and we can solve for all the unknown currents using Ohm's Law.

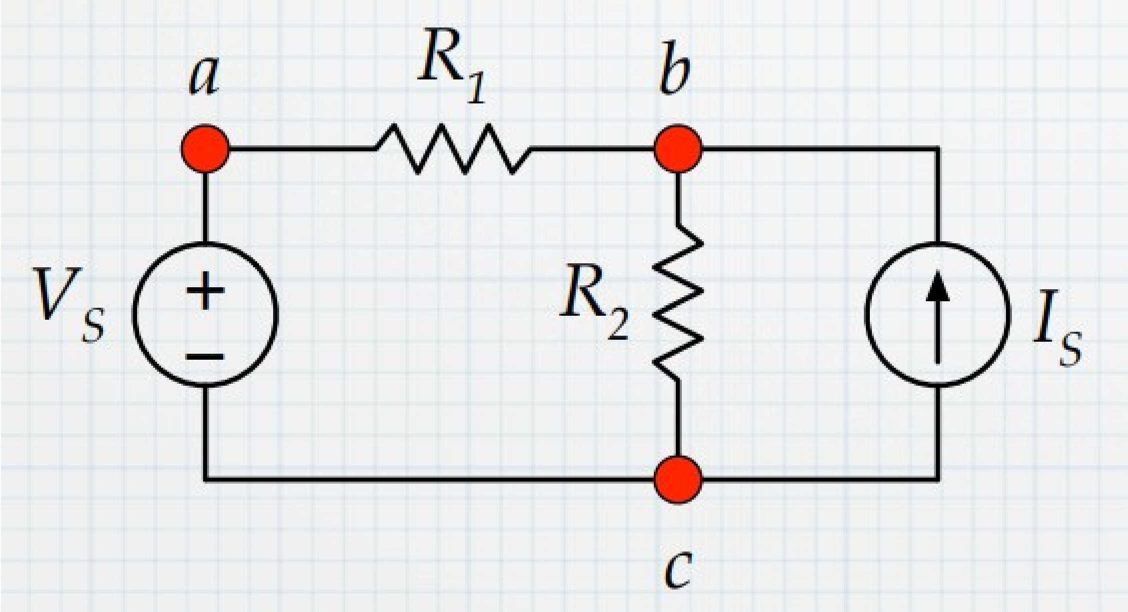
$$i_{20\,\Omega} = \frac{(v_a - v_b)}{20} = \frac{(140 - 60)}{20} = 4\text{ A}$$

$$i_{6\,\Omega} = \frac{v_b}{6} = \frac{60}{6} = 10\text{ A}$$

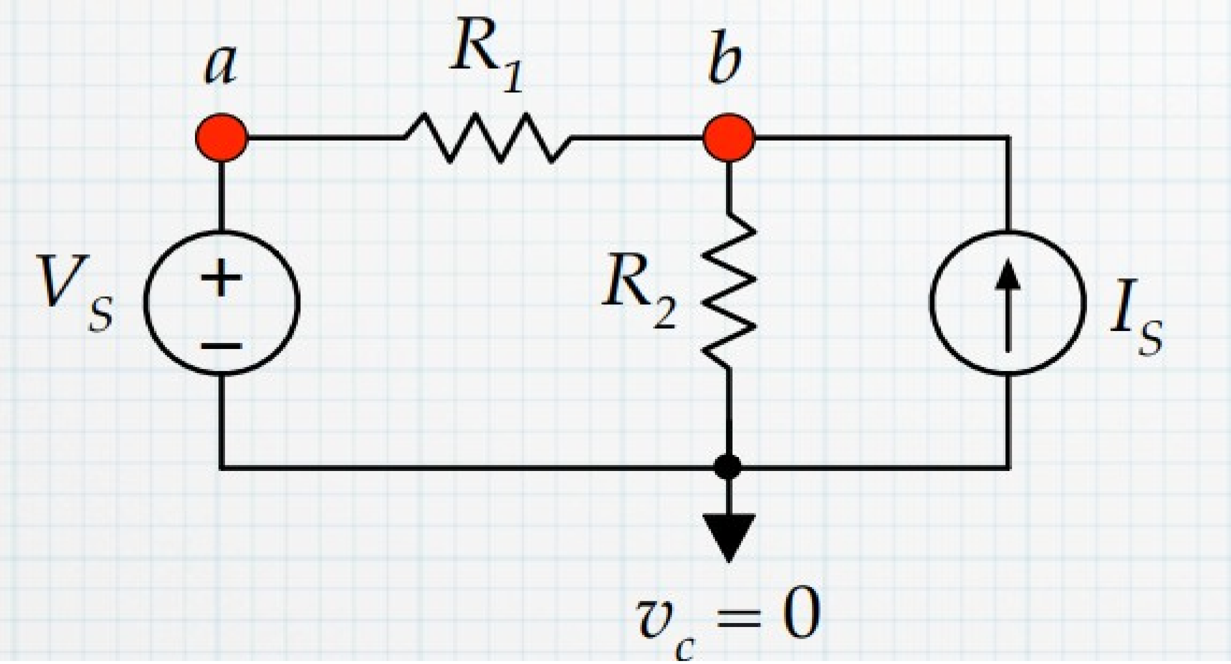
$$i_{5\,\Omega} = \frac{v_b}{5} = \frac{60}{5} = 12\text{ A}$$



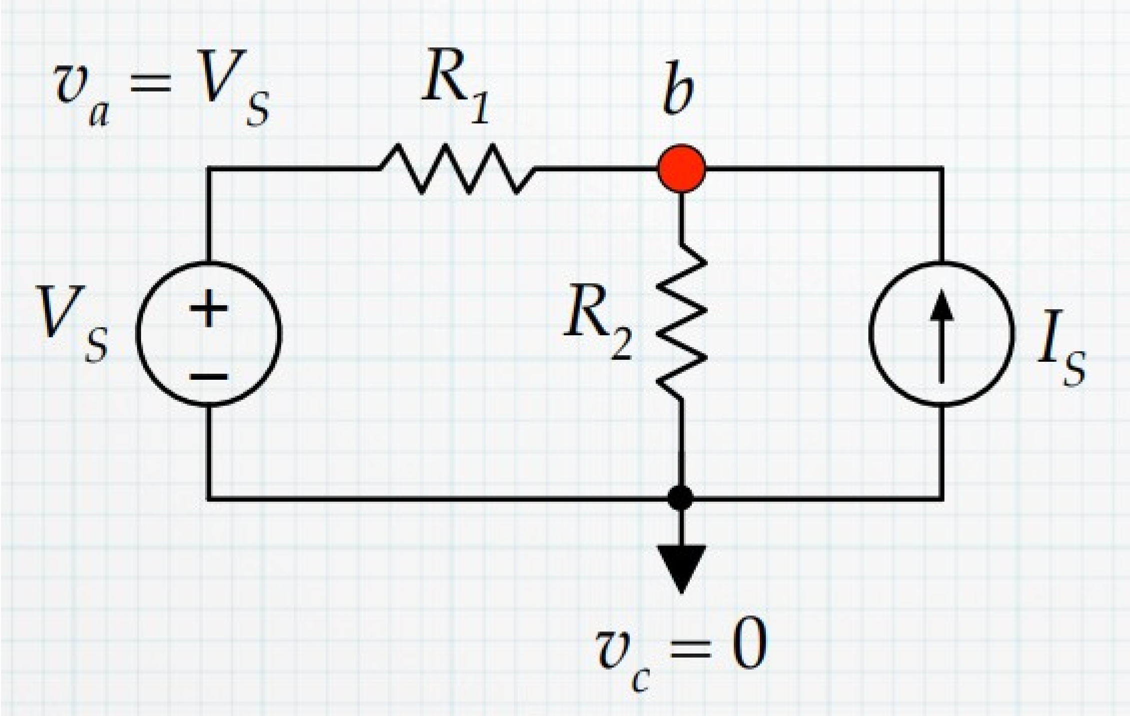
R1
10 Ω
VS
10 V
R2
5 Ω
IS 1 A



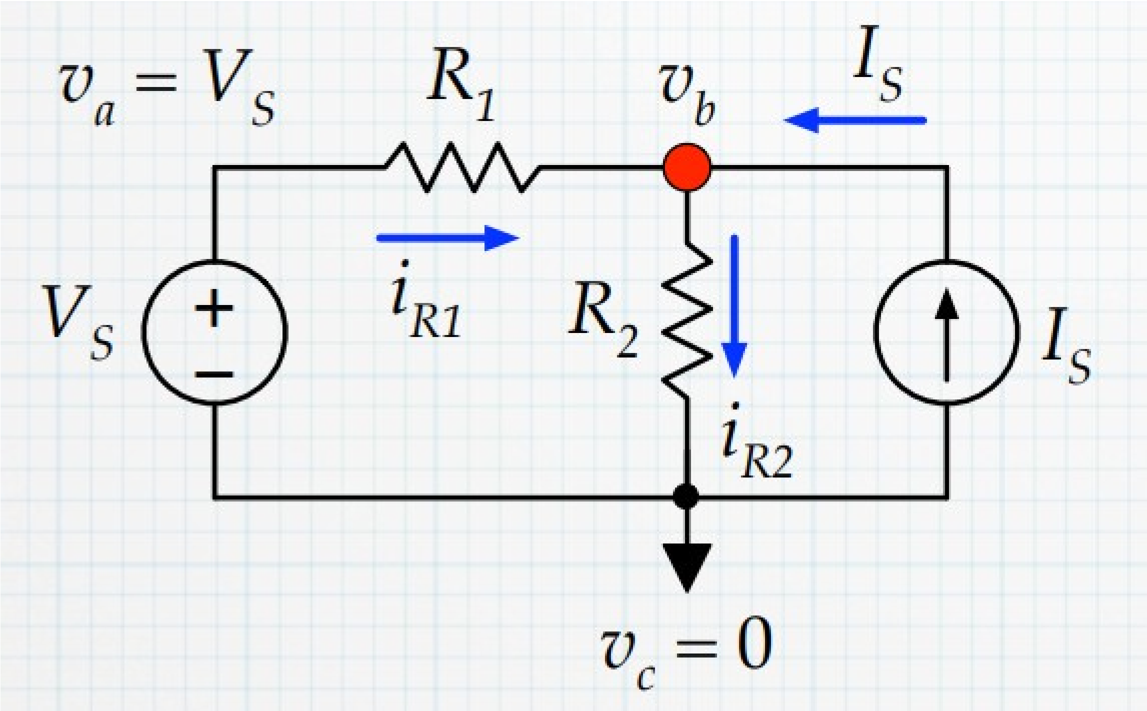
a
R1
b
VS
R2
IS
c



a
R1
b
VS
R2
IS
vc = 0



va = VS
R1
b
VS
R2
IS
vc = 0



$$i_{R1} + I_S = i_{R2}$$

$$i_{R1} = \frac{V_S - v_b}{R_1}$$

$$i_{R2} = \frac{v_b - 0}{R_2}$$

$$\frac{V_S - v_b}{R_1} + I_S = \frac{v_b}{R_2}$$

$$V_S - v_b + R_1 I_S = \frac{R_1}{R_2} v_b$$

$$v_b = \frac{V_S + R_1 I_S}{1 + \frac{R_1}{R_2}}$$

$$V_S + R_1 I_S = \left(1 + \frac{R_1}{R_2}\right) v_b$$

$$v_b = \frac{10\text{V} + (10\Omega)(1\text{A})}{\left(1 + \frac{10\Omega}{5\Omega}\right)} = \boxed{6.67\text{V}}$$

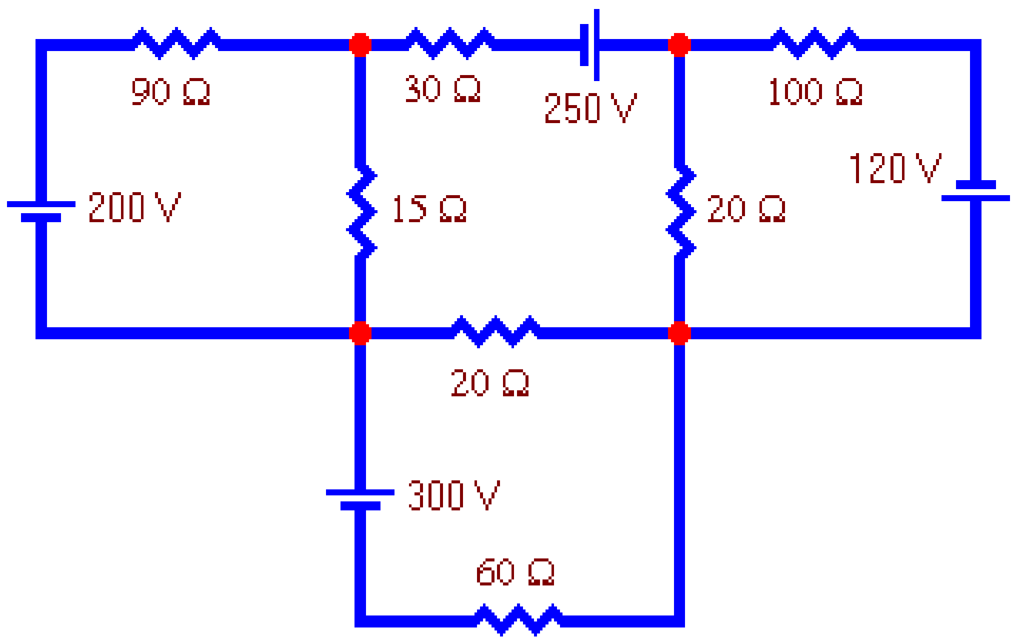
$$v_{R1} = V_S - v_b = 10\text{ V} - 6.67\text{ V} = 3.33\text{ V}$$

$$v_{R2} = v_b - 0 = 6.67\text{ V}$$

$$i_{R1} = \frac{v_{R1}}{R_1} = \frac{3.33\text{ V}}{10\Omega} = 0.333\text{ A}$$

$$i_{R2} = \frac{v_{R2}}{R_2} = \frac{6.67\text{ V}}{5\Omega} = 1.33\text{ A}$$

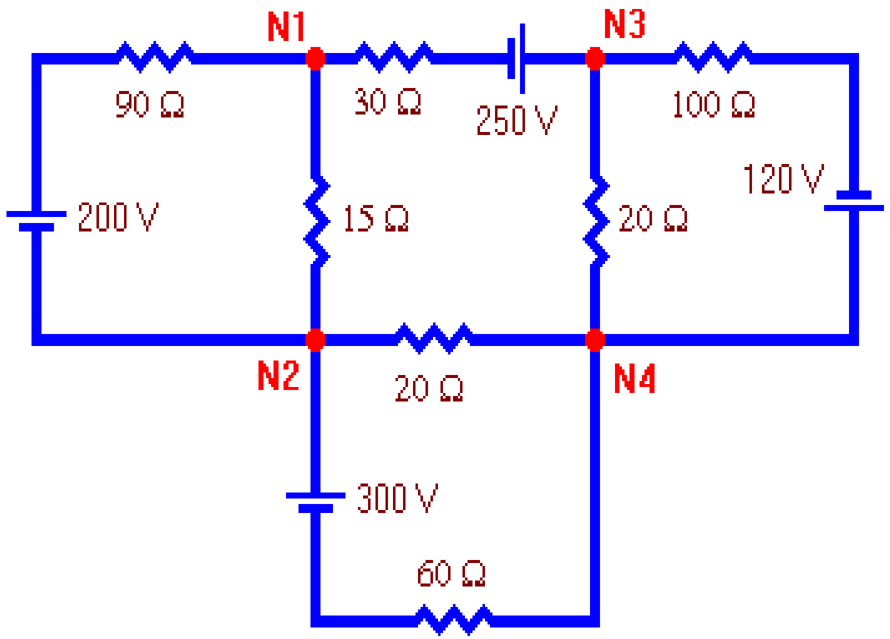
Example Use nodal analysis to find the voltage at each node of this circuit.



Solution:

- The number of nodes is 4.

- We will number the nodes as shown to the right.



- We will choose node 2 as the reference node and assign it a voltage of zero.

- Write down Kirchoff's Current Law for each node. Call V_1 the voltage at node 1, V_3 the voltage at node 3, V_4 the voltage at node 4, and remember that $V_2 = 0$. The result is the following system of equations:

$$\frac{V1}{15} + \frac{V1-200}{90} + \frac{V1-V3+250}{30} = 0$$

$$\frac{V3-V1-250}{30} + \frac{V3-V4}{20} + \frac{V3-V4+120}{100} = 0$$

$$\frac{V4-V3}{20} + \frac{V4}{20} + \frac{V4-V3-120}{100} + \frac{V4+300}{60} = 0$$

The first equation results from KCL applied at node 1, the second equation results from KCL applied at node 3 and the third equation results from KCL applied at node 4. Collecting terms this becomes:

$$\left(\frac{1}{15}+\frac{1}{90}+\frac{1}{30}\right)V1-\left(\frac{1}{30}\right)V3=\frac{200}{90}-\frac{250}{30}$$

$$-\left(\frac{1}{30}\right)V1+\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{100}\right)V3-\left(\frac{1}{20}+\frac{1}{100}\right)V4=\frac{250}{30}-\frac{120}{100}$$

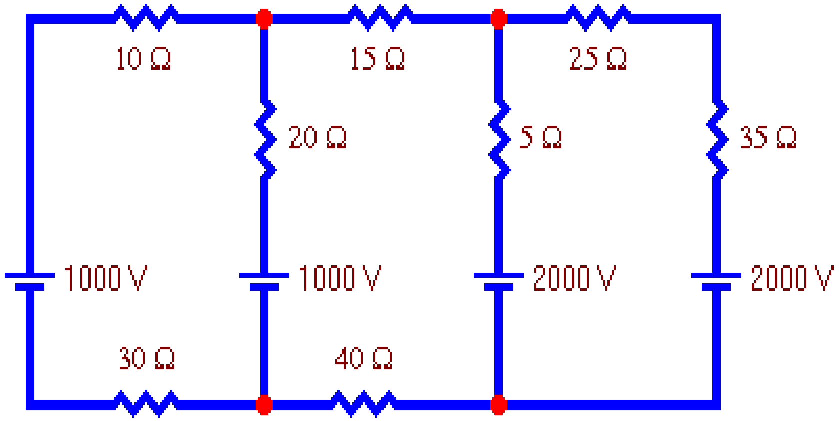
$$-\left(\frac{1}{20}+\frac{1}{100}\right)V3+\left(\frac{1}{20}+\frac{1}{20}+\frac{1}{100}+\frac{1}{60}\right)V4=\frac{120}{100}-\frac{300}{60}$$

This form for the system of equations could have been gotten immediately by using the inspection method.

- Solving the system of equations using Gaussian elimination or some other method gives the following voltages:

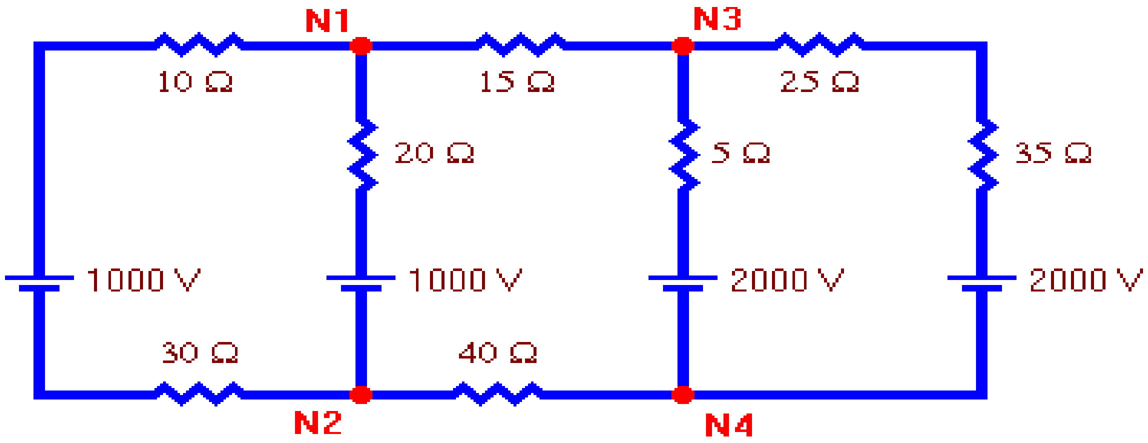
V_1 = -35.88 volts, V_3 = 63.74 volts and V_4 = 0.19 volts

Example Use nodal analysis to find the voltage at each node of this circuit.



Solution:

- The number of nodes is 4.
- We will number the nodes as shown to the right.



- We will choose node 4 as the reference node and assign it a voltage of zero.
- Write down Kirchoff's Current Law for each node. Call V_1 the voltage at node 1, V_2 the voltage at node 2, V_3 the voltage at node 3, and remember that $V_4 = 0$. The result is the following system of equations:

$$\frac{V1-V2-1000}{20} + \frac{V1-V2-1000}{40} + \frac{V1-V3}{15} = 0$$

$$\frac{V2-V1+1000}{20} + \frac{V2-V1+1000}{40} + \frac{V2}{40} = 0$$

$$\frac{V3-V1}{15} + \frac{V3-2000}{5} + \frac{V3-2000}{60} = 0$$

The first equation results from KCL applied at node 1, the second equation results from KCL applied at node 2 and the third equation results from KCL applied at node 3. Collecting terms this becomes:

$$\left(\frac{1}{20}+\frac{1}{40}+\frac{1}{15}\right)V1-\left(\frac{1}{20}+\frac{1}{40}\right)V2-\left(\frac{1}{15}\right)V3=\frac{1000}{20}+\frac{1000}{40}$$

$$-\left(\frac{1}{20}+\frac{1}{40}\right)V1+\left(\frac{1}{20}+\frac{1}{40}+\frac{1}{40}\right)V2=-\frac{1000}{20}-\frac{1000}{40}$$

$$-\left(\frac{1}{15}\right)V1+\left(\frac{1}{15}+\frac{1}{5}+\frac{1}{60}\right)V3=\frac{2000}{5}+\frac{2000}{60}$$

This form for the system of equations could have been gotten immediately by using the inspection method.

- Solving the system of equations using Gaussian elimination or some other method gives the following voltages:

V_1 = 1731 volts, V_2 = 548 volts and V_3 = 1937 volts

COM234 ELECTRONICS

The Mesh Current Method

Mesh Currents

The mesh method uses the **mesh currents** as the circuit variables. The procedure for obtaining the solution is similar to that followed in the Node method and the various steps are given below.

1.Clearly label all circuit parameters and distinguish the unknown parameters from the known.

2. Identify all meshes of the circuit.

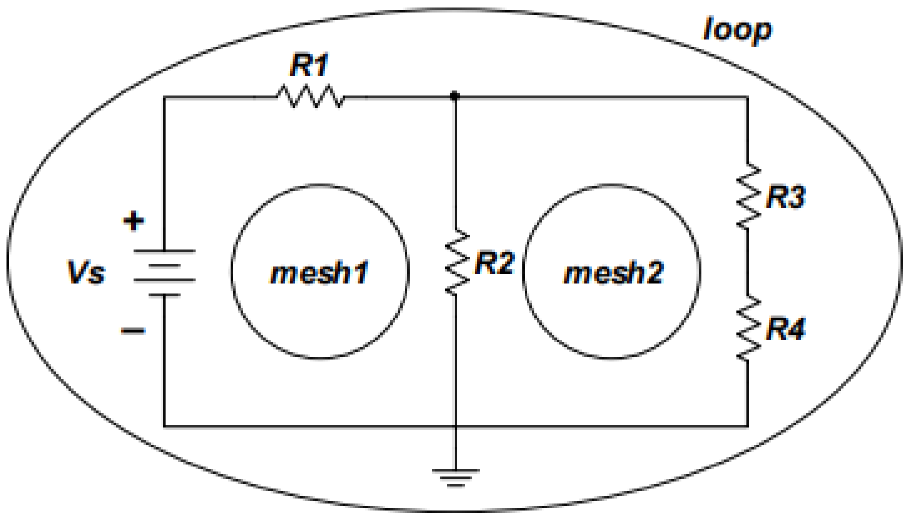
3. Assign mesh currents and label polarities.

4. Apply KVL at each mesh and express the voltages in terms of the mesh currents.

5. Solve the resulting simultaneous equations for the mesh currents.

The Mesh Method 1. Step

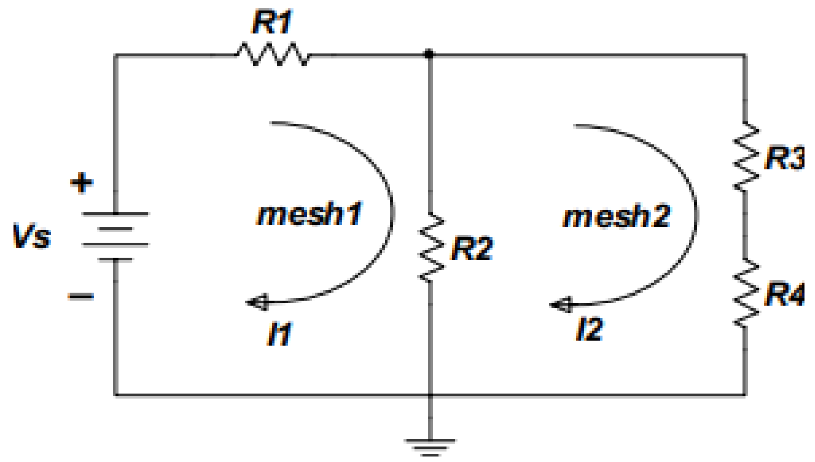
A mesh is defined as a loop which does not contain any other loops. Our circuit example has three loops but only two meshes as shown on Figure. Note that we have assigned a ground potential to a certain part of the circuit. Since the definition of ground potential is fundamental in understanding circuits this is a good practice and thus will continue to designate a reference (ground) potential as we continue to design and analyze circuits regardless of the method used in the analysis.



The Mesh Method 2. Step

For the next step we will assign mesh currents, define current direction and voltage polarities.

The direction of the mesh currents I1 and I2 is defined in the clockwise direction as shown on Figure. This definition for the current direction is arbitrary but it helps if we maintain consistence in the way we define these current directions. Note that in certain parts of the mesh the branch current may be the same as the current in the mesh. The branch of the circuit containing resistor R2 is shared by the two meshes and thus the branch current (the current flowing through R2) is the difference of the two mesh currents. (Note that in order to distinguish between the mesh currents and the branch currents by using the symbol **I** for the mesh currents and the symbol **i** for the branch currents.)



The Mesh Method 3. Step

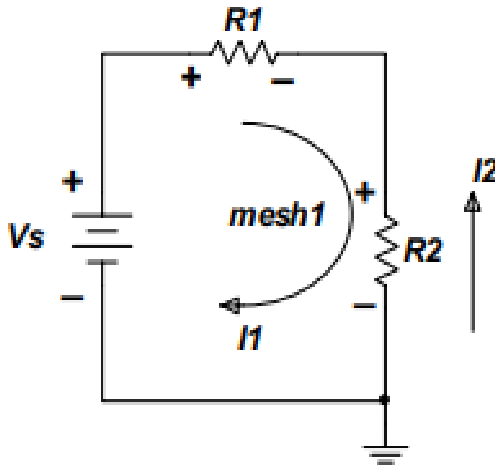
Now let's turn our attention in labeling the voltages across the various branch elements.

We choose to assign the voltage labels to be consistent with the direction of the indicated mesh currents. In the case where a certain branch is shared by two meshes as is the case in our example with the branch that contains resistor R2 the labeling of the voltage is done for each mesh consistent with the assigned direction of the mesh current.

In this, our first encounter with mesh analysis let's consider the each mesh separately and apply KVL around the loop following the defined direction of the mesh current. Considering mesh1. For clarity we have separated mesh1 from the circuit on Figure 11. In doing this, care must be taken to carry all the information of the shared branches. Here we indicate the direction of mesh current I2 on the shared branch.

Considering mesh1.

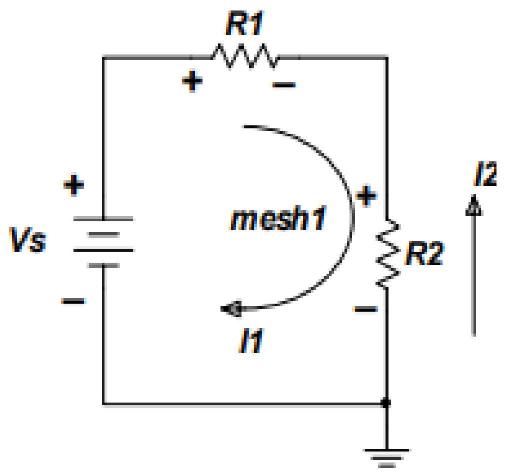
For clarity we have separated mesh1 from the circuit on Figure. In doing this, care must be taken to carry all the information of the shared branches. Here we indicate the direction of mesh current I2 on the shared branch.



The Mesh Method 4. Step

Apply KVL to mesh1.

Starting at the upper left corner and proceeding in a clock-wise direction the sum of voltages across all elements encountered is:



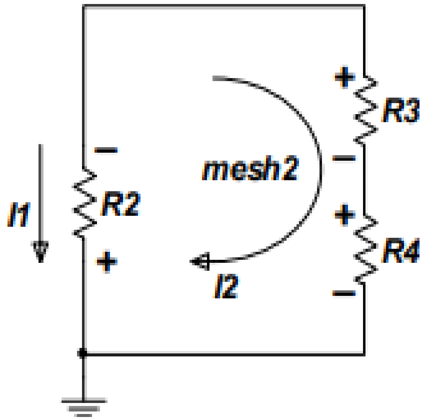
$$I1R1 + (I1 - I2)R2 - Vs = 0$$

The Mesh Method 4. Step

Apply KVL to mesh2.

Similarly, consideration of mesh2 is shown on Figure. Note again that we have indicated the direction of the mesh current I1 on the shared circuit branch.

Starting at the upper right corner and proceeding in a clock-wise direction the sum of voltages across all elements encountered is:



$$I2(R3 + R4) + (I2 - I1)R2 = 0$$

The Mesh Method 5. Step

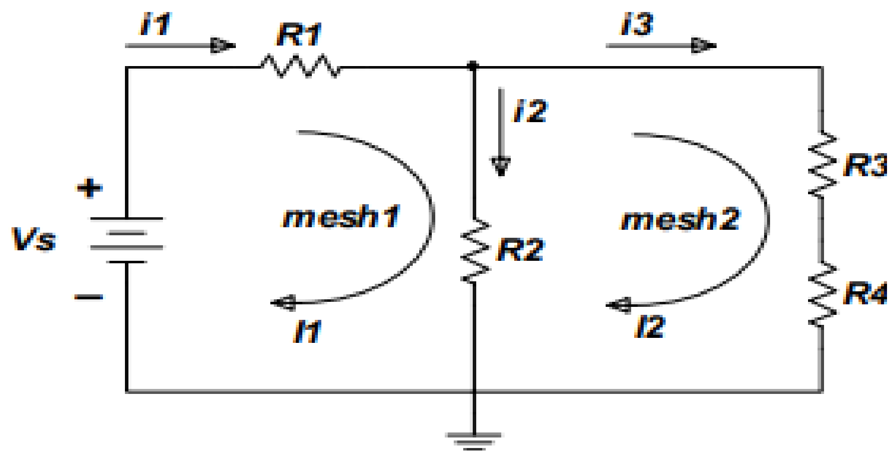
Solve equations

Keeping in mind that the unknowns of the problem are the mesh currents I1 and I2 we rewrite the mesh equations as:

$$I1(R1+R2)-I2R2=Vs$$

$$-I1R2+I2(R2+R3+R4)=0$$

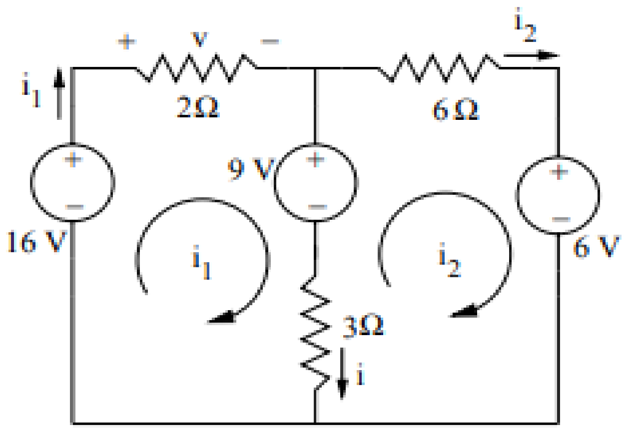
$$\begin{bmatrix} R1+R2 & -R2 \\ -R2 & R2+R3+R4 \end{bmatrix}\begin{bmatrix} I1 \\ I2 \end{bmatrix}=\begin{bmatrix} Vs \\ 0 \end{bmatrix}$$



$$i1 = I1$$
$$i2 = I1 - I2$$
$$i3 = I2$$

Example 1.

Example Find i and v.
Using mesh-current method:



Mesh 1: $2i_1 + 9 + 3(i_1 - i_2) - 16 = 0$

Mesh 2: $6i_2 + 6 + 3(i_2 - i_1) - 9 = 0$

$$\begin{cases} 5i_1 - 3i_2 = 7 \\ -3i_1 + 9i_2 = 3 \end{cases} \rightarrow \begin{cases} i_1 = 2\ \text{A} \\ i_2 = 1\ \text{A} \end{cases}$$

The problem unknowns, i and v can now be found from the mesh currents:

$$i = i_1 - i_2 = 1\ \text{A}$$

$$v = 2i_1 = 4\ \text{V}$$

Node-voltage or mesh-current?

Deciding which approach to take in a particular circuit usually boils down to determining which method leads to easier math – fewest number of simultaneous equations.

node number (N)

1. Count number of nodes in the circuit.
2. Subtract 1 for ground.
3. Subtract 1 for each voltage source which has a connection (+ or –) to ground.
4. Add 1 for each voltage source which has no connection to ground.

mesh number (M)

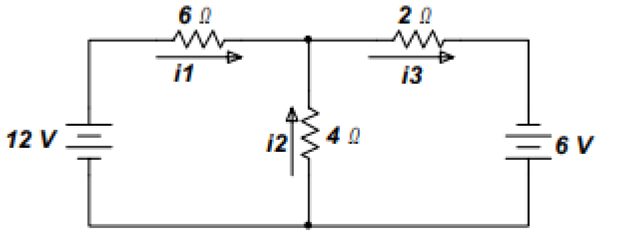
1. Count number of meshes in the circuit.
2. Subtract 1 for each current source which is located in an outside branch of a mesh.
3. Add 1 for each current source which is located in an interior branch (shared between two meshes). (More on this later.)

If $N < M$, the node-voltage method should have less math.

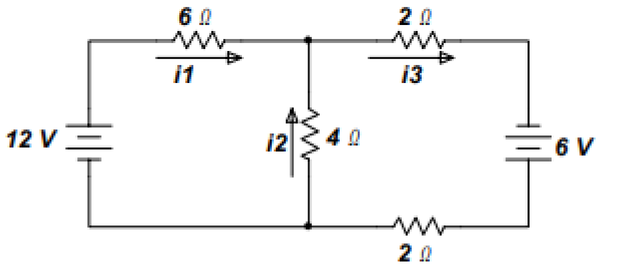
If $M < N$, the mesh-current method should have less math.

Practice problems with answers.

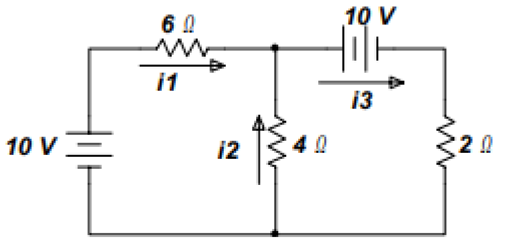
Determine the currents in the following circuits with reference to the indicated direction.



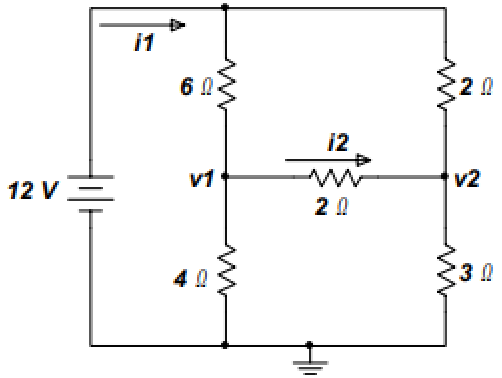
Answer: $i1 = 2.180A$, $i2 = 0.270A$, $i3 = 2.450A$



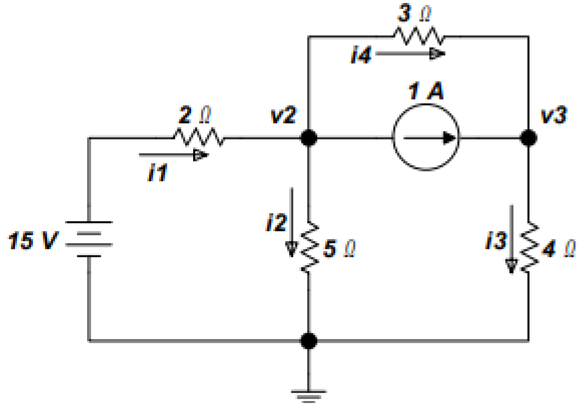
Answer: $i1 = 1.877A$, $i2 = -0.187A$, $i3 = 1.690A$



Answer: $i1 = 0.455A$, $i2 = -1.820A$, $i3 = -1.36A$



Answer: $i1 = 3.690A$, $i2 = -0.429A$, $v1 = 5.83V$, $v2 = 6.69V$



Answer: $i1 = 3.31A, i2 = 1.68A, i3 = 1.63A, i4 = 0.627A, v2 = 8.39V, v3 = 6.51V$

COM234 ELECTRONICS

Superposition

Thevenin’s and Norton’s Theorems

SUPERPOSITION

The idea of superposition rests on the linearity property.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables. With these in mind, we apply the superposition principle in three steps:

SUPERPOSITION

Steps to Apply Super position Principle:

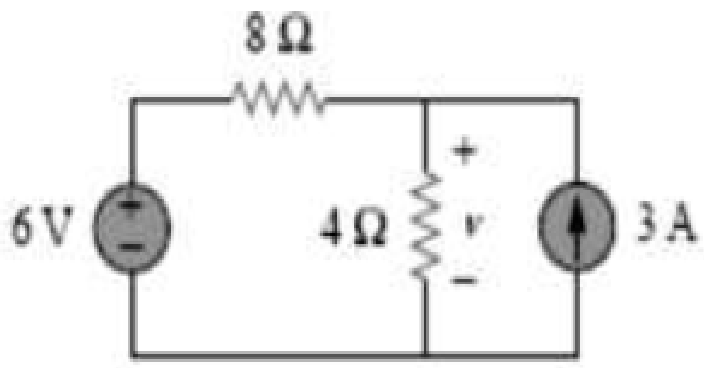
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. Keep in mind that superposition is based on linearity.

Example 1: Use the superposition theorem to find v in the circuit.



Solution:

Since there are two sources, let

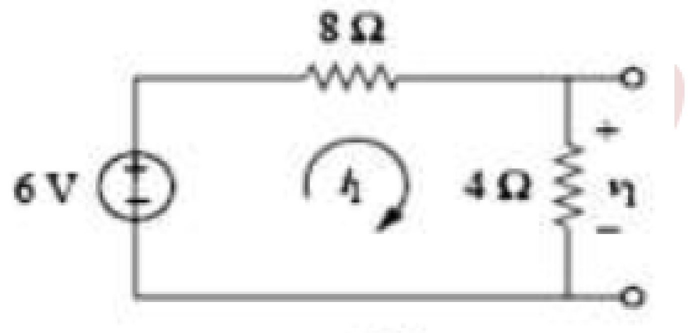
$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6V voltage source and the 3A current source, respectively. To obtain v_1, we set the current source to zero, as shown in **Fig (a)**. Applying **KVL** to the loop in **Fig.(a)** gives

$$12i_1 - 6 = 0 \Rightarrow i_1 = 0.5\ A$$

Thus:

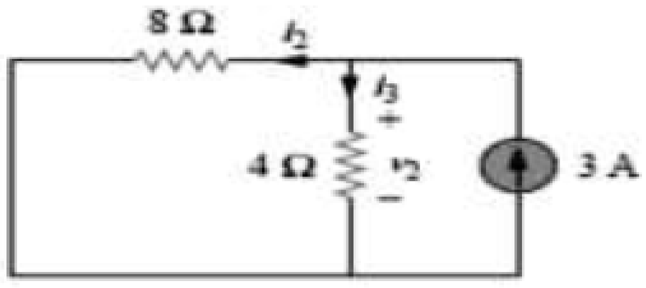
$$v_1 = 4i_1 = 2\ V$$



(a)

To get v_2, we set the voltage source to zero, as in **Fig. (b)**. Using current division,

$$i_3 = \frac{8}{4+8} 3A = 2A$$



(b)

Hence,

$$v_2 = 4i_3 = 8\ V$$

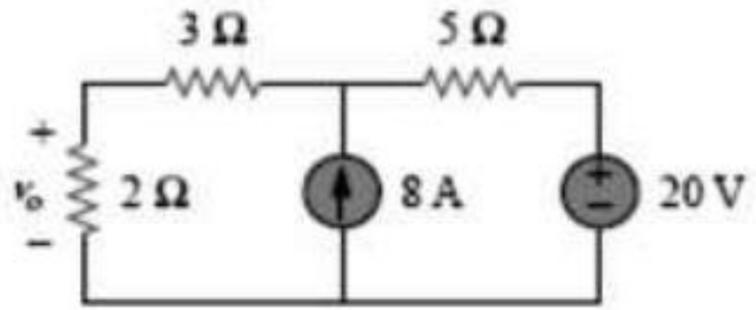
And we find

$$v = v_1 + v_2 = 2 + 8 = 10\ V$$

Practice 1:

Using the superposition theorem, find $\mathbf{v_o}$ in the circuit in Figure below.

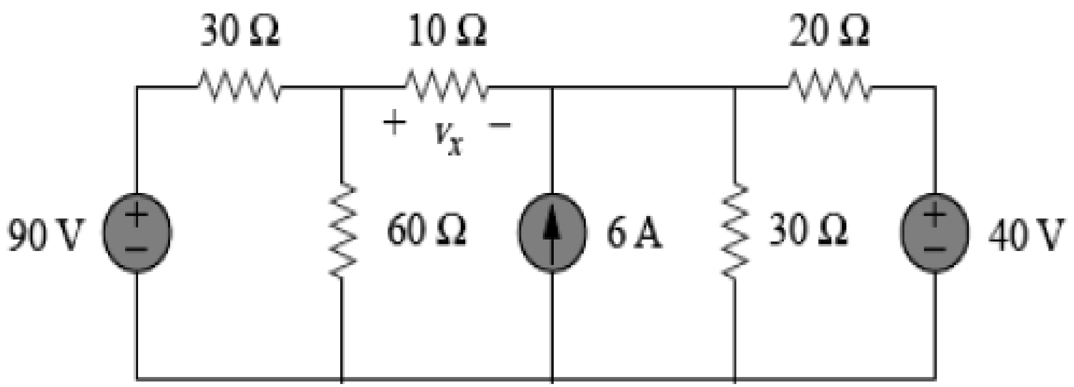
Answer:12 V



Practice 2:

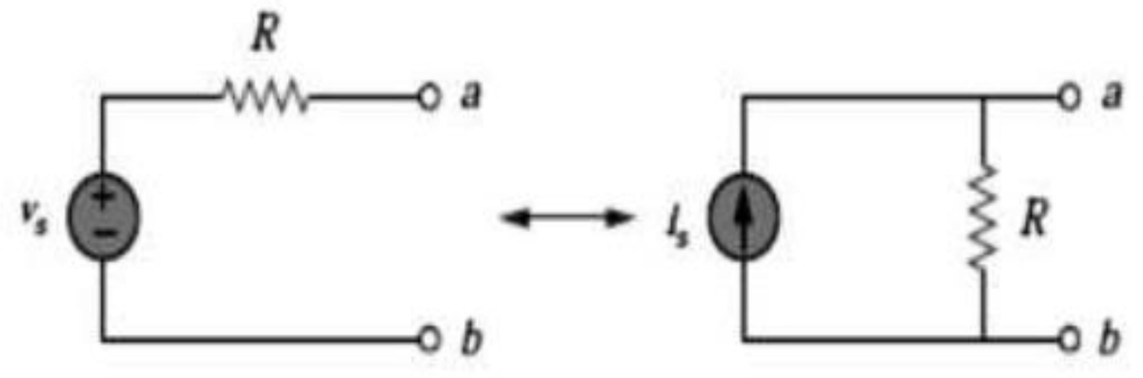
Use superposition to obtain v_x in the circuit of Figure below..

Answer: 0.75 A



SOURCE TRANSFORMATION

We have noticed that series-parallel combination. Source transformation is another tool for simplifying circuits. We can substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in **Fig.** below. Either substitution is known as a ***source transformation***.



Key Point: *A source transformation is the process of replacing a voltage source vs in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.*

SOURCE TRANSFORMATION

We need to find relationship between v_s and i_s that guarantees the two configurations in Fig below are equivalent with respect to nodes a, b.

Suppose R_L, is connected between nodes a, b in Fig. Using Ohm Law, the Current in R_L is.

$$i_L = \frac{v_s}{(R+R_L)}$$

R and R_L in series

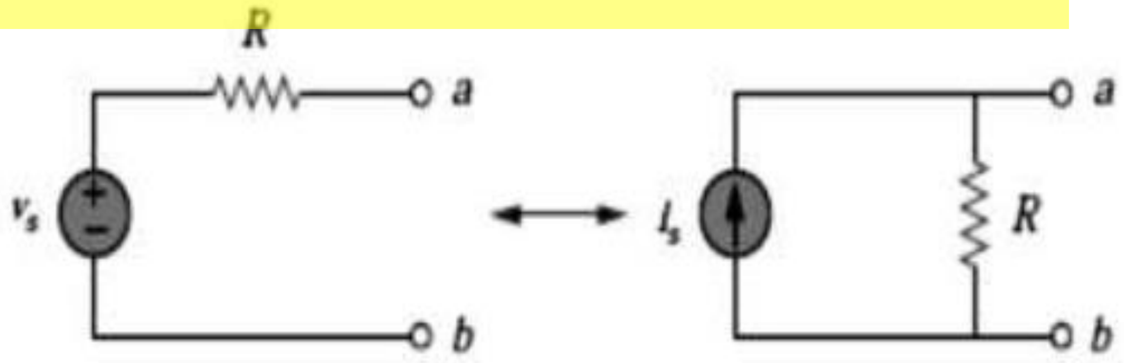
If it is to be replaced by a current source then load current must be $\frac{V}{(R+R_L)}$

Now suppose the same resistor **R_L**, is connected between nodes **a**, **b** in **Fig. (b)**. Using current division, the current in **R_L**, is

$$i_L = i_s \frac{R}{(R+R_L)}$$

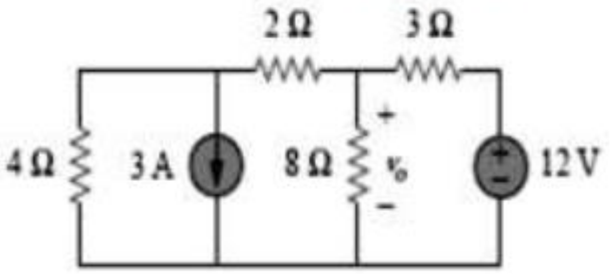
If the two circuits in Fig are equivalent, these resistor and currents must be the same. Equating the right hand sides of equations and simplifying

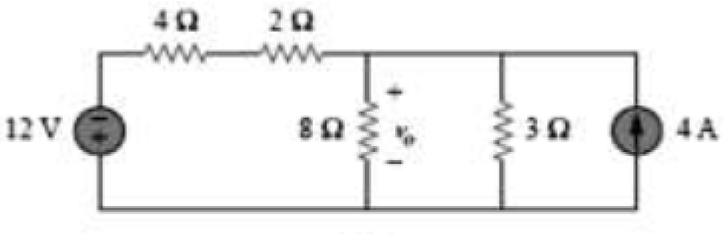
$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R$$



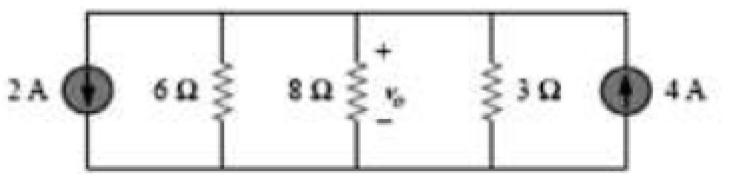
EXAMPLE: Use source transformation to find v_o in the circuit in Fig

Solution: We first transform the current and voltage sources to obtain the circuit in **Fig. (a)**. Combining the 4Ω and 2Ω resistors in series and transforming the 12V voltage source gives us **Fig. (b)**. We now combine the 3Ω and 6Ω resistors in parallel to get 2Ω. We also combine the 2A and 4A current sources to get a 2A source. Thus, by repeatedly applying source transformations, we obtain the circuit in **Fig. (c)**.





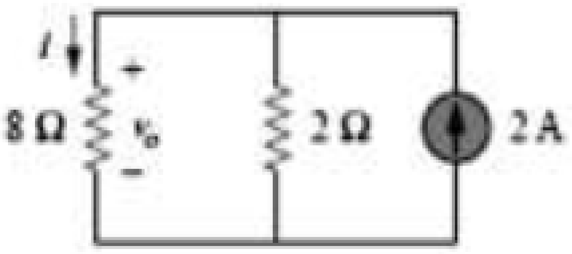
(a)



(b)

Alternatively, since the 8Ω and 2Ω resistors in **Fig. (c)** are in parallel, they have the same voltage $\boldsymbol{v_o}$ across them. Hence,

$$v_o = (8||2)(2\text{ A}) = ((8\times2)/10)*2 = 3.2\text{ V}$$

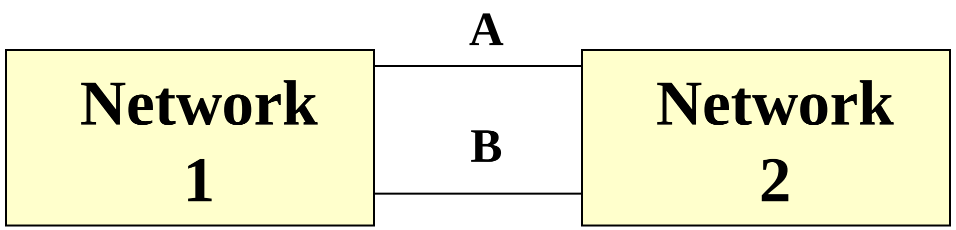


(c)

THEVENIN AND NORTON

THEVENIN'S THEOREM

Consider the following:

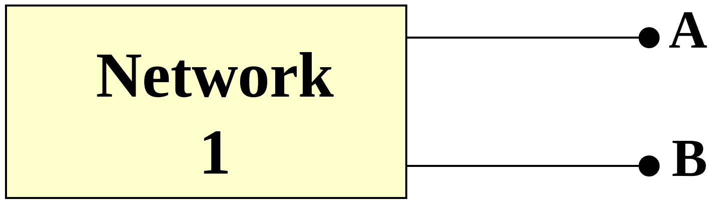


For purposes of discussion, at this point, we consider that both networks are composed of resistors and independent voltage and current sources

THEVENIN & NORTON

THEVENIN'S THEOREM:

Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.



Network 1 can be as complicated in structure as one can imagine. Maybe 45 meshes, 387 resistors, 91 voltage sources and 39 current sources.

COM234 ELECTRONICS

Superposition

Thevenin’s and Norton’s Theorems

SUPERPOSITION

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Thus:

$$v_1 = 4i_1 = 2\ \text{V}$$

To get v_2, we set the voltage source to zero, as in **Fig. (b)**. Using current division,

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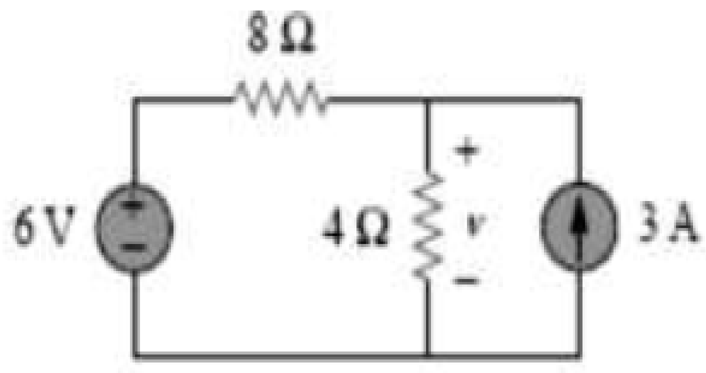
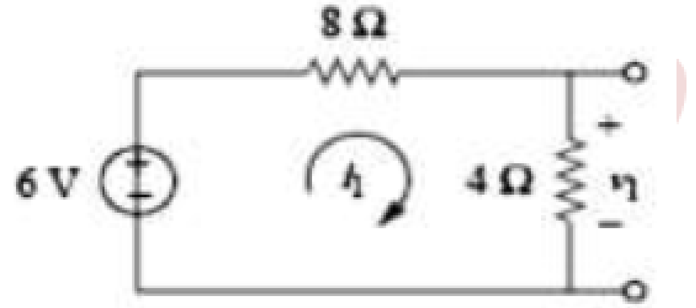
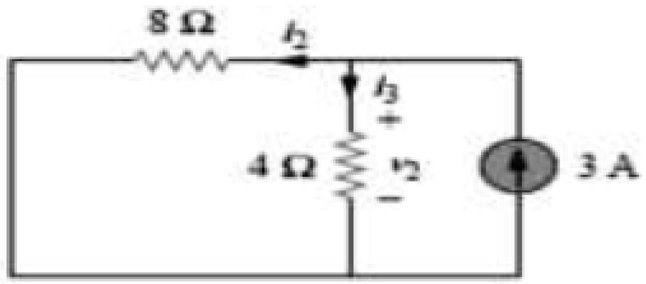




(a)



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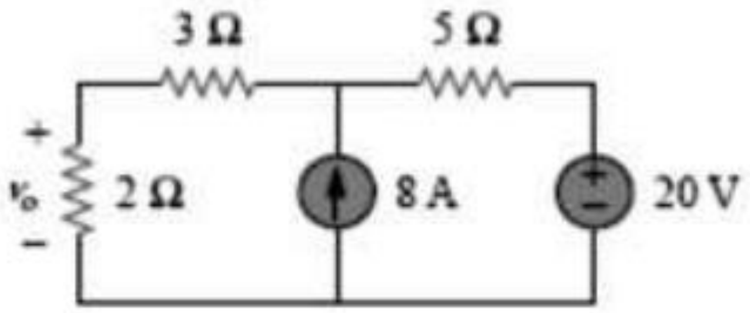

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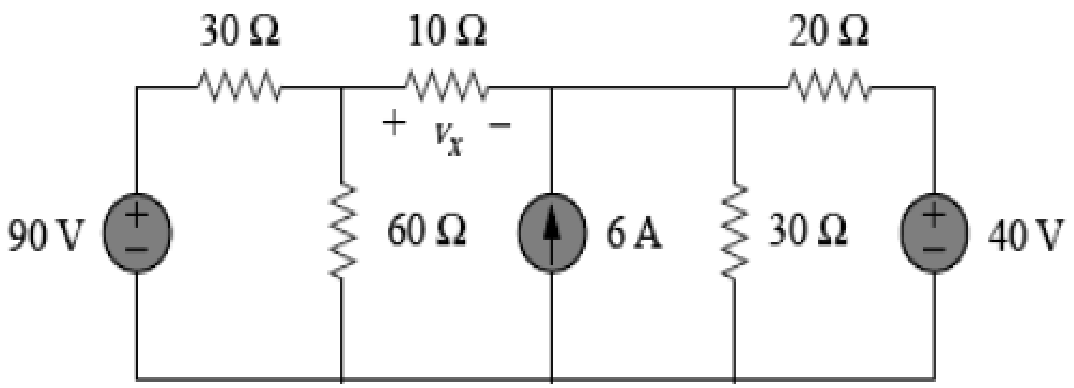

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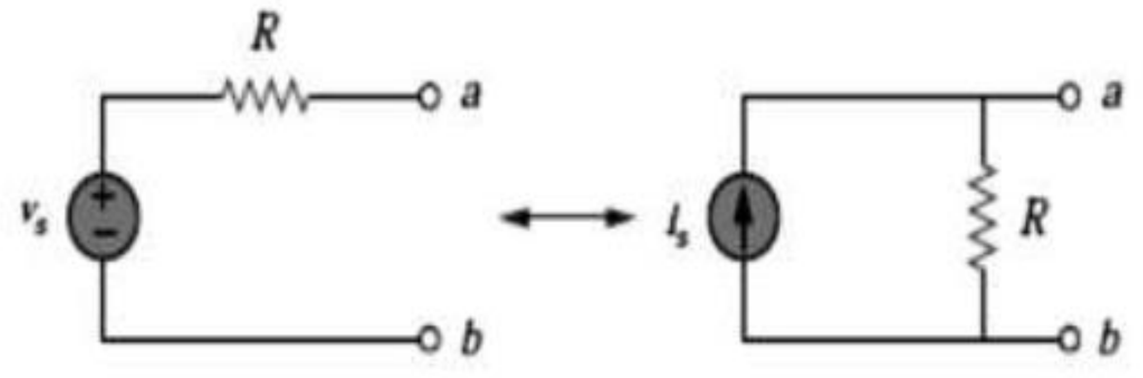

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$$i_L = \frac{v_s}{(R+R_L)}$$

R and R_L in series

If it is to be replaced by a current source then load current must be

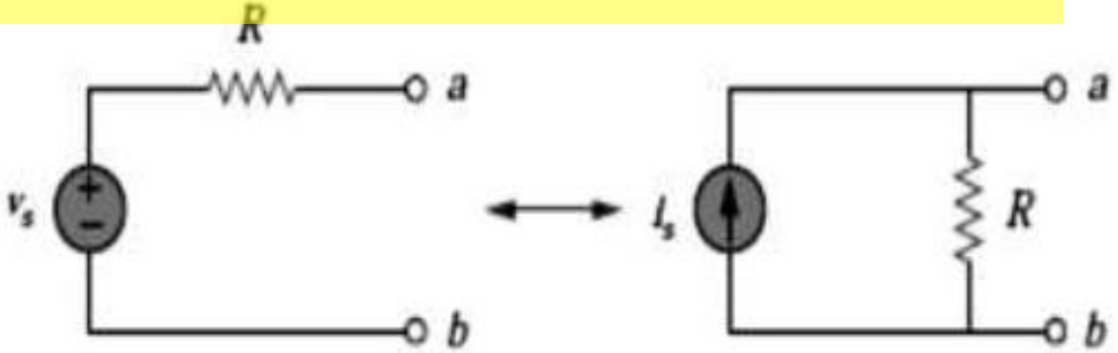
$$\frac{V}{(R+R_L)}$$

Now suppose the same resistor $\mathbf{R_L}$, is connected between nodes **a**, **b** in **Fig. (b)**. Using current division, the current in $\mathbf{R_L}$, is

$$i_L = i_s \frac{R}{(R+R_L)}$$

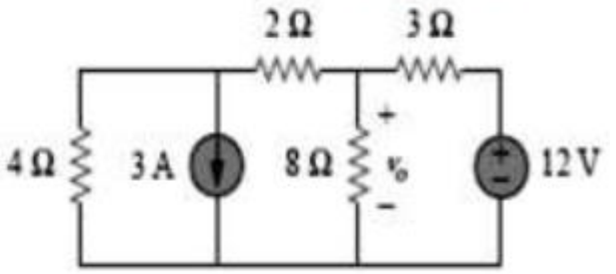
If the two circuits in Fig are equivalent, these resistor and currents must be the same. Equating the right hand sides of equations and simplifying

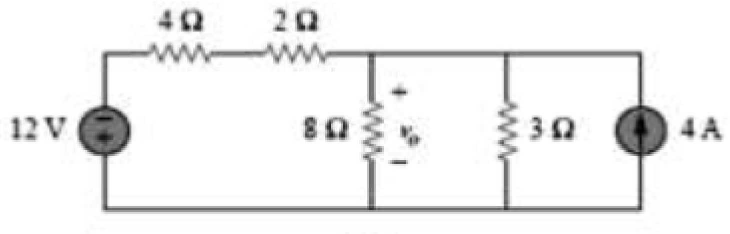
$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R$$



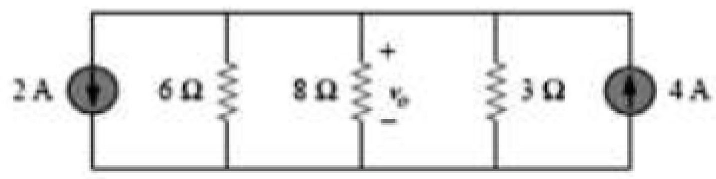
EXAMPLE: Use source transformation to find v_o in the circuit in Fig

Solution: We first transform the current and voltage sources to obtain the circuit in **Fig. (a)**. Combining the 4Ω and 2Ω resistors in series and transforming the 12V voltage source gives us **Fig. (b)**. We now combine the 3Ω and 6Ω resistors in parallel to get 2Ω. We also combine the 2A and 4A current sources to get a 2A source. Thus, by repeatedly applying source transformations, we obtain the circuit in **Fig. (c)**.





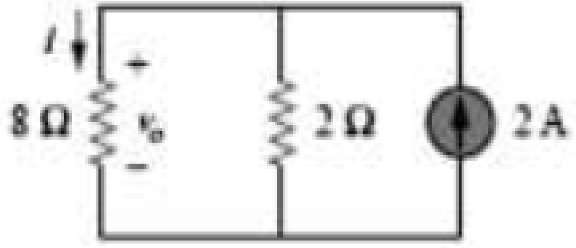
(a)



(b)

Alternatively, since the 8Ω and 2Ω resistors in **Fig. (c)** are in parallel, they have the same voltage $\boldsymbol{v_o}$ across them. Hence,

$$v_o = (8||2)(2\text{ A}) = ((8\times 2)/10)*2 = 3.2\text{ V}$$

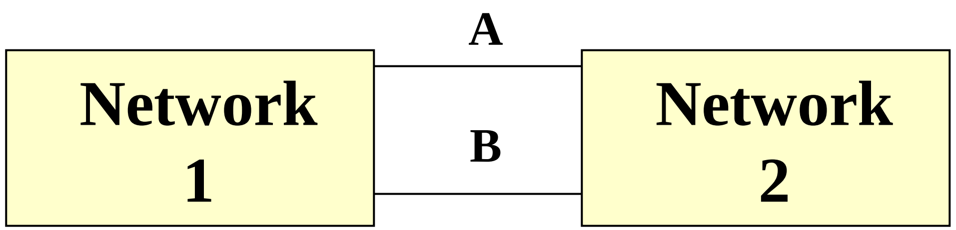


(c)

THEVENIN AND NORTON

THEVENIN'S THEOREM

Consider the following:

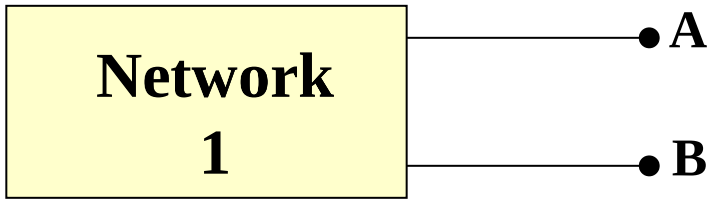


For purposes of discussion, at this point, we consider that both networks are composed of resistors and independent voltage and current sources

THEVENIN & NORTON

THEVENIN'S THEOREM:

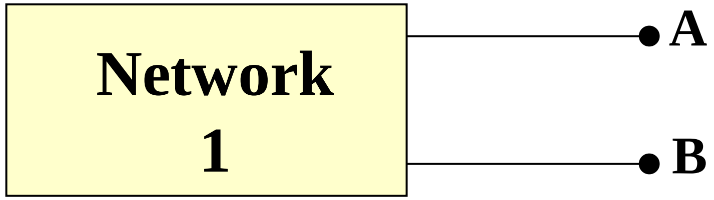
Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.



Network 1 can be as complicated in structure as one can imagine. Maybe 45 meshes, 387 resistors, 91 voltage sources and 39 current sources.

THEVENIN & NORTON

THEVENIN'S THEOREM:



Now place a voltmeter across terminals A-B and read the voltage. We call this the open-circuit voltage.

No matter how complicated Network 1 is, we read one voltage. It is either positive at A, (with respect to B) or negative at A.

We call this voltage V_{os} and we also call it $V_{THEVENIN} = V_{TH}$

THEVENIN & NORTON

THEVENIN'S THEOREM:

- We now deactivate all sources of Network 1.

- To deactivate a voltage source, we remove the source and replace it with a short circuit.

- To deactivate a current source, we remove the source.

THEVENIN & NORTON

THEVENIN'S THEOREM:

Consider the following circuit.

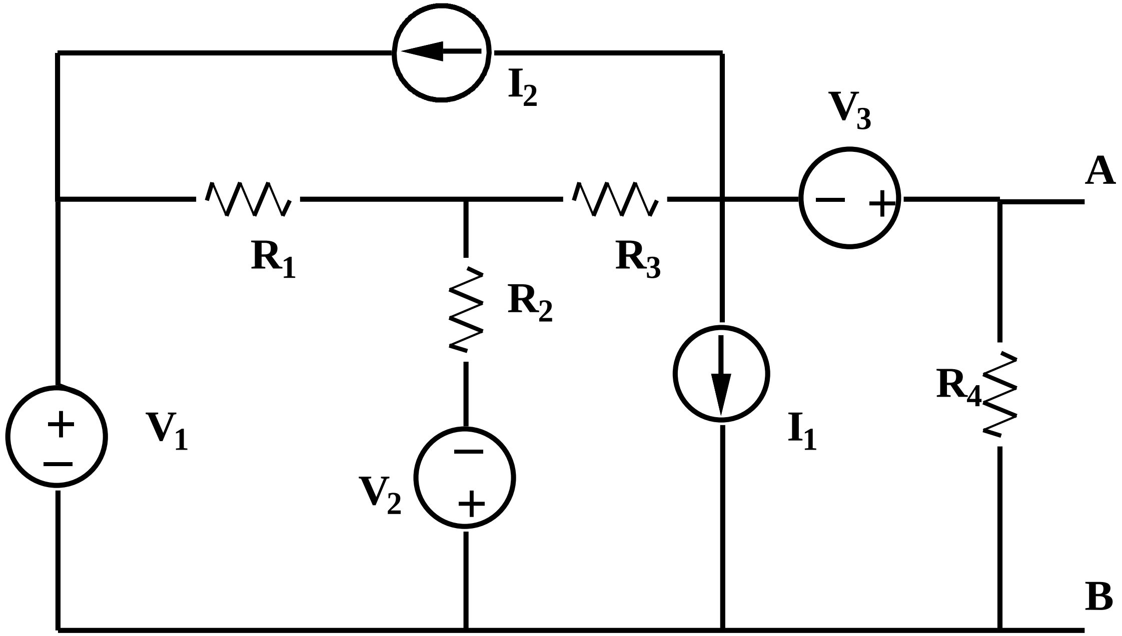


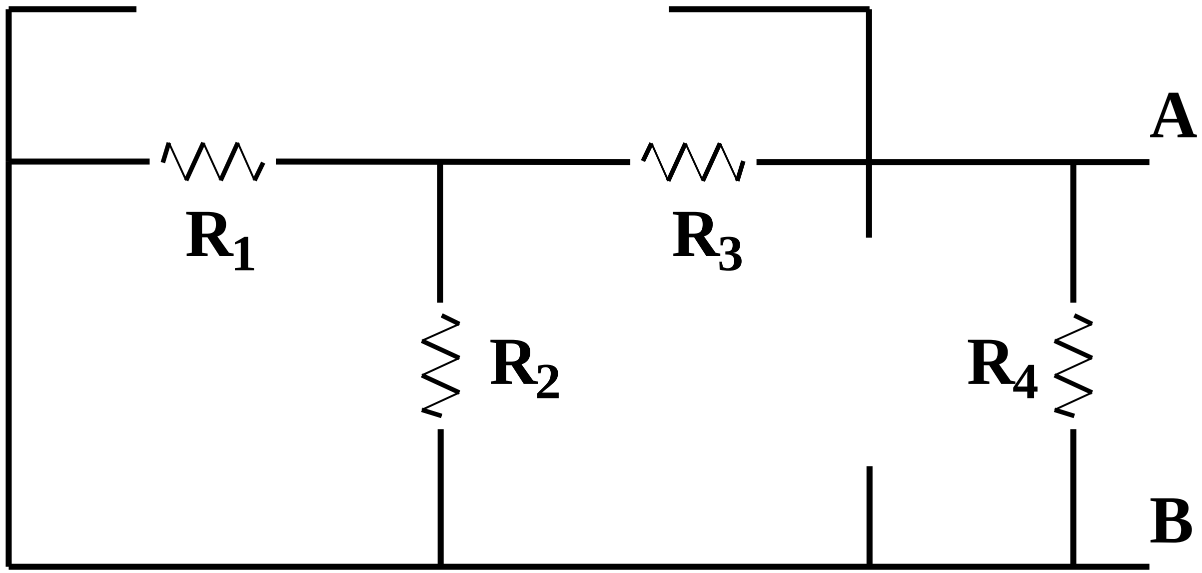
Figure : A typical circuit with independent sources

How do we deactivate the sources of this circuit?

THEVENIN & NORTON

THEVENIN'S THEOREM:

When the sources are deactivated the circuit appears as in Figure below.



Now place an ohmmeter across A-B and read the resistance.
If $R_1 = R_2 = R_4 = 20\ \Omega$ and $R_3 = 10\ \Omega$ then the meter reads $10\ \Omega$.

THEVENIN & NORTON

THEVENIN'S THEOREM:

We call the ohmmeter reading, under these conditions, $R_{THEVENIN}$ and shorten this to R_{TH}. Therefore, the important results are that we can replace Network 1 with the following network.

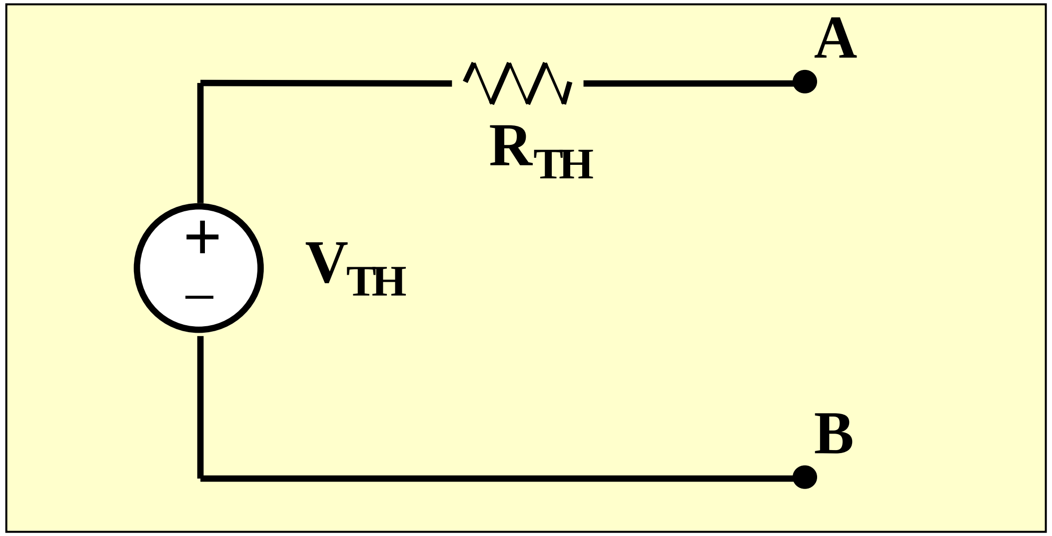
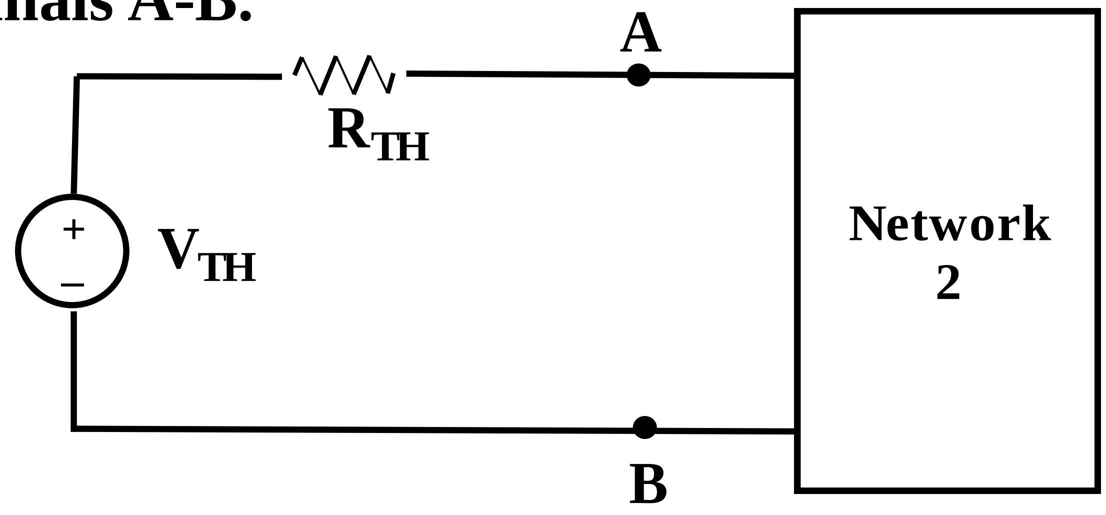


Figure : The Thevenin equivalent structure.

THEVENIN & NORTON

THEVENIN'S THEOREM:

We can now tie (reconnect) Network 2 back to terminals A-B.

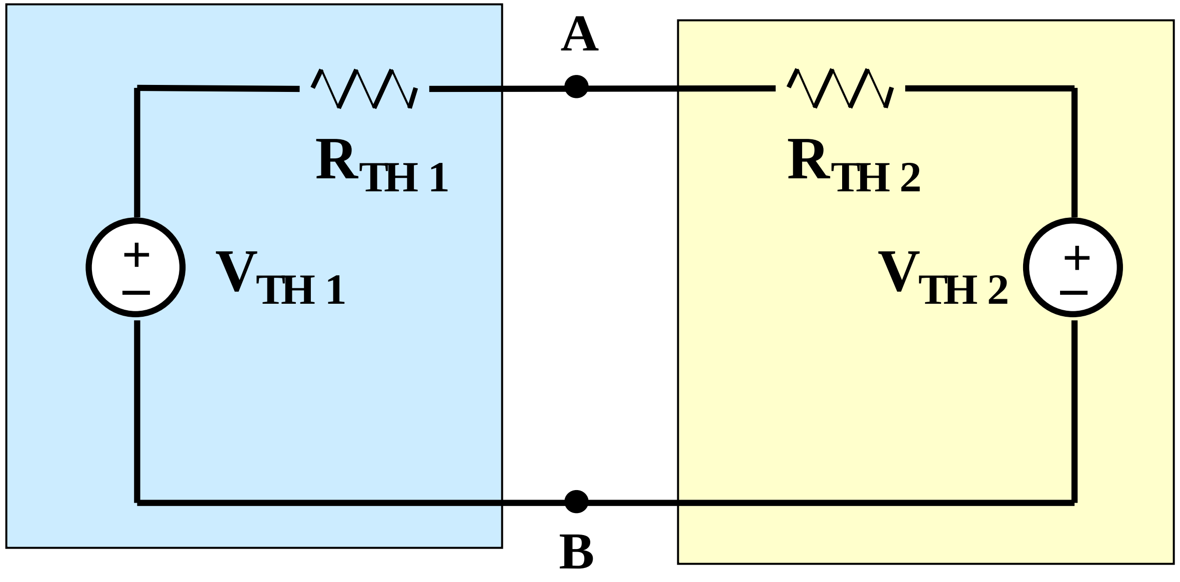


We can now make any calculations we desire within Network 2 and they will give the same results as if we still had Network 1 connected.

THEVENIN & NORTON

THEVENIN'S THEOREM:

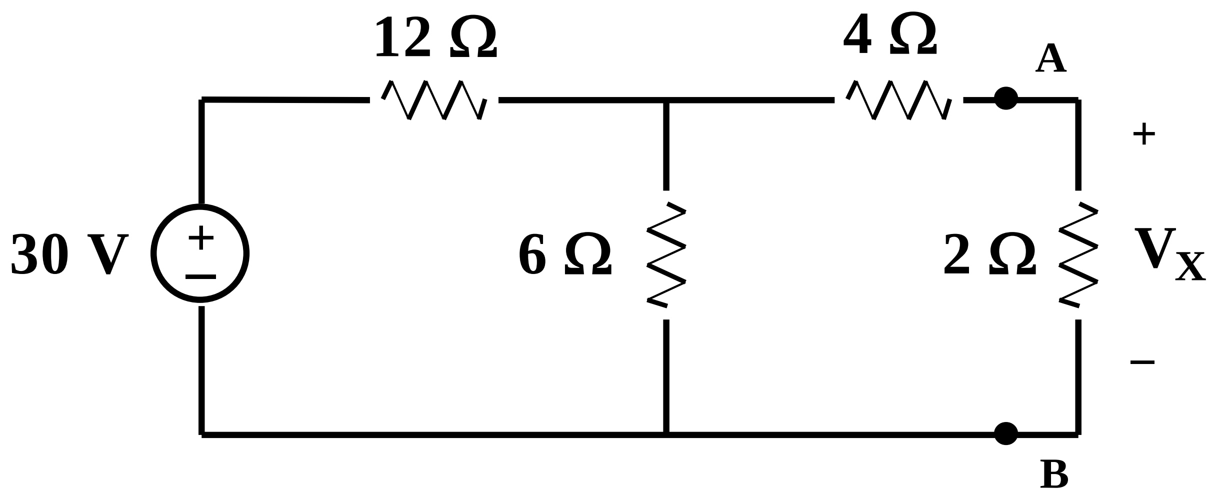
It follows that we could also replace Network 2 with a Thevenin voltage and Thevenin resistance. The results would be as shown in Figure below.



THEVENIN & NORTON

THEVENIN'S THEOREM: Example.

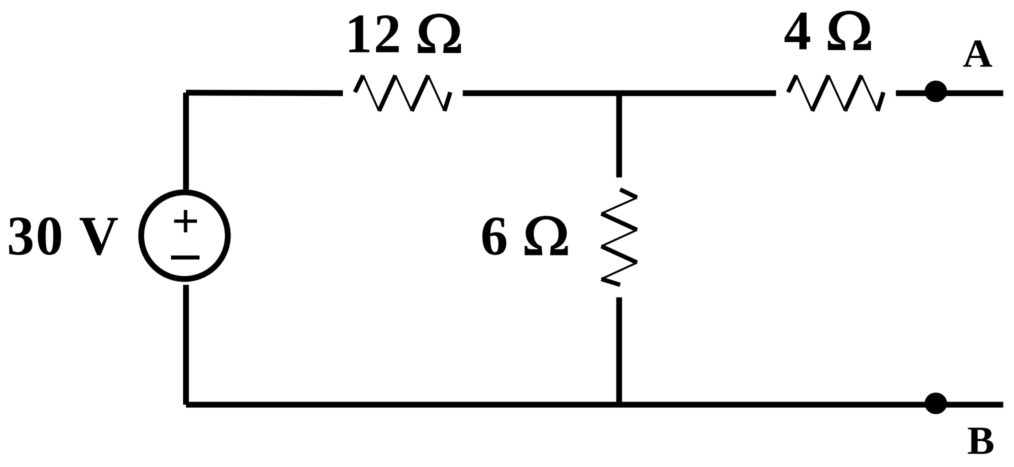
Find V_X by first finding V_{TH} and R_{TH} to the left of A-B.



First remove everything to the right of A-B.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued



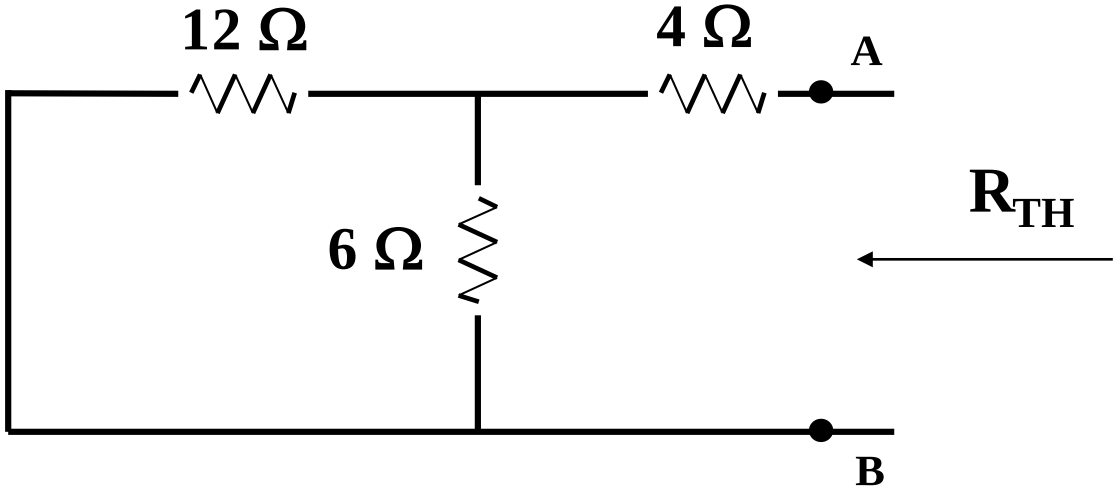
$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus there can be no voltage across the resistor.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.



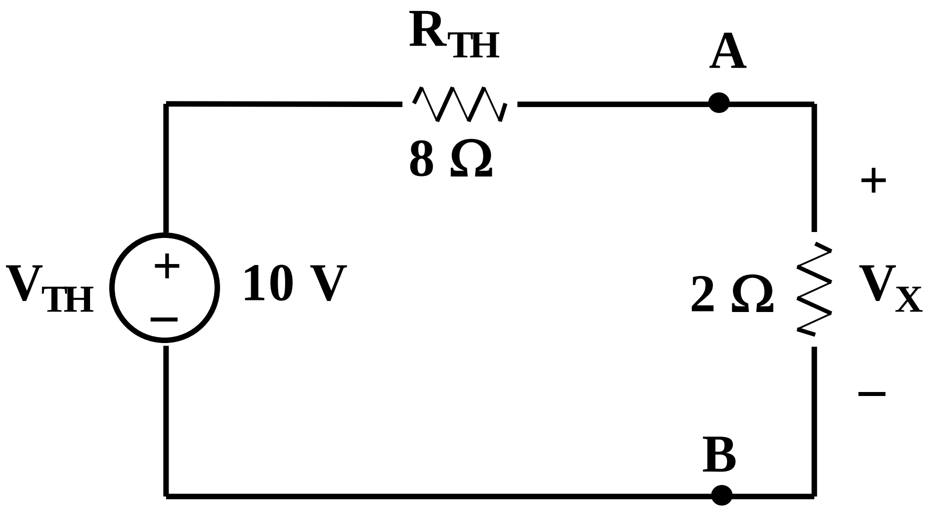
We see,

$$R_{TH} = 12\|6 + 4 = 8\ \Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

After having found the Thevenin circuit, we connect this to the load in order to find V_X.



$$V_X = \frac{(10)(2)}{2+8} = 2V$$

THEVENIN & NORTON

THEVENIN'S THEOREM:

In some cases it may become tedious to find R_{TH} by reducing the resistive network with the sources deactivated. Consider the following:

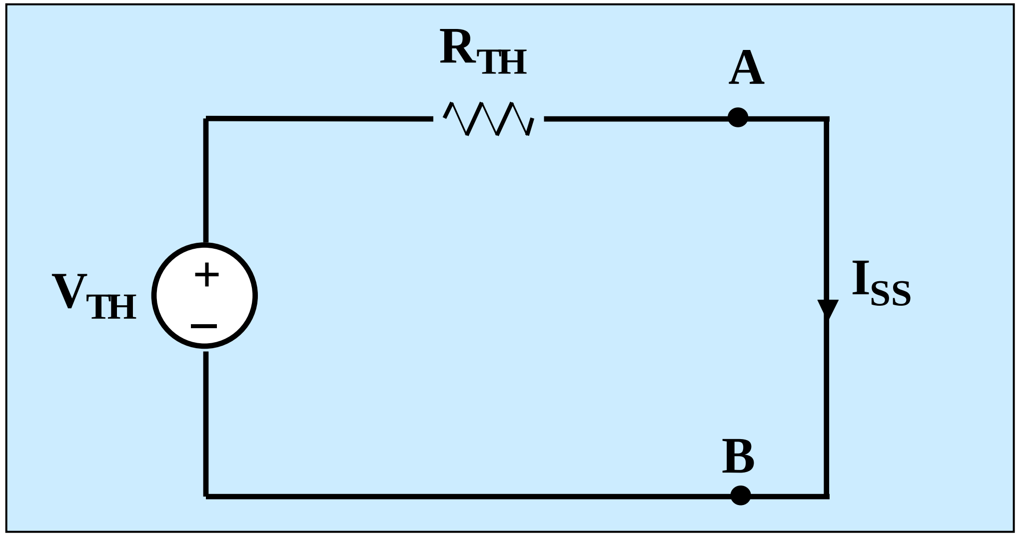


Figure : A Thevenin circuit with the output shorted.

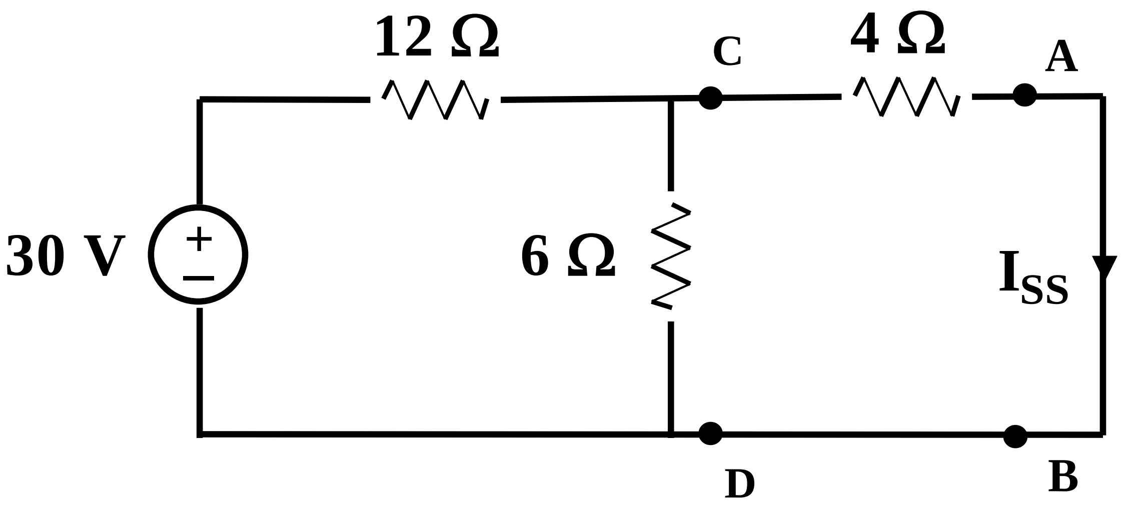
We see;

$$R_{TH} = \frac{V_{TH}}{I_{SS}}$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.2.

For the circuit in Figure, find R_{TH}.

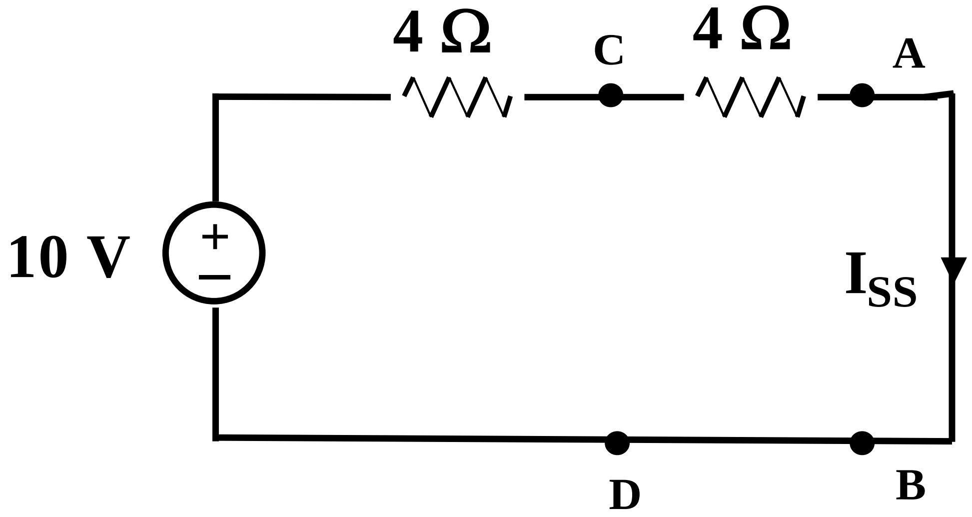


The task now is to find I_{SS}. One way to do this is to replace the circuit to the left of C-D with a Thevenin voltage and Thevenin resistance.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

Applying Thevenin's theorem to the left of terminals C-D and reconnecting to the load gives,



$$R_{TH} = \frac{V_{TH}}{I_{SS}} = \frac{10}{10/8} = 8\Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example

For the circuit below, find V_{AB} by first finding the Thevenin circuit to the left of terminals A-B.

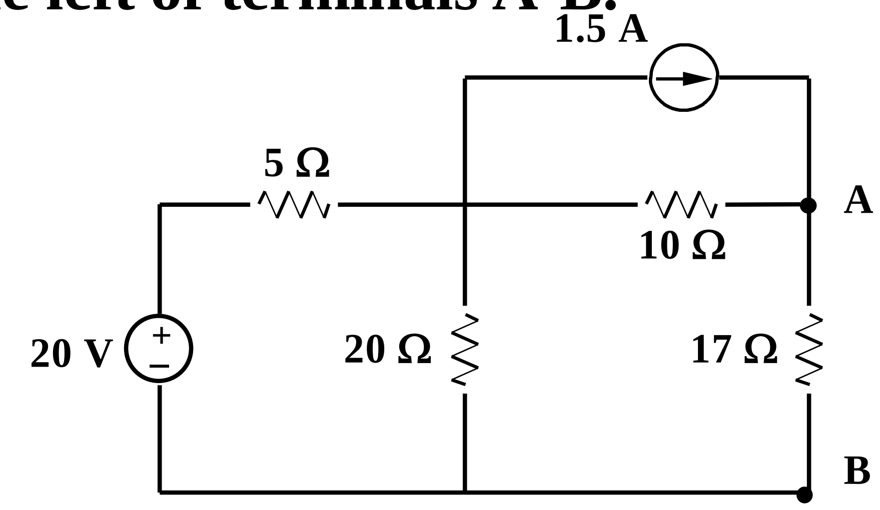


Figure : Circuit for Example .

We first find V_{TH} with the 17 Ω resistor removed.
Next we find R_{TH} by looking into terminals A-B with the sources deactivated.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

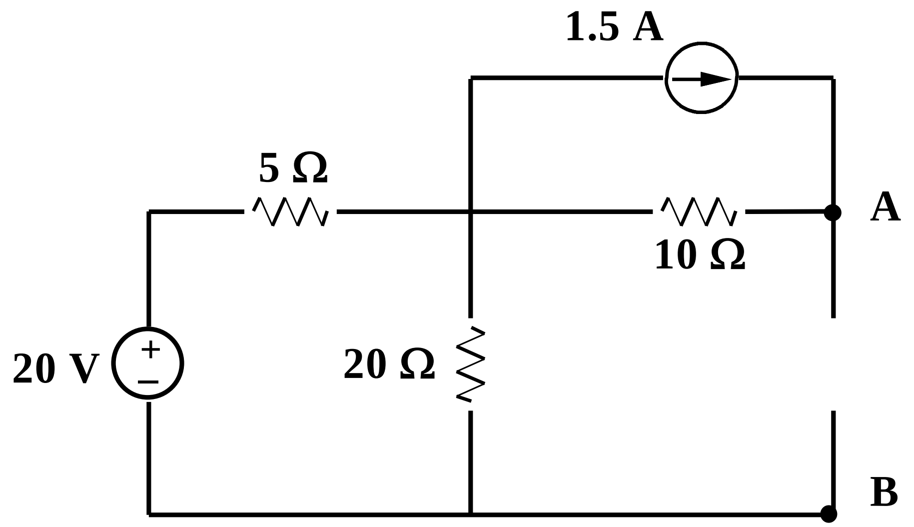


Figure : Circuit for finding V_{OS} for Example

$$V_{OS} = V_{AB} = V_{TH} = (1.5)(10) + \frac{20(20)}{(20+5)}$$

$$\therefore \quad V_{TH} = 31V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.3 continued

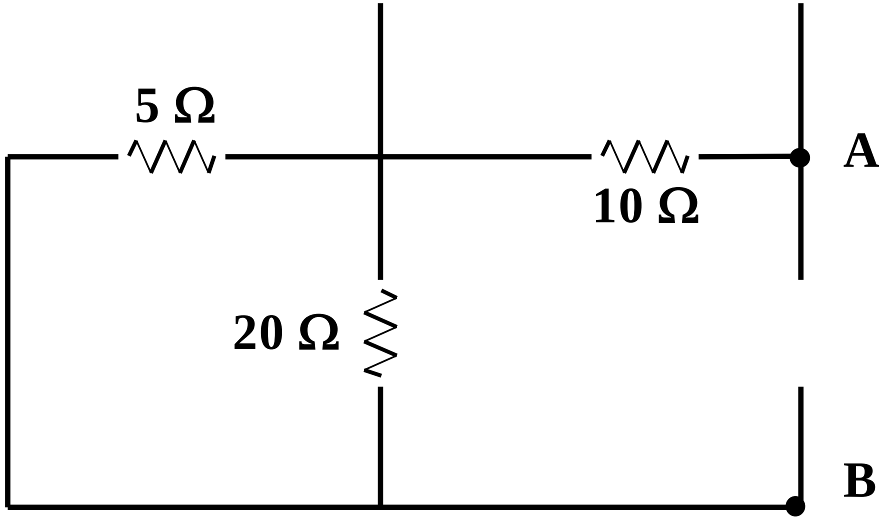


Figure : Circuit for find R_{TH} for Example.

$$R_{TH} = 10 + \frac{5(20)}{(5+20)} = 14\Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example continued

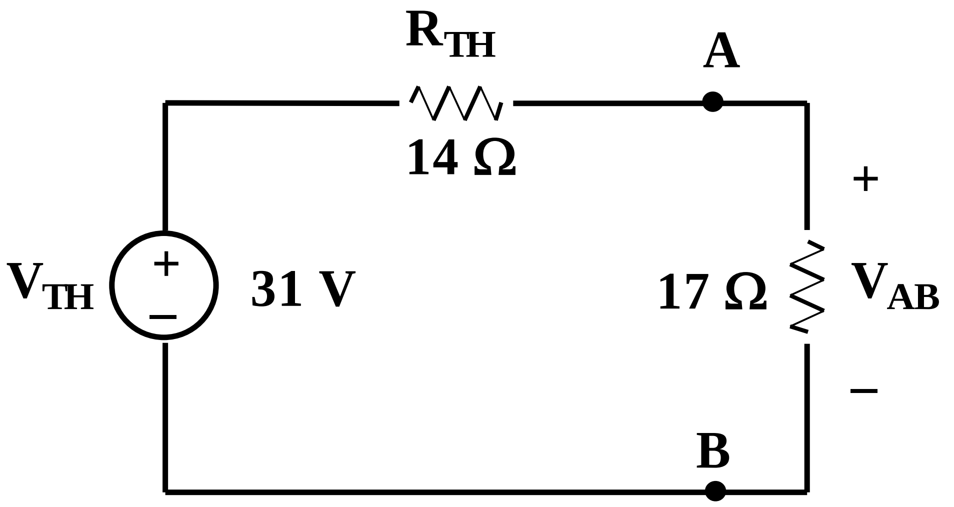


Figure : Thevenin reduced circuit for Example

We can easily find that,

$$V_{AB} = 17V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example : Working with a mix of independent and dependent sources.

Find the voltage across the 100 Ω load resistor by first finding the Thevenin circuit to the left of terminals A-B.

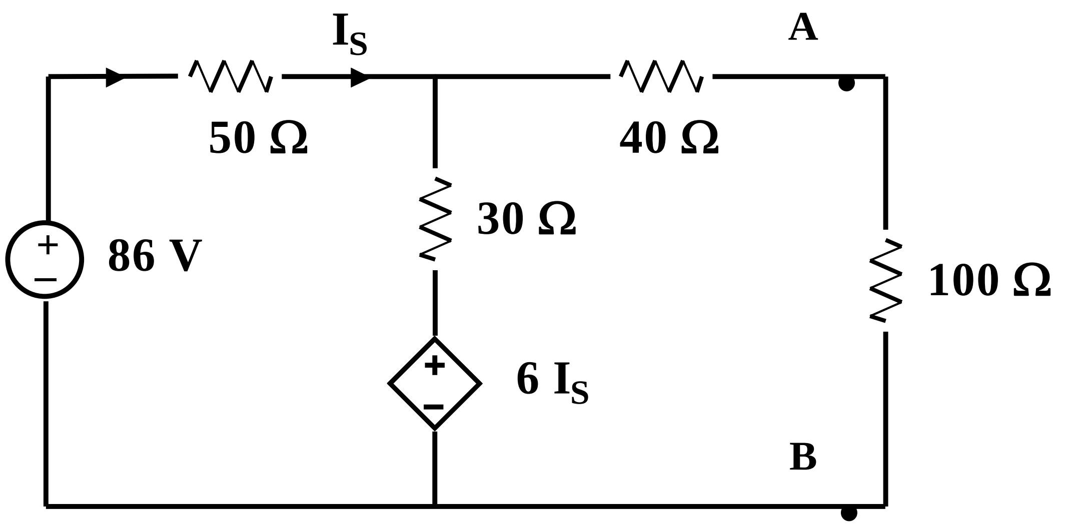


Figure : Circuit for Example

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.4: continued

First remove the 100 Ω load resistor and find $V_{AB} = V_{TH}$ to the left of terminals A-B.

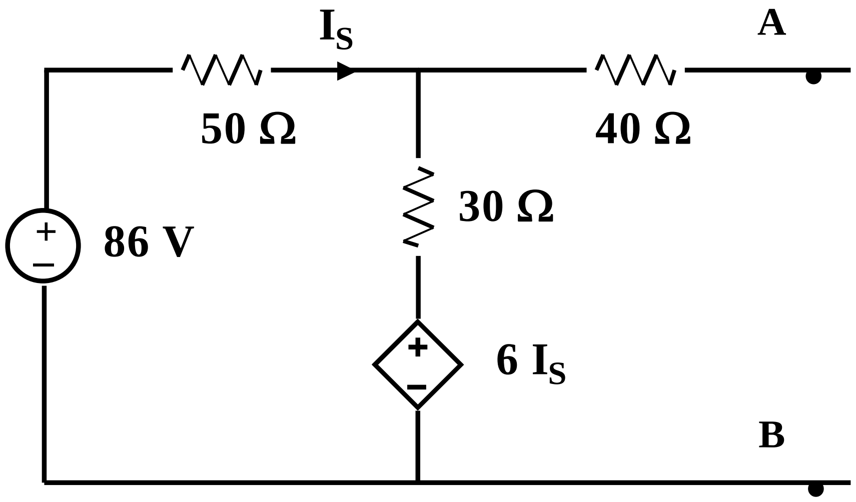


Figure 10.20: Circuit for find V_{TH}, Example 10.4.

$$-86 + 80I_S + 6I_S = 0 \;\rightarrow\; I_S = 1\,A$$

$$V_{AB} = 6I_S + 30I_S = \;\rightarrow\; 36V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example : continued

To find R_{TH} we deactivate all independent sources but retain all dependent sources as shown in Figure .

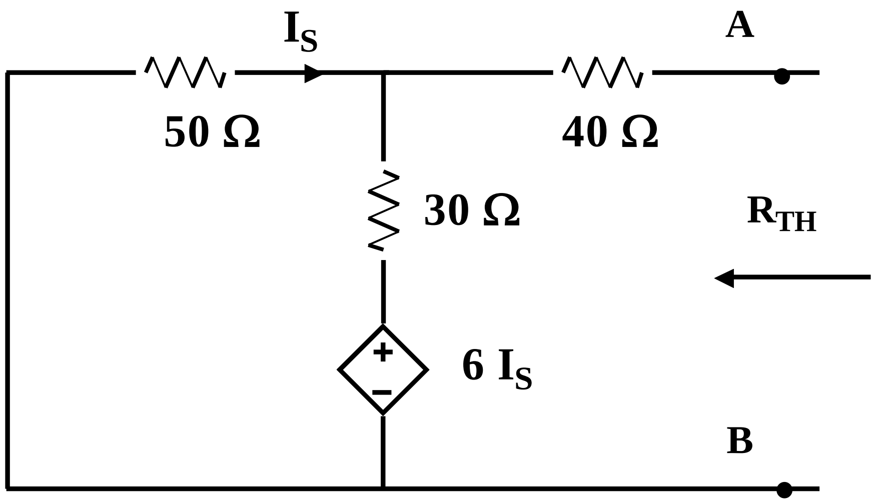


Figure : Example , independent sources deactivated.

We cannot find R_{TH} of the above circuit, as it stands. We must apply either a voltage or current source at the load and calculate the ratio of this voltage to current to find R_{TH}.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.4: continued

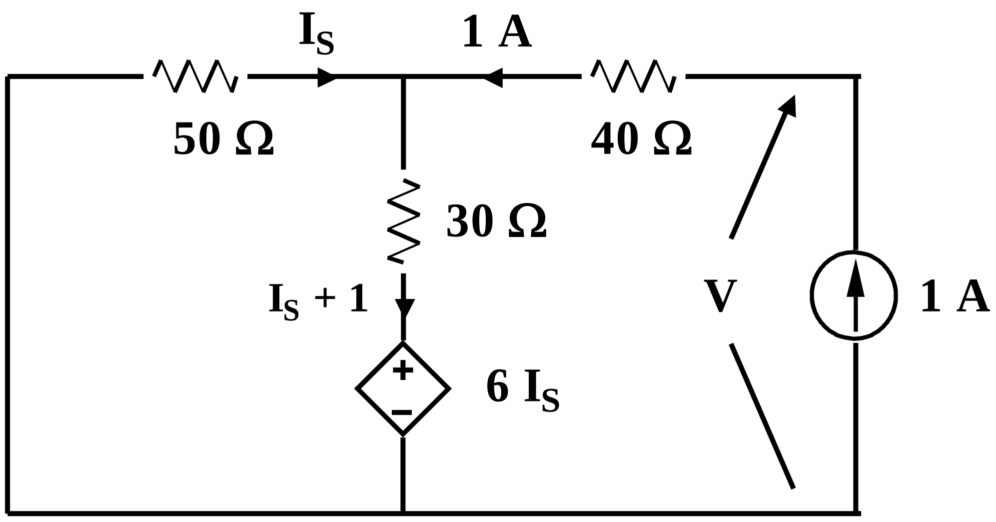


Figure 10.22: Circuit for find R_{TH}, Example 10.4.

Around the loop at the left we write the following equation:

$$50I_S + 30(I_S + 1) + 6I_S = 0$$

From which

$$I_S = \frac{-15}{43}\ A$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.4: continued

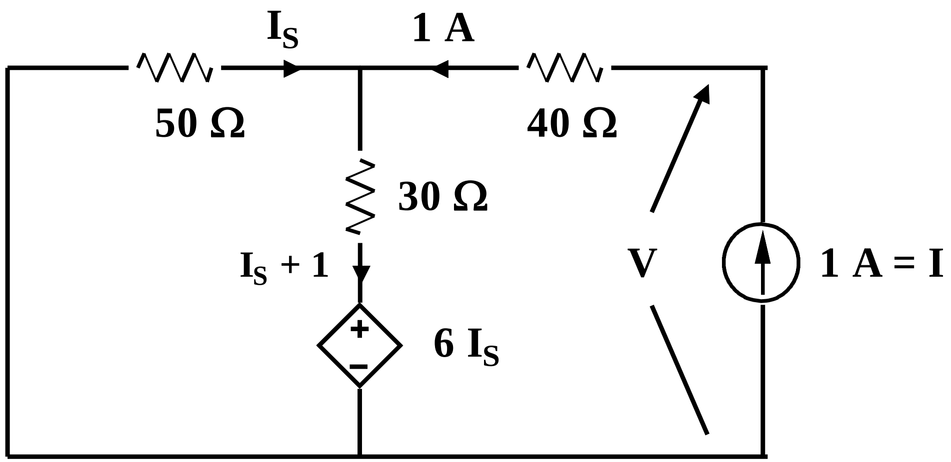


Figure 10.23: Circuit for find R_{TH}, Example 10.4.

Using the outer loop, going in the cw direction, using drops;

$$50\left(\frac{-15}{43}\right) - 1(40) + V = 0 \qquad \text{or} \qquad V = 57.4 \; volts$$

$$R_{TH} = \frac{V}{I} = \frac{V}{1} = 57.4\ \Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.4: continued

The Thevenin equivalent circuit tied to the 100 Ω load resistor is shown below.

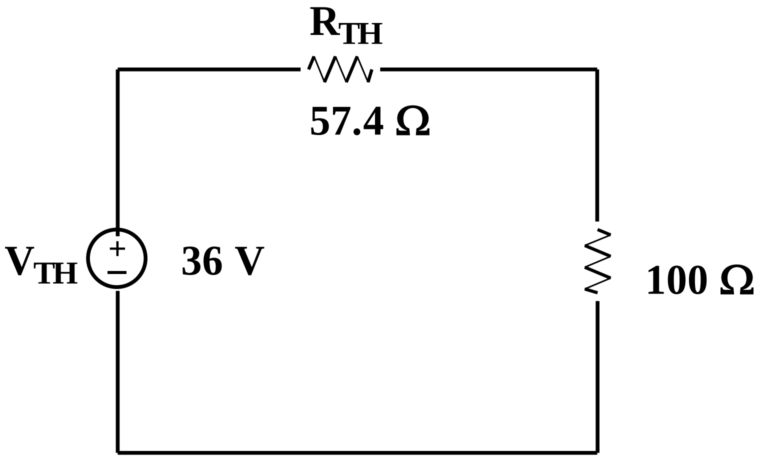


Figure 10.24: Thevenin circuit tied to load, Example 10.4.

$$V_{100} = \frac{36 x 100}{57.4 + 100} = 22.9\ V$$

THEVENIN & NORTON

THEVENIN'S THEOREM: **Example 10.5: Finding the Thevenin circuit when only resistors and dependent sources are present. Consider the circuit below. Find V_{xy} by first finding the Thevenin circuit to the left of x-y.**

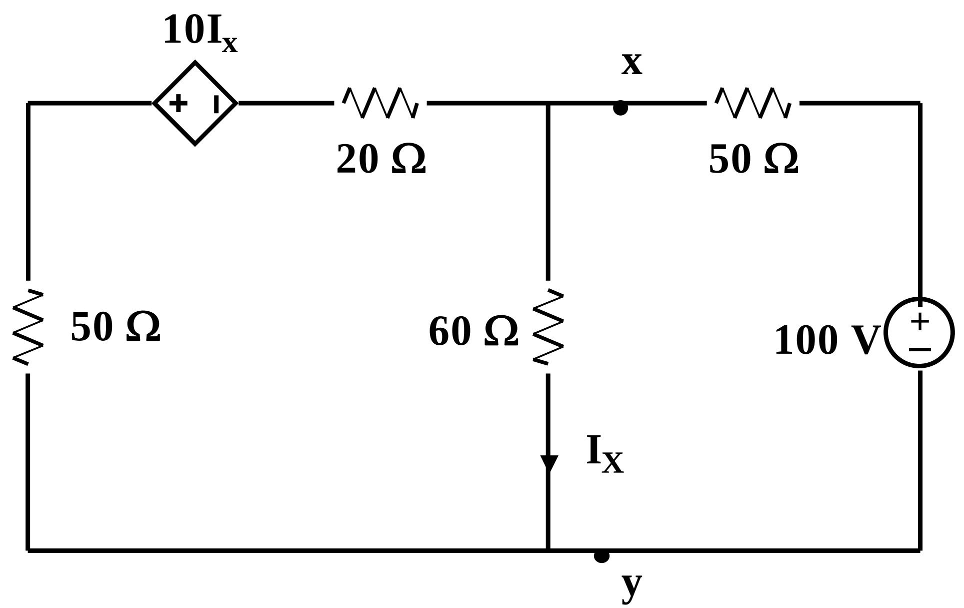


Figure 10.25: Circuit for Example 10.5.

For this circuit, it would probably be easier to use mesh or nodal analysis to find V_{xy}. However, the purpose is to illustrate Thevenin's theorem.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

We first reconcile that the Thevenin voltage for this circuit must be zero. There is no "juice" in the circuit so there cannot be any open circuit voltage except zero. This is always true when the circuit is made up of only dependent sources and resistors.

To find R_{TH} we apply a 1 A source and determine V for the circuit below.

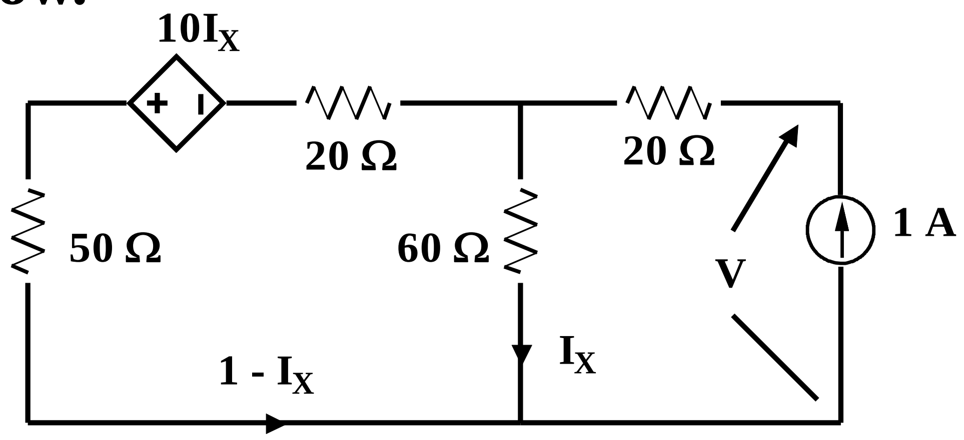


Figure 10.26: Circuit for find R_{TH}, Example 10.5.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

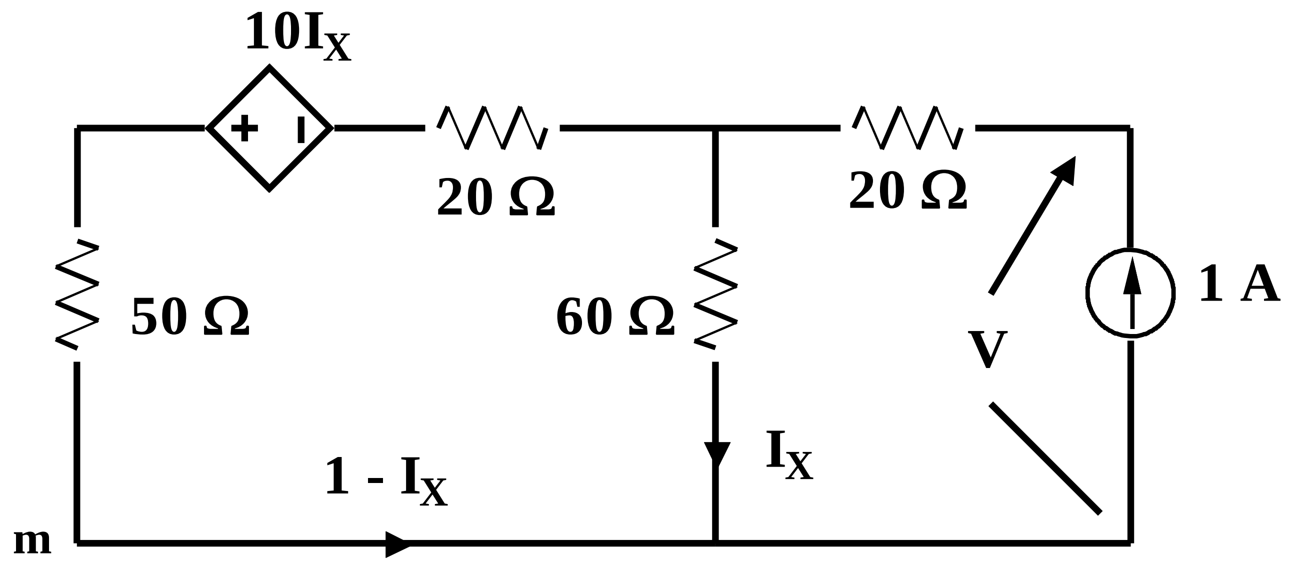


Figure 10.27: Circuit for find R_{TH}, Example 10.5.

Write KVL around the loop at the left, starting at "m", going cw, using drops:

$$-50(1-I_X)+10I_X-20(1-I_X)+60I_X = 0$$

$$I_X = 0.5\ A$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

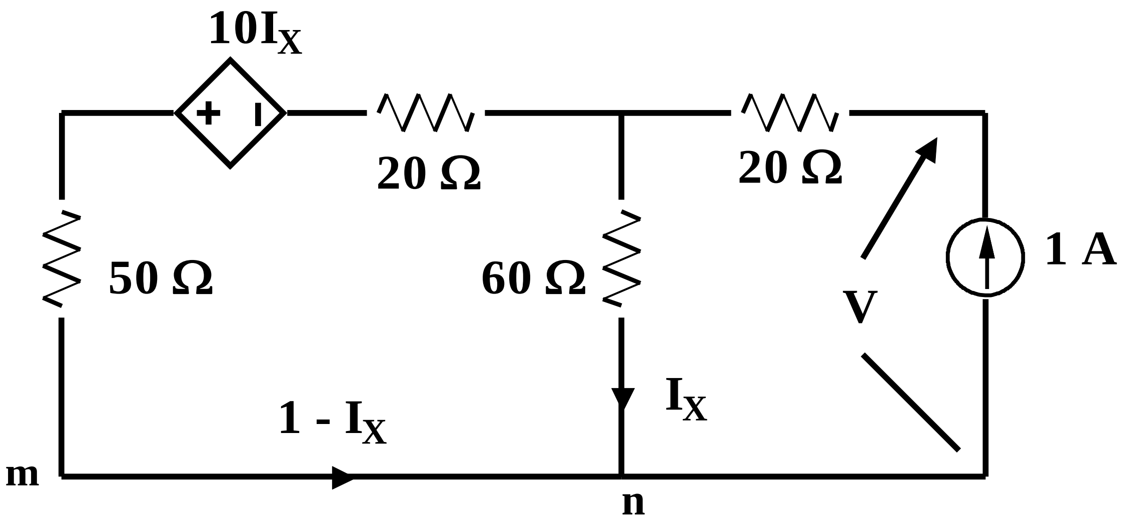


Figure 10.28: Determining R_{TH} for Example 10.5.

We write KVL for the loop to the right, starting at n, using drops and find;

$$-60(0.5) - 1x20 + V = 0$$

or

$$V = 50 \ volts$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.5: continued

We know that, $R_{TH} = \dfrac{V}{I}$, **where V = 50 and I = 1.**

Thus, R_{TH} = 50 Ω. The Thevenin circuit tied to the load is given below.

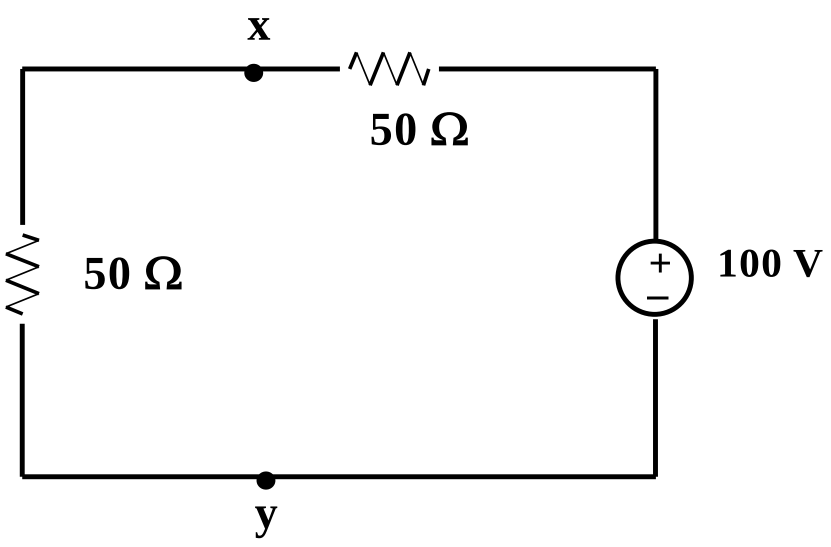


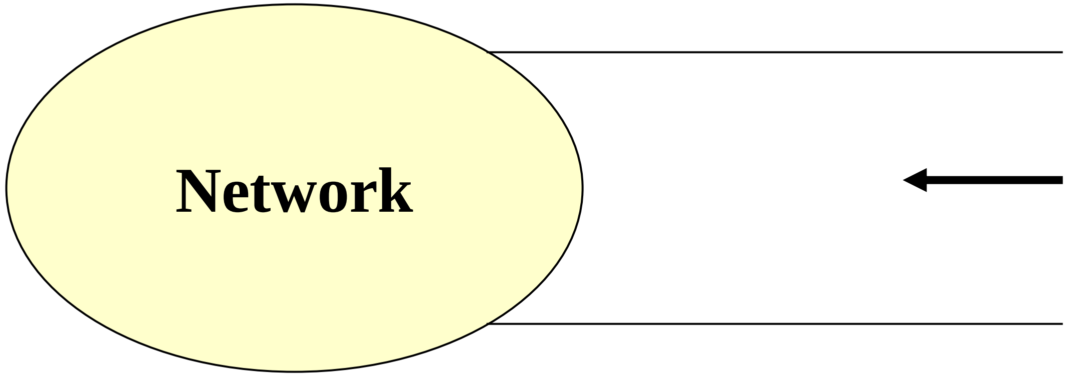
Figure 10.29: Thevenin circuit tied to the load, Example 10.5.

Obviously, V_{XY} = 50 V

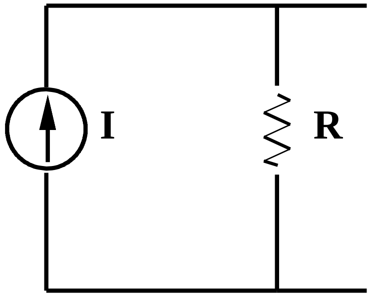
THEVENIN & NORTON

NORTON'S THEOREM:

Assume that the network enclosed below is composed of independent sources and resistors.



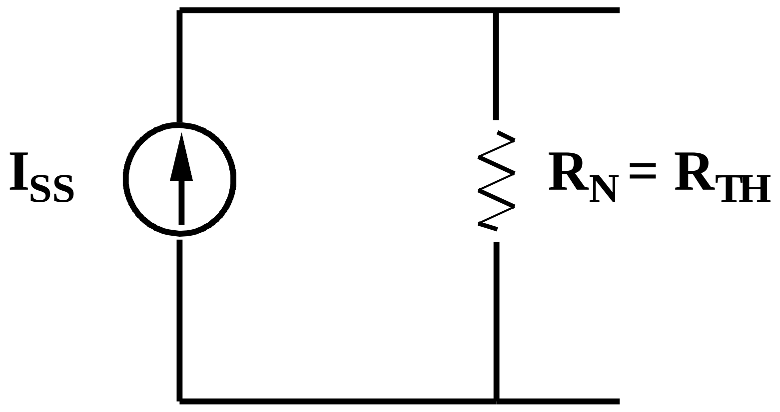
Norton's Theorem states that this network can be replaced by a current source shunted by a resistance R.



THEVENIN & NORTON

NORTON'S THEOREM:

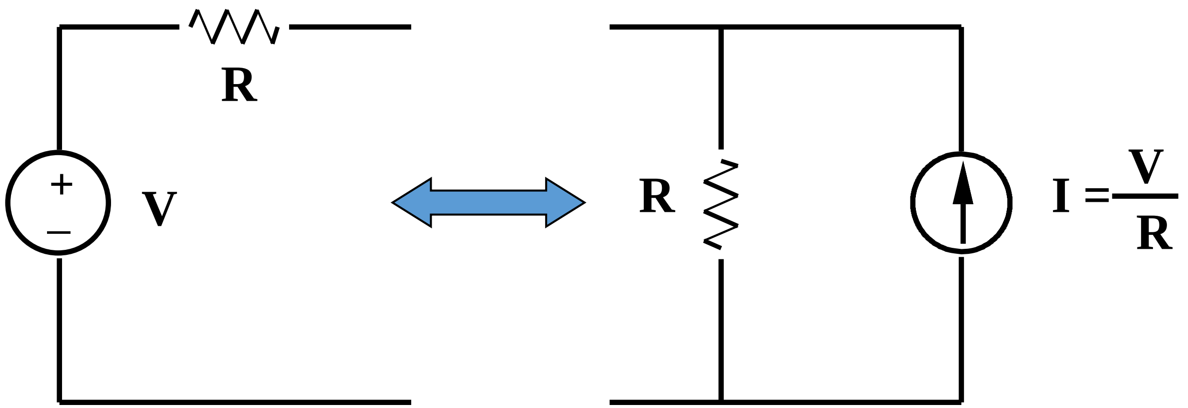
In the Norton circuit, the current source is the short circuit current of the network, that is, the current obtained by shorting the output of the network. The resistance is the resistance seen looking into the network with all sources deactivated. This is the same as R_{TH}.



THEVENIN & NORTON

NORTON'S THEOREM:

We recall the following from source transformations.



In view of the above, if we have the Thevenin equivalent circuit of a network, we can obtain the Norton equivalent by using source transformation.

However, this is not how we normally go about finding the Norton equivalent circuit.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6.

Find the Norton equivalent circuit to the left of terminals A-B for the network shown below. Connect the Norton equivalent circuit to the load and find the current in the 50 Ω resistor.

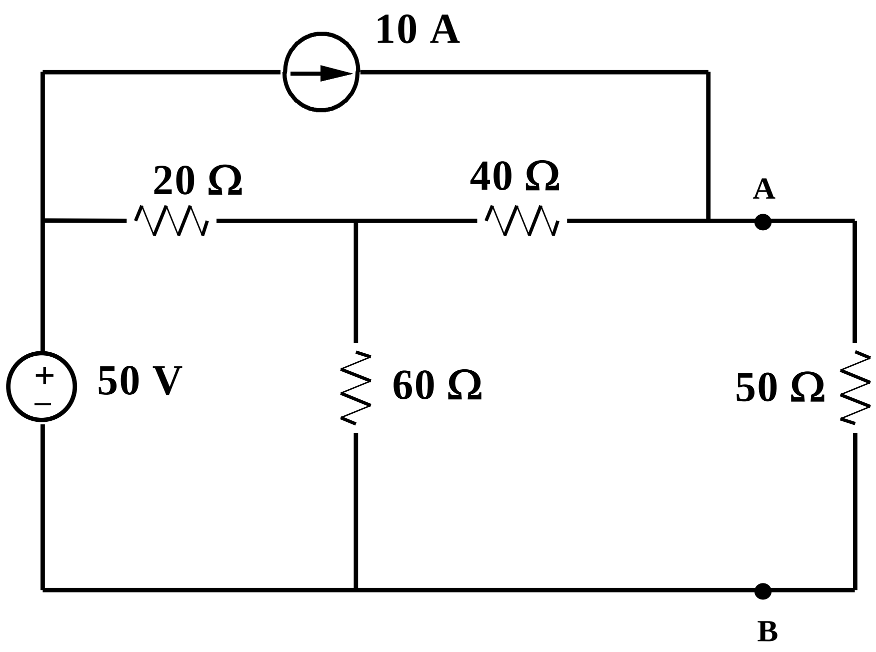


Figure 10.30: Circuit for Example 10.6.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6. continued

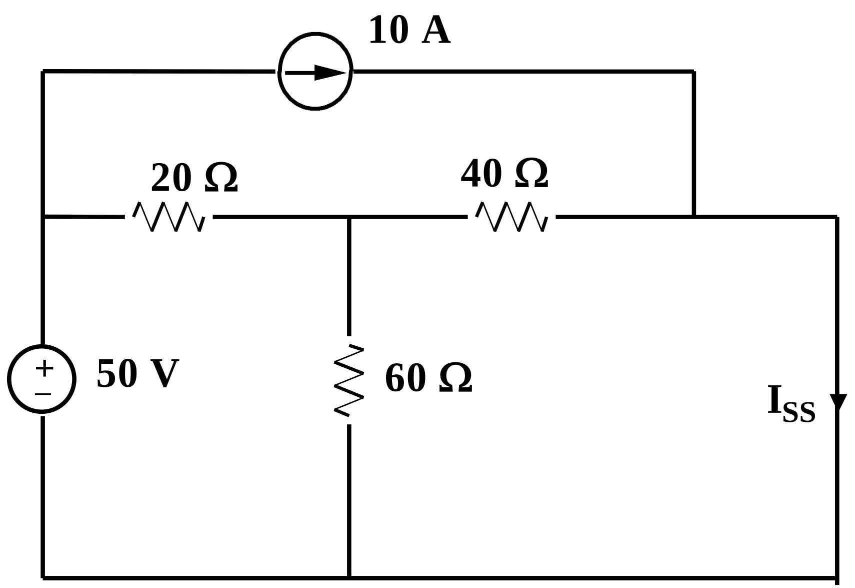


Figure 10.31: Circuit for find I_{NORTON}.

It can be shown by standard circuit analysis that

$$I_{SS} = 10.7\ A$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.6. continued

It can also be shown that by deactivating the sources,
We find the resistance looking into terminals A-B is

$$R_N = 55\ \Omega$$

R_N and R_{TH} will always be the same value for a given circuit.
The Norton equivalent circuit tied to the load is shown below.

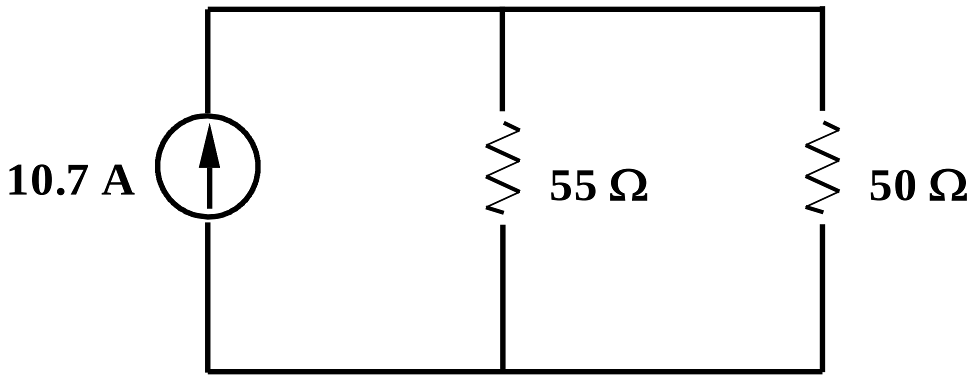


Figure 10.32: Final circuit for Example 10.6.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. **This example illustrates how one might use Norton's Theorem in electronics. the following circuit comes close to representing the model of a transistor.**

For the circuit shown below, find the Norton equivalent circuit to the left of terminals A-B.

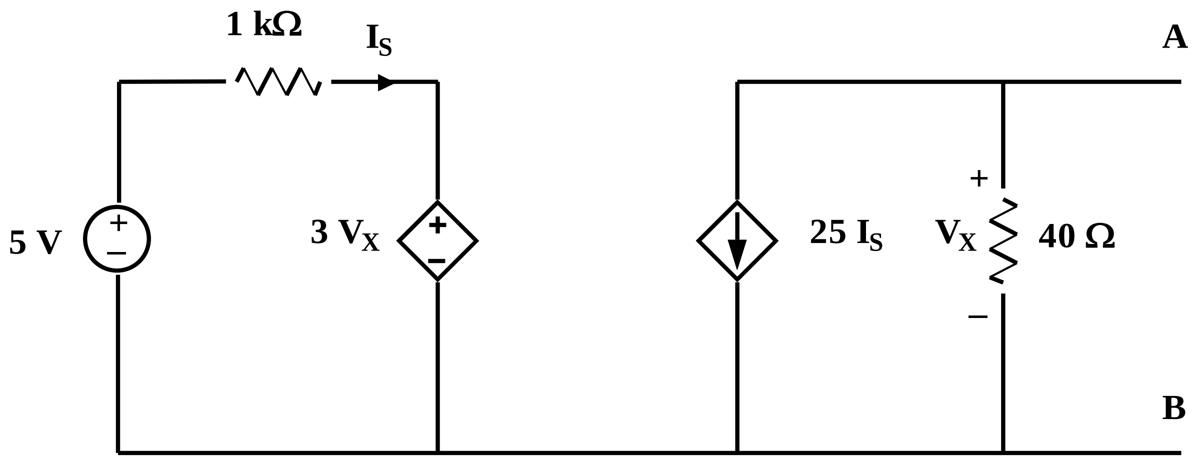
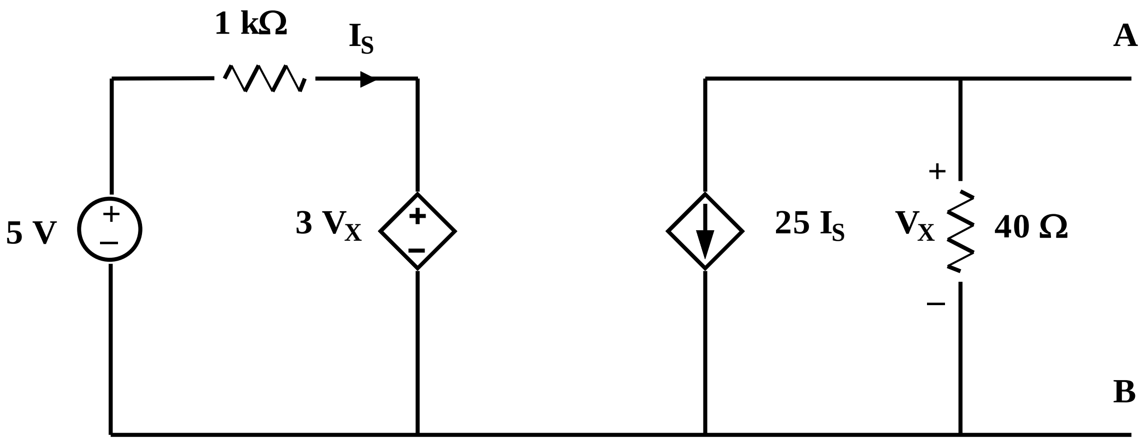


Figure 10.33: Circuit for Example 10.7.

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued



We first find;

$$R_N = \frac{V_{OS}}{I_{SS}}$$

We first find V_{OS}:

$$V_{OS} = V_X = (-25I_S)(40) = -1000I_S$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued

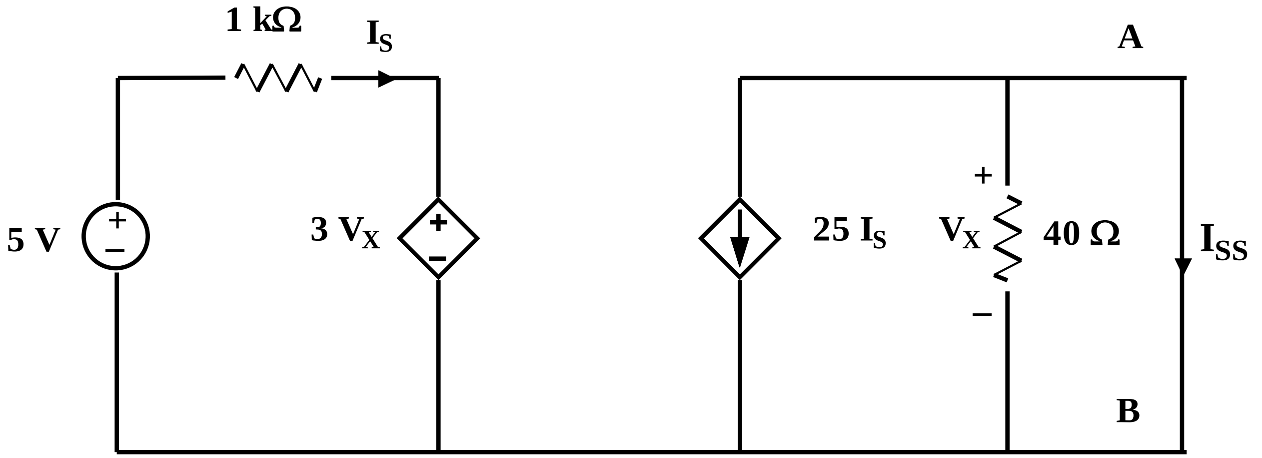


Figure 10.34: Circuit for find I_{SS}, Example 10.7.

We note that $I_{SS} = -25I_S$. Thus,

$$R_N = \frac{V_{OS}}{I_{SS}} = \frac{-1000 I_S}{-25 I_S} = 40\ \Omega$$

THEVENIN & NORTON

NORTON’S THEOREM: Example 10.7. continued

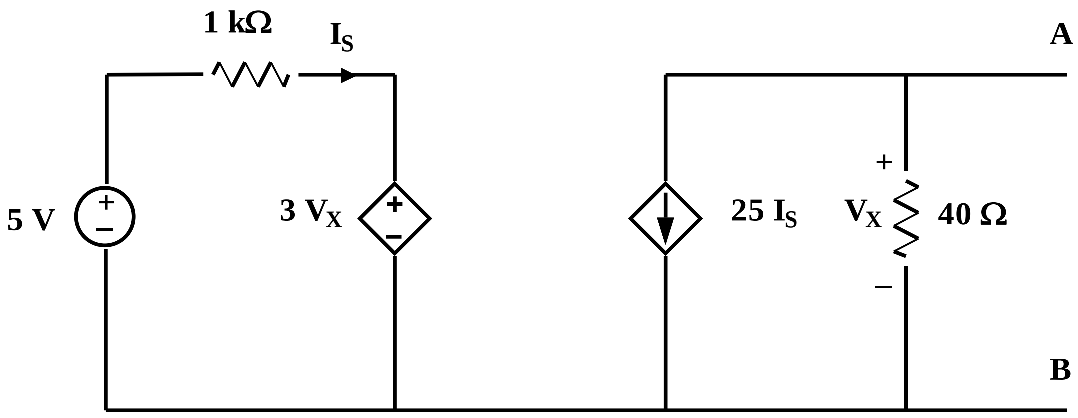


Figure 10.35: Circuit for find V_{OS}, Example 10.7.

From the mesh on the left we have;

$$-5 + 1000 I_S + 3(-1000 I_S) = 0$$

From which,

$$I_S = -2.5\ mA$$

THEVENIN & NORTON

NORTON'S THEOREM: Example 10.7. continued

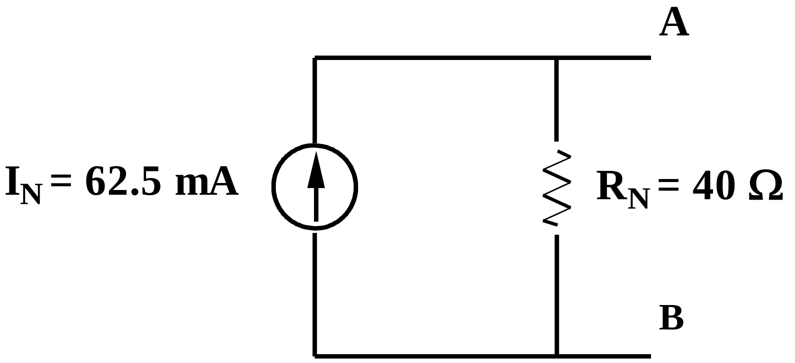
We saw earlier that,

$$I_{SS} = -25 I_S$$

Therefore;

$$I_{SS} = 62.5\ mA$$

The Norton equivalent circuit is shown below.



Norton Circuit for Example 10.7

THEVENIN & NORTON

Extension of Example 10.7:

Using source transformations we know that the Thevenin equivalent circuit is as follows:

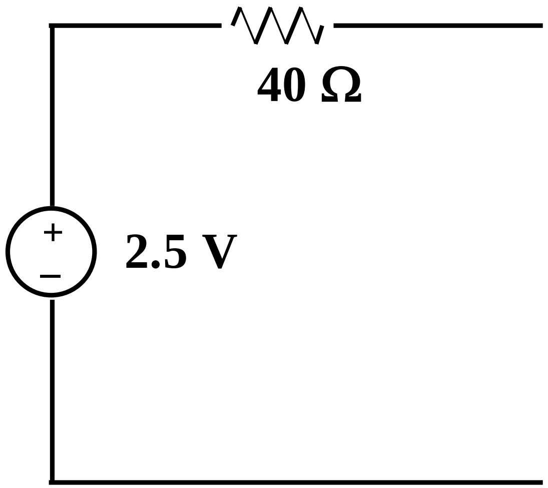


Figure 10.36: Thevenin equivalent for Example 10.7.