

Sample Quations for Final

1. For the functions $f(x) = x^2 + 3x^2 + 1$ and $g(x) = x^3$

- a) Determine whether $f(x) = O(g(x))$ or not.
- b) Determine whether $g(x) = O(f(x))$ or not.

2. A committee of 10 people (four women and six men) is to be seated around a circular table. In how many ways can the seats be assigned so that no two men are seated next to each other?

3. Find the number of permutations of the letters a b c d e ... x y z (26 letters) in which none of the patterns 'spin' or 'net' occurs.

4. Let R be the relation defined on $A = \mathbb{Z} \times \mathbb{Z}$ in the following way :

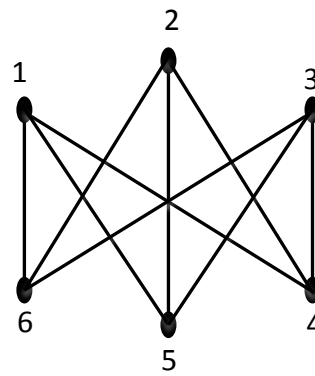
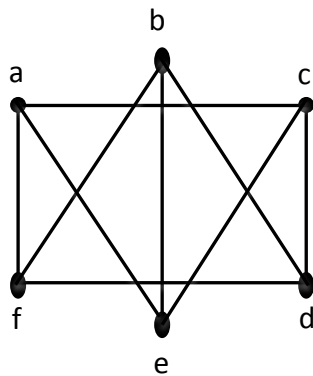
$$((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1 + y_1 = x_2 + y_2$$

Determine whether the relation R is an equivalence relation on A or not.

5. Let S be the set of all positive divisors of p^2q^2 where p and q are primes. Consider the poset (S, R) where R is the 'divides' relation, i.e. $\forall x, y \in S, (x, y) \in R$ if x divides y.

- a) Find a topological sorting for the elements of S.
- b) What is the greatest element of the poset if there is any ?
- c) What is the least element of the poset if there is any ?

6.



Determine whether two given graphs are isomorphic or not.

7. (20p) Find the postorder traversal of the tree T whose preorder traversal and inorder traversal are given as follows:

preorder : A B D G L H C E K F

inorder : B L G D H A E K C F

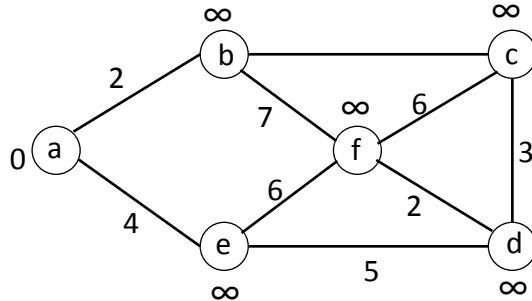
8. Give a big-O estimate for each of the following functions:

a) $(x^3 + x^2 \log x)(\log x + 5) + (15 \log x + 7)(x^3 + 3)$

b) $x \log(x^2 + 1) + x^2 \log x$

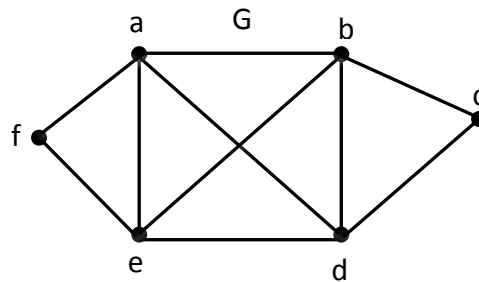
(For the function g of your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order)

9.



Assume you are running Dijkstra's algorithm to find the shortest path from a source to all other nodes. If the algorithm begins with the given graph, then what will be the view of the graph after three iterations?

10.



a) What is the Chromatic number $\chi(G)$ of G ?

b) What is $\kappa(G)$, the minimum number of vertices in a vertex cut of G ?

b) What is $\lambda(G)$, the minimum number of edges in an edge cut of G ?

11. For the given procedures below, find the asymptotic running time in terms of big-O notation.

PROCEDURE A

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for i = 1 to N
  j = i
  while (j < N)
    j = j + 3
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PROCEDURE B

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for i = 1 to N
  j = N
  while (i * i < j)
    j = j - 1
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