MODULE *Utilities*

EXTENDS *FiniteSetsExt*, *Sequences*, *SequencesExt*, *Functions*, *Bags*, *Relation*, *Naturals*, *Integers*

Utility helpers for Sets

$ChooseOne(S,\ P(\_)) \triangleq \text{CHOOSE } x \in S : P(x) \wedge \forall\, y \in S : P(y) \Rightarrow y = x$

$ChooseOrDefault(S,\ P(\_),\ D) \triangleq \text{IF } \exists\, s \in S : P(s)$
THEN
CHOOSE $s \in S : P(s)$
ELSE
D

$AnyOf(S) \triangleq \text{CHOOSE } s \in S : \text{TRUE}$

$Min(S) \triangleq \text{CHOOSE } min \in S : \forall\, other \in S : min \leq other$
$Max(S) \triangleq \text{CHOOSE } max \in S : \forall\, other \in S : max \geq other$

$Abs(x) \triangleq Max(\{x,\ -x\})$

Utility helpers for *Functions*

$DOM(f) \triangleq \text{DOMAIN } f$
$RAN(f) \triangleq \{f[x] : x \in \text{DOMAIN } f\}$

$RestrictRange(f,\ S) \triangleq$
LET
$DomainForRange \triangleq \{x \in \text{DOMAIN } f : f[x] \in S\}$
IN
$Restrict(f,\ DomainForRange)$

$RestrictRangeWithPredicate(f,\ P(\_)) \triangleq$
LET
$DomainForRange \triangleq \{x \in \text{DOMAIN } f : P(f[x])\}$
IN
$Restrict(f,\ DomainForRange)$

Utility helpers for *Sequences*

$Indexes(xs,\ x) \triangleq \{i \in \text{DOMAIN } xs : xs[i] = x\}$

$Index(xs,\ x) \triangleq$
IF
$IsInjective(xs)$ index non-sensical in non-injective sequences
THEN
CHOOSE $i \in 1\,..\,Len(xs) : xs[i] = x$
ELSE
CHOOSE $b \in \text{BOOLEAN} : b \notin \text{BOOLEAN}$

$$
\begin{array}{l}
\mathit{FirstIndex}(xs,\ x) \triangleq \\
\quad \text{IF} \\
\qquad \exists\, i \in 1\,..\,\mathit{Len}(xs) : xs[i] = x \quad \text{index non-sensical iff } x \text{ not in } xs \\
\quad \text{THEN} \\
\qquad \text{CHOOSE } i \in 1\,..\,\mathit{Len}(xs) : \\
\qquad\quad \land \quad xs[i] = x \\
\qquad\quad \land \quad \forall\, j \in 1\,..\,\mathit{Len}(xs) : xs[j] = x \Rightarrow j \geq i \\
\quad \text{ELSE} \\
\qquad \text{CHOOSE } b \in \text{BOOLEAN} \ : b \notin \text{BOOLEAN}
\end{array}
$$

$$
\begin{array}{l}
\mathit{LastIndex}(xs,\ x) \triangleq \\
\quad \text{IF} \\
\qquad \exists\, i \in 1\,..\,\mathit{Len}(xs) : xs[i] = x \quad \text{index non-sensical iff } x \text{ not in } xs \\
\quad \text{THEN} \\
\qquad \text{CHOOSE } i \in 1\,..\,\mathit{Len}(xs) : \\
\qquad\quad \land \quad xs[i] = x \\
\qquad\quad \land \quad \forall\, j \in 1\,..\,\mathit{Len}(xs) : xs[j] = x \Rightarrow j \leq i \\
\quad \text{ELSE} \\
\qquad \text{CHOOSE } b \in \text{BOOLEAN} \ : b \notin \text{BOOLEAN}
\end{array}
$$

$$
\begin{array}{l}
\mathit{Count}(xs,\ x) \triangleq \\
\quad \text{LET} \\
\qquad \mathit{Cnt}[i \in 1\,..\,\mathit{Len}(xs)] \triangleq \\
\qquad\quad \text{LET} \\
\qquad\qquad \mathit{Eq} \triangleq \text{IF } xs[i] = x \text{ THEN } 1 \text{ ELSE } 0 \\
\qquad\quad \text{IN} \\
\qquad\qquad \text{IF } i = 1 \text{ THEN } \mathit{Eq} \text{ ELSE } \mathit{Eq} + \mathit{Cnt}[i-1] \\
\quad \text{IN} \\
\qquad \text{IF } xs = \langle\rangle \text{ THEN } 0 \text{ ELSE } \mathit{Cnt}[\mathit{Len}(xs)]
\end{array}
$$

$$
\mathit{SumSeq}(xs) \triangleq \mathit{ReduceSeq}(\text{LAMBDA } x,\ acc : acc + x,\ xs,\ 0)
$$

$$
\begin{array}{l}
\mathit{ReduceSeqPairs}(op(\_,\ \_),\ xs,\ acc) \triangleq \\
\quad \text{LET } \mathit{ReduceSeqPairs}[i \in 1\,..\,\mathit{Len}(xs)] \triangleq \\
\qquad \text{IF } i = 2 \\
\qquad\ \text{THEN } op(\langle xs[i-1],\ xs[i]\rangle,\ acc) \\
\qquad\ \text{ELSE } op(\langle xs[i-1],\ xs[i]\rangle,\ \mathit{ReduceSeqPairs}[i-2]) \\
\quad \text{IN} \\
\qquad \text{IF } xs = \langle\rangle \lor \mathit{Len}(xs)\%2 \neq 0 \text{ THEN } acc \text{ ELSE } \mathit{ReduceSeqPairs}[\mathit{Len}(xs)]
\end{array}
$$

see https://*learntla.com*/tla/functions/

$$
\begin{array}{l}
\mathit{PermutationKey}(n) \triangleq \{key \in [1\,..\,n \to 1\,..\,n] : \mathit{Range}(key) = 1\,..\,n\} \\
\mathit{PermutationsOf}(T) \triangleq \{[x \ \in 1\,..\,\mathit{Len}(T) \mapsto T[P[x]]] : P \in \mathit{PermutationKey}(\mathit{Len}(T))\}
\end{array}
$$

Utility helpers for *Bags*

$$SeqToBag(xs) \triangleq [x \in RAN(xs) \mapsto Count(xs, x)]$$

Utility helpers for Relations

$$Is2Acyclic(R, S) \triangleq \forall x \in S : \neg(\exists y \in S : x \neq y \wedge R[x, y] \wedge R[y, x])$$