Three Triangles Integer Factorization Algorithm

Abstract:

Factor the composite number $C=(a+1)(a+b)$ by finding the solution to $C=T(a)+T(a+b)-T(b-1)$ where $T(n)$ generates the nth triangular number.

Definitions:

- a, b := natural numbers
- T(n) := n(n+1)/2
- S(a, b) := T(a)+T(a+b)-T(b-1)
- make_even(n) := if n is odd return n-1 else return n

Theorem:

For every composite number $C=(a+1)(a+b)$ there exist three triangular numbers such that $C=T(a)+T(a+b)-T(b-1)$.

Proof:

1. $(a+1)(a+b)=T(a)+T(a+b)-T(b-1)$
2. $a^2+ab+a+b=a(a+1)/2+(a+b)(a+b+1)/2-(b-1)b/2$
3. $2a^2+2ab+2a+2b=a(a+1)+(a+b)(a+b+1)-(b-1)b$
4. $2a^2+2ab+2a+2b=a^2+a+a(a+b+1)+b(a+b+1)-b^2+b$
5. $2a^2+2ab+2a+2b=a^2+a+a^2+ab+a+ba+b^2+b-b^2+b$
6. $2a^2+2ab+2a+2b=2a^2+2ab+2a+2b$
7. $0=0$

Theorem:

For every natural number $a >= 2$ and $b > 1$ it holds that $S(a, b) > S(a-2, b+2)$.

Proof:

1. $S(a, b) > S(a-2, b+2)$
2. $(a+1)(a+b) > (a+1-2)(a-2+b+2)$
3. $a^2+a+b > (a-1)(a+b)$
4. $a^2+a+b > a^2-(a+b)$
5. $a+b > -(a+b)$
6. $1 > -1$($a+b > 0$ since $a >= 2$ and $b > 1$)

Theorem:

For every natural number $a, b > 0$ it holds that $S(a, b) < S(a, b+max(1, ceil((C-S(a, b))/(a+1))))$.

Proof:

1. $S(a, b) < S(a, b+max(1, ceil((C-S(a, b))/(a+1))))$
2. $(a+1)(a+b) < (a+1)(a+b+max(1, ceil((C-S(a, b))/(a+1))))$
3. $a+b < a+b+max(1, ceil((C-S(a, b))/(a+1)))$
4. $0 < max(1, ceil((C - S(a, b))/(a+1)))$
5. $0 < 1$

Algorithm:

input: C => integer greater than 2
output: found factors, or 1 and C if C is a prime number

1. let a = make_even(floor(sqrt(C)) - 1)
2. let b = 1
3. if $S(a, b) > C$ then $a = a - 2, b = b + 2$
4. if $S(a, b) < C$ then $b = b + max(1, ceil((C - S(a, b)) / (a + 1)))$
5. if $S(a, b) == C$ then exit: found factors $(a + 1)$ and $(a + b)$
6. if $a == 0$ then exit: C is a prime number

7. goto step 3.

Examples

Factor 51

1. a = 6, b = 1
   S(6, 1) = 49
   => b = b + max(1, ceil(2 / 7))
2. a = 6, b = 2
   S(6, 2) = 56
   => a = a - 2, b = b + 2
3. a = 4, b = 4
   S(4, 4) = 40
   => b = b + max(1, ceil(11 / 5))
4. a = 4, b = 7
   S(4, 7) = 55
   => a = a - 2, b = b + 2
5. a = 2, b = 9
   S(2, 9) = 33
   => b = b + max(1, ceil(18 / 3))
6. a = 2, b = 15
   S(2, 15) = 51
   => 51 = 3 * 17

Factor 23

1. a = 2, b = 1
   S(2, 1) = 9
   => b = b + max(1, ceil(14 / 3))
2. a = 2, b = 6
   S(2, 6) = 24
   => a = a - 2, b = b + 2
3. a = 0, b = 8
   => 23 = 1 * 23

Conclusion

To my knowledge this is not based on any existing solutions. I do not claim it to be efficient or useful, I'm only concerned with its correctness and completeness. One potential benefit I see is the reduced magnitude of dividends used in divisions. Hope it inspires some ideas.