

# Three Triangles Integer Factorization Algorithm

"Discovery consists of seeing what everybody has seen and thinking what nobody has thought."

— Albert Szent-Györgyi

## Abstract:

Factor the composite number  $C=(a+1)(a+b)$  by finding the solution to  $C=T(a)+T(a+b)-T(b-1)$  where  $T(n)$  is the nth triangular number.

## Definitions:

- $T(n) := n(n+1)/2$
- $S(a, b) := T(a)+T(a+b)-T(b-1)$

## Theorem:

For every composite number  $C=(a+1)(a+b)$  there exist three triangular numbers such that  $C=T(a)+T(a+b)-T(b-1)$ .

## Proof:

1.  $(a+1)(a+b)=T(a)+T(a+b)-T(b-1)$
2.  $a^2+ab+a+b=a(a+1)/2+(a+b)(a+b+1)/2-(b-1)b/2$
3.  $2a^2+2ab+2a+2b=a(a+1)+(a+b)(a+b+1)-(b-1)b$
4.  $2a^2+2ab+2a+2b=a^2+a+a(a+b+1)+b(a+b+1)-b^2+b$
5.  $2a^2+2ab+2a+2b=a^2+a+a^2+ab+a+ba+b^2+b-b^2+b$
6.  $2a^2+2ab+2a+2b=2a^2+2ab+2a+2b$
7.  $0=0$

## Theorem:

For natural numbers  $a, b$  it holds that  $S(a, b) > S(a-1, b+1)$ .

## Proof:

1.  $S(a, b) > S(a-1, b+1)$
2.  $(a+1)(a+b) > (a+1-1)(a-1+b+1)$
3.  $a(a+b)+a+b > a(a+b)$
4.  $a+b > 0$

## Theorem:

For natural number  $a, b$  it holds that  $S(a, b) < S(a, b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$ .

## Proof:

1.  $S(a, b) < S(a, b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
2.  $(a+1)(a+b) < (a+1)(a+b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
3.  $a+b < a+b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
4.  $0 < \max(1, \text{ceil}((C-S(a, b))/(a+1))))$
5.  $0 < 1$

## Algorithm:

*input:* natural number C

*output:* found factors, or 1 and C if C is a prime number

1. let  $a = \text{floor}(\sqrt{C}) - 1$
2. let  $b = 1$
3. if  $S(a, b) > C$  then  $a=a-1, b=b+1$
4. else if  $S(a, b) < C$  then  $b=b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
5. else if  $S(a, b) = C$  then exit: found factors  $(a+1)$  and  $(a+b)$
6. if  $a = 0$  then exit: C is a prime number
7. go to step 3

## Examples

### 51

1.  $S(6, 1) = 49 \Rightarrow b = b + \max(1, \text{ceil}(2/7))$
2.  $S(6, 2) = 56 \Rightarrow a = a - 1, b = b + 1$
3.  $S(5, 3) = 48 \Rightarrow b = b + \max(1, \text{ceil}(3/6))$
4.  $S(5, 4) = 54 \Rightarrow a = a - 1, b = b + 1$
5.  $S(4, 5) = 45 \Rightarrow b = b + \max(1, \text{ceil}(6/5))$
6.  $S(4, 7) = 55 \Rightarrow a = a - 1, b = b + 1$
7.  $S(3, 8) = 44 \Rightarrow b = b + \max(1, \text{ceil}(7/4))$
8.  $S(3, 10) = 52 \Rightarrow a = a - 1, b = b + 1$
9.  $S(2, 11) = 39 \Rightarrow b = b + \max(1, \text{ceil}(12/3))$
10.  $S(2, 15) = 51 \Rightarrow 51 = 3 * 17$

### 23

1.  $S(3, 1) = 16 \Rightarrow b = b + \max(1, \text{ceil}(7/4))$
2.  $S(3, 3) = 24 \Rightarrow a = a - 1, b = b + 1$
3.  $S(2, 4) = 18 \Rightarrow b = b + \max(1, \text{ceil}(5/3))$
4.  $S(2, 6) = 24 \Rightarrow a = a - 1, b = b + 1$
5.  $S(1, 7) = 16 \Rightarrow b = b + \max(1, \text{ceil}(7/2))$
6.  $S(1, 11) = 24 \Rightarrow a = a - 1, b = b + 1$
7.  $a = 0 \Rightarrow 23 = 1 * 23$

### 221

1.  $S(13, 1) = 196 \Rightarrow b = b + \max(1, \text{ceil}(25/14))$
2.  $S(13, 3) = 224 \Rightarrow a = a - 1, b = b + 1$
3.  $S(12, 4) = 208 \Rightarrow b = b + \max(1, \text{ceil}(13/13))$
4.  $S(12, 5) = 221 \Rightarrow 221 = 13 * 17$

### 36

1.  $S(5, 1) = 36 \Rightarrow 36 = 6 * 6$

## Conclusion

To my knowledge this is not based on any existing solutions. I do not claim it to be efficient or useful, I'm only concerned with its correctness and completeness. Some optimizations have been omitted for simplicity. Hope it inspires some ideas.