

Three Triangles Integer Factorization Algorithm

"Discovery consists of seeing what everybody has seen and thinking what nobody has thought."

— Albert Szent-Györgyi

Abstract:

Factor the natural number $C=(a+1)(a+b)$ by finding the solution to $C=T(a)+T(a+b)-T(b-1)$ where $T(n)$ is the nth triangular number.

Definitions:

- $T(n) := n(n+1)/2$
- $S(a, b) := T(a)+T(a+b)-T(b-1)$

Theorem:

For every natural number $C=(a+1)(a+b)$ there exist three triangular numbers such that $C=T(a)+T(a+b)-T(b-1)$.

Proof:

1. $(a+1)(a+b)=T(a)+T(a+b)-T(b-1)$
2. $a^2+ab+a+b=a(a+1)/2+(a+b)(a+b+1)/2-(b-1)b/2$
3. $2a^2+2ab+2a+2b=a(a+1)+(a+b)(a+b+1)-(b-1)b$
4. $2a^2+2ab+2a+2b=a^2+a+a(a+b+1)+b(a+b+1)-b^2+b$
5. $2a^2+2ab+2a+2b=a^2+a+a^2+ab+a+ba+b^2+b-b^2+b$
6. $2a^2+2ab+2a+2b=2a^2+2ab+2a+2b$
7. $0=0$

Theorem:

For all natural numbers a, b it holds that $S(a, b) > S(a-1, b+1)$.

Proof:

1. $S(a, b) > S(a-1, b+1)$
2. $(a+1)(a+b) > (a+1-1)(a-1+b+1)$
3. $a(a+b)+a+b > a(a+b)$
4. $a+b > 0$

Theorem:

For all natural numbers a, b it holds that $S(a, b) < S(a, b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$.

Proof:

1. $S(a, b) < S(a, b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
2. $(a+1)(a+b) < (a+1)(a+b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
3. $a+b < a+b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
4. $0 < \max(1, \text{ceil}((C-S(a, b))/(a+1))))$
5. $0 < 1$

Algorithm:

input: natural number C

output: found factors

1. let $a = \text{floor}(\sqrt{C}) - 1$
2. let $b = 1$
3. if $S(a, b) > C$ then $a=a-1, b=b+1$
4. else if $S(a, b) < C$ then $b=b+\max(1, \text{ceil}((C-S(a, b))/(a+1))))$
5. else if $S(a, b) = C$ then exit: found factors $(a+1)$ and $(a+b)$
6. go to step 3

Examples

51

1. $S(6, 1) = 49 \Rightarrow b = b + \max(1, \text{ceil}(2/7))$
2. $S(6, 2) = 56 \Rightarrow a = a - 1, b = b + 1$
3. $S(5, 3) = 48 \Rightarrow b = b + \max(1, \text{ceil}(3/6))$
4. $S(5, 4) = 54 \Rightarrow a = a - 1, b = b + 1$
5. $S(4, 5) = 45 \Rightarrow b = b + \max(1, \text{ceil}(6/5))$
6. $S(4, 7) = 55 \Rightarrow a = a - 1, b = b + 1$
7. $S(3, 8) = 44 \Rightarrow b = b + \max(1, \text{ceil}(7/4))$
8. $S(3, 10) = 52 \Rightarrow a = a - 1, b = b + 1$
9. $S(2, 11) = 39 \Rightarrow b = b + \max(1, \text{ceil}(12/3))$
10. $S(2, 15) = 51 \Rightarrow 51 = 3 * 17$

23

1. $S(3, 1) = 16 \Rightarrow b = b + \max(1, \text{ceil}(7/4))$
2. $S(3, 3) = 24 \Rightarrow a = a - 1, b = b + 1$
3. $S(2, 4) = 18 \Rightarrow b = b + \max(1, \text{ceil}(5/3))$
4. $S(2, 6) = 24 \Rightarrow a = a - 1, b = b + 1$
5. $S(1, 7) = 16 \Rightarrow b = b + \max(1, \text{ceil}(7/2))$
6. $S(1, 11) = 24 \Rightarrow a = a - 1, b = b + 1$
7. $S(0, 12) = 12 \Rightarrow b = b + \max(1, \text{ceil}(11/1))$
8. $S(0, 23) = 23 \Rightarrow 23 = 1 * 23$

221

1. $S(13, 1) = 196 \Rightarrow b = b + \max(1, \text{ceil}(25/14))$
2. $S(13, 3) = 224 \Rightarrow a = a - 1, b = b + 1$
3. $S(12, 4) = 208 \Rightarrow b = b + \max(1, \text{ceil}(13/13))$
4. $S(12, 5) = 221 \Rightarrow 221 = 13 * 17$

36

1. $S(5, 1) = 36 \Rightarrow 36 = 6 * 6$

Conclusion

To my knowledge this is not based on any existing solutions. I do not claim it to be efficient or useful, only correct and complete. I hope it inspires more discovery.