Maximum Visual Efficiency of Colored Materials

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Tristimulus values have been computed for hypothetical spectrophotometric curves of the type found to give the maximum visual reflectance factor (or transmission factor) for specified chromaticities. These computations have been based on the I.C.I. 1931 data for the normal observer for colorimetry, and on the I.C.I. Illuminants "A" and "C." By plotting the results on the I.C.I. color mixture diagram, the loci of points characterized by equal maximum efficiencies have been established. Tables have been prepared showing the maximum visual efficiency as a function of excitation purity for twenty-four dominant wave-lengths.

ONE of the most compelling objectives of pigment and dye chemists has been to synthesize materials which could be used to produce colors of ever greater color purity without the sacrifice of brightness. The steady progress in this endeavor has led industrial users of colored materials to expect almost limitless improvements in these directions. This paper gives data which indicate that such improvements are possible in many directions, that not more than a few percent of the color possibilities have been realized to date. It also reveals that a physical limit has already been closely approached in some pigments and dyes, and that marked improvement is not to be expected in colors lying in these regions. These data will be most useful in indicating promising regions for research, and in preventing unreasonable demands for unattainable colors. In some cases, after specifying the dominant wave-length and purity desired in conjunction with a certain light source, designers of filters have requested visual transmission factors in excess of physically possible values. In other cases, opaque colored materials having visual reflection factors closely approaching the attainable limit for their chromaticity, have been criticized because these reflection factors were small compared to the unit reflection factor characteristic of a perfect white. Such criticism, and such unreasonable specifications are as unsound as criticism of the seemingly low efficiencies of well-designed heat engines operating between assigned temperatures that can be realized in practice. The purpose of a previous article[1] was to determine the characteristics of the most efficient colored materials. That work was analogous to the introduction of the concept of the perfect heat engine operating on the Carnot cycle as a standard of comparison against which engineers may make reasonable evaluations of the efficiencies of real heat engines. This paper presents data against which color technologists may make reasonable comparisons of the visual efficiencies of colored materials. It is analogous to the thermodynamic formula for the efficiency of a Carnot engine. However, the complicated behavior of this maximum attainable visual efficiency for various chromaticities makes it necessary to describe it by the use of tables and diagrams rather than by a simple formula such as occurs in heat engine theory.

The previous paper established the fact that the maximum attainable brightness corresponding to any desired chromaticity of a colored material under a given illuminant is secured if the material has a rather simple type of spectrophotometric curve. One condition for maximum brightness is that the reflection factor (or transmission factor) be either zero or unity at every wave-length of the visible spectrum. As a second condition, there must be no more than two transitions between zero and unity within the limits of the visible spectrum. The two classes of curves permitted are illustrated by the full lines in Figs. 1a and 1b. The wave-lengths of the transitions are determined by the quality of the illuminant and by the chromaticity desired. The brightness of a sample having such a spectrophotometric curve can be compared with the brightness of a perfect reflector under the same illumination by computation with the aid of standard visibility data and of data on the

[1] D. L. MacAdam, *Theory of the Maximum Visual Efficiency of Colored Materials*, J. O. S. A. 25, 249–252 (1935).

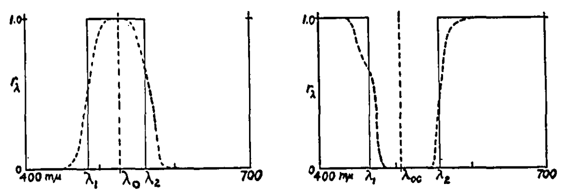
FORM A. *Sample Calculation for Illuminant A.*
R = Visual efficiency = 0.60 (assigned).

	Column 1		Column 2	Column 3	
$\lambda_1=485.5$ (assigned)	$\sum_{\lambda_1}^{\lambda_2} yE = Y$ $= R\sum_{380}^{780}\bar{y}E = 6473.73$ $\sum_{380}^{\lambda_1}\bar{y}E = (+)158.00$ $\sum_{380}^{\lambda_2}\bar{y}E =$ $\sum_{380}^{\lambda_1}\bar{y}E+\sum_{\lambda_1}^{\lambda_2}\bar{y}E = 6631.73$	$\lambda_2=583.87$ (interpolated from $\sum_{380}^{\lambda}\bar{y}E$ table)	$\sum_{380}^{\lambda_2}\bar{z}E = 3833.99$ $\sum_{380}^{\lambda_1}\bar{z}E = (-)3202.09$ $\sum_{\lambda_1}^{\lambda_2}\bar{z}E = Z = 631.90$	$\sum_{380}^{\lambda_2}\bar{x}E = 3756.45$ $\sum_{380}^{\lambda_1}\bar{x}E = (-)560.22$ $\sum_{\lambda_1}^{\lambda_2}\bar{x}E = X = 3196.23$ $Y = 6473.73$ $Z = 631.90$ $S = X+Y+Z = 10301.86$	$x = X/S = 0.3103$ $y = Y/S = 0.6284$

FORM B. *Sample Calculation for Illuminant A.*
R = 0.70.

$\lambda_1=485.5$	$\sum_{\lambda_1}^{\lambda_2}\bar{y}E =$ $(1-R)\sum_{380}^{780}\bar{y}E = 3236.86$ $\sum_{380}^{\lambda_1}\bar{y}E = (+)158.00$ $\sum_{380}^{\lambda_2}\bar{y}E = 3394.86$ $Y = R\sum_{380}^{780}\bar{y}E = 7552.67$	$\lambda_2=551.43$	$\sum_{380}^{780}\bar{z}E = 3837.57$ $\sum_{380}^{\lambda_2}\bar{z}E = (-)3823.93$ $\sum_{\lambda_2}^{780}\bar{z}E = 13.64$ $\sum_{380}^{\lambda_1}\bar{z}E = (+)3202.09$ $\sum_{380}^{\lambda_1}\bar{z}E+\sum_{\lambda_2}^{380}\bar{z}E = 3215.73$ $= Z$	$\sum_{380}^{780}\bar{x}E = 11851.77$ $\sum_{380}^{\lambda_2}\bar{x}E = (-)1259.05$ $\sum_{\lambda_2}^{780}\bar{x}E = 10592.72$ $\sum_{380}^{\lambda_1}\bar{x}E = (+)560.22$ $X = 11152.94$ $Y = 7552.67$ $Z = 3215.73$ $S = X+Y+Z = 21921.34$	$x = X/S = 0.5088$ $\bar{y} = Y/S = 0.3445$

spectral distribution of energy in the illumination. This ratio of brightnesses has been called visual efficiency in the earlier communication. This term permits the simultaneous consideration of transparent and opaque reflecting materials and is used to represent interchangeably the concepts of visual transmission factors and of visual reflection factors.



a b

FIG. 1a. Spectrophotometric curves for two green samples having dominant wave-length about 525 mμ and visual efficiency about 0.50. Box shaped curve, full line, has maximum attainable purity.

FIG. 1b. Spectrophotometric curves for two purple samples complementary to 505 mμ and having visual efficiency about 0.50.

Calculations intended for a general survey of the maximum visual efficiencies for all chromaticities might be planned in several ways since any two of five quantities may be considered as independent variables determining the values of the other three. For convenience of computation the visual efficiency to be attained and the shorter of the two transition wave-lengths have been assigned arbitrarily within limits imposed

TABLE I. *Points on loci of equal maximum efficiency.*

Results of computation for Illuminant A. Upper half of table is for curves similar to Fig. 1a. Lower half of table is for curves similar to Fig. 1b.

λ_1	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y
	$R=0.95$			$R=0.90$			$R=0.80$			$R=0.70$			$R=0.60$		
380.5	639.7	.4251	.4183	625.1	.4016	.4289	607.2	.3509	.4458	593.8	.3039	.4617	582.3	.2576	.4663
430.5	639.8	.4297	.4257	625.2	.4063	.4371	607.2	.3570	.4582	593.8	.3076	.4740	582.3	.2606	.4811
450.5	640.3	.4443	.4469	625.5	.4208	.4609	607.4	.3709	.4881	594.0	.3200	.5113	582.5	.2707	.5267
465.5	641.5	.4593	.4691	626.2	.4376	.4844	607.8	.3876	.5185	594.3	.3358	.5505	582.8	.2847	.5764
475.5	643.1	.4726	.4784	627.1	.4502	.4976	608.4	.4006	.5361	594.8	.3487	.5739	583.2	.2969	.6077
480.5							608.8	.4072	.5421	595.2	.3564	.5825	583.5	.3027	.6203
485.5	646.2	.4855	.4849	628.7	.4598	.5070	609.4	.4137	.5462	595.5	.3616	.5890	583.9	.3103	.6284
490.5							610.1	.4207	.5477	596.2	.3695	.5916	584.4	.3178	.6338
495.5				631.6	.4760	.5038	611.1	.4284	.5471	597.0	.3775	.5922	585.2	.3268	.6354
500.5				634.0	.4839	.5007	611.6	.4332	.5479	598.0	.3867	.5900	586.0	.3358	.6352
505.5							614.2	.4474	.5385	599.4	.3978	.5848	587.2	.3472	.6309
515.5							620.0	.4747	.5179	603.7	.4271	.5636	590.9	.3781	.6104
380.5	507.8	.5189	.4719	520.9	.5343	.4614	537.2	.5603	.4381	549.7	.5852	.4145	560.9	.6091	.3904
430.5	507.8	.5122	.4628	521.0	.5270	.4522	537.2	.5520	.4287	549.8	.5754	.4048	560.9	.5980	.3805
450.5	508.3	.4947	.4390	521.3	.5104	.4306	537.4	.5305	.4050	549.9	.5508	.3808	561.1	.5697	.3560
465.5	509.3	.4795	.4199	521.8	.4917	.4091	537.8	.5120	.3854	550.3	.5295	.3611	561.4	.5454	.3361
475.5	510.5	.4709	.4096	522.5	.4823	.3986	538.3	.5011	.3751	550.7	.5175	.3509	561.8	.5318	.3260
485.5	512.5	.4651	.4031	523.7	.4757	.3919	539.1	.4937	.3684	551.4	.5088	.3445	562.5	.5217	.3197
500.5	517.3	.4599	.3983	527.1	.4694	.3871	541.6	.4860	.3640	553.6	.4996	.3407	564.5	.5104	.3163
515.5				533.7	.4649	.3860	546.9	.4789	.3639	558.3	.4912	.3408	569.0	.4982	.3187
525.5										563.6	.4828	.3448	574.1	.4872	.3240
535.5													581.0	.4721	.3325
545.5										578.4	.4607	.3581	589.6	.4510	.3452
550.5				561.6	.4544	.3912	562.4	.4739	.3644	583.0	.4528	.3632	594.6	.4361	.3544
570.5	575.8	.4490	.4009	581.2	.4444	.3983	592.6	.4319	.3934	605.7	.4035	.3957			
590.5				603.2	.4282	.4096	619.4	.3904	.4221						

λ_1	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y
	$R=0.50$			$R=0.40$			$R=0.30$			$R=0.20$			$R=0.10$		
380.5	571.6	.2151	.4589	560.9	.1770	.4370	549.7	.1442	.3920	537.2	.1200	.3190	520.9	.1120	.1990
430.5	571.6	.2169	.4763	560.9	.1780	.4560	549.8	.1428	.4140	537.2	.1160	.3400	521.0	.1070	.2160
450.5	571.7	.2239	.5311	561.1	.1800	.5210	549.9	.1400	.4870	537.4	.1060	.4180	521.3	.0870	.2750
465.5	572.0	.2350	.5936	561.4	.1870	.5980	550.3	.1420	.5920	537.8	.0960	.5290	521.8	.0660	.3940
475.5	572.4	.2458	.6348	561.8	.1950	.6520	550.7	.1440	.6530	538.3	.0920	.6220	522.5	.0480	.5090
480.5	572.7	.2503	.6526												
485.5	573.1	.2588	.6639	562.5	.2070	.6940	551.4	.1530	.7100	539.1	.0940	.7070	523.7	.0390	.6340
490.5	573.6	.2663	.6724												
495.5	574.3	.2749	.6771												
500.5	575.1	.2850	.6780	564.5	.2340	.7180	553.6	.1800	.7510	541.6	.1220	.7790	527.1	.0550	.7750
505.5	576.3	.2972	.6750												
515.5	579.7	.3293	.6560	569.0	.2810	.7000	558.3	.2240	.7490	546.9	.1790	.7820	533.7	.1190	.8150
380.5	571.6	.6332	.3664	582.3	.6565	.3427	593.8	.6793	.3195	607.2	.7100	.3030	625.1	.7190	.2820
430.5	571.6	.6200	.3560	582.3	.6405	.3315	593.8	.6592	.3065	607.2	.6710	.2830	625.2	.6540	.2540
450.5	571.7	.5868	.3305	582.5	.6001	.3043	594.0	.6080	.2762	607.4	.6020	.2460	625.5	.5530	.2000
465.5	572.0	.5587	.3102	582.8	.5675	.2833	594.3	.5675	.2538	607.8	.5500	.2200	626.2	.4790	.1680
475.5	572.4	.5430	.3000	583.2	.5490	.2728	594.8	.5450	.2427	608.4	.5210	.2080	627.1	.4410	.1550
485.5	573.1	.5312	.2939	583.9	.5350	.2667	595.5	.5285	.2368	609.4	.4970	.2020	628.7	.4160	.1475
500.5	575.1	.5175	.2913	586.0	.5170	.2650	598.0	.5045	.2363	611.6	.4690	.2010	634.0	.3485	.1525
515.5	579.7	.5009	.2955	590.9	.4938	.2720	603.7	.4638	.2473	620.0	.4020	.2200			
525.5	585.0	.4841	.3033												
535.5	592.4	.4603	.3165												
545.5	602.1	.4243	.3370												
550.5	608.2	.3998	.3512												

by the nature of the problem. The longer transition wave-length has been calculated from these assigned quantities. The spectrophotometric curve is completely determined by the two transitions. Consequently the chromaticity can be computed from the knowledge of these transitions. The chromaticity may be described interchangeably in two manners, each involving two parameters. The first mode is the location by Cartesian coordinates of a point representing the sample in the color mixture diagram. The second mode is the location of this point by a species of polar coordinates. The polar coordinates are called dominant wave-length and purity. The angular coordinate, dominant wave-length, is measured by the wave-length of the spectral light whose chromaticity is represented by a point on the line through the sample point and the pole (the white point). If the points representing the spectral light and the sample are on opposite sides of the white point, the sample is said to be complementary to that dominant wave-length. This fact is indicated in this paper by a lower case "*c*" immediately following the wave-length of spectral light complementary to the color of the sample. The radial coordinate, purity, has been measured in

TABLE II. *Points on loci of equal maximum efficiency.*

Results of computation for Illuminant C. Upper half of table is for curves similar to Fig. 1a. Lower half of table is for curves similar to Fig. 1b.

λ_1	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y
	$R=0.95$			$R=0.90$			$R=0.80$			$R=0.70$			$R=0.60$		
380.5	627.3	.2857	.3185	611.7	.2631	.3192	592.1	.2236	.3120	578.4	.1932	.3005	567.1	.1694	.2797
430.5	627.7	.2943	.3395	612.0	.2697	.3410	592.3	.2282	.3382	578.5	.1953	.3263	567.2	.1698	.3065
450.5	630.2	.3226	.4055	613.4	.2956	.4111	593.2	.2465	.4183	579.3	.2064	.4136	567.8	.1732	.3995
465.5	635.0	.3608	.4679	616.0	.3302	.4827	594.8	.2743	.5056	580.4	.2261	.5163	568.8	.1847	.5156
475.5	642.1	.3907	.5025	619.2	.3590	.5232	596.6	.2991	.5591	581.8	.2495	.5835	570.0	.2011	.5982
480.5	648.7	.4055	.5126	621.6	.3742	.5364	598.0	.3136	.5784				570.8	.2117	.6316
485.5	662.7	.4209	.5180	624.8	.3896	.5438	599.6	.3284	.5913	584.0	.2733	.6282	571.8	.2238	.6567
490.5				629.3	.4020	.5493									
495.5				635.9	.4221	.5430	604.4	.3570	.5932	587.3	.3063	.6432	574.6	.2525	.6823
500.5				647.8	.4397	.5350	607.8	.3785	.5912	589.6	.3213	.6415	576.4	.2694	.6840
505.5										592.6	.3408	.6316			
515.5							628.1	.4493	.5433	601.2	.3876	.5999	585.1	.3344	.6502
525.5													594.9	.3908	.6016
535.5													611.0	.4605	.5364
380.5	488.5	.4300	.5195	504.8	.4555	.5235	523.1	.4901	.5038	535.8	.5187	.4780	546.7	.5470	.4514
430.5	489.0	.4070	.4720	505.1	.4295	.4741	523.2	.4562	.4505	535.9	.4795	.4243	546.8	.5004	.3963
450.5	491.3	.3630	.3855				524.0	.3966	.3584	536.6	.4107	.3319	547.3	.4217	.3042
465.5							525.4	.3631	.3103				548.3	.3803	.2593
475.5										538.9	.3566	.2675			
480.5	500.8	.3270	.3172	512.3	.3330	.3080									
485.5							529.0	.3391	.2815	540.7	.3460	.2578	551.1	.3500	.2330
500.5	512.2	.3160	.3069	520.7	.3230	.2975	533.9	.3304	.2754	545.0	.3356	.2525	555.2	.3376	.2284
515.5				530.0	.3180	.2958	541.6	.3229	.2756				562.1	.3238	.2294
525.5										558.5	.3185	.2544	568.9	.3132	.2322
550.5							571.2	.3035	.2802	583.2	.2875	.2651	598.5	.2593	.2497
570.5	576.2	.3053	.3096	582.4	.2980	.3030	597.4	.2747	.2926	620.6	.2290	.2868			
590.5				609.1	.2813	.3106	654.6	.2276	.3119						

λ_1	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y	λ_2	x	y
	$R=0.50$			$R=0.40$			$R=0.30$			$R=0.20$			$R=0.10$		
380.5	556.8	.1510	.2520	546.7	.1383	.2180	535.8	.1302	.1764	523.1	.1289	.1268	504.8	.1363	.0692
430.5	556.9	.1497	.2785	546.8	.1350	.2425	535.9	.1255	.1980	523.2	.1230	.1438	505.1	.1308	.0792
450.5	557.5	.1462	.3736	547.3	.1246	.3363	536.6	.1092	.2845	524.0	.1027	.2152			
465.5	558.4	.1490	.5017	548.3	.1179	.4720	537.0	.0909	.4178	525.4	.0762	.3420	508.1	.0808	.2132
475.5	559.5	.1589	.5990				538.9	.0855	.5500	526.8	.0572	.4775			
480.5	560.3	.1677	.6411				539.0	.0836	.6110				512.0	.0371	.4135
485.5	561.2	.1782	.6750	551.1	.1343	.6800	540.7	.0911	.6700	529.0	.0500	.6250	513.7	.0251	.5007
490.5	562.4	.1913	.6980				541.3	.0975	.7140				515.6	.0181	.5893
495.5				553.6	.1596	.7377	542.7	.1100	.7487	532.0	.0637	.7410	517.8	.0181	.6718
500.5	565.4	.2222	.7185	555.2	.1766	.7470	545.0	.1294	.7700	533.9	.0787	.7747	520.7	.0276	.7416
505.5				557.1	.1952	.7500	546.3	.1462	.7806	536.1	.0992	.7975	523.2	.0434	.7890
510.5							548.7	.1698	.7793	538.7	.1239	.8055	526.4	.0687	.8178
515.5	572.8	.2867	.6936	562.1	.2437	.7305	551.4	.1957	.7696	541.6	.1518	.7983	530.0	.0996	.8252
525.5	580.5	.3412	.6493	568.9	.2964	.6903									
535.5	591.6	.4066	.5890	578.0	.3200	.6357									
380.5	556.8	.5759	.4231	567.1	.6065	.3925	578.4	.6390	.3613	592.1	.6717	.3273	611.7	.7040	.2946
430.5	556.9	.5207	.3655	567.2	.5395	.3320	578.5	.5530	.2950	592.3	.5542	.2513	612.0	.5126	.1913
450.5	557.5	.4304	.2737	567.8	.4347	.2410	579.3	.4300	.2040	593.2	.4077	.1603	613.4	.3424	.1028
465.5	558.4	.3844	.2309	568.8	.3833	.2000	580.4	.3733	.1658	594.8	.3463	.1263	616.0	.2813	.0771
475.5				570.0	.3607	.1851	581.8	.3485	.1528	596.6	.3195	.1150	619.2	.2518	.0693
480.5				570.8	.3527	.1807				598.0	.3075	.1122	621.6	.2378	.0674
485.5	561.3	.3489	.2071	571.8	.3453	.1777	584.0	.3300	.1462	599.6	.2968	.1104	624.8	.2230	.0663
495.5				574.6	.3325	.1752	587.3	.3140	.1443				635.9	.1868	.0664
500.5	565.4	.3347	.2026	576.4	.3260	.1750	589.6	.3045	.1447	607.8	.2586	.1104	647.8	.1628	.0676
515.5	572.5	.3175	.2046	585.1	.3003	.1783	601.2	.2643	.1503	628.1	.1918	.1182			
525.5	580.5	.3000	.2092	594.9	.2727	.1844									
535.5	591.7	.2746	.2162	611.0	.2276	.1955									
550.5	622.6	.2024	.2373												

several ways.[2] In this paper it will be measured in units of excitation purity which is simply the length of the line from the white point to the sample point divided by the length of the line through the sample point from the white point to the intersection with the locus bounding realizable chromaticities. This bounding locus consists of the spectral locus and the straight line joining the extremities of the spectral locus.

[2] D. B. Judd, *General Formula for Computation of Colorimetric Purity*, J. O. S. A. **21**, 729–747 (1931).

The computations were carried out according to the following forms.[3] Form A is for spectrophotometric curves of the type exhibited as a full line in Fig. 1a. Form B is for spectrophotometric curves of the type exhibited as a full line in Fig. 1b. In either case, arbitrary values of R and λ_1 were entered in the indicated spaces. The first two quantities in column 1 were secured from tables giving $\sum_{380\,m\mu}^{\lambda} E_\lambda \bar{y}_\lambda$ for I.C.I.

[3] For complete explanation of notation see J. O. S. A. **25**, 250 (1935).

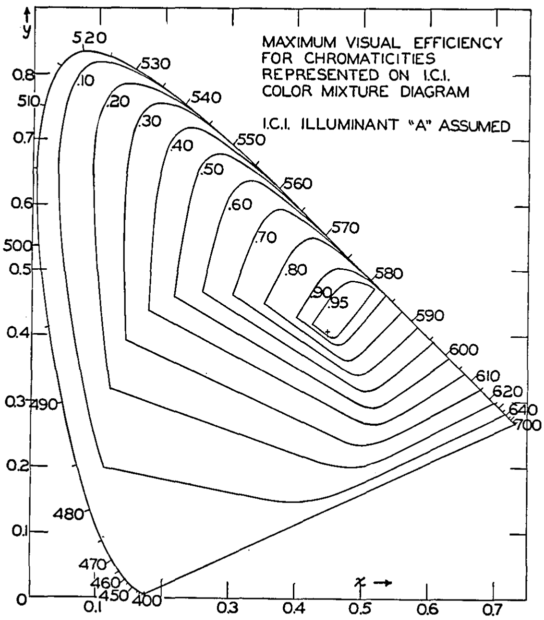


FIG. 2. Loci in I.C.I. color mixture diagram of points characterized by equal maximum visual efficiencies in Illuminant A. The values of maximum visual efficiency are indicated by numbers placed within the diagram near the corresponding locus. Numbers just outside the bounding locus indicate wave-lengths in millimicrons of spectral light represented by the associated points on that locus.

Illuminants A, B, or C as desired for λ at intervals of one millimicron from 380 mμ to 780 mμ. The third entry in this column was obtained by adding the first two entries. This total was then sought in the table for $\sum_{380}^{\lambda} E_\lambda \bar{y}_\lambda$

TABLE III. *Maximum visual efficiency (%) of colored materials having indicated dominant wave-length and excitation purity for I.C.I. observer and Illuminant A.*

	Excitation Purity								
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
λ =									
400mμ	61.0	41.0	29.5	20.5	14.5	10.0	7.0	4.5	2.0
470	63.0	43.0	31.5	23.0	17.0	12.5	9.0	5.0	3.0
480	66.0	46.0	34.5	27.0	21.0	16.0	13.0	10.0	6.0
490	73.5	55.0	43.5	36.5	31.0	27.5	24.0	21.0	12.0
495	80.0	64.5	54.0	47.0	41.5	37.0	33.0	22.0	10.0
500	86.0	75.0	67.0	61.5	54.5	43.5	32.5	22.0	11.0
505	90.0	81.0	71.5	61.5	50.5	40.0	29.5	20.0	12.0
510	90.0	80.0	70.0	60.5	50.0	40.0	31.0	22.0	13.0
520	89.5	80.5	72.0	63.5	54.5	46.0	37.0	27.5	17.5
540	93.0	86.5	80.0	74.0	68.0	62.0	55.0	47.5	37.5
550	95.0	90.0	85.0	80.5	76.0	71.5	66.5	61.0	49.5
560	96.5	93.5	89.5	86.5	83.0	80.5	77.5	73.5	65.5
570	98.5	97.0	95.5	94.0	92.0	90.0	87.5	84.5	78.5
575	99.5	99.0	89.5	98.0	97.0	95.5	94.0	91.5	87.5
580	100.0	100.0	100.0	100.0	99.5	99.5	99.0	98.5	96.5
585	98.5	97.0	95.5	94.0	92.5	91.0	90.0	88.5	86.5
590	97.5	94.5	92.0	89.5	86.5	84.0	82.0	79.5	77.0
600	95.5	90.5	85.0	80.0	75.5	71.0	66.0	61.0	56.5
700	91.0	81.0	71.0	61.0	51.0	41.5	33.0	23.5	12.5
510*c*	91.5	81.5	72.5	64.0	54.0	44.5	35.5	25.5	15.0
550*c*	89.0	78.0	68.0	58.5	51.0	44.0	35.5	25.5	14.5
570*c*	80.5	64.0	51.5	42.5	35.5	29.5	23.5	17.0	12.0
575*c*	72.0	53.0	41.0	33.0	25.5	20.0	15.5	11.5	6.5
579*c* = 400	61.0	41.0	29.5	20.5	14.5	10.0	7.0	4.5	2.0

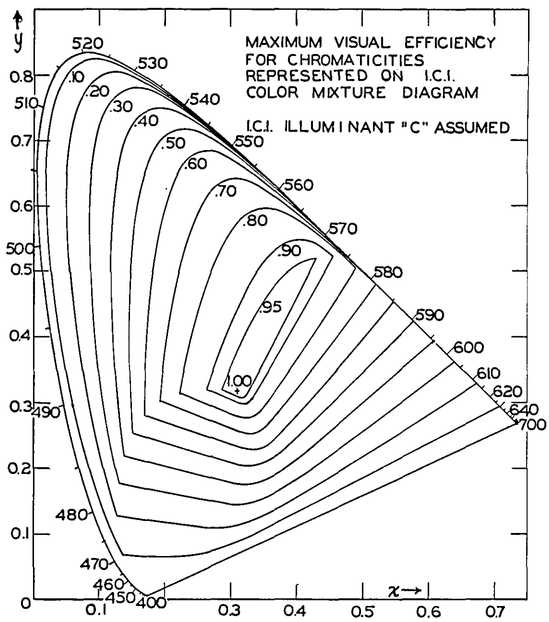


FIG. 3. Loci in I.C.I. color mixture diagram of points characterized by equal maximum visual efficiencies in Illuminant C. The values of maximum visual efficiency are indicated by numbers placed within the diagram near the corresponding locus. Numbers just outside the bounding locus indicate wave-lengths in millimicrons of spectral light represented by the associated points on that locus.

and λ_2 was interpolated and entered in the form. The entries in columns 2 and 3 were secured from similar tables for $\sum_{380}^{\lambda} E_\lambda \bar{z}_\lambda$ and $\sum_{380}^{\lambda} E_\lambda \bar{x}_\lambda$, respectively, and by carrying out the operations

TABLE IV. *Maximum visual efficiency (%) of colored materials having indicated dominant wave-length and excitation purity for I.C.I. observer and Illuminant C.*

	Excitation Purity								
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
λ =									
400mμ	81.0	65.5	53.0	41.0	32.0	24.0	17.0	11.0	5.5
470	82.5	68.5	57.5	47.0	37.5	30.5	24.0	18.0	12.5
480	86.0	74.0	64.5	56.0	48.5	41.0	34.5	29.5	16.0
490	92.5	85.5	80.0	73.0	62.0	50.0	38.0	26.0	14.5
495	93.0	86.0	79.0	69.0	58.0	46.0	35.5	25.0	14.5
500	92.0	84.0	75.5	66.5	55.5	44.5	34.0	24.5	15.0
505	91.0	82.5	73.0	63.5	53.5	43.0	33.5	24.0	15.5
510	91.0	83.0	74.0	64.0	54.0	44.5	35.5	26.5	17.5
520	92.0	84.0	76.0	68.0	59.5	51.5	43.0	34.0	23.0
530	93.0	87.0	81.0	74.5	68.5	62.5	56.0	48.5	37.5
540	95.0	90.5	86.0	82.0	77.5	73.0	68.0	62.5	54.0
550	97.0	94.0	91.0	88.0	85.5	82.0	78.5	74.0	64.0
560	98.5	97.0	95.5	94.0	92.5	91.0	88.5	85.5	80.5
570	99.5	99.0	98.5	98.0	97.5	97.0	96.0	95.0	88.5
580	96.5	93.5	90.5	87.5	84.5	81.5	78.5	76.0	73.5
590	93.5	87.0	81.0	75.0	69.5	64.5	60.5	57.0	53.5
600	91.0	81.5	72.5	64.5	57.5	51.5	46.0	41.5	38.0
620	87.0	74.0	62.0	52.5	43.5	36.0	29.5	24.0	19.0
700	84.0	70.0	56.5	44.5	34.5	26.5	20.5	14.0	7.0
495*c*	86.5	73.0	60.5	49.5	39.5	30.5	22.5	15.0	7.5
500*c*	89.0	77.5	66.5	55.5	45.0	35.0	26.0	16.5	7.0
510*c*	90.5	80.0	70.0	60.0	50.0	40.0	30.0	20.0	10.0
550*c*	88.0	76.5	66.0	55.5	47.0	38.5	30.5	22.5	13.0
560*c*	84.0	70.5	59.5	49.0	39.0	31.0	24.0	17.5	10.0
567*c* = 400	81.0	65.5	53.0	41.0	32.0	24.0	17.0	11.0	5.5

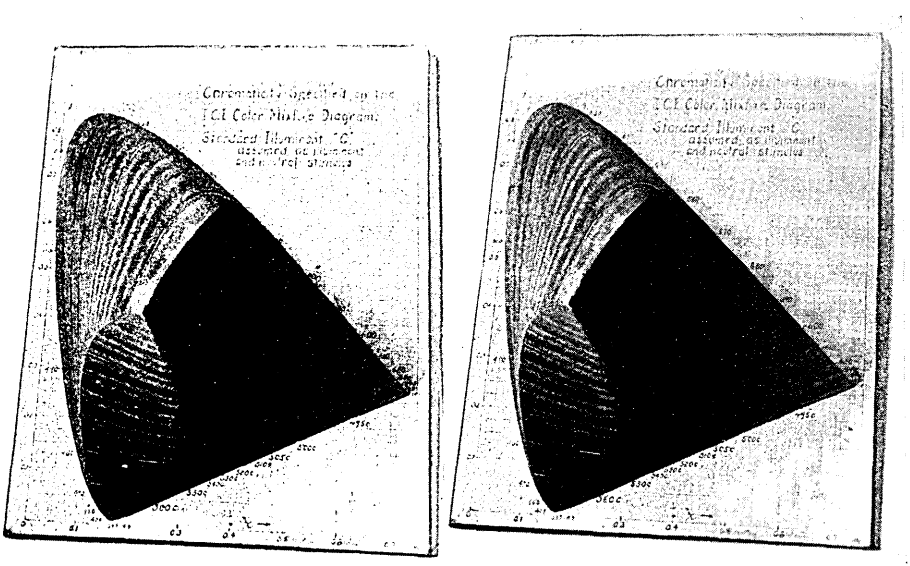


FIG. 4. Stereoscopic photograph of surface bounding all attainable colors in Illuminant C. Base of this solid is I.C.I. color mixture diagram. The vertical axis is visual efficiency. This surface is fully defined by Fig. 3, which may be regarded as a contour map.

indicated by the algebra and the symbols in parentheses. The values of x and y were then determined as indicated.

Table I shows the results of about two hundred such computations for Illuminant A. Table II shows similar results for Illuminant C. These tables are published with the above computation forms for the guidance of workers who may desire to investigate more fully small regions of chromaticity having special interest and who require a denser distribution of points in such regions. The tables of the running sums used as the basis for these computations are easily prepared from tables to be published soon.

Figs. 2 and 3 exhibit these results as loci in the color mixture diagram of points having constant maximum visual efficiencies. For some purposes it is convenient to represent colors by points in a three-dimensional space consisting of a visual efficiency coordinate axis perpendicular to the usual chromaticity axes of the color mixture diagram. Figs. 2 and 3 may be regarded as contour maps of surfaces enclosing points representing in this three-dimensional space all attainable colors of materials. Fig. 4 is a stereoscopic photograph of this surface for Illuminant C.

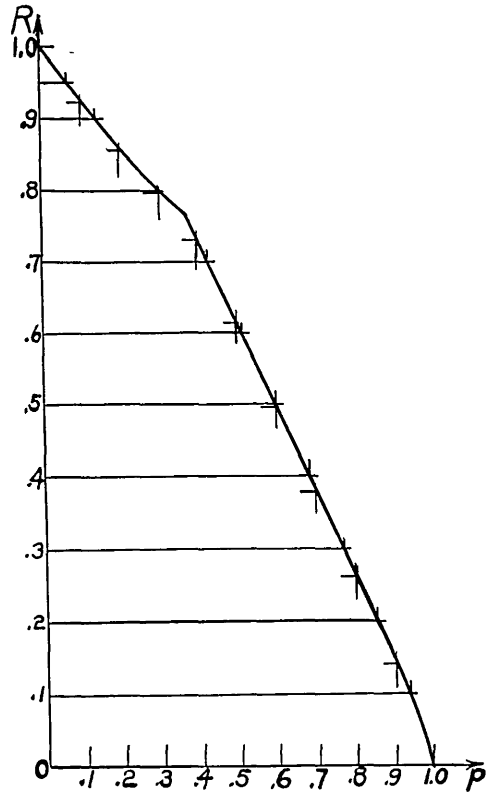


FIG. 5. Maximum visual efficiency plotted against excitation purity for one representative dominant wavelength, 490 mμ. This plot was prepared graphically from Fig. 3, i.e., for Illuminant C.

Tables III and IV give maximum attainable visual efficiency as a function of the chromaticity variables, namely, dominant wave-length and excitation purity for Illuminants A and C. These values were secured by graphical construction from Figs. 2 and 3. The dominant wave-length loci were drawn on these figures. The lengths of the segments of each of these loci from the white point to the intersections with the successive efficiency loci were plotted against the corresponding value of the visual efficiency. Smooth curves were fitted appropriately through these observed points, discontinuous slopes being allowed corresponding to the two ridges in the three-dimensional surface shown above. The maximum visual efficiency corresponding to desired values of excitation purity was then found by locating the point on these curves at the corresponding fraction of the distance from the white point to the spectral locus. Fig. 5 illustrates this method for Illuminant C and for dominant wave-length 490 mμ.

Examination of the sources of error in these calculations will show that the values given in Tables I and II should be accurate to ± 0.001. Figs. 2 and 3 have been drawn carefully and it is unlikely that any points on the loci are in error by greater than ± 0.005 in either of the coordinates. The double chance for errors in graphical construction to enter in the preparation of Tables III and IV makes it seem unwise to rely on these values of maximum visual efficiency to better than ± 0.5 percent.

The author wishes to express his sincere appreciation of the interest and assistance of Professor A. C. Hardy during the preparation of this material.