(Geometric) Camera Calibration

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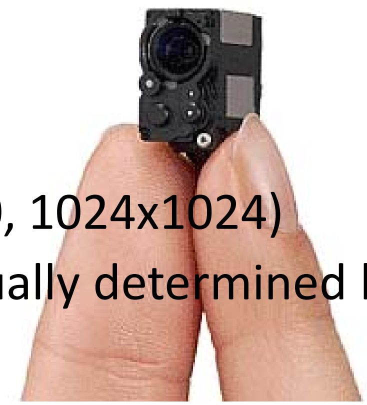
Camera Calibration

- **Digital Cameras and CCDs**
- Aberrations
- Perspective Projection
- Calibration

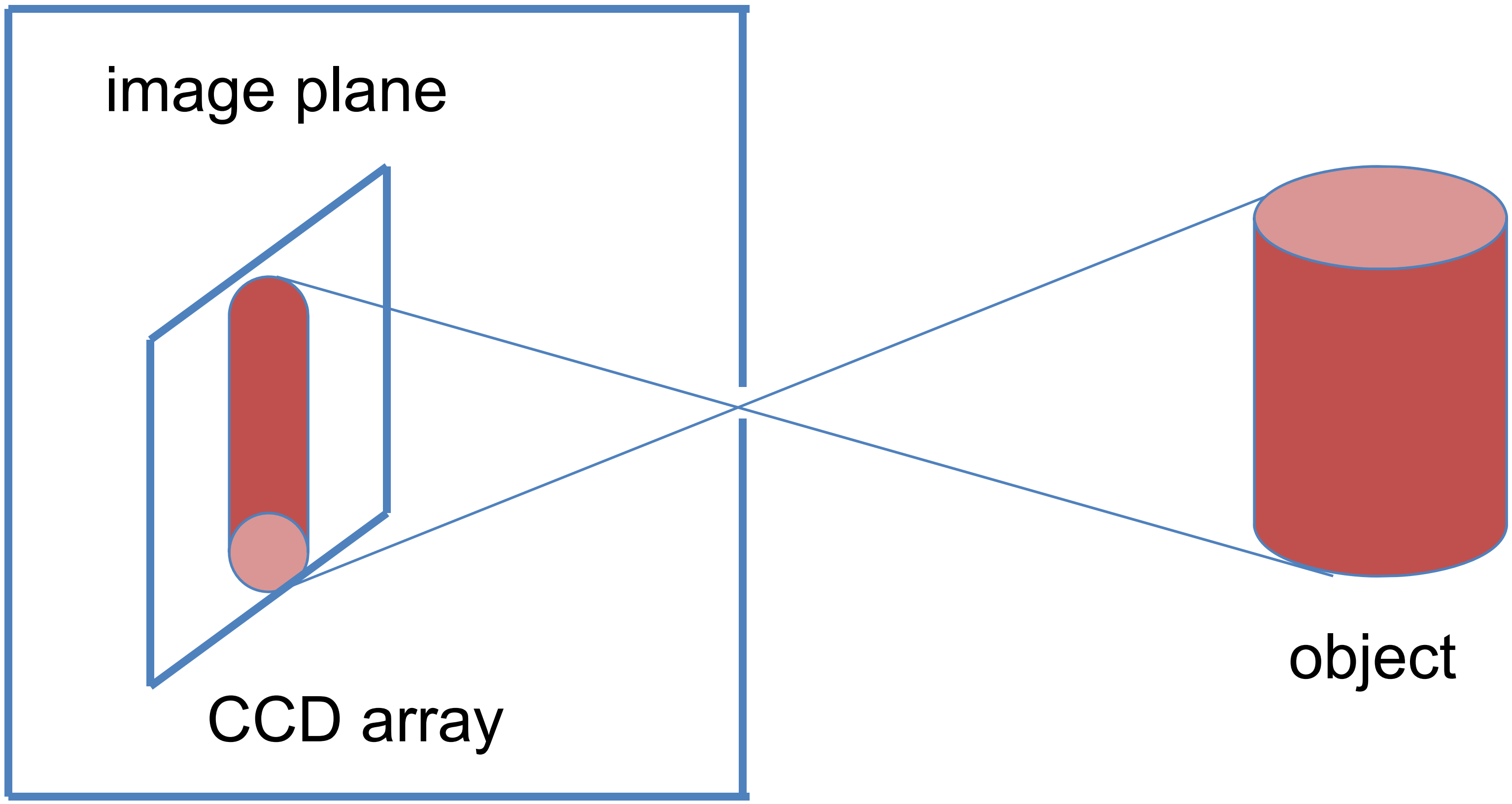
Digital Camera vs. "Film" Camera

- Charge-Coupled Device (CCD)
  - Image plane is a CCD array instead of film
  - CCD arrays are typically ¼ or ½ inch in size
  - CCD arrays have a pixel resolution (e.g., 640x480, 1024x1024)
  - CCD Cameras have a maximum "frame rate", usually determined by the hardware and bandwidth



- Number of CCDs
  - 3: each CCD captures only R, G, or B wavelengths
  - 1: the single CCD captures RGB simultaneously, reducing the resolution by 1/3 (kinda)
- Video
  - Interlaced: only "half" of the horizontal lines of pixels are present in each frame
  - Progressive scan: each frame has a full-set of pixels

The simplest 1-CCD camera in town



Exposures

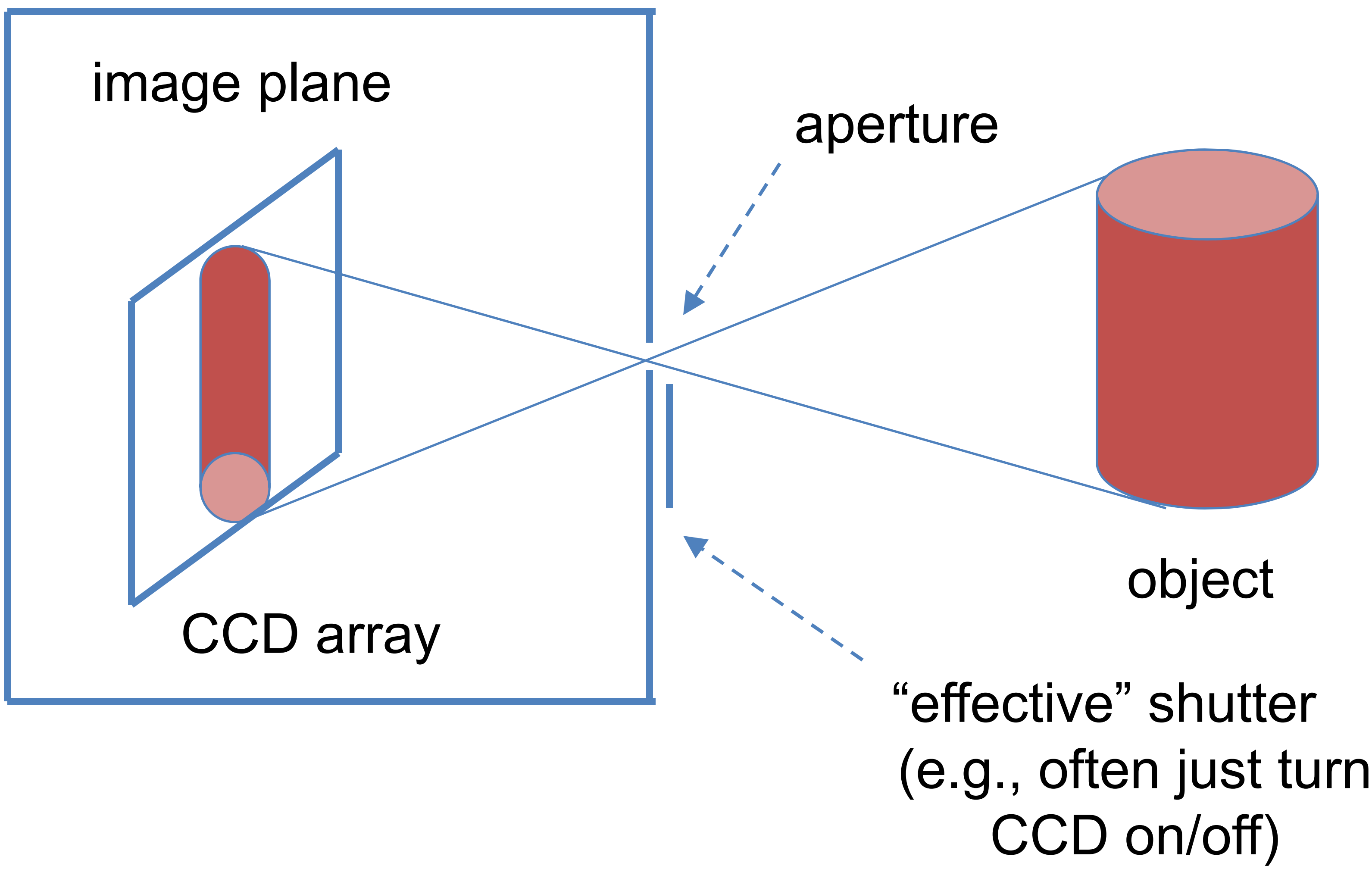


image plane
aperture
object
CCD array
“effective” shutter
(e.g., often just turn
CCD on/off)

Exposures

- An "exposure" is when the CCD is exposed to the scene, typically for a brief amount of time and with a particular set of camera parameters

- The characteristics of an "exposure" are determined by multiple factors, in particular:
  - Camera aperture
    - Determines amount of light that shines onto CCD
  - Camera shutter speed
    - Determines time during which aperture is "open" and light shines on CCD

Camera Calibration

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Aberrations

- A "real" lens system does not produce a perfect image

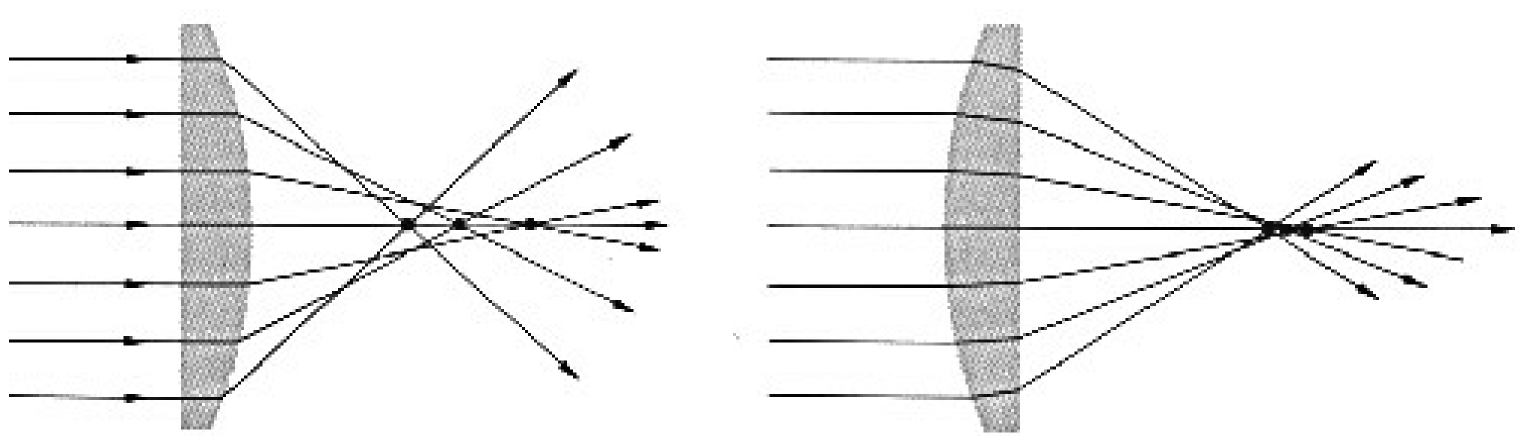
- Aberrations are caused by imperfect manufacturing and by our approximate models
  - Lenses typically have a spherical surface
    - Aspherical lenses would better compensate for refraction but are more difficult to manufacture
  - Typically 1st order approximations are used
    - Remember $\sin \Omega = \Omega - \Omega^3/3! + \Omega^5/5! - \ldots$
    - Thus, thin-lens equations only valid iff $\sin \Omega \approx \Omega$

Aberrations

- Most common aberrations:
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Chromatic aberration
  - **Distortion**

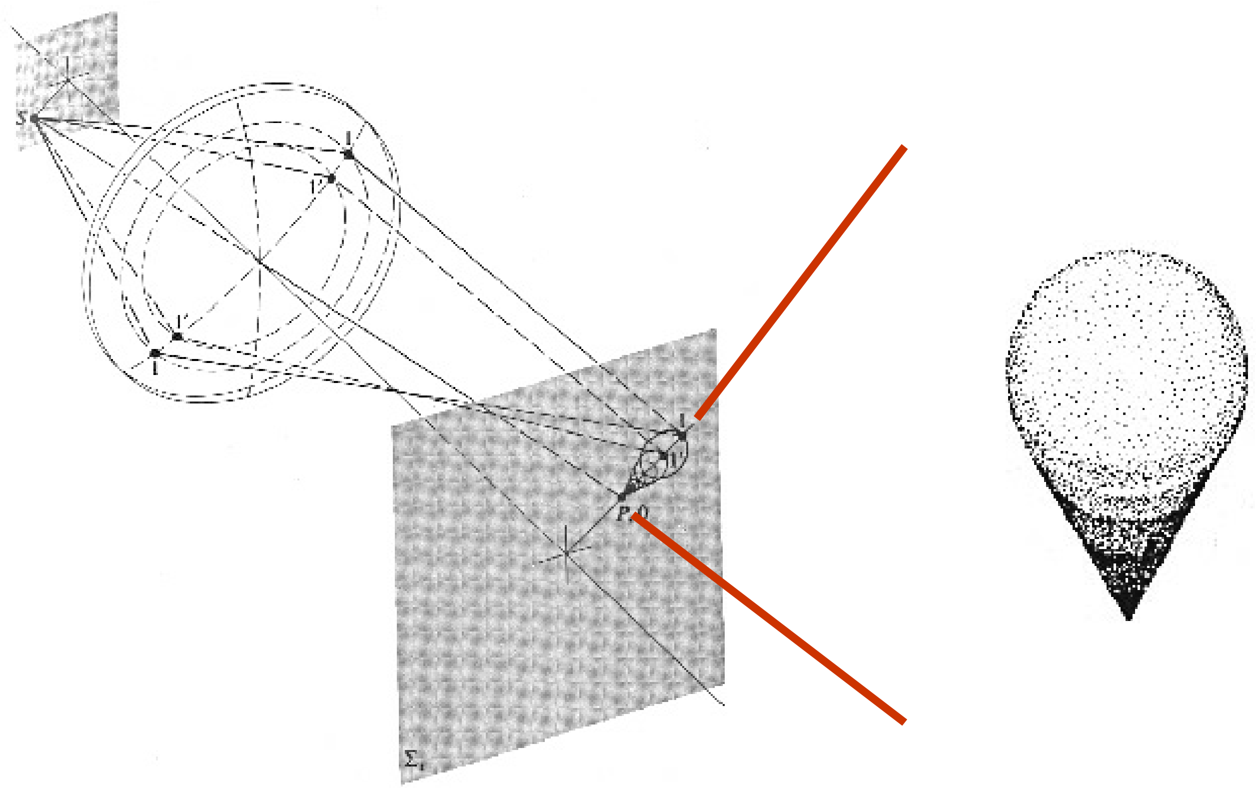
Spherical Aberration

- Deteriorates axial image



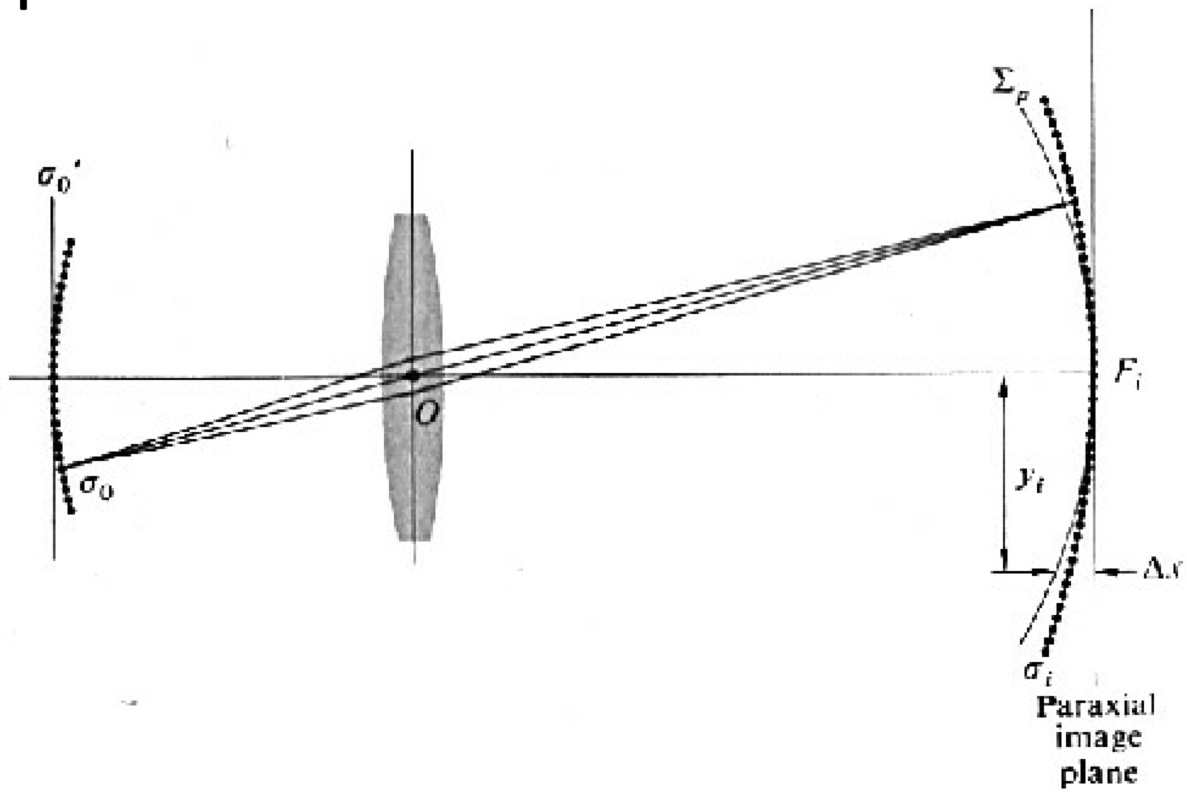
Coma

- Deteriorates off-axial bundles of rays



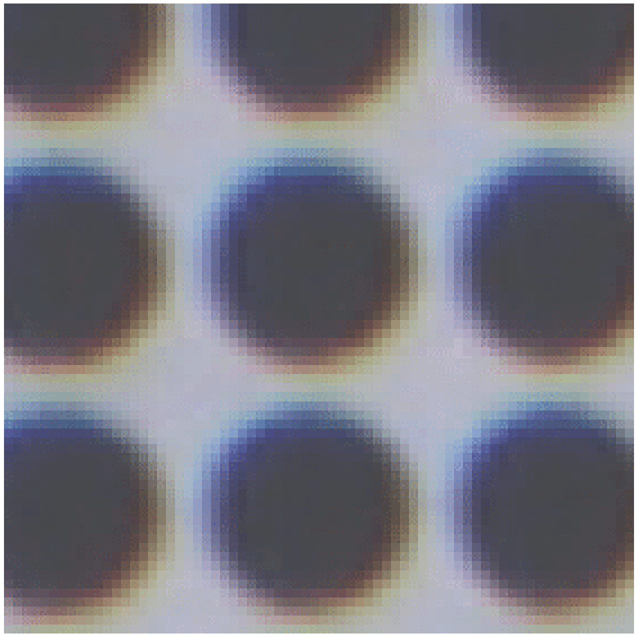
Astigmatism and Curvature of Field

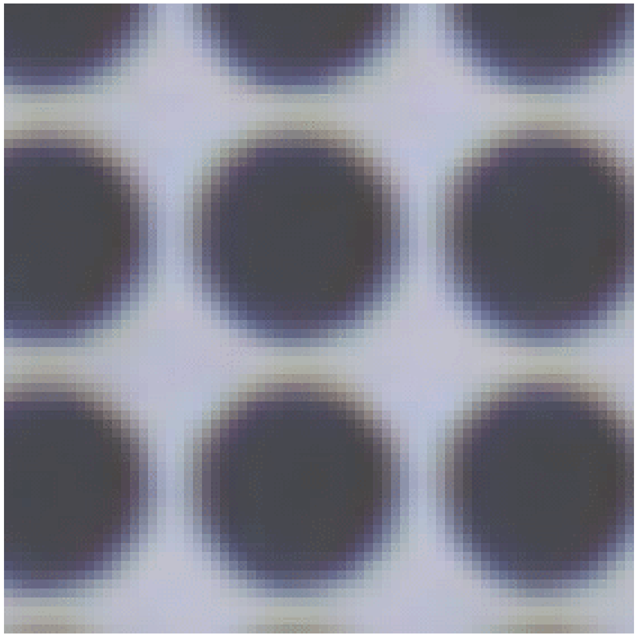
- Produces multiple (two) images of a single object point



Chromatic Aberration

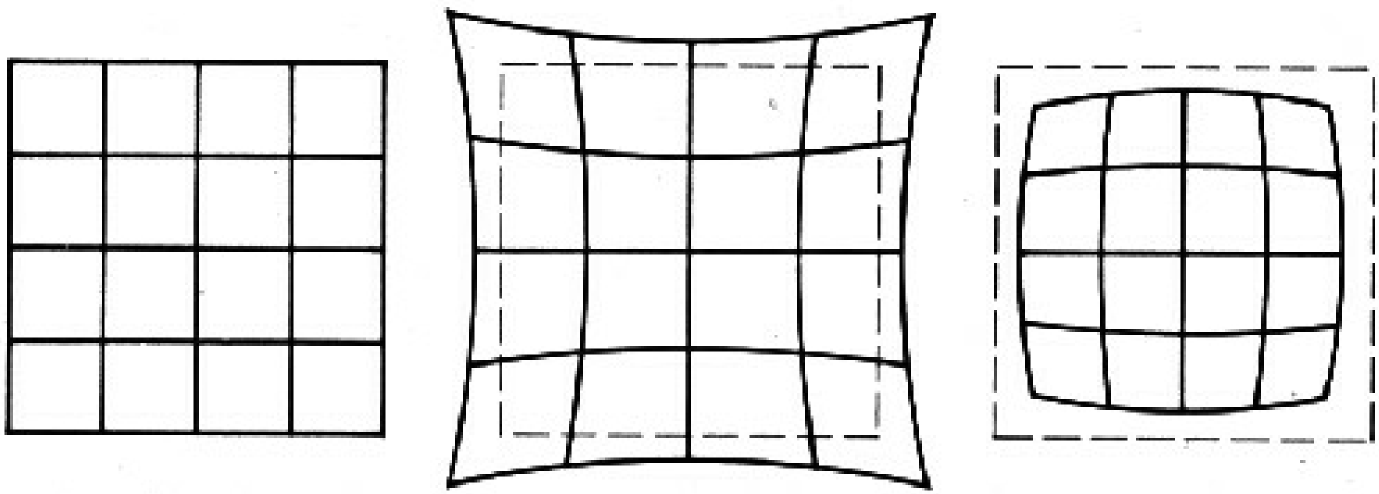
- Caused by wavelength dependent refraction
  - Apochromatic lenses (e.g., RGB) can help





Distortion

- Radial (and tangential) image distortions



Orthoscopic

Pin–cushion distortion

Barrel distortion

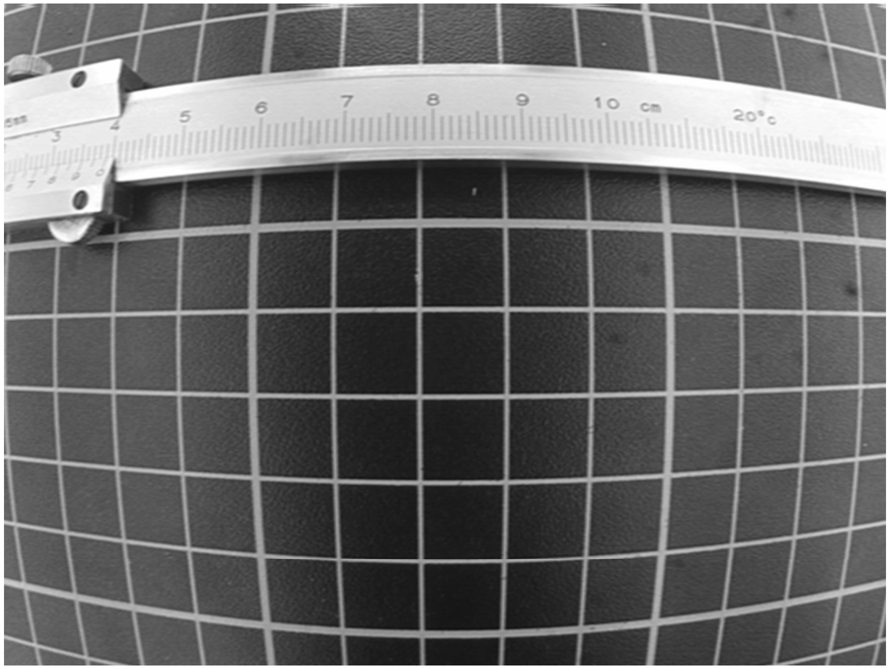
Radial Distortion

- (x, y) pixel before distortion correction
- (x', y') pixel after distortion correction
- Let $r = (x^2 + y^2)^{-1}$

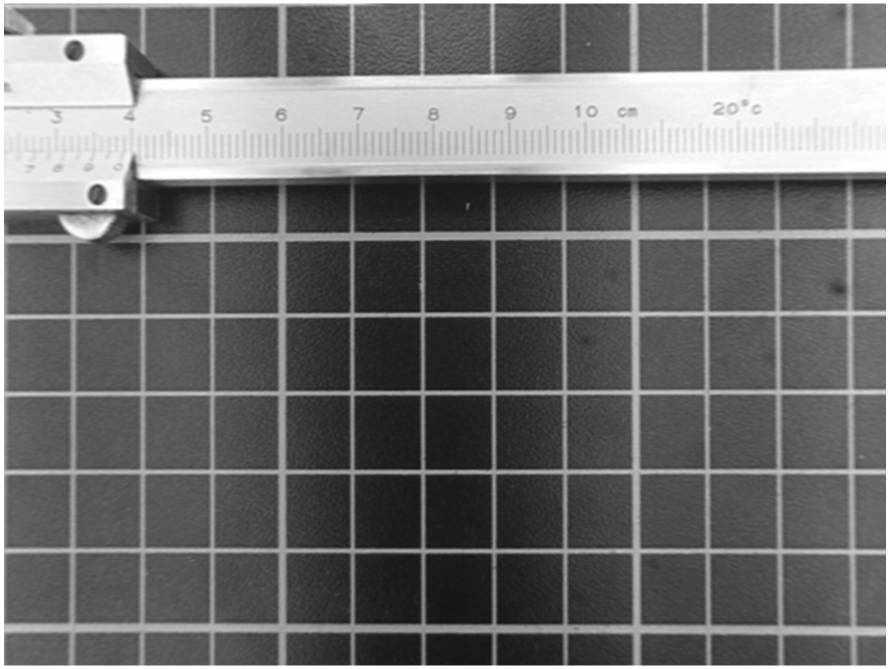
- Then
  - $x' = x(1 - \Delta r/r)$
  - $y' = y(1 - \Delta r/r)$
  - where $\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + \ldots$

- Finally,
  - $x' = x(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)$
  - $y' = y(1 - k_0 - k_1 r^2 - k_2 r^4 - \ldots)$

Radial Distortion



before



after

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- **Perspective Projection**
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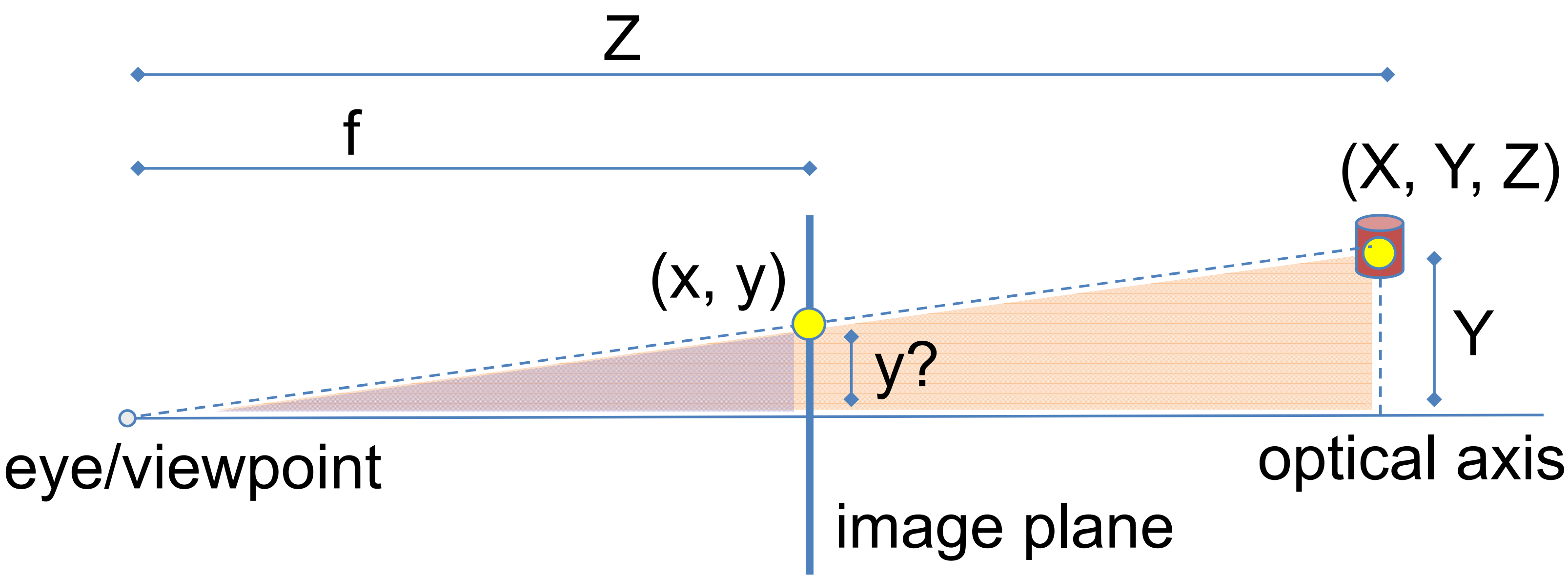
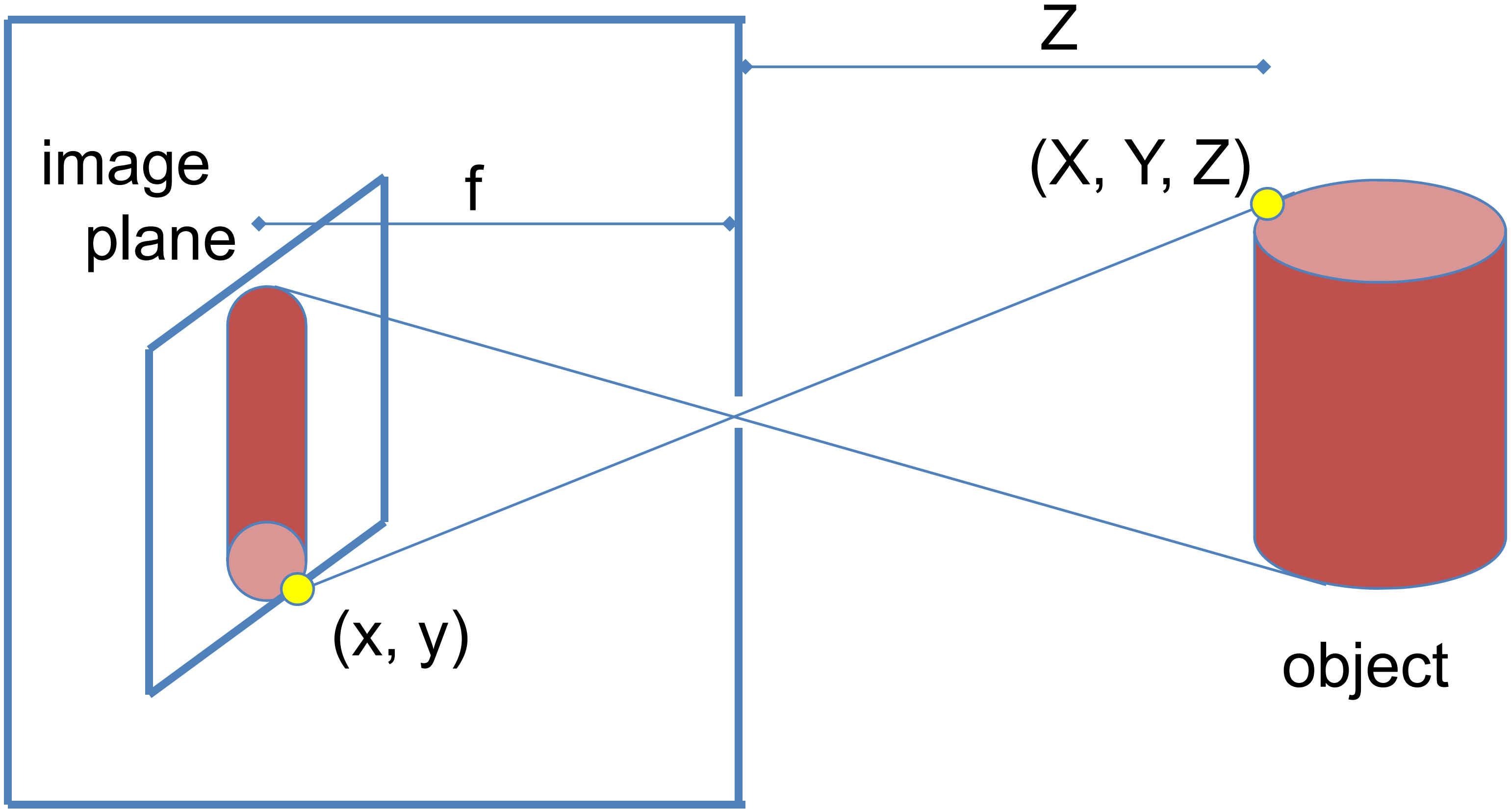
Perspective Projection



$$\frac{y}{f} = \frac{Y}{Z} \Rightarrow y = f\frac{Y}{Z} \quad \& \quad x = f\frac{X}{Z}$$

Perspective Projection



$$x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z}$$

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Tsai Camera Model and Calibration

- A widely used camera model to calibrate conventional cameras based on a pinhole camera

- Reference
  - "A Versatile Camera Calibration Technique for High-Accuracy 3D Machine Vision Metrology Using Off-the-Shelf TV Cameras and Lenses", Roger Y. Tsai, IEEE Journal of Robotics and Automation, Vol. 3, No. 4, August 1987

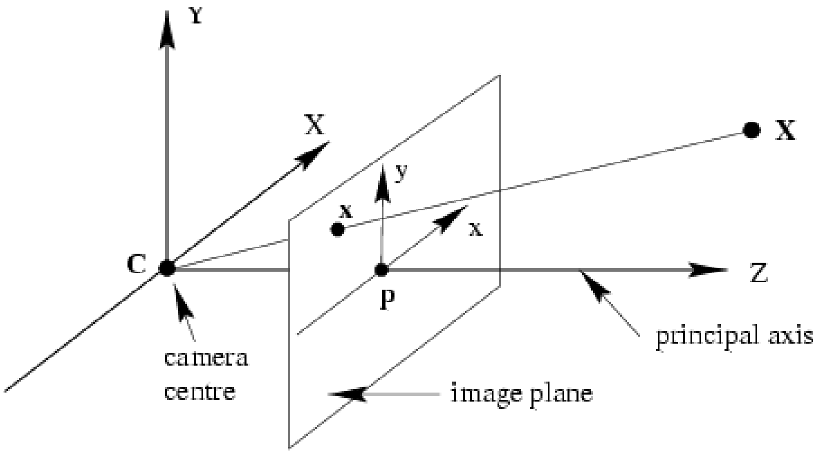
Calibration Goal

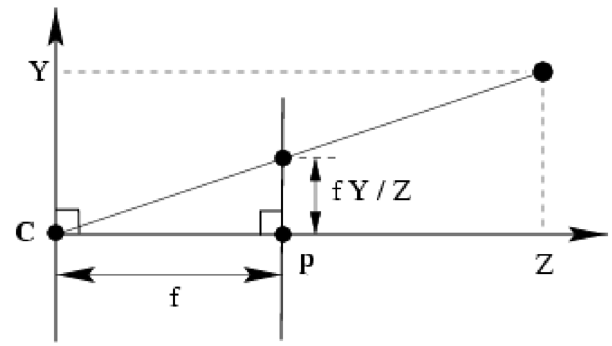
- Determine the intrinsic (and extrinsic) parameters of a camera (with lens)

Camera Parameters

- Intrinsic/Internal
  - Focal length f
  - Principal point (center) p_x, p_y
  - Pixel size s_x, s_y
  - (Distortion coefficients) $k_1, \ldots$
- Extrinsic/External
  - Rotation ϕ, φ, ψ
  - Translation t_x, t_y, t_z

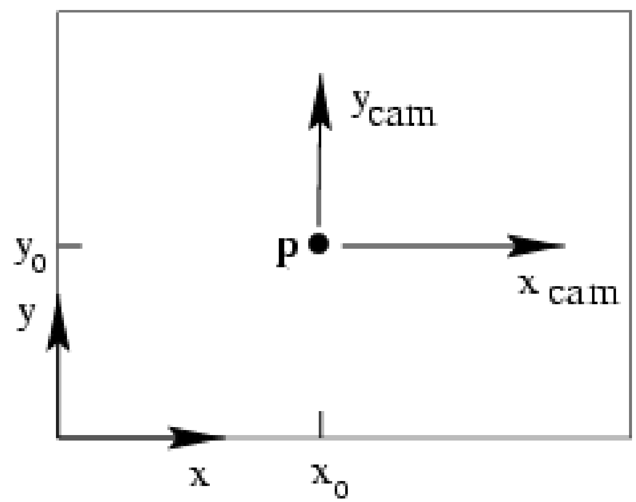
Focal Length





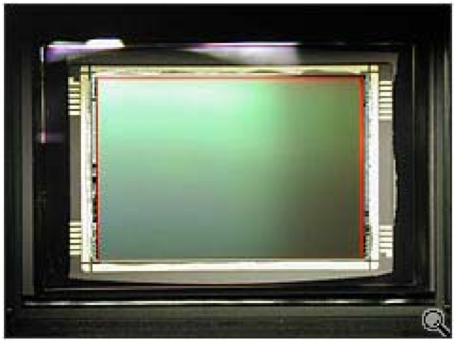
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \Leftarrow \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

CCD Camera: Pixel Size

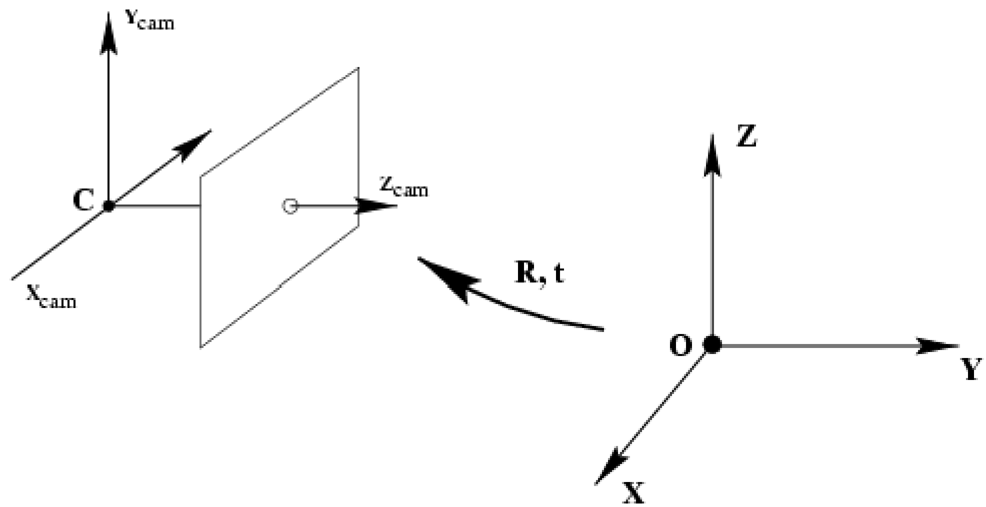


$$K = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha_x & 0 & p_x & 0 \\ 0 & \alpha_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsic) calibration matrix

Translation & Rotation



$$\tilde{x}_{cam} = R(\tilde{X} - C)$$

$$\tilde{x}_{cam} = R\tilde{X} - \underbrace{RC}_{-t}$$

$$\tilde{x}_{cam} = \underbrace{\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}}_{P} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$R = R_\phi R_\varphi R_\psi$$

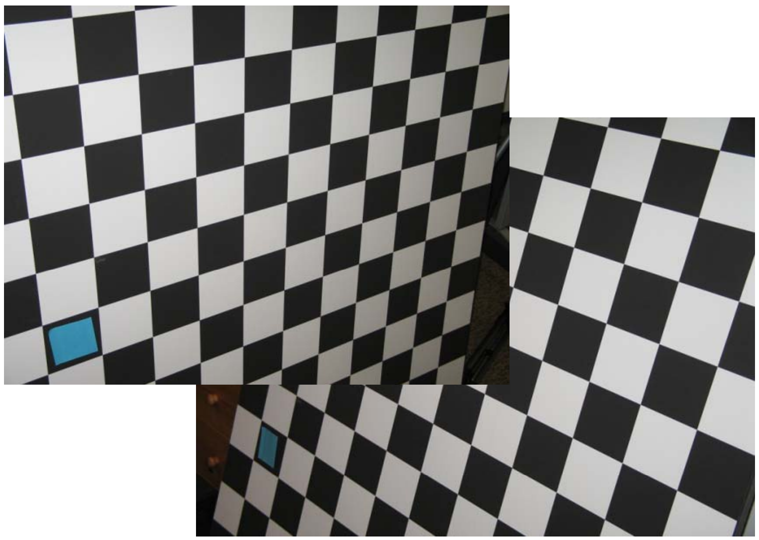
3x3 rotation matrices

$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

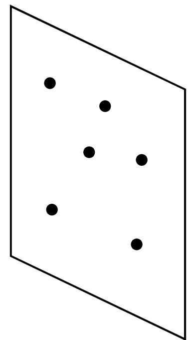
translation vector

(extrinsic) calibration matrix P

Calibration Task

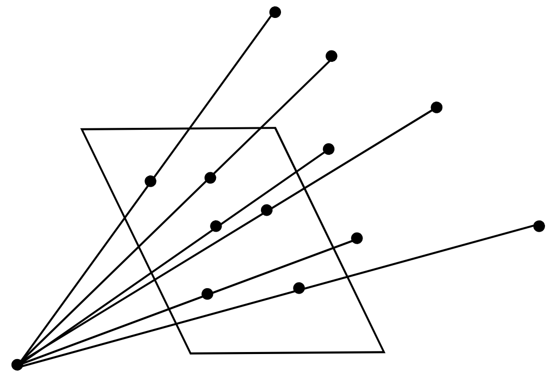


physical arrangement
(calibration pad)



observation
(camera with initial parameters)

points in space



calibration result
(camera with calibrated parameters)

Given $\tilde{X}_i \leftrightarrow \tilde{x}_i$ What is K? P ?

A Linear Formulation

Let $M = KP$

$$\tilde{x}_{cam} = M\tilde{X}$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x'/w' \\ y'/w' \end{pmatrix}$$

$$x = (m_1 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

A Linear Formulation

$$x = (m_1 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$
$$y = (m_2 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

for $i = 1 \ldots n$ observations

$$(m_1 - x_i m_3) \cdot \tilde{X}_i = 0$$
$$(m_2 - y_i m_3) \cdot \tilde{X}_i = 0$$

$2n$ homogeneous linear equations and 12 unknowns (coefficients of M)

Thus, given $n \geq 6$ can solve for M; namely $Qm = 0$

$$Q = \begin{bmatrix} \tilde{X}_1^T & 0^T & -x_1 \tilde{X}_1^T \\ 0^T & \tilde{X}_1^T & -y_1 \tilde{X}_1^T \\ \ldots & \ldots & \\ \tilde{X}_n^T & 0^T & -x_n \tilde{X}_n^T \\ 0^T & \tilde{X}n^T & -y_n \tilde{X}_n^T \end{bmatrix} \qquad m = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

Decomposing M into Camera Parameters

$$M = \rho\begin{bmatrix} A & b \end{bmatrix} = K\begin{bmatrix} R & t \end{bmatrix}$$

$$A = \rho\begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = \begin{bmatrix} \alpha_x r_1^T - \alpha \cot(\theta) r_2^T + p_x r_3^T \\ (\alpha_y / \sin\theta) r_2^T + p_y r_3^T \\ r_3^T \end{bmatrix}$$

…function of $\alpha_x, \alpha_y, p_x, p_y$ and skew θ

(often we assume skew is non-existent, which means $\theta = \pi / 2$)

Decomposing M into Camera Parameters

$$\rho = \varepsilon / \|a_3\|$$

$$r_3 = \rho a_3$$

$$p_x = \rho^2 (a_1 \cdot a_3)$$

$$p_y = \rho^2 (a_2 \cdot a_3)$$

$$\cos\theta = -\frac{(a_1 \times a_3) \cdot (a_2 \times a_3)}{\|a_1 \times a_3\| \|a_2 \times a_3\|}$$

$$\alpha_x = \rho^2 \|a_1 \times a_3\| \sin\theta$$

$$\alpha_y = \rho^2 \|a_2 \times a_3\| \sin\theta$$

$$r_1 = \frac{a_2 \times a_3}{\|a_2 \times a_3\|}$$

$$r_2 = r_3 \times r_1$$

$$t = \rho K^{-1} b$$

$$K = \begin{bmatrix} \alpha_x & 0 & p_x & 0 \\ 0 & \alpha_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, P = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

A Nonlinear Formulation

- Bundle Adjustment
  - Given initial guesses, use nonlinear least squares to refine/compute the calibration parameters
  - Simple but good convergence depends on accuracy of initial guess

A Nonlinear Formulation

Recall

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$E = \frac{1}{mn} \sum_{ij} \left[(x_{ij} - \frac{m_{i1} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 + (y_{ij} - \frac{m_{i2} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 \right]$$

Goal is $E \to 0$

A Nonlinear Formulation

Option A:

Define M as a matrix of 11 unknowns (i.e., $m_{34} = 1$)

And solve for m_{ij}

Can be made very efficient, especially for sparse matrices

Option B:

Define M as function of intrinsic and extrinsic parameters so that it is "recomputed" during each loop of the optimization