Fixed Point Iteration

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March 27, 2023

Definition

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- A fixed point of a function $f(x)$ is a point x where $f(x) = x$.
  This is true only when $f(x)$ is continuous

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- This is what we will apply in code

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- More Information - here

Finding the roots of an equation

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- Define function $g(x)$ which is obtained from $f(x) = 0$ such that $x = g(x)$ and $|g\prime(x) < 1|$

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- The root will be at x_n.

Sample equations

- $x^3 - x - 1$
- $x^3 - 2x - 5$
- $x^4 - 2x^3 - 5x - 2$