Subcarrier Weighting: A Method for Sidelobe Suppression in OFDM Systems

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***Abstract*— In this letter, a method for sidelobe suppression in OFDM systems is proposed and investigated. The proposed method is based on the multiplication of the used subcarriers with subcarrier weights. The subcarrier weights are determined in such a way that the sidelobes of the transmission signal are minimized according to an optimization algorithm which allows several optimization constraints. As a result, sidelobe suppression by subcarrier weighting reduces OFDM sidelobes by more than 10 dB in the average without requiring the transmission of any side information.**

***Index Terms*— Orthogonal frequency-division multiplexing (OFDM), sidelobe suppression.**

I. Introduction

THE ever increasing demand for wireless communications with high data rates entails a substantial need for additional spectral resources through more flexible and efficient use of the available spectrum. Orthogonal frequency-division multiplexing (OFDM) has high levels of out-of-band radiation due to the high sidelobes inherent in OFDM modulation. To enable higher spectral efficiencies and/or co-existence with legacy systems [1], this letter proposes a method to reduce the out-of-band radiation.

Existing methods for sidelobe suppression are based on the insertion of guard bands, i.e., subcarriers lying at the borders of the OFDM spectrum are deactivated, and/or windowing of the transmission signal in time domain [2]–[4]. However, both methods lead to a reduction in system throughput. The insertion of guard bands sacrifices bandwidth, whereas windowing expands the signal in time domain.

In this letter, a new method to significantly suppress the OFDM sidelobes is proposed and analyzed. This technique, referred to as subcarrier weighting (SW), is based on the multiplication of the used subcarriers with subcarrier weights which are chosen such that sidelobes are suppressed. Furthermore, it overcomes the problems of existing techniques as it decreases the waste of additional scarce spectral resources and avoids expanding the signal in time domain.

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II. OFDM Signal Model

An OFDM system with a total number of N subcarriers is considered. The input bits are symbol-mapped applying phase-shift keying (PSK) or quadrature amplitude modulation (QAM) and N complex-valued data symbols $d_n,\ n = 1, 2, \ldots, N$, are generated and stacked into a data symbol array $\mathbf{d} = (d_1, d_2, \ldots, d_N)^{\mathrm{T}}$, where $(.)^{\mathrm{T}}$ denotes transposition. The array $\mathbf{d}$ is fed into the sidelobe suppression unit which outputs $\bar{\mathbf{d}} = (\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_N)^{\mathrm{T}}$. The sidelobe suppression unit performs the multiplication of each symbol d_n with a subcarrier weight g_n. Thus, the entries of $\bar{\mathbf{d}}$ are given by

$$\bar{d}_n = g_n d_n,\ n = 1, 2, \ldots, N. \tag{1}$$

Finally, the vector $\bar{\mathbf{d}}$ is modulated on N subcarriers using the inverse discrete Fourier transform, parallel-to-serial conversion is performed, and a guard interval of length T_Δ exceeding the delay spread of the multipath channel is added as cyclic prefix. Note, throughout this letter it is assumed that the guard interval is much shorter than the length T_0 of the useful part of an OFDM symbol, and thus, its influence is neglected ($T_\Delta \approx 0$).

In the following, we design a weighting array $\mathbf{g} = (g_1, g_2, \ldots, g_N)^{\mathrm{T}}$, according to an optimization criterion for the sidelobe suppression.

III. Sidelobe Suppression by Subcarrier Weighting

A single non-weighted subcarrier $s_n(x),\ n = 1, 2, \ldots, N$, is represented in frequency domain as

$$s_n(x) = d_n \frac{\sin(\pi(x - x_n))}{\pi(x - x_n)},\ n = 1, 2, \ldots, N. \tag{2}$$

In (2), x is a normalized frequency given by

$$x = (f - f_0)T_0, \tag{3}$$

where f denotes the frequency and f_0 is the center frequency. In addition, x_n is the normalized center frequency of the nth subcarrier. In (2), rectangular time domain windowing at the transmitter with window length T_0 is implicitly assumed. When a different type of windowing, e.g., raised-cosine windowing, is applied, or a non-negligible guard interval length $T_\Delta > 0$ is assumed, (2) has to be adapted according to the Fourier transform of the chosen window.

As our goal is to suppress the sidelobes in a certain frequency range, we consider $s_n(x)$ only in that range. We observe M samples at the normalized frequencies $y_m,\ m = 1, 2, \ldots, M$, lying in the frequency range where the optimization of the sidelobes is performed. With that, (2) reduces to

$$s_{n,m} = s_n(y_m) = d_n \frac{\sin(\pi(y_m - x_n))}{\pi(y_m - x_n)}, \quad n = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, M. \tag{4}$$

Collecting $s_{n,m},\ m = 1,2,\ldots,M$, into a vector we obtain $\mathbf{s}_n = (s_{n,1}, s_{n,2}, \ldots, s_{n,M})^{\mathrm{T}},\ n = 1,2,\ldots,N$. Finally, stacking the vectors $\mathbf{s}_n$ into a matrix we get $\mathbf{S} = (\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_N)$.

To minimize the sidelobes of the weighted transmission signal $\bar{\mathbf{d}}$, we have to solve the following optimization problem

$$\mathbf{g} = \arg\min_{\tilde{\mathbf{g}}} \|\mathbf{S}\tilde{\mathbf{g}}\|^2 \tag{5}$$

where $\tilde{\mathbf{g}}$ is a trial value of $\mathbf{g}$.

In addition, we include two constraints on the weighting vector $\mathbf{g}$. The first constraint ensures that the data symbols $\bar{\mathbf{d}}$ after SW require just the same transmission power as the data symbols $\mathbf{d}$ before SW, i.e.,

$$\|\bar{\mathbf{d}}\|^2 = \|\mathbf{d}\|^2, \tag{6}$$

which in the case of PSK reduces to $\|\mathbf{g}\|^2 = N$.
The second constraint ensures that the elements of $\mathbf{g}$ are real-valued and lie between pre-defined limits, i.e.,

$$0 < g_{\min} \le g_n \le g_{\max}, \qquad g_{\min}, g_{\max}, g_n \in \mathcal{R}, \quad n = 1,2,\ldots,N. \tag{7}$$

Such a constraint guarantees that each subcarrier receives a certain amount of the transmission power which is inherently controlled through the ratio $\rho = g_{\max}/g_{\min}$. Furthermore, $g_{\min}$ and $g_{\max}$ can be selected such that a weighted symbol $\bar{d}_n$ remains in the same decision region as the original symbol d_n. In such case no signalling from transmitter to receiver is required, e.g., for PSK this is valid for $g_{\min} > 0$.

The optimization problem given in (5), together with the constraints in (6) and (7), can be generalized to a nonlinear programming problem with a quadratic equality (6) and a linear inequality constraint (7). To solve such an optimization problem many effective and reliable numerical algorithms exist, e.g., the projected Lagrangian method [5].

The principle of the SW technique is illustrated in Fig. 1 for the parameters $N = 5$, $g_{\max}/g_{\min} = \sqrt{4}$, and $\mathbf{d} = (1,1,1,1,1)^{\mathrm{T}}$. The optimization range spans 6 sidelobes at each side of the useful OFDM bandwidth and it starts at the first sidelobe outside the transmission bandwidth. To keep the dimensions of the matrix $\mathbf{S}$ low only one normalized frequency sample $s_n(y_m)$ per sidelobe is considered in the optimization range, i.e., M=12. In addition, these normalized frequencies y_m are chosen such that they correspond to the frequencies which lie in the middle of a sidelobe between two zero crossings. Note, considering more samples in the optimization range can lead to a slightly better sidelobe suppression, but increases the dimension of the matrix $\mathbf{S}$ and thus, the complexity. For simplicity, in Fig. 1 only the optimization range at the right-hand side of the transmission bandwidth is displayed, whereas the optimization is performed on both sides. The spectra of the individual subcarriers as well as the spectrum of the sum signal of all subcarriers are shown in frequency domain. From Fig. 1(a)-(b) it can be seen that in the case of SW the signals of the individual subcarriers are adapted so as to mainly cancel each other in the optimization range.

A possible drawback of the SW method is a degradation in bit-error-rate (BER) versus signal-to-noise ratio (SNR) performance as, due to the weighting, the subcarriers do not receive equal amounts of transmission power. Assuming a Rayleigh fading channel and BPSK modulation the mean BER P_{b} can be calculated according to

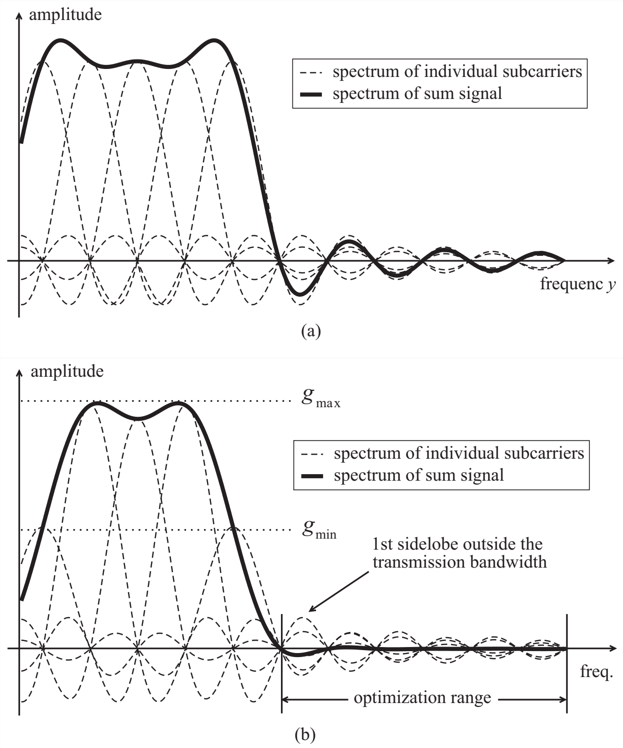

Fig. 1. Illustration of the SW technique: (a) standard OFDM signal without SW and (b) OFDM signal with SW.

$$P_{\mathrm{b}} = \int_0^\infty \int_0^\infty f_{yz}(y,z) \cdot P_{\mathrm{AWGN}}(y \cdot z \cdot \gamma_b)\, \mathrm{d}y\, \mathrm{d}z \tag{8}$$

where $\gamma_b = E_{\mathrm{b}}/N_0$ and E_{b} and N_0 are the energy per transmission bit and the noise spectral density, respectively. In addition, $f_{yz}(y,z)$ is the joint probability density function (PDF) that describes the influence of Rayleigh fading and SW on the SNR γ_b. In particular, the random variable z describes the received power after transmission over a Rayleigh fading channel and the random variable y the power fluctuations due to SW. The BER formula $P_{\mathrm{AWGN}}(y \cdot z \cdot \gamma_b)$ for AWGN is given by

$$P_{\mathrm{AWGN}}(y \cdot z \cdot \gamma_b) = \frac{1}{2}\mathrm{erfc}(\sqrt{y \cdot z \cdot \gamma_b}). \tag{9}$$

As y and z are statistically independent, the joint PDF $f_{yz}(y,z)$ separates into two PDFs and we get

$$P_{\mathrm{b}} = \int_0^\infty f_y(y) \cdot \left(\int_0^\infty f_z(z) \cdot P_{\mathrm{AWGN}}(y \cdot z \cdot \gamma_b)\, \mathrm{d}z \right) \mathrm{d}y. \tag{10}$$

The inner integral is well-known and its calculation leads to the BER in the case of Rayleigh fading [6]. Thus, we have

$$P_{\mathrm{b}} = \frac{1}{2} \int_0^\infty f_y(y) \cdot \left(1 - \sqrt{\frac{y\gamma_b}{1 + y\gamma_b}} \right) \mathrm{d}y. \tag{11}$$

To the best of our knowledge a closed-form solution for the PDF $f_y(y)$ is not available. Hence, we have resorted to numerical simulations which have shown that $f_y(y)$ can be well approximated by

$$f_y(y) = \alpha \cdot \delta(y - g_{\min}) + (1 - \alpha) \cdot \delta(y - g_{\max}) \tag{12}$$

where $\alpha \in (0,1)$ and $\delta(.)$ is the Dirac's delta function. Substituting (12) in (11) we finally obtain

$$P_{\mathrm{b}}=\frac{1}{2}\left(1-\alpha\sqrt{\frac{|g_{\min}|^2\gamma_b}{1+|g_{\min}|^2\gamma_b}}-(1-\alpha)\sqrt{\frac{|g_{\max}|^2\gamma_b}{1+|g_{\max}|^2\gamma_b}}\right). \quad (13)$$

The analytical approximation given in (13) is validated by Monte-Carlo simulations which are done in the next section.

IV. Numerical Results

In this section, numerical results are given that illustrate the effectiveness of the proposed sidelobe suppression method.

BPSK modulation is applied and the number of used subcarriers is set to $N = 12$. The optimization range consists of 16 sidelobes at each side of the spectrum and starts from the first sidelobe outside the OFDM transmission bandwidth. A single normalized frequency sample $s_n(y_m)$ is considered per sidelobe in the optimization range, i.e., $M = 32$, and is chosen as discussed in the previous section.

The spectra of the OFDM signals with and without SW are illustrated in Fig. 2 for the symbol vector $\mathbf{d} = (1, 1, \ldots, 1)^{\mathrm{T}}$. In the case of SW the ratio of $g_{\max}/g_{\min} = \sqrt{4}$ is used. The benefits of the SW technique are clearly visible. In comparison to OFDM without SW the sidelobes are suppressed by more than 10 dB in the optimization range.

The sidelobe suppression results as well as BER performances at a fixed SNR of $\gamma_b = 14$ dB averaged over all possible data symbol sequences, i.e., $2^N{=}2^{12}$ sequences, for OFDM applying SW with different ratios $g_{\max}/g_{\min}$ are given in Table I. It is noticeable that already for $g_{\max}/g_{\min}{=}\sqrt{4}$ a remarkable average suppression of more than 10 dB is achieved. A further increase of the ratio $g_{\max}/g_{\min}$ enables even better suppression. The reason for this lies in the fact that as this ratio grows the constraint from (7) becomes looser, thus allowing more degrees of freedom to find a solution of (5). However, SW results in an BER loss, since the subcarriers do not receive equal amounts of the transmission power. As $g_{\max}/g_{\min}$ grows, some subcarriers receive very small amounts of transmission power and cannot be decoded properly at the receiver resulting in a performance degradation. To evaluate the corresponding SNR loss, we consider OFDM transmission with $(g_{\max}/g_{\min}{>}1)$ and without $(g_{\max}/g_{\min}{=}1)$ SW over a mobile radio channel modelled as a Rayleigh fading channel. At the receiver we assume perfect channel knowledge and perform detection applying matched filtering. Both the BER as obtained by simulations and the BER calculated in (13) are given in Table I and they agree almost perfectly. Note, for the chosen simulation parameters, the parameter α from (13) is determined to be 0.63. As can be seen from Table I, with increasing $g_{\max}/g_{\min}$ the BER further degrades. Thus, there is a trade-off between the additional sidelobe suppression obtained by enlarging the ratio $g_{\max}/g_{\min}$ and the increased degradation in BER. Setting $g_{\max}/g_{\min} = \sqrt{4}$ seems to be a good compromise since a further increase of $g_{\max}/g_{\min}$ leads to a relatively high BER degradation with only moderate improvement in sidelobe suppression. Although not explicitly shown due to space limitations, we also note that for $g_{\max}/g_{\min}{=}\sqrt{4}$ and $\sqrt{6}$ this BER degradation translates into an SNR loss of 2.2 and 3.2 dB, respectively.

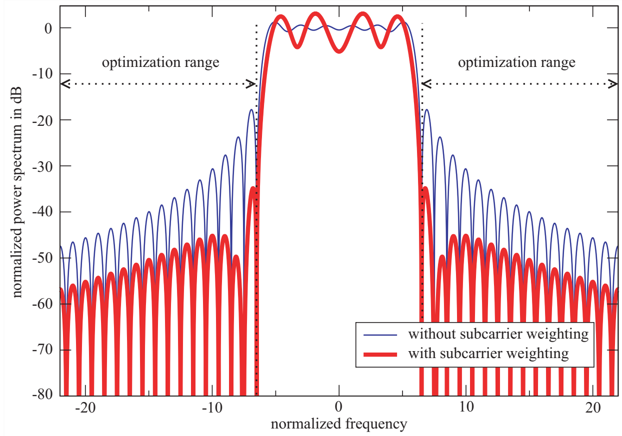

Fig. 2. Spectrum of OFDM signal with and without SW; $N = 12$; $M = 32$; $\mathbf{d} = (1, 1, \ldots, 1)^{\mathrm{T}}$; $g_{\max}/g_{\min} = \sqrt{4}$.

TABLE I
Average sidelobe suppression and BER for OFDM applying SW for different ratios $g_{\max}/g_{\min}$.

ratio $g_{\max}/g_{\min}$	1	$\sqrt{2}$	$\sqrt{4}$	$\sqrt{6}$	$\sqrt{8}$
average sidelobe suppression in dB	0.0	4.9	10.21	13.45	15.76
simulated BER at $\gamma_b = 14$ dB	0.0095	0.0116	0.0158	0.02	0.0231
approximated BER at $\gamma_b = 14$ dB	0.0095	0.0114	0.0153	0.0189	0.0224

V. Conclusions

In this letter, a new method to suppress sidelobes of OFDM transmission signals is introduced. Besides using this method to improve the spectral efficiency of OFDM based transmission systems, it can be applied to OFDM based overlay systems to avoid interference towards the legacy systems sharing the same frequency band. The proposed sidelobe suppression scheme does not require the transmission of any side information and is capable of reducing the sidelobes of OFDM signals by more than 10 dB. The price to pay for this achievement is a moderate loss in BER performance. Currently, the proposed sidelobe suppression method is extended to the application in OFDM based overlay systems, and a combination with existing sidelobe suppression methods is investigated.

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