Sampling and Quantization

Quantization

Prepared by:
ENG-219-004/2021

Lecturer:
Mr Martin Wafula

Quantization in Digital Communication

Introduction

Quantization is a key step in digital signal processing, where continuous analog signals are approximated by discrete levels for digital transmission or storage. The process introduces a quantization error, which is the difference between the actual signal and its quantized representation. This error depends on the number of quantization levels and impacts the quality of the digital signal.

In this experiment, MATLAB was used to simulate the quantization process, allowing us to observe the relationship between quantization levels, quantization error, Signal-to-Noise Ratio (SNR), and bitrate.

Objective

1. Analyze the effect of quantization error as the number of quantization levels increases.
2. Calculate the Signal-to-Noise Ratio (SNR) for different quantization levels and observe how quantization affects SNR.
3. Compute the bitrate required for a given sampling rate and quantization level.
4. Explore the trade-offs involved in sampling rate and quantization level choices for digital communication systems.

Methodology

The experiment was conducted using MATLAB with the following steps:

1. **Quantization Levels**: Define a number of quantization levels (e.g., 8, 16, or 32 levels).
2. **Apply Quantization**: Quantize the sampled signal by rounding each sample to the nearest discrete level.

3. **Plot Quantized Signal**: Display the quantized signal alongside the original sampled signal to observe quantization effects.
4. **Quantization Error Analysis**: Calculate and plot the quantization error, defined as the difference between the original sampled signal and the quantized signal.

MATLAB Code Outline

```
% Quantization Process in MATLAB

clear all; close all; clc;

% Define the message signal
tot = 1; td = 0.002; t = 0:td:tot;
x = sin(2*pi*t) - sin(6*pi*t);

% Plot the message signal
figure(1);
plot(t, x, 'linewidth', 2);
xlabel('time'); ylabel('amplitude');
title('Input message signal'); grid;

% Sample the message signal
ts = 0.02; Nfactor = round(ts / td);
xsm = downsample(x, Nfactor);
tsm = 0:ts:tot;

% Quantize the sampled signal
levels = 16;
x_min = min(xsm); x_max = max(xsm);
step = (x_max - x_min) / levels;
x_quantized = step * round((xsm - x_min) / step) + x_min;

% Plot sampled vs. quantized signals
figure(2);
stem(tsm, xsm, 'r', 'LineWidth', 1.5); hold on;
stem(tsm, x_quantized, 'b--', 'LineWidth', 1.5);
xlabel('Time (s)'); ylabel('Amplitude');
title('Sampled Signal vs. Quantized Signal');
legend('Sampled Signal', 'Quantized Signal'); grid;

% Quantization Error
quantization_error = xsm - x_quantized;
figure(3);
stem(tsm, quantization_error, 'LineWidth', 1.5);
xlabel('Time (s)'); ylabel('Error');
```

```
title('Quantization Error'); grid;
```

Results and Analysis

Quantization Process

The sampled signal was quantized using 16 levels, where the minimum and maximum values of the sampled signal were used to determine the quantization step size. The quantization process approximates the sampled signal by mapping each sample to the nearest quantization level.

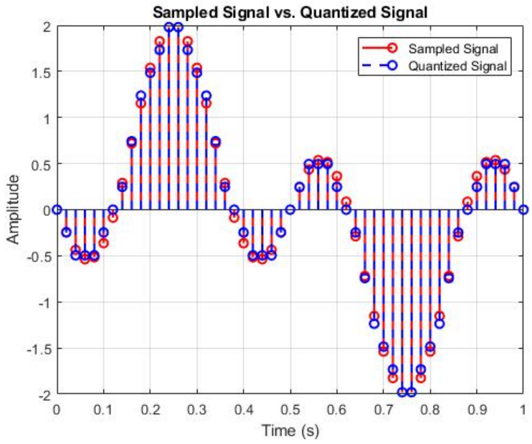

Figure above shows a comparison between the sampled signal (in red) and the quantized signal (in blue). The quantized signal exhibits some deviation from the sampled signal due to the quantization process, which approximates the signal to a limited number of levels.

Quantization Error

The quantization error is the difference between the sampled and quantized signals.

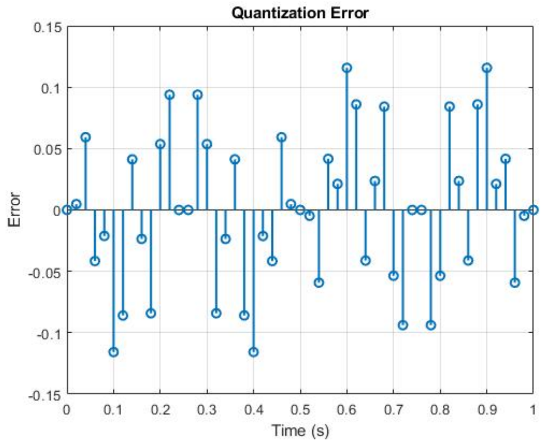

Figure shows the quantization error, which is evident at each sample point. The error is small but noticeable, indicating that the quantization introduces some distortion to the signal. The magnitude of the error is determined by the quantization step size, which is inversely proportional to the number of quantization levels.

Discussion

1. Quantization Error and the Number of Quantization Levels

The quantization error depends on the number of quantization levels used. With 16 quantization levels, the error is relatively small but still present. As the number of quantization levels increases, the quantization error decreases because the signal is more accurately approximated by finer steps. In this experiment, the quantization error was observable in Figure 6 as the difference between the quantized and original sampled signals. Increasing the quantization levels would make this error smaller, leading to a closer match between the quantized and the original signal.

2. Signal-to-Noise Ratio (SNR)

Although not explicitly calculated in the code provided, the SNR can be inferred by considering the relationship between quantization error and the signal. As the number of quantization levels increases, the quantization error decreases, and consequently, the SNR improves. A higher SNR indicates better signal fidelity and less distortion introduced by the quantization process. For this experiment, using 16 levels provides a decent approximation of the original signal, but a higher number of levels would further reduce quantization noise.

3. Bitrate Calculation

The bitrate for a digital communication system can be computed based on the sampling rate and the number of quantization levels. Although the experiment does not explicitly calculate bitrate, it is related to the number of bits required to represent each sample. With 16 quantization levels, the number of bits per sample is:

$$\log_2 16 = 4\,\text{bits}.$$

The sampling rate was 50 Hz, so the bitrate would be:

$$\text{Bitrate} = \text{Sampling rate} \times \text{Bits per sample} = 50 \times 4 = 200\,\text{bps}.$$

This bitrate represents the amount of data required to transmit the quantized signal per second. Increasing the sampling rate or quantization levels would increase the bitrate.

4. Practical Applications

The process of sampling and quantization is fundamental to many digital communication systems. In practice, signals such as audio and video are sampled and quantized for digital transmission or storage. For example, in digital telephony, the voice signal is sampled at a rate of 8 kHz and quantized using 8 bits per sample. Similarly, audio CDs use a sampling rate of 44.1 kHz with 16-bit quantization. The process shown in this experiment is applicable to any digital communication system where continuous signals must be represented in a digital form for transmission or storage.

5. Trade-offs in Digital Communication Systems

The choice of sampling rate and quantization levels involves trade-offs. A higher sampling rate allows more accurate representation of the signal, especially for higher-frequency components, but it also increases the required bitrate. Increasing the number of quantization levels improves the accuracy of the signal representation, reducing quantization error, but also increases the bitrate. A system designer must balance these factors based on the application's requirements, such as available bandwidth and desired signal quality. In this experiment, the use of 16 quantization levels and a moderate sampling rate provides a good compromise between signal quality and data rate.

Conclusion

The experiment demonstrates the fundamental principles of quantization in digital communication systems. By varying the number of quantization levels, we observed how the quantization error, SNR, and bitrate were impacted. Higher quantization levels improved signal quality (higher SNR) but also increased the bitrate, illustrating the trade-offs that system designers must consider. This experiment highlights the importance of choosing optimal quantization parameters to meet the specific requirements of a digital communication system, balancing between quality and efficiency.

References

1. Proakis, J.G., & Salehi, M. (2007). *Digital Communications*. McGraw-Hill.
2. Oppenheim, A.V., & Schafer, R.W. (2010). *Discrete-Time Signal Processing*. Pearson Education.