My bs notes

Makus

November 21, 2023

Contents

1 Introduction

just random ass notes, separted by sections.

1.1 forenotes

for the entire part of Computer Science, Chapter 3, the variable naming standard will be as follows:

$$\text{i is input}$$
$$\text{o is output}$$

unlesss stated otherwise.

2 math

2.1 general

2.1.1 bases

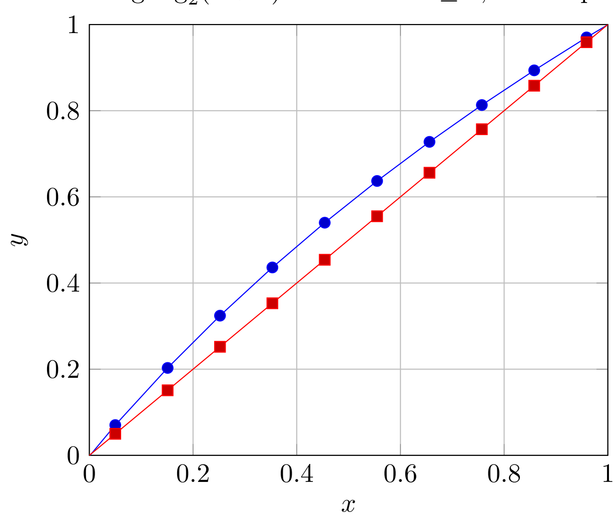
given

$$n = \text{base of number representation}$$
$$x = \text{number that is represented in base } n,$$

Then $\frac{x}{n}$ shiftes everything in x towards the left by one and $x \cdot n$ shiftes everything to the right.

2.1.2 logrithmic base 2

when taking $\log_2(1+x)$ where $0 < x \leq 1$, the output is approx. x.



but by adding μ to this, we can lower the amount of error to the average. to find μ, we can do a mean value theorem stated in calculus portion to find μ, which can be gotten from

$$\text{if} f(x) = \log_2(x+1), \text{ then}$$

$$f'(x) = \frac{1}{(x+1)\ln(2)}$$

since the average rate of change is

$$\frac{f(1) - f(0)}{1 - 0} = 1$$

then to get μ, the steps are

$$f'(c) = \frac{1}{(c+1)\ln(2)} = 1$$

$$1 = (c+1)\ln(2)$$

$$c = \frac{1}{\ln(2)} - 1$$

solving for c gives us the point on $f(x)$ where $f'(c) = 1$

given $c = \dfrac{1}{\ln(2)} - 1$, then we can graph the tangent line with the point slope form.

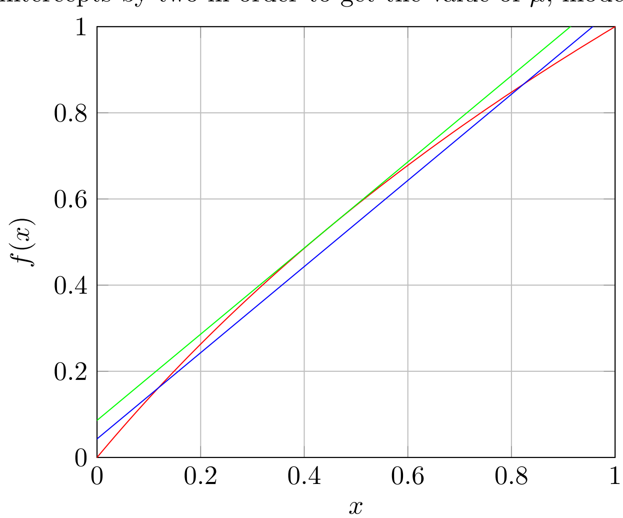
$y - f(c) = f'(c)(x - c)$, or

$$y = f(c) + f'(c)(x - c)$$

or

$$y = f(c) - c + x$$

since $f'(c) = 1$. since $f(c) - c$ is constant, it would be a straight line, modeled by the green line in the following the green line shows the tangent line with least error throughout. In order to find the overall average error, we can divide the sum of y intercepts by two in order to get the value of μ, modeled by the



blue line. red is $\log_2(x+1)$

green is $x + c - f(c)$, where $f(c) = \log_2(x+1)$

blue is $x + \mu$, where $\mu = \dfrac{f(c) - c}{2}$

2.2 calculus

this section will follow the order in which Calculus BC is taught, with first half from start of school to optimization problems, and second half being from integration to everything else.

2.2.1 First Half

2.2.1.1 Mean Value Theorem

Mean Value Theorem, or MVT, states that if f(x) is continuous and differentiable within the range $a \leq x \leq b$, then there exists a point c where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

2.2.2 Second Half

2.2.2.1 Integration

it is the inverse of differentiation, much like addition and subtraction. This process is denoted with the equation below.

$$\int f'(x)dx = f(x)$$

This shows the process of integration. If something isn't integratable, then U-substitution can be used.

2.2.2.2 U substitution

A variable can be used to replace a portion of a function. How this will work is:

1. Substitute a part of the function as u.
2. Where $u = f(x)$, the derivative would be $\dfrac{du}{dx} = f'(x)$, or $du = f'(x)dx$
3. Isolate dx an whatever way needed, then replace it back into the original function, so that the integral will be in terms of u (meaning it would be $\int du$)
4. After integrating, replace u with $f(x)$

Following these steps would look something like:

$$\int \frac{x}{\sqrt{1-4x^2}}dx \qquad u = 1-4x^2$$

$$\frac{du}{dx} = 8x$$

$$du = 8xdx$$

$$\int \frac{1}{\sqrt{u}}du$$

$$\int u^{-1/2}du$$

$$\int u^{-\frac{1}{2}}du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$2(1-4x^2)^{\frac{1}{2}} + c$$

2.2.2.3 Definite Integrals

definite integrals output the area beneath the curve of the function, with the bounds of α and β, where $\alpha < \beta$. Notation denoted below:

$$\int_{\alpha}^{\beta} f'(x)dx = [f(x)]_{x=\beta}^{x=\alpha}$$

if α and β are switched, then:

$$\int_{\beta}^{\alpha} f'(x)dx = -[f(x)]_{x=\beta}^{x=\alpha}$$

some more properties include:

1. $\int_{\alpha}^{\alpha} f(x)dx = 0$
2. $\int_{\alpha}^{\beta} cf(x)dx = c\int_{\alpha}^{\beta} f(x)dx$(including negative numbers)
3. $\int_{\alpha}^{\beta}(f(x) \pm g(x))dx = \int_{\alpha}^{\beta} f(x)dx \pm \int_{\alpha}^{\beta} g(x)dx$
4. $\int_{\alpha}^{\beta}(f(x)dx) + \int_{\alpha}^{\gamma}(f(x)dx) = \int_{\alpha}^{\gamma}(f(x)dx)$
5. $\int_{-\alpha}^{\alpha} f(x)dx = 2\int_{0}^{\alpha} f(x)dx$ assuming f(x) is even
6. $\int_{-\alpha}^{\alpha} f(x)dx = 0$ assuming f(x) is odd

2.2.2.4 Fundamental Theorem of Calculus part 1

If f is continuous on $[a, b]$, then the following would be true:

$$\int_{\alpha}^{\beta} f'(x)dx = [f(x)]_{x=\alpha}^{x=\beta} = f(b) - f(a) \tag{1}$$

3 Computer Science

will be using c++ for every example

3.1 bit shit

3.1.1 bitwise operators to stop forgetting

& is and
| is or
∧ is xor
∼ for not
>> or << for moving bits left and right respectively

3.1.1.1 bitwise operator quirks?

In C++, every number is represented in binary, or base 2. Thus if anyone were to divide a number by two, then

```
x=x>>1;//shifting towards the right
```

works the same, but is faster. it also rounds up. Conversely, if anyone were to multiply by two, then

```
x=x<<1;//shifting towards the left
```

does the same.

3.1.2 IEEE 754

3.1.2.1 overview

This is how floats in C++ are stored. Floats are stored in 32 bit. the first bit is sign. The second to ninth bit is exponent, and the 10th to 32th bit is the mantissa. the exponent has an offset of 127, because it needs negative exponents. The mantissa, 23 bits, is the representation of the number(scientific notation). The equation to get from float to the final number is given by:

$$(1 + \frac{M}{2^{23}}) \cdot 2^{E-127} \tag{2}$$

3.1.2.2 nitty griddy

The layout of this(the float) is

sign bit:1 bit
exponent:1 byte(8 bits)
mantissa:23 bits

So if we were to follow this and try to extract exponents, we can do

```
o = ((0xff<<23)&i)-127;
```

and extracting the mantissa would be just

```
o = ((~(0xff<<23))&i);
```

assuming the variables i and o are unsigned longs.

3.2 optimizations

4 physics

4.1 Unit 4

4.1.1 Work

4.1.1.1 definition

A force that causes an object to move. Units is J, $Joules$.

4.1.1.2 rules

there are three rules for work to exist.

- force must be applied
- there must be a displacement
- there must be a direction

4.1.1.3 equations

$$W = fd$$
$$W = fd\cos(\theta)$$
$$P = \frac{\Delta w}{t}$$
$$\text{where } P \text{ is power}$$
$$K = \frac{mv^2}{2}$$

4.1.2 Energy

4.1.2.1 Energy

the ability to do work. Units is J, $Joules$. $w = e$

4.1.2.2 kinetic energy

object with velocity. K represents kinetic energy.

$$K = \frac{mv^2}{2}$$

4.1.2.3 gravitational potential energy

stored energy associated with an object's height above some reference point.

$$U_g = mgy$$