ID	Model	Type	Original Equation	Lyapunov Functions	Source	Notes
			Category I: Voltage-Constant / Motion-Equation-Based Models			
01	Multi-machine swing equation model	Classical power system (lossless swing equations)	Kinetic / swing equation for each generator: $$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_j B_{kj}V_kV_j\sin(\delta_k-\delta_j) - P_k = 0$$ k=1…, n. This is the classical multi-machine swing-equation model with constant voltage magnitudes and static impedance loads.	Classic energy function, which corresponds to the traditional direct method for swing-equation systems $$E = \sum_k \frac{m_k\dot{\delta}_k^2}{2} - \sum_{\{k,j\}} B_{kj}V_kV_j\cos\delta_{kj} - \sum_k P_k\delta_k$$ The first term represents the kinetic energy of the generators, and the second term the potential energy stored in the network inductances. Because of damping d_k, this energy is monotonically decreasing along trajectories and acts as the traditional Lyapunov/energy function for swing-equation systems.	T. L. Vu and K. Turitsyn, "Lyapunov Functions Family Approach to Transient Stability Assessment," in *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1269-1277, March 2016, doi: 10.1109/TPWRS.2015.2425885.	1. The paper generalises **classical energy methods** and extends the concept of an energy function to a Lyapunov function family constructed via semidefinite programming. 2. The proposed method builds a convex set of Lyapunov functions that can certify broader regions of transient stability than closest-UEP energy methods, without requiring fault-on trajectories.
02	Compact state-space representation of the same multi-machine swing-equation model	Classical power system (compact state-space form)	Compact State Space $$\dot{x} = Ax - BF(Cx),$$ with the matrix A given by the following expression: $$A = \begin{bmatrix} O_{n\times n} & I_{n\times n} \\ O_{n\times n} & -M^{-1}D \end{bmatrix},$$ $$B = \begin{bmatrix} O_{n\times\lvert\mathcal{E}\rvert} \\ M^{-1}E^TB \end{bmatrix}, C = [E \;\; O_{\lvert\mathcal{E}\rvert\times n}].$$	Lyapunov Function Family, constructed through LMI conditions to guarantee $\dot{V}(x) < 0$ over a polytope region. $$V(x) = \frac{1}{2}x^TQx - \sum_{\{k,j\}\in E} K_{\{k,j\}}\left(\cos\delta_{kj} + \delta_{kj}\sin\delta_{kj}^*\right)$$ where $Q \succ 0$ and diagonal matrices $K, H \succ 0$ satisfy the LMI $$\begin{bmatrix} A^\top Q + QA & R \\ R^\top & -2H \end{bmatrix} \preceq 0,$$ with $R = QB - C^\top H - (KCA)^\top$. Every triple (Q, K, H) satisfying this LMI defines a Lyapunov function whose derivative is non-positive inside the polytope $P = \{x: \lvert \delta_{kj} + \delta_{kj}^* \rvert < \pi\}$		
03	3-machine classical system without transfer conductances	Classical power system (non-polynomial swing equations)	Four-dimensional swing equations for a 3-machine classical power system without transfer conductances, written in shifted coordinates with $x_1 = \delta_1, x_2 = \omega_1, x_3 = \delta_2, x_4 = \omega_2$. This is the **original non-polynomial system** $\dot{x} = f(x)$ for which the Lyapunov function is constructed. $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\sin(x_1) - 0.5\sin(x_1 - x_3) - 0.4x_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -0.5\sin(x_3) - 0.5\sin(x_3 - x_1) - 0.5x_4 + 0.05 \end{aligned}$$	Polynomial Lyapunov function $V(x)$expressed in the original variables $x_1, \dots, x_4$. It is obtained as the solution of an SOS program (cast as an SDP) on the recast polynomial DAE. $$\begin{aligned} V(x) = {} & 0.0030\sin(x_1) - 0.00008x_4 - 0.2683\cos(x_1) \\ & - 0.2649\cos(x_3) - 0.0030x_2 + 0.0044\sin(x_3) \\ & - 0.2377\cos(x_1)\cos(x_3) + 0.0008\cos(x_1)\sin(x_1) \\ & + 0.0047\cos(x_1)\sin(x_3) - 0.0037\cos(x_3)\sin(x_1) \\ & - 0.0092\cos(x_3)\sin(x_3) - 0.1588\sin(x_1)\sin(x_3) \\ & - 0.0109\cos(x_1)^2 + 0.0203\cos(x_3)^2 - 0.0004x_2x_4 \\ & - 0.0016x_2\cos(x_1) + 0.0047x_2\cos(x_3) \\ & + 0.0011x_4\cos(x_1) - 0.0010x_4\cos(x_3) \\ & + 0.0579x_2\sin(x_1) + 0.0219x_2\sin(x_3) \\ & + 0.0195x_4\sin(x_1) + 0.0972x_4\sin(x_3) \\ & + 0.1461x_2^2 + 0.1703x_4^2 + 0.7614. \end{aligned}$$	M. Anghel, F. Milano and A. Papachristodoulou, "Algorithmic Construction of Lyapunov Functions for Power System Stability Analysis," in IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 60, no. 9, pp. 2533-2546, Sept. 2013, doi: 10.1109/TCSI.2013.2246233.	1. The paper develops an algorithmic framework that recasts non-polynomial power-system swing equations into a polynomial DAE and constructs **polynomial Lyapunov functions** via SOS-based semidefinite programming to certify transient stability. 2. Through two test cases (a three-machine lossless system and a two-machine–infinite-bus system with transfer conductances), the method provides inner approximations of the region of attraction that can
04	2-machine vs infinite-bus system with transfer conductances	Classical power system (non-polynomial swing equations with conductances)	Four-dimensional swing equations of a **two-machine versus infinite-bus** classical power system **with transfer conductances**, written in shifted coordinates around the equilibrium. The states are $x_1 = \delta_1, x_2 = \omega_1, x_3 = \delta_2, x_4 = \omega_2$; the infinite bus has no rotor dynamics. This four-state model is the **original non-polynomial system** $\dot{x} = f(x)$used in the Lyapunov construction. $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 33.5849 - 1.8868\cos(x_1 - x_3) - 5.2830\cos(x_1) \\ &\quad - 16.9811\sin(x_1 - x_3) - 59.6226\sin(x_1) \\ &\quad - 1.8868x_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= 11.3924\sin(x_1 - x_3) - 1.2658\cos(x_1 - x_3) \\ &\quad - 3.2278\cos(x_3) - 1.2658x_4 - 99.3671\sin(x_3) \\ &\quad + 48.4810 \end{aligned}$$	In the presence of transfer conductances, $V(x)$ serves as a Lyapunov function for the **original non-polynomial system** and provides an inner approximation of its region of attraction. It is obtained by solving the same SOS-based semidefinite program on the polynomial DAE corresponding to the model. $$\begin{aligned} V(x) = {} & 0.0036x_2 - 0.0026x_4 - 0.7007\cos(x_1) \\ & - 0.7866\cos(x_3) - 0.2762\sin(x_1) - 0.2702\sin(x_3) \\ & - 0.1905\cos(x_1)\cos(x_3) + 0.2072\cos(x_1)\sin(x_1) \\ & + 0.0467\cos(x_1)\sin(x_3) + 0.0690\cos(x_3)\sin(x_1) \\ & + 0.2235\cos(x_3)\sin(x_3) - 0.0559\sin(x_1)\sin(x_3) \\ & - 0.0744\cos(x_1)^2 - 0.1044\cos(x_3)^2 + 0.0015x_2x_4 \\ & - 0.0076x_2\cos(x_1) + 0.0040x_2\cos(x_3) \\ & + 0.0042x_4\cos(x_1) - 0.0016x_4\cos(x_3) \\ & + 0.0138x_2\sin(x_1) - 0.0018x_2\sin(x_3) \\ & + 0.0056x_4\sin(x_1) + 0.0091x_4\sin(x_3) \\ & + 0.0075x_2^2 + 0.0059x_4^2 + 1.8567. \end{aligned}$$		

						go beyond classical closest-UEP energy methods and remain applicable even when no exact energy function is available.
05	Adaptive Lyapunov Function (ALF) power-system model	Structure-preserving multimachine power system in Lur’e form	Nonlinear system in Lur’e form $\dot{x} = Ax + B\phi(Cx),$ which is later instantiated by a **structure-preserving multimachine power-system model** with first-order dynamic loads and second-order generator swing equations (see Eqs. (15a)–(15b)). $$r_i\dot{\delta}_i + \sum_{k=1}^{n_l} \sigma_k \left[f_{ki} \sin(\bar{\boldsymbol{c}}_k \boldsymbol{\delta} + \varphi_k) + t_{ki} \sin(-\bar{\boldsymbol{c}}_k \boldsymbol{\delta} + \varphi_k) \right] = -P_{di}, i = 1, 2, \cdots, n_c, \quad (15a)$$ $$m_i\ddot{\delta}_i + d_i\dot{\delta}_i + \sum_{k=1}^{n_l} \sigma_k \left[f_{ki} \sin(\bar{\boldsymbol{c}}_k \boldsymbol{\delta} + \varphi_k) + t_{ki} \sin(-\bar{\boldsymbol{c}}_k \boldsymbol{\delta} + \varphi_k) \right] = P_{mi}, i = n_c + 1, n_c + 2, \cdots, n_b, \quad (15b)$$	**Adaptive quadratic Lyapunov function:** $V(x) = x^{\mathrm{T}}Px,\ P \geq 0.$ where $P \in \mathbb{S}^n, P \geq 0$. The matrix P, the adjustable neighborhood $\Omega(\gamma)$, and the derivative bound $\dot{V}(x) \leq \eta + \alpha V(x)$ in $\Omega(\gamma)$ are co-designed via the robust LMI (9), giving an adaptive local Lyapunov function used to estimate the critical clearing time (CCT).	Z. Qiu, C. Duan, W. Yao, P. Zeng and L. Jiang, "Adaptive Lyapunov Function Method for Power System Transient Stability Analysis," in IEEE Transactions on Power Systems, vol. 38, no. 4, pp. 3331-3344, July 2023, doi: 10.1109/TPWRS.2022.3199448.	Adaptive Lyapunov Function (ALF) method that co-designs a quadratic Lyapunov function, its neighbourhood and derivative bounds via LMIs for structure-preserving multimachine power-system models.
06	Multi-machine structure-preserving power system (lossless network)	Classical structure-preserving DAE (generators + detailed loads)	For each generator bus i: $\dot{\theta}_i = w_i - w_i$ $$M_i\dot{\omega}_i = T_{Mi} - \sum_{j=1}^{n+m} V_i V_j B_{ij} \sin(\theta_i - \theta_j),$$ $i = n + 1, \ldots, n + m$ and for each load bus i: $$P_{Li}(V_i) = \sum_{j=1}^{n+m} V_i V_j B_{ij} \sin(\theta_i - \theta_j), i = 1, \ldots, n$$ $$Q_{Li}(V_i) = -\sum_{j=1}^{n+m} V_i V_j B_{ij} \cos(\theta_i - \theta_j), i = 1, \ldots, n.$$ with $G_{ij} = 0$(lossless network).	Structure-preserving energy function (TEF) for the multi-machine lossless network $$V(\theta, \tilde{\omega}) = V_{KE}(\tilde{\omega}) + V_p(\theta) + V_d(\theta)$$ where the kinetic-energy term is $V_{KE}(\tilde{\omega}) = \frac{1}{2}\sum_i M_i\tilde{\omega}_i^2$ and $V_p(\theta), V_d(\theta)$collect the potential energy stored in the network and the path-dependent term due to conductances, for example $$V_d(\theta) = \sum_{i=1}^{m-1}\sum_{j=i+1}^{m} I_{ij}$$ $$V_p(\theta) = -\sum_{i=1}^{m} P_i(\theta_i - \theta_i^s) - \sum_{i=1}^{m}\sum_{j=i+1}^{m} C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s)$$ In the **lossless** case $\mathrm{G_{ij}} = 0$and with fixed voltage magnitudes, this reduces to the classical Hill-type TEF: $$V(\delta, \omega) = \sum_i \frac{M_i}{2}\omega_i^2 + \sum_{(i,j)\in\mathcal{E}} E_i E_j B_{ij}\big(1 - \cos(\delta_i - \delta_j)\big),$$ where the rotor angles $\delta_i$are measured relative to a synchronous reference (e.g. the centre-of-inertia angle).	P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*, rev. printing, Stipes Publishing, 2007, Chapter 9, Section 9.8 “Structure-Preserving Energy Functions”, eqs. (9.49), (9.52)–(9.53), (9.69)–(9.72).	Sauer–Pai’s multimachine TEF; in the lossless COI frame, it becomes the standard Hill-type energy function widely used for rotor-angle stability studies.
07	Polynomial surrogate of a multi-machine power-system swing model (6-state approximation)	Polynomial swing-system model with a rational Lyapunov function obtained via optimization	**A double-machine versus infinite bus power system with transfer conductances is defined by: (where x1 and x3 denote the generator phase angles, x2 and x4 denote the angular velocities. A stable equilibrium point can be found at (0.4680,0, 0.4630, 0))** $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 33.5849 - 1.8868\cos(x_1 - x_3) - 5.2830\cos(x_1) \\ &\quad - 59.6226\sin(x_1) - 16.9811\sin(x_1 - x_3) - 1.8868x_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= 48.4810 + 11.3924\sin(x_1 - x_3) - 3.2278\cos(x_3) \\ &\quad - 99.3761\sin(x_3) - 1.2658\cos(x_1 - x_3) - 1.2658x_4 \end{aligned}$$.	**Optimal rational Lyapunov function** $V_1(y)$computed via convex optimisation $$V_1(y) = \frac{y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_1^4 - y_1^2y_3^2 + y_3^4}{1 + 2y_1 + y_2 - 2y_3 + 8y_1^2 + 4y_2^2 + 4y_3^2},$$	D. Han, A. El-Guindy and M. Althoff, "Power systems transient stability analysis via optimal rational Lyapunov functions," *2016 IEEE Power and Energy Society General Meeting (PESGM)*, Boston, MA, USA, 2016, pp. 1-5, doi: 10.1109/PESGM.2016.7741322.	Uses optimised rational Lyapunov functions to verify transient stability for a polynomial surrogate of the multi-machine system, providing larger certified regions of attraction than classical quadratic forms.

			Six-dimensional polynomial swing-system approximation $\dot{y}=f(y)$used for rational Lyapunov construction: $y=(y_1,y_2,y_3,y_4)^T=(x_1-0.4680,x_2,x_3-0.4630,x_4)^T$ (This is the reduced-order polynomial surrogate model used for optimising the rational Lyapunov function.) $\dot{y}_1 = y_2$ $\dot{y}_2 = 33.5849-1.8868y_2-1.8868\eta_1-5.2830\eta_2 -59.6226\eta_3-16.9811\eta_4$ $\dot{y}_3 = y_4$ $\dot{y}_4 = 48.4810-1.2658y_4-1.2658\eta_1+11.3924\eta_4 -3.2278\eta_5-99.3761\eta_6$			
08	Multi-machine structure-preserving power system with UPFC–SMES	Structure-preserving SPM with direct Lyapunov energy function	A power system with N buses and M generators without an exciter and governor is considered. The system is assumed to be lossless. The equations governing the system are: $\dot{\delta}_i=\omega_i$ $M_i\dot{\omega}_i=P_{mi}-P_{Gi}-D_i\omega_i i=1..M$ $T'_{doi}\dot{E}'_{qi}=\frac{x'_{di}-x'_{di}}{x'_{di}}V_{M+i}\cos(\delta_l-\theta_{M+i})+E_{fdi}-\frac{x_{di}}{x_{di}}E'_{qi}$ Generated active and reactive electric powers are given by $P_{Gi}=\frac{1}{X'_{di}}\prime E'_{qi}V_{M+i}\sin(\delta_i-\theta_{M+i}) -\frac{x'_{di}-x_{qi}}{2x'_{di}x_{qi}}V^2_{M+i}\sin(2(\delta_i-\theta_{M+i}))$ $Q_{Gi}=\frac{1}{X'_{di}}[E'_{qi}V_{M+i}\cos(\theta_{M+i}-\delta_j)-V^2_{M+i}] +\frac{x'_{di}-x_{qi}}{2x'_{di},x_n}V^2_{M+i}[\cos(2(\delta_i-\theta_{M+i})-1)]$ The injected real and reactive powers into k−th node are $P_k=\sum_{i=M+1}^{M+N}\mathrm{X}B_{ki}V_kV_l\sin(\theta_k-\theta_i)$ $Q_k=-\sum_{i=M+1}^{M+N}B_{ki}V_kV_i\cos(\theta_k-\theta_i)$ The equilibrium of powers at load buses gives the load flow equations as $P_k+P_{Jik}-P_{Gk}=0$ $Q_k+Q_{Lk}-Q_{Gk}=0$	With $\dot{x}_i$assumption of =0, the direct Lyapunov function for the SPM power system without control, UPFC-SMES, is given by $w(\omega,\delta,E'_q,V,\theta)=w_1+w_2+C_0$ $w_1=\frac{1}{2}\sum_{k=1}^{M}M_k\omega_k^2$ $w_2=\sum_{i=1}^{8}w_{2i}$ Where w_1 is kinetic energy and w_2 is overall potential energy which is defined as $w_{21}=-\sum_{k=1}^{M}P_{mk}\delta_k$ $w_{22}=\sum_{k=M+1}^{M+N}P_{Lk}\theta_k$ $w_{23}=\sum_{k=M+1}^{M+N}\int\frac{Q_{Lk}}{V_k}dV_k$ $w_{24}=\sum_{k=M+1}^{2M}\frac{1}{2x'_{dk-M}}[E'^2_{qk-M}+V_k^2-2E'_{qk-M}V_k\cos(\delta_{k-M}-\theta_k)]$ $w_{25}=-\frac{1}{2}\sum_{k=M+1l}^{M+N}\sum_{M=M+1}^{M+N}B_{kl}V_kV_l\cos(\theta_k-\theta_l)$ $w_{26}=\sum_{k=M+1}^{2M}\frac{x'_{dk-M}-x_{qk-M}}{4x'_{dk-M}x_{qk-M}}[V_k^2-V_k^2\cos\left(2(\delta_{k-M}-\theta_k)\right)]$ $w_{27}=-\sum_{k=1}^{M}\frac{E_{fdk}E'_{qk}}{x_{dk}-x'_{dk}},$ $w_{28}=\sum_{k=1}^{M}\frac{E'^2_{qk}}{2(x_{dk}-x'_{dk})}$ C_0 is a constant, such that at the post-fault equilibrium point, the total energy is equal to zero.	A. Bidadfar, M. Abedi, M. Karari and Chia-Chi Chu, "Power swings damping improvement by control of UPFC and SMES based on direct Lyapunov method application," *2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, Pittsburgh, PA, USA, 2008, pp. 1-7, doi: 10.1109/PES.2008.4596175.	Uses the structure-preserving energy function $w(x)$as a Lyapunov function and designs UPFC–SMES supplementary controls that make $\dot{w}$more negative, thereby improving damping of power swings and enlarging the transient stability margin of the multimachine system.
09	swing dynamics, reduced network model	Motion-Equation-Based / Analysis (quasi-gradient structure)	Traditional equations of the Multi-machine Swing Model $\dot{\delta}=\omega$ $\dot{\omega}=-M^{-1}(D\omega+f(\delta)-P)$ where $M=\mathrm{diag}(M_1,M_2,\ldots,M_n)$ $D=\mathrm{diag}(D_1,D_2,\ldots,D_n)$ and $f_i(\delta)=E_i^2G_{ii}+\sum_{j=1,\ j\neq i}E_iE_jB_{ij}\sin(\delta_i-\delta_j)$	$V(\omega,\delta)=\sum_{k=1}^{n}\frac{1}{2}M_k\omega_k^2+(E_k^2G_{kk}-P_k)$ $+\sum_{k=1}^{n}\sum_{j=k+1}^{n}E_kE_jB_{kj}[\cos(\delta_k^0-\delta_j^0)-\cos(\delta_k-\delta_j)]$	C. L. DeMarco, "A new method of constructing Lyapunov functions for power systems," *1988., IEEE International Symposium on Circuits and Systems*, Espoo, Finland, 1988, pp. 905-908 vol.1, doi: 10.1109/ISCAS.1988.15070.	Introduces the **quasi-gradient** concept and shows that the reduced swing-equation model admits a Lyapunov function when the dynamics can be written in quasi-gradient form.
10	swing dynamics & flux decay dynamics, reduced network model	Motion-Equation-Based / Analysis (quasi-gradient extension with				Extends the quasi-gradient Lyapunov construction to **swing dynamics**

		voltage dynamics)	$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = \sum_{j=1}^{n} B_{ij} (E_i^0 E_j^0 \sin\delta_{ij}^0 - E_i E_j \sin\delta_{ij})$$ $$\frac{dE_i}{dt} = -\alpha_i (E_i - E_i^0) - \beta_i \sum_{j=1;\ j\neq i}^{n} B_{ij} E_j (\cos\delta_{ij}^0 - \cos\delta_{ij})$$	$$V(\delta, \omega, \Delta E) = 1/2\omega^T M\omega + 1/2\Delta E^T \alpha\beta^{-1}\Delta E + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_i E_j(\cos\delta_{ij}^0 - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^0) E_i^0 E_j^0 \sin\delta_{ij}^0] \quad (9)$$		**with flux-decay (voltage) states**, yielding an explicit Lyapunov function that includes both rotor-angle and internal voltage dynamics.
11	swing dynamics, structure-preserving network model	Motion-Equation-Based / Analysis (structure-preserving, integral-form Lyapunov)	$$\dot{\alpha} = T_1\omega_1 - T_2 D_2^{-1} T_2^T [f(\alpha) - P^0]$$ $$\dot{\omega}_1 = -M^{-1} D_1 \omega_1 - M^{-1} T_1^T [f(\alpha) - P^0]$$	$$V(\alpha, \omega_1) = \frac{1}{2}\omega^T M\omega + \int_{\alpha^0}^{\alpha} [f(\xi) - f(\alpha^0)]^T d\xi$$		Develops a Lyapunov function for **structure-preserving swing dynamics** using an **integral (potential-energy) form**, and shows that the quasi-gradient framework applies to network models with additional control dynamics.
Category II: Excitation and Voltage-Dynamic Models						
01	Lossy two-machine classical flux-decay model	Classical flux-decay synchronous generator model with excitation control (extends to lossy multi-machine systems)	**Classical flux-decay model of an n-machine system** $$\dot{\delta}_i = \omega_i,$$ $$\dot{\omega}_i = -D_i\omega_i + P_i - G_{ii}E_i^2 - E_i \sum_{j=1, j\neq i}^{n} E_j Y_{ij} \sin(\delta_{ij} + \alpha_{ij}),$$ $$\dot{E}_i = -a_i E_i + \sum_{j=1, j\neq i}^{n} b_{ij} E_j \cos(\delta_{ij} + \alpha_{ij}) + e_{f_i}^* + u_i,$$ specialised in the paper to a **two-machine system** $$\dot{\delta}_1 = \omega_1,$$ $$\dot{\omega}_1 = -D_1\omega_1 + P_1 - [G_{11}E_1^2 + E_1E_2Y_{12}\sin(\delta_{12} + \alpha_{12})],$$ $$\dot{E}_1 = -a_1E_1 + b_{12}E_2\cos(\delta_{12} + \alpha_{12}) + e_{f_1}^* + u_1,$$ $$\dot{\delta}_2 = \omega_2,$$ $$\dot{\omega}_2 = -D_2\omega_2 + P_2 - [G_{22}E_2^2 + E_2E_1Y_{21}\sin(\delta_{21} + \alpha_{21})],$$ $$\dot{E}_2 = -a_2E_2 + b_{21}E_1\cos(\delta_{21} + \alpha_{21}) + e_{f_2}^* + u_2.$$ (equations (2)).	**Control Lyapunov function (CLF) candidate:** $$V(\delta, \omega, E, \xi) = \psi(\delta) + \frac{1}{2}\sum_{i=1}^{2} \eta_i \omega_i^2 + \frac{1}{2}\sum_{i=1}^{2} \mu_i (E_i - \lambda_i(\delta, \xi)E_i^*)^2 + \frac{1}{2}\sum_{i=1}^{2} \rho_i (\lambda_i(\delta, \xi) - \xi_i)^2,$$ where $\psi(\delta) \geq 0$ is a potential-energy-like term, $\lambda(\delta, \xi)$ is a cross-term depending on an auxiliary state ξ, and $\eta_i, \mu_i, \rho_i > 0$ are weighting coefficients. *In the case study, $\psi(\delta)$ is chosen as* $$\psi(\delta) = \sigma(1 - \cos(w(\delta_{21} - \delta_{21}^*)))$$	J. Gao, B. Chaudhuri and A. Astolfi, "Lyapunov-based Transient Stability Analysis," *2022 IEEE 61st Conference on Decision and Control (CDC)*, Cancun, Mexico, 2022, pp. 5099-5104, doi: 10.1109/CDC51059.2022.9992811.	Proposes a new explicit **control Lyapunov function with a flexible potential-energy-like term and an auxiliary state**, which generates a cross-term and guarantees a negative-definite time derivative for lossy multi-machine models.
02	Multi-machine power system with controllable series devices (UPFC / CSC / QBT) using the injection model	Structure-preserving / reduced-network model with flux-decay (one-axis) generators and constant-real-power loads; energy-function-based CLF control	The dynamics of the generators are described by the following differential equations (with respect to the COI reference frame). For k=1⋯n, $$\dot{\tilde{\delta}}_k = \tilde{\omega}_k$$ $$M_k\dot{\tilde{\omega}}_k = P_{mk} - P_{ek} - D_k\tilde{\omega}_k - \frac{M_k}{M_T}P_{COI}$$ $$T'_{dok}\dot{E}'_{qk} = \frac{x_{dk} - x'_{dk}}{x'_{dk}} V_{k+n}\cos(\delta_k - \theta_{k+n}) + E_{f\,dk} - \frac{x_{dk}}{x'_{dk}}E'_{qk}$$ where $P_{COI} = \sum_{k=1}^{n}(P_{mk} - P_{ek})$ and P_{ek} is the generated electrical power. For the lossless system, the following equations can be written at bus k, where P_k is the real power, and Q_k is the reactive power injected into the system from bus k. $$P_k = \sum_{l=n+1}^{2n+N} B_{kl}V_kV_l\sin(\theta_k - \theta_l)$$ $$Q_k = -\sum_{l=n+1}^{2n+N} B_{kl}V_kV_l\cos(\theta_k - \theta_l).$$	$$\mathcal{V}(\tilde{\omega}, \tilde{\delta}, E'_q, V, \tilde{\theta}) = \mathcal{V}_1 + \sum_{k=1}^{8}\mathcal{V}_{2k} + C_o$$ $$\mathcal{V}_1 = \frac{1}{2}\sum_{k=1}^{n} M_k\tilde{\omega}_k^2, \quad \mathcal{V}_{21} = -\sum_{k=1}^{n} P_{mk}\tilde{\delta}_k$$ $$\mathcal{V}_{22} = \sum_{k=n+1}^{2n+N} P_{Lk}\tilde{\theta}_k, \quad \mathcal{V}_{23} = \sum_{k=n+1}^{2n+N}\int\frac{Q_{Lk}}{V_k}dV_k$$ $$\mathcal{V}_{24} = \sum_{k=n+1}^{2n}\frac{1}{2x'_{dk-n}} \cdot [E'^2_{qk-n} + V_k^2 - 2E'_{qk-n}V_k\cos(\delta_{k-n} - \theta_k)]$$ $$\mathcal{V}_{25} = -\frac{1}{2}\sum_{k=n+1}^{2n+N}\sum_{l=n+1}^{2n+N} B_{kl}V_kV_l\cos(\theta_k - \theta_l)$$ $$\mathcal{V}_{26} = \sum_{k=n+1}^{2n}\frac{x'_{dk-n} - x_{qk-n}}{4x'_{dk-n}x_{qk-n}} \cdot [V_k^2 - V_k^2\cos(2(\delta_{k-n} - \theta_k))]$$ $$\mathcal{V}_{27} = -\sum_{k=1}^{n}\frac{E_{f\,dk}E'_{qk}}{x_{dk} - x'_{dk}}, \quad \mathcal{V}_{28} = \sum_{k=1}^{n}\frac{E'^2_{qk}}{2(x_{dk} - x'_{dk})}.$$ $\mathcal{V}_1$ is known as the kinetic energy and $\sum\mathcal{V}_{2k}$ as the potential energy. C_0 is a constant such that at the post-fault stable equilibrium point, the energy function is zero.	M. Ghandhari, G. Andersson and I. A. Hiskens, "Control Lyapunov functions for controllable series devices," in *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 689-694, Nov. 2001, doi: 10.1109/59.962414.	Uses the classical multi-machine energy function $V = E_{\text{kin}} + E_{\text{pot}}$ as a control Lyapunov function, and derives simple local feedback laws for UPFC, CSC and QBT that guarantee $\dot{V} < 0$and add effective damping to electromechanical oscillations.

Category III: Grid-Forming (GFM) Inverter-Based Systems						
01	Virtual Synchronous Generator (VSG) using virtual inertia + Q–V droop control, grid-connected through an RL line	Constructed Lyapunov function for a reduced second-order VSG model (non-energy, parameter-dependent)	After shifting to the equilibrium point $(\delta_{s'}\, \omega_g)$, define $\begin{cases} x_1 = \delta - \delta^s \\ x_2 = \omega - \omega_g \end{cases}$ The reduced dynamic model of the VSG (Eq. (11)) becomes: $\dot{x}_1 = x_2,$ $J\dot{x}_2 = T_0 - \frac{1}{\omega_0}\left(EV_g B \sin(\delta_s + x_1) + EV_g G \cos(\delta_s + x_1)\right) - Dx_2,$ Here • J is the virtual inertia, • D is the damping coefficient, T_0 is the mechanical torque reference, • E is the internal voltage determined by the Q–V droop constraint, • B, G are the line susceptance and conductance.	By neglecting the damping term Dx_2, the Lyapunov function V(x1 , x2) of the system can be expressed in the form as $\mathrm{V}(x_1, x_2) = \frac{1}{2}Jx_2^2 - Tx_1 + \frac{EV_g B}{\omega_0}[\cos(\delta^s + x_1) - \cos\delta^s]$ $- \frac{EV_g G}{\omega_0}[\sin(\delta^s + x_1) - \sin\delta^s]$ $+ D\lambda x_1 x_2 + \frac{D^2}{2J}\lambda x_1^2$	Z. Shuai, C. Shen, X. Liu, Z. Li and Z. J. Shen, "Transient Angle Stability of Virtual Synchronous Generators Using Lyapunov's Direct Method," in *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 4648-4661, July 2019, doi: 10.1109/TSG.2018.2866122.	Analyses VSG transient angle stability by building a Lyapunov function tailored to the reduced model, which depends on the operating point instead of representing physical energy.
02	PLL-dominated GFL converter represented as a single–machine swing system	Classical power-system swing dynamics with angle-dependent damping (non-polynomial)	Nonlinear swing equations for the PLL-dominated GFL converter, written in state form $\dot{x} = F(x)$with $x = [\delta_c, \omega_c]^\top$: $\boldsymbol{F} = \begin{cases} \frac{d\delta_c}{dt} = \omega_c \\ \frac{d\omega_c}{dt} = P_{m,c} - P_{e,c}\sin(\delta_c - \theta_1) - D_c'\omega_c \end{cases}$ $P_{m,c} = K_i Z_{eq2} I_c \sin(\theta_2 + \varphi_I)$ $P_{e,c} = K_i Z_{eq1} U_g$ $D_c' = D_c \cos(\delta_c - \theta_1) = K_p Z_{eq1} U_g \cos(\delta_c - \theta_1)$	The traditional Lyapunov function Vtr for the GFL converter with damping energy included can be expressed by: $V_{tr} = \frac{1}{2}\omega_c^2 - P_{m,c}(\delta_c - \delta_{c,se}) + \int_{\delta_{c,se}}^{\delta_c} D_c'\omega_c d\delta_c$ $- P_{e,c}\left(\cos(\delta_c - \theta_1) - \cos(\delta_{c,se} - \theta_1)\right)$	F. Sun, R. Diao, R. Zeng, J. Li, and W. Tang, "Improved PLL design for transient stability enhancement of grid-following converters based on Lyapunov method," arXiv:2509.00489, 2025, doi: 10.48550/arXiv.2509.00489.	The GFL converter behaves as a nonlinear swing system whose phase-angle–dependent damping can turn negative for $\delta_c < \theta_1$, motivating the extended Lyapunov function that includes damping energy.
03		High-order nonlinear converter model with a polynomial reformulation; Lyapunov function constructed via sum-of-squares (SOS) optimisation	**Nonlinear SRF dynamic equations of the GFM converter** Lyapunov_stability_analysis_of_… The system equations of synchronous reference frame (SRF): $v_{od} = v_{cd} - r_1 i_{cd} - \frac{l_1}{\omega_b}\frac{di_{cd}}{dt} + \omega_g l_1 i_{cq},$ $v_{oq} = v_{cq} - r_1 i_{cq} - \frac{l_1}{\omega_b}\frac{di_{cq}}{dt} - \omega_g l_1 i_{cd},$ $i_{od} = i_{cd} - \frac{c_f}{\omega_b}\frac{dv_{od}}{dt} + \omega_g c_f v_{oq},$ $i_{oq} = i_{cq} - \frac{c_f}{\omega_b}\frac{dv_{oq}}{dt} - \omega_g c_f v_{od}.$ The equation of Grid-side dynamics: $v_{gd} = v_{od} - r_g i_{od} - \frac{l_g}{\omega_b}\frac{di_{od}}{dt} + \omega_g l_g i_{oq},$ $v_{gq} = v_{oq} - r_g i_{oq} - \frac{l_g}{\omega_b}\frac{di_{oq}}{dt} - \omega_g l_g i_{od}.$ The VSM and the reactive power droop can be defined by: $T_a\frac{d\omega_{\mathrm{VSM}}}{dt} = p^* - k_d(\omega_{\mathrm{VSM}} - \omega^*) - p_0,$ $T_f\frac{dq_m}{dt} = q_0 - q_m, p_0 = v_{od}i_{od} + v_{oq}i_{oq}.$	SOS-generated polynomial Lyapunov function: $V(\boldsymbol{x}) = 1989.67\sin^2(\delta\theta)\,(1 - \cos(\delta\theta))^2 - 1298.69\varepsilon_{\mathrm{d}}\sin^2(\delta\theta) - 1281.98\varepsilon_{\mathrm{d}}(1 - \cos(\delta\theta))^2 - 3950.41\sin^2(\delta\theta)\,(1 - \cos(\delta\theta)) - 3949.37(1 - \cos(\delta\theta)) + 3760.53\varepsilon_{\mathrm{d}}^2 + 2446.3\varepsilon_{\mathrm{d}}\sigma_{\mathrm{d}} + 2522.84\varepsilon_{\mathrm{d}}(1 - \cos(\delta\theta)) + 4227.60\varepsilon_{\mathrm{q}}^2 + 2308.68\varepsilon_{\mathrm{q}}\sigma_{\mathrm{q}}\cdots$ (Polynomial of degree ≤4; over 3,000 terms—only dominant terms shown in paper.)	S. Li, X. Xiao, A. Hebing, Y. Jia, S. Choudhury and J. Hanson, "Lyapunov stability analysis of grid forming converters using sum of squares optimization," *22nd IET International Conference on AC and DC Power Transmission (ACDC Global 2025)*, Birmingham, UK, 2025, pp. 245-251, doi: 10.1049/icp.2025.1213.	Uses a full high-order GFM model and constructs a non-energy Lyapunov function via SOS optimization to estimate the ROA, enabling stability certification for all control loops simultaneously.

			Angle dynamics equation: $\frac{d\theta_{\mathrm{VSM}}}{dt} = \frac{\omega_{\mathrm{VSM}}}{\omega_b}$. Because the VSM angle appears in trigonometric terms, the system is converted to a **polynomial form** using $z_1 = \sin\ \delta, z_2 = 1 - \cos\ \delta$ with algebraic constraint $z_1^2 + z_2^2 - 2z_2 = 0$. This yields a **polynomial state-space model** $\dot{x} = f(x)$suitable for SOS optimisation.			
04	PLL-based grid-tied VSC GSS model (2D nondimensional PLL dynamics)	Nonlinear 2D PLL swing-like model with analytically constructed Lyapunov function	Nondimensionalized PLL-based GSS model: $\begin{cases} \delta'_{\tau} = \gamma(m - \sin\delta) + x \\ x'_{\tau} = (m - \sin\delta) + hx \end{cases}$ $\gamma = \frac{k_{\mathrm{p,n}}\sqrt{\bar{U}_{\mathrm{g}}}}{\sqrt{k_{\mathrm{i,n}}}},\ h = \frac{\sqrt{k_{\mathrm{i,n}}}}{\omega_{\mathrm{g}}} \frac{\bar{X}_{\mathrm{g}} \bar{I}_{\mathrm{sd}}}{\sqrt{\bar{U}_{\mathrm{g}}}},\ m = \frac{\bar{X}_{\mathrm{g}} \bar{I}_{\mathrm{sd}} + \bar{R}_{\mathrm{g}} \bar{I}_{\mathrm{sq}}}{\bar{U}_{\mathrm{g}}}$ $x = \frac{x_{\mathrm{int}}}{\sqrt{k_{\mathrm{i,n}} \bar{U}_{\mathrm{g}}}},\quad \tau = \frac{\sqrt{k_{\mathrm{i,n}} \bar{U}_{\mathrm{g}}} \cdot t}{(1 - \gamma h)}$	**Lyapunov function for the nondimensional PLL system (Eqs. (3), (9)/(A6)):** $V(\delta, x) = V_0 + \frac{1}{2}[x - h(\delta - \delta_s)]^2 - (1 - \gamma h)(m\delta + \cos\ \delta)$, with $\delta_s = \arcsin\ m$, $V_0 = (1 - \gamma h)(m\delta_s + \cos\ \delta_s)$.	Y. Zhang, C. Zhang and X. Cai, "Large-Signal Grid-Synchronization Stability Analysis of PLL-Based VSCs Using Lyapunov's Direct Method," in *IEEE Transactions on Power Systems*, vol. 37, no. 1, pp. 788-791, Jan. 2022, doi: 10.1109/TPWRS.2021.3089025.	Using a two-state nondimensional PLL model, the paper derives an explicit Lyapunov function $V(\delta, x)$and employs it to **explicitly quantify the large-signal synchronisation margin** of PLL-based grid-tied VSCs.
05	Heterogeneous Second-Order Forced Kuramoto Oscillator Networks	Consensus-based phase oscillator model (second-order Kuramoto with inertia and external forcing); analytical synchronisation conditions via Lyapunov method and differential inequalities.	Normalised second-order forced Kuramoto dynamics for an oscillator i: the inertia–damping term $m_i\ddot{\theta}_i + \dot{\theta}_i$is driven by its natural frequency ω_i, the coupling with neighbouring phases $\sum_j k_{ij} \sin{(\theta_j - \theta_i)}$, and the external forcing term $-f_i \sin\ \theta_i$. $m_i\ddot{\theta}_i + \dot{\theta}_i = \omega_i + \sum_{j=1}^{N} k_{ij} \sin(\theta_j - \theta_i) - f_i \sin\theta_i$	Energy identity obtained by multiplying the dynamics by $\dot{\theta}_i$, summing over all oscillators and integrating from $0$to t; it equates the change of total kinetic energy plus the accumulated damping dissipation to the combined contributions of natural frequencies, external forcing and coupling potentials. $\sum_{i=1}^{N} \frac{M_i}{2}(\dot{\theta}_i(t)^2 - \dot{\theta}_i(0)^2) + \int_0^t \sum_{i=1}^{N} D_i \dot{\theta}_i(s)^2\, ds$ $= \sum_{i=1}^{N} \Omega_i(\theta_i(t) - \theta_i(0)) + \sum_{i=1}^{N} F_i \left(\cos\theta_i(t) - \cos\theta_i(0)\right)$ $+ \sum_{1 \le i < j \le N} K_{ij} \left(\cos(\theta_i(t) - \theta_j(t)) - \cos(\theta_i(0) - \theta_j(0))\right).$ Lyapunov-type energy function reconstructed from the above identity: $V(\theta, \dot{\theta}) = \sum_i \frac{M_i}{2}\dot{\theta}_i^2 - \sum_i \Omega_i\theta_i - \sum_i F_i \cos\theta_i - \sum_{i<j} K_{ij} \cos(\theta_i - \theta_j).$ so that along system trajectories one has $\dot{V} = -\sum_i D_i\, \dot{\theta}_i^2 \le 0$. This explicit form of V is not written in the paper, but is consistent with its energy equality.	S. -H. Chen, C. -C. Chu, C. -H. Hsia and S. Moon, "Frequency Synchronization of Heterogeneous Second-Order Forced Kuramoto Oscillator Networks: A Differential Inequality Approach," in *IEEE Transactions on Control of Network Systems*, vol. 10, no. 2, pp. 530-543, June 2023, doi: 10.1109/TCNS.2022.3219767	Using a heterogeneous second-order forced Kuramoto oscillator network model, the paper studies the frequency synchronisation problem and, via a second-order differential inequality approach that directly involves inertia, damping, natural frequencies, coupling strengths and external forcing parameters, derives explicit sufficient conditions for frequency synchronisation.
06	High-order grid-forming inverter system represented in TS fuzzy multimodel form	GFM / Algorithmic Lyapunov Construction (LMI &	$\dot{V}_{\mathrm{sys}} = A(R)V_{\mathrm{sys}}$ where V_{sys} are the matrix of state variables of the system. Dimension of V_{sys} is m. The A is a nonlinear matrix function with state variables. R represents a matrix of nonlinear	$E(X_{sys}) = [X_{sys}]^{\mathrm{T}} M_{\mathrm{best}} X_{sys}$	F. Zhao, Z. Shuai, W. Wang, Y. Liang, L. He and D. Wang, "An Active Attraction-Domain-Search-Based Lyapunov Construction Method in Power Electronic Systems," in IEEE Transactions on Circuits and Systems I:	Proposes an active attraction-domain-search-based (AADSB) Lyapunov construction

		Attraction-Domain Search)	variables of the studied system. The dimension of the R is H (H ≤ m), and R can be expressed as: $R = [r_1, r_2, \ldots, r_H]$ where the r_H represents the Hth nonlinear variable of the R.		Regular Papers, doi: 10.1109/TCSI.2025.3590009.	method for power electronic systems. The method constructs quadratic Lyapunov functions via LMI feasibility and maximizes the region of attraction using an attraction domain index (ADI) combined with PSO. Applicable to high-order grid-forming inverter systems for large-signal stability analysis.
				Others		
01	COA structure-preserving multimachine power system with dynamic reactive loads	Motion-Equation-Based / Analysis (explicit energy/Lyapunov function)	By combining the machine swing equations, the dynamic load model, and the real/reactive power-balance equations, the paper forms a singularly perturbed COA structure-preserving multimachine model: $\dot{\underline{\tilde{\omega}}}_g = -M_g^{-1}(\underline{P}_g(\underline{\theta}, \underline{V}) - \underline{P}_M)$ $\dot{\underline{\theta}} = R_2\underline{\tilde{\omega}}_g - R_1 D_l^{-1}(\underline{P}_l(\underline{\theta}, \underline{V}) + \underline{P}_d^0)$ $\dot{\underline{x}}_q = -T_q^{-1}\underline{x}_q - T_q^{-1}Q_t^0\underline{k}(\underline{V})$ $\dot{\underline{V}} = -E^{-1}[V]^{-1}(\underline{Q}_l(\underline{\theta}, \underline{V}) + \underline{Q}_d(\underline{x}_q, \underline{V}))$ where $M_g = \mathrm{diag}\{M_i\}$ $D_l = \mathrm{diag}\{D_{l_i}\}$ $T_q = \mathrm{diag}\{T_{q_i}\}$ $Q_t^0 = \mathrm{diag}\{Q_{t_i}^0\}$ $E = \mathrm{diag}\{\epsilon_i\}$ $[V] = \mathrm{diag}\{V_i\}$ $R_1 = \begin{bmatrix} I_{n_0} \\ 0_{m\times n_0} \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0_{n_0\times m} \\ I_m \end{bmatrix}$ $k_i(V_i) = \ln\left(\frac{V_i}{\gamma_i}\right)$ $Q_{d_i}(\underline{x}_q, \underline{V}) = \begin{cases} x_{q_i} + Q_{t_i}^0 \ln\left(\frac{V_i}{\mu_i}\right) & i = 1, \cdots, n_0 - m \\ 0 & i = n_0 - m + 1, \cdots, n_0 \end{cases}$	Lyapunov function of the Energy function: $\mathcal{V}(\underline{\tilde{\omega}}_g, \underline{\theta}, \underline{x}_q, \underline{V}, \underline{\theta}^s, \underline{x}_q^s, \underline{V}^s)$ $= \frac{1}{2}\sum_{i=1}^{m} M_i \tilde{\omega}_{g_i}^2 + \frac{1}{2}\sum_{i=1}^{n_0-m} \frac{(x_{q_i} - x_{q_i}^s)^2}{Q_{t_i}^0}$ $- \sum_{i=n_0+1}^{n} P_{M_{i-n_0}}(\theta_i - \theta_i^s) + \sum_{i=1}^{n_0} P_{d_i}^0(\theta_i - \theta_i^s)$ $- \frac{1}{2}\sum_{i=1}^{n} B_{ii}(V_i^2 - V_i^{s2})$ $- \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} B_{ij}(V_iV_j\cos\theta_{ij} - V_i^sV_j^s\cos\theta_{ij}^s)$ $+ \sum_{i=1}^{n_0-m}\left(x_{q_i}\ln\left(\frac{V_i}{V_i^s}\right)\right.$ $\left. + \frac{Q_{t_i}^0}{2}\left(\left(\ln\left(\frac{V_i}{\mu_i}\right)\right)^2 - \left(\ln\left(\frac{V_i^s}{\mu_i}\right)\right)^2\right)\right).$	R. J. Davy and I. A. Hiskens, "Lyapunov functions for multimachine power systems with dynamic loads," in *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, no. 9, pp. 796-812, Sept. 1997, doi: 10.1109/81.622983.	The model couples COA swing dynamics with voltage and a dynamic reactive-load state. The Lyapunov function is an energy balance: generator kinetic energy + network coupling energy (cos terms) + load-voltage recovery energy, where the logarithmic load assumption makes the voltage-related potential terms explicit.
02	Same COA model recast in Lur'e form for Popov analysis: $\dot{x} = Fx - G\psi(H^T x)$	Motion-Equation-Based / Analysis (Popov criterion; Lur'e–Postnikov integral Lyapunov)	Using the Popov/ Lur'e stability criterion, the original function becomes: $\dot{\underline{x}} = F\underline{x} - G\underline{\psi}(H^t\underline{x}).$	The corresponding Lyapunov function: $\mathcal{V}(\underline{x} - \underline{x}^s) = \frac{1}{2}(\underline{x} - \underline{x}^s)^t P(\underline{x} - \underline{x}^s)$ $+ \int_{H^t\underline{x}^s}^{H^t\underline{x}} [\underline{\psi}(\underline{\xi}) - \underline{\psi}(H^t\underline{x}^s)]^t Q\, d\underline{\xi}$ Or		The same COA dynamics are rewritten in Lur'e form to apply Popov theory. The Lyapunov candidate has a quadratic storage term plus an integral "potential"

				$\mathcal{V}(\underline{x}-\underline{x}^s) = \frac{1}{2}(\underline{x}-\underline{x}^s)^t \begin{bmatrix} qM_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q(Q_t^0)^{-1} & 0 \\ 0 & 0 & 0 & n_2 E \end{bmatrix} (\underline{x}-\underline{x}^s) + q\int_{H^t\underline{x}^s}^{H^t\underline{x}} [\underline{\psi}(\underline{\xi}) - \underline{\psi}(H^t\underline{x}^s)]^t \, d\underline{\xi}. \quad (41)$		of the nonlinearity; specialising it to the power-flow and load nonlinearities leads back to the explicit energy function.
03	COA structure-preserving multimachine power system with dynamic reactive loads	Motion-Equation-Based / Analysis (energy-function Lyapunov with dynamic loads)	Classical machine + swing equation: $M_i\dot{\omega}_{gi} + D_{gi}\omega_{gi} + P_{\mathrm{ELEC},i} = P_{Mi}$ $P_{d_i}(\omega_i) = P_{d_i}^o + \omega_i D_{l_i}$ $T_{q_i}\dot{x}_{q_i} = -x_{q_i} + Q_{s_i}^o - Q_{t_i}(V_i)$ $Q_{l_i}(\underline{V},\underline{\alpha}) = x_{q_i} + Q_{t_i}(V_i)$ $Q_{t_i}(V_i) = Q_{t_i}^o \ln\left(\frac{V_i}{\mu_i}\right)$	$\mathcal{V}(\underline{\omega}_g,\underline{\alpha},\underline{x}_q,\underline{V},\underline{\alpha}^s,\underline{x}_q^s,\underline{V}^s) = \frac{1}{2}\underline{\omega}_g^t\underline{M}_g\underline{\omega}_g + \frac{1}{2}(\underline{x}_q-\underline{x}_q^s)^t[Q_t^o]^{-1}(\underline{x}_q-\underline{x}_q^s) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(V_iV_j\cos\alpha_{ij} - V_i^sV_j^s\cos\alpha_{ij}^s) - \sum_{i=1}^{n-1} P_i(\alpha_i-\alpha_i^s) + \sum_{i=1}^{n_o-m} x_{q_i}\ln\left(\frac{V_i}{V_i^s}\right) + \sum_{i=1}^{n_o-m}\frac{Q_{t_i}^o}{2}\left(\ln^2\left(\frac{V_i}{\mu_i}\right) - \ln^2\left(\frac{V_i^s}{\mu_i}\right)\right)$	I. A. Hiskens and R. J. Davy, "Lyapunov function analysis of power systems with dynamic loads," *Proceedings of 35th IEEE Conference on Decision and Control*, Kobe, Japan, 1996, pp. 3870-3875 vol.4, doi: 10.1109/CDC.1996.577267.	Reviews and applies a Lyapunov (energy) function that combines generator swing dynamics with dynamic reactive load behaviour, enabling direct assessment of stability and voltage collapse under load and capacitor switching.